A Presupposition of Bell's Theorem

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The most prominent version of Bell's theorem consists of the Bell-CHSH inequality and a quantum-mechanical example violating it. The inequality is shown to rest on the non-trivial presupposition that the values of elementary spin quantities are scalars, not, e.g., vectors. In the version considered, the theorem's argument succeeds for scalars and fails for vectors. However, the reference to vector values can be motivated from the physics of spin. Hence, recognizing the presupposition suggests a critical reassessment of the theorem.

Keywords: Bell's theorem, Bell-CHSH inequality, locality

I. INTRODUCTION

Bell's theorem [1-2] shows that an assumption of locality is incompatible with the predictions of quantum mechanics (QM). The theorem consists of two parts: an inequality, derived by means of a locality assumption, and a quantum-mechanical example exceeding the bound given by that inequality. Conjoined, both parts yield a contradiction, i.e., QM violates the inequality. There are different versions of the theorem using different inequalities and QM counterexamples. Here, we consider the best-known version: the Bell-CHSH inequality (named after its originators Clauser, Horne, Shimony, and Holt [3]), and show that it rests on the non-trivial presupposition that the values of elementary spin observables are scalars, not, e.g., vectors. Accordingly, in the version considered, the theorem's argument succeeds for scalars but fails for vectors. Moreover, the use of vector values can be motivated from the physics of spin. Hence, recognizing the presupposition suggests a critical reassessment of the theorem.

The Bell-CHSH inequality can be presented in the form:

$$E\,(AB) + E\,(AB') + E\,(A'B) - E\,(A'B') \leq 2, \tag{1}$$

where E (AB), etc., are correlations, i.e., expectation values of products of physical quantities A and B, belonging to two different systems. The two most important derivations of inequality (1) are well-known; so it will suffice to sketch them. The first derivation begins with the sum of expectation values that is the LHS of (1) and combines it with a locality assumption to derive an inequality $E\,(AB) + E\,(AB') + E\,(A'B) - E\,(A'B') \leq |b + b'| + |b - b'|$ (where $b = E\,(B)$ and $b' = E\,(B')$). From the inequality, we derive (1) by noting this simple algebraic fact:

$$|b + b'| + |b - b'| \leq 2, \tag{2}$$

where $b, b' \in [-1, 1]$. Locality in this derivation manifests as the equation $E\,(AB) = E\,(A)\,E\,(B)$, etc., used in the first step (see [4]).

The second derivation of (1) takes a different course. It begins with the following fact:

$$a\,b + a\,b' + a'\,b - a'\,b' = \pm 2, \tag{3}$$

where $a, a', b, b' \in \{-1, 1\}$. We derive (1) from (3) and properties of expectations in three steps: the LHS of (1) = $E\,(AB + AB' + A'B - A'B') = E\,(\pm 2) \leq 2$, where the first and third step follow from the definition of an expectation and the second step is (3). As a result, we have an inequality $E\,(AB) + E\,(AB') + E\,(A'B) - E\,(A'B') \leq 2$, which is (1). Locality here is present in the assumption that the value, say, a is the same whether we are measuring b or b′, possibly far away. Formally, this is represented in the triviality that in (3) the two instances of 'a' refer to the same real number a (similarly for a′, b, b′) (see [5]).

In the literature, many proofs of the first kind (e.g., [4, 6-8]) or the second kind (e.g., [5, 9-10]) can be found. Arguably, every proof of (1) uses either fact (2) or fact (3). Below, we will refer to these two facts repeatedly.

The systems to which (1) refers are copies of a system M, with subsystems M_A and M_B, taken from a suitable set of systems. Both derivations of (1) refer to a classical, not a QM description of system M. Accordingly, the expectations used in (1) have the usual statistical definitions. E.g., the expectation $E\,(AB)$ is defined as $\sum_{a,b} ab\, p\,(ab\,|A, B)$. Here A and B are physical quantities representing properties of subsystems M_A and M_B, with a, b as their respective values,

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such that, e.g., E (AB) is the sum of the products of values a and b, weighted by the respective probabilities. As we see, these explanations refer to physical systems M_A and M_B but contain no QM. So, we may call expectations defined in this way classical. Note that, for such a classical expectation, e.g., E (AB), the constraint E (AB) = E (A) E (B) follows iff every function p (ab| A, B) within E(AB) factorizes into p (a|A) and p (b|B), i.e., the expectations factorize iff the respective probabilities do.

For future reference, we also sketch the second part of Bell's theorem, i.e., the QM example violating inequality (1). Bell's proposal, originating from Bohm, was to consider a two-particle spin-½ system in the singlet state $\mathbf{\Psi_S} = 1/\sqrt{2}\ (|01\rangle - |10\rangle)$. (Here we have abbreviated $|\mathbf{ab}\rangle := |\mathbf{a}\rangle \otimes |\mathbf{b}\rangle$, and defined $|0\rangle$ and $|1\rangle$ as the eigenstates of Pauli matrix $\boldsymbol{\sigma}_\mathbf{z}$ for the eigenvalues +1 and −1 respectively.) Let the quantities A and B be associated with vectors **a** and **b** corresponding to measurements of $\mathbf{a} \cdot \boldsymbol{\sigma}$ on subsystem M_A and of $\mathbf{b} \cdot \boldsymbol{\sigma}$ on subsystem M_B, where $\boldsymbol{\sigma} = (\boldsymbol{\sigma}_\mathbf{x}, \boldsymbol{\sigma}_\mathbf{y}, \boldsymbol{\sigma}_\mathbf{z})$ is the Pauli vector with respect to either M_A or M_B. According to QM, we then have the expectations $\langle AB\rangle := \langle(\mathbf{a} \cdot \boldsymbol{\sigma}) \otimes (\mathbf{b} \cdot \boldsymbol{\sigma})\rangle_{\Psi S} = - \mathbf{a} \cdot \mathbf{b} = - \cos \theta_{ab}$, where θ_{ab} is the angle between **a** and **b** ([11]).

For suitably chosen vectors, we now can produce a violation of (1). Consider four vectors $\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}$ in the 1, 2-plane ($\mathbf{R}^2$). Let { $\mathbf{e}_1$, $\mathbf{e}_2$ } be an orthonormal basis of the plane. Let the four vectors be fixed by the following identities: $\mathbf{a} = \mathbf{e}_1$, $\mathbf{a'} = \mathbf{e}_2$, $\mathbf{b} = 1/\sqrt{2}\ (-\mathbf{e}_1 - \mathbf{e}_2)$, $\mathbf{b'} = 1/\sqrt{2}\ (-\mathbf{e}_1 + \mathbf{e}_2)$. Then we obtain:

$$\langle AB\rangle + \langle AB'\rangle + \langle A'B\rangle - \langle A'B'\rangle = 2\sqrt{2}. \tag{4}$$

Finally, we assume the classical and QM sums of expectations, i.e., the LHSs of (1) and (4), to be identical. Given this assumption, we have $2 = 2\sqrt{2}$ – a contradiction.

II. SCALAR AND VECTOR VALUES OF QUANTITIES

We return to the classical expectations $E\ (AB) = \sum_{a,b} ab\ p\ (ab\ |A, B)$, etc. In these expressions, the values a, a′, b, b′ are the values of elementary physical quantities A, A′, B, B′, and the values of products of quantities are products of the values, such that, e.g., the value of AB is the product of a and b, i.e., ab. But what are the elementary values a, a′, b, b′, mathematically? The obvious suggestion is that they are scalars.

But this is not the only possible interpretation. Instead of taking the values to be scalars, we can interpret them, e.g., as vectors $\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'} \in \mathbf{R}^2$. If so, we can (employing the inner product of vectors in $\mathbf{R}^2$) assume that, while the elementary values themselves are vectors, their inner products and sums of products, i.e., the expectations, are scalars as before. We thus have an alternative way to define classical expectations, one that, for whatever reason, we might prefer to the original one. We can assume that these new expectations have the form: $E\ (\mathbf{AB}) = \sum_{\mathbf{a,b}} \mathbf{ab}\ p\ (\mathbf{ab}\ |\mathbf{A}, \mathbf{B})$ (etc.), where $\mathbf{A}, \mathbf{A'}, \mathbf{B}, \mathbf{B'}$ (written in boldface!) are elementary quantities whose values are vectors and $\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}$ (again in boldface) are the vectors themselves. In a similar fashion, we can define vector expectations, e.g., $E\ (\mathbf{A}) = \sum_{\mathbf{a}} \mathbf{a}\ p\ (\mathbf{a}\ |\mathbf{A})$, where, as just defined, **A** is an elementary quantity that can take on vectorial values and **a** is a vectorial value of **A** (and similarly for $\mathbf{A'}, \mathbf{B}, \mathbf{B'}$).

We should take this alternative option seriously and include it in our analysis of Bell's theorem. Recall that above, in equations (2) and (3), the values a, a′, b, b′ were assumed to be scalars (elements of $[-1, 1]$ or $\{-1, 1\}$) without any discussion. We immediately see the unexplored possibility here. Instead of taking the values to be scalars, we could interpret them as vectors. And so, where earlier, in a *classical* approach to the elementary physical quantities A, A′, B, B′, we took their values a, a′, b, b′ to be scalars, we now – again in a *classical* approach – consider elementary physical quantities $\mathbf{A}, \mathbf{A'}, \mathbf{B}, \mathbf{B'}$, with vector values $\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}$, and explore the implications.

In the context of Bell's theorem, the question whether physical quantities have scalar or vector values is crucially important. For no premise of the two derivations in sec.1 does it matter whether the values involved are scalars or vectors – excepting the two algebraic facts reported in (2) and (3). If we exchange the scalars on the LHSs of (2) and (3) by vectors, then: (i) the exchange *does not* affect the locality assumptions, but (ii) the exchange *does* affect the RHSs of (2) and (3).

To see (i), we replace the scalars a, a′, b, b′ in the definitions of expectation values by vectors $\mathbf{a}, \mathbf{a'}, \mathbf{b}, \mathbf{b'}$, such that the definitions become $E\ (\mathbf{AB}) = \sum_{\mathbf{a,b}} \mathbf{ab}\ p\ (\mathbf{ab}\ |\mathbf{A}, \mathbf{B})$, etc., as just introduced. This choice allows us to derive E (**AB**) = E (**A**) E (**B**) iff every function p (**ab**| **A**, **B**) within E(**AB**) factorizes into p (**a**|**A**) and p (**b**|**B**) – as was the case with scalars. Hence, the switch from scalars to vectors does not alter the role of the locality assumption used to derive (1) from (2).

Similarly for locality in the derivation of (1) from (3). Beginning with the LHS of (3), the first step is, say, a b + a b′ + a′ b – a′ b′ = a (b + b′) + a′ (b – b′), which presupposes that the two instances of 'a' on the LHS of (3) refer to the same real number a, and likewise for a′, b, b′. This structure recurs in a vector version (3′) of (3), where the equation $\mathbf{a\ b} + \mathbf{a\ b'} + \mathbf{a'\ b} - \mathbf{a'\ b'} = \mathbf{a}\ (\mathbf{b} + \mathbf{b'}) + \mathbf{a'}\ (\mathbf{b} - \mathbf{b'})$ presupposes that the two instances of '**a**' on the LHS of (3′)

refer to the same unit vector **a**, and likewise for **a′**, **b**, **b′**. Again, the switch from scalars to vectors leaves the role of the locality assumption in (3′) untouched.

To see (**ii**), we again replace scalars a, a′, b, b′ by vectors **a**, **a′**, **b**, **b′** on the LHSs of (2) and (3). As a result, both the inequality (2) and the equations (3) are no longer true in all cases. Indeed, if we choose the specific angles assumed in (4), then, instead of the LHS of (2) (i.e., |b + b′| + |b – b′|), we have |**b** + **b′**| + |**b** – **b′**| and thus, instead of (the RHS of) (2), we get:

$$|\mathbf{b} + \mathbf{b'}| + |\mathbf{b} - \mathbf{b'}| \;=\; 2\sqrt{2}. \qquad (5)$$

But this means that a vector version of (1) that yields a bound of 2 cannot be derived. Instead, a vector version of (1), yielding a less rigid bound, follows. In analogy with the derivation of (1) from (2), we have: $E\,(\mathbf{AB}) + E\,(\mathbf{AB'}) + E\,(\mathbf{A'B}) - E\,(\mathbf{A'B'}) \le |\mathbf{b} + \mathbf{b'}| + |\mathbf{b} - \mathbf{b'}|$. From this, using (5), we obtain:

$$E\,(\mathbf{AB}) + E\,(\mathbf{AB'}) + E\,(\mathbf{A'B}) - E\,(\mathbf{A'B'}) \;\le\; 2\sqrt{2}. \qquad (6)$$

Equation (6) illustrates how a classical system can *transcend* the classical bound in (1) with classical means (i.e., respecting locality). This becomes possible if we assume that the values of the relevant physical quantities are vectors. As we have seen, locality is not at issue here because, depending on whether we consider scalar values or vector values, either E (AB) = E (A) E (B) can be assumed for quantities with scalar values, or E (**AB**) = E (**A**) E (**B**) for quantities with vector values.

A similar result follows if we reconsider the derivation of (1) from (3). Suppose that we replace the LHS of (3) (i.e., a b + a b′ + a′ b − a′ b′) by **a b** + **a b′** + **a′ b** – **a′ b′**. Assume again that **a**, **a′**, **b**, **b′** are unit vectors defined as in (4). Then, we immediately get, instead of (3):

$$\mathbf{a\,b} + \mathbf{a\,b'} + \mathbf{a'\,b} - \mathbf{a'\,b'} = -\,2\sqrt{2}. \qquad (7)$$

From (7), we get:

$$\mathbf{a\,b} + \mathbf{a\,b'} + \mathbf{a'\,b} - \mathbf{a'\,b'} = \pm\,2\sqrt{2}. \qquad (8)$$

For given unit vectors **a**, **a′**, **b**, **b′**, the sum **AB** + **AB′** + **A′B** – **A′B′** is a constant such that the expectation $E\,(\mathbf{AB} + \mathbf{AB'} + \mathbf{A'B} - \mathbf{A'B'}) = E\,(\mathbf{a\,b} + \mathbf{a\,b'} + \mathbf{a'\,b} - \mathbf{a'\,b'})$. Using (8) plus this last equation and the linearity of expectations, we get, as before: $E\,(\mathbf{AB}) + E\,(\mathbf{AB'}) + E\,(\mathbf{A'B}) - E\,(\mathbf{A'B'}) = E\,(\mathbf{AB} + \mathbf{AB'} + \mathbf{A'B} - \mathbf{A'B'}) = E\,(\pm 2\sqrt{2}) \le 2\sqrt{2}$. Thus, we obtain a second derivation of (6). Indeed, just as we earlier derived (1) from either (2) or (3), two algebraic facts, we now have derived (6) from either (5) or (7), two geometric facts. Moreover, neither from (5) nor from (7) a contradiction with the QM equation (4) arises because (6), the vector version of (1), exhibits a more relaxed bound than (1) itself does.

We have argued for the following claim: *Bell's theorem, in the CHSH version, is based on the tacit presupposition that the values of the quantities A, A′, B, B′ are scalars, not vectors.* Mentioning this presupposition would be superfluous if it were a triviality. But as we just saw, this is not the case. We have a reasonable alternative that is motivated by an elementary physical assumption: that the values of the components of spin are indeed vectors. Thereby, the alternative avoids the fatal contradiction of Bell's theorem.

III. DISCUSSION (values and outcomes)

Bell's theorem proves that QM and locality are incompatible. This result is of great significance for the foundations of physics. But if the theorem's proof is based on a tacit presupposition, then it is not as general as we assumed. A quick reply to this challenge might be to reject the presupposition as superfluous. The assumptions of scalar values in (2) and (3) suffice for the contradiction and additional assumptions are simply unnecessary. In particular, the argument of Bell's theorem can be presented for measurement outcomes that surely can be viewed as scalars. This was arguably Bell's own line of thought. He writes that the "difficulty" (i.e., the contradiction between (1) and (4)) is "created by the predictions about the correlations in the visible outputs of certain conceivable experimental set-ups." [12] In effect, this means that the Bell-CHSH inequality can be presented in terms of "visible outputs" and we can identify the outputs with scalar values. What sense does it make to add vectorial values to this picture? None, apparently. But of course, this reply misses the point. We have not assumed that vector values can be added gratuitously to the theorem's set-up but have argued that if they are added, as replacements of scalar values, then the contradiction of Bell's theorem will not come about.

As far as outcomes are concerned, we should keep in mind that in real experiments we are interested not in outcomes but in what can be concluded from them about the target systems. Thus, we are not interested in the "visible outputs" but in the invisible system properties that can be inferred from them. The system quantities and values, inferred from the outcomes, may well be very different from these outcomes. To clarify this possibility, we consider three options:

Option 1: the values a, a′, b, b′ of quantities A, A′, B, B′ are outcomes labelled by arbitrary labels, say '↑' and '↓'. From these labels we cannot derive (1) for a trivial reason. The classical expectations introduced above are defined as E (AB) = $\sum_{a,b}$ ab p (ab |A, B), etc., so cannot incorporate outcomes '↑' and '↓', as for them addition and multiplication are undefined.

Option 2: the values a, a′, b, b′ of quantities A, A′, B, B′ are outcomes labelled by scalars ± 1. In this case, we can derive the Bell-CHSH inequality (1), using (2) or (3), without any reference to "a quantum-mechanical system", which is just what Bell originally intended [13]. Here, the relation between outcomes and system values remains open.

Option 3: the values a, a′, b, b′ of quantities A, A′, B, B′ are inferred from outcomes labelled by arbitrary labels but are not identical with them. (An example would be the inference from an outcome labelled '↑' (or '↓') to 'the left electron has a positive (or negative) spin in the Z-direction'.) Here, by assumption, outcomes and system values are related by an inference but are not identical. So, it is possible to assume that these entities are of different types. The system values (a, a′, b, b′ of A, A′, B, B′) are scalars while the outcomes are arbitrary labels. Similarly, for the alternative theory from sec.2, the system values (**a**, **a′**, **b**, **b′** of **A**, **A′**, **B**, **B′**), are vectors, while the outcomes again are mere labels.

It seems strange to consider spin without directly addressing QM, the only serious theory of spin. But of course, we can imagine a primitive classical theory of spin and must be able to do so to have a classic theory competing with QM. If QM and the competitor theory did not address the very systems and values referred to in the definitions leading to eq. (4), then the desired contradiction between (1) and (4) would not arise. The expressions in (1-3) refer to such a classic competitor theory. The values a, a′, b, b′ of quantities A, A′, B, B′ are elements of this theory, and it is understood that the elements are unspecified entities but might be values of spin. Similarly for vector values **a**, **a′**, **b**, **b′** of vector quantities **A**, **A′**, **B**, **B′**, which represent a second classical theory competing with QM. In sec.2 it was argued that the first competitor theory fails (produces the contradiction of Bell's theorem) but the second does not (avoids the contradiction).

IV. DISCUSSION (spin)

The interpretation of values of spin quantities as vectors is by no means far-fetched; instead, it is suggested by the physics of spin. This can be seen from the careful explication of spin as a *vector quantity*:

> "Spin in QM is a vector quantity **S** associated with the 'internal' degrees of freedom of a system. We denote the observables corresponding to the X-, Y-, and Z-components of spin relative to a Cartesian reference frame by S_x, S_y, and S_z, respectively … We assume that the observables S_x, S_y, S_z (more correctly their associated self-adjoint operators) obey the same commutation relations as those applying in the case of 'orbital' angular momentum …" [14]

Here we learn that, on the one hand, "the X-, Y-, and Z-components" of spin **S** are vectors because **S** itself is a vector, and on the other hand, that to every spin component there corresponds an observable "S_x, S_y, and S_z", or its "associated self-adjoint operator". Note that "the X-, Y-, and Z-components of spin" are physical entities that every theory of spin must address. By contrast, the observables S_x, S_y, and S_z and the associated operators are creatures of QM and its mathematics. It is unclear whether their values or eigenvalues are physical entities that every theory of spin must address.

Consider a system M and let the X-component of spin **S**, a vector, be a property of M. We assume that this component is a unit vector $\mathbf{e_x}$. Consider the corresponding observable S_x. The value of S_x is a property of M. Is that value a scalar property of M? If so, then M has two different properties related to direction X: a vector property $\mathbf{e_x}$ and a scalar property that is the value of S_x. This seems implausible. Instead, the value of S_x should be viewed as a vector that can be interpreted as being identical with the vector $\mathbf{e_x}$.

By definition, the operator associated with S_x has a scalar eigenvalue. Does this fact carry over to observable S_x (in the sense that the value of S_x is a scalar)? If so, the earlier problem remains: M has two different properties for direction x: the vector $\mathbf{e_x}$ and the scalar value of S_x. Can we give up the idea that the X-component of **S** is a vector? If so, we have renounced our starting point that spin in QM "is a *vector quantity* **S**" with *vector components*. It is much more plausible to relinquish the identification of values of spin components with certain scalars, e.g., the eigenvalues of self-adjoined operators. Instead, we can identify the eigenvalues (of the operator associated with observable S_x) with the *signs of vectors* $\pm \mathbf{e_x}$ and the vectors $\pm \mathbf{e_x}$ themselves with the values of S_x. We thus distinguish mathematical entities (eigenvalues) from physical ones (values of observables); the latter can still be identified with the components of spin **S**, and in the case of component S_x with the vector $\mathbf{e_x}$.

These considerations are internal to QM; they concern the above-quoted explication of spin in QM and its consequences. The underlying problem is how to harmonize the operator formalism of QM with the QM (!) requirement that the components of **S** are vectors. But the explication of spin as "a vector quantity" is the first step into the QM of spin and the operator formalism is built on it as a second step. Every theory of spin – QM as well as a classic competitor – should address this first step: that **S** is a vector quantity with vectorial components that can be vectorial properties of systems.

With respect to this criterion, our two classic theories of spin differ markedly. The scalar values theory from sec.1 addresses spin by just introducing scalar values a, a′, b, b′, while the vector values theory from sec.2 introduces vector values **a**, **a′**, **b**, **b′** (both theories do so to construct their different expectation values). But the theory that is faithful to the vectorial character of spin **S** and its components, the one that incorporates the physical nature of spin, is the vector values theory.

V. CONCLUSION

We have reconsidered Bell's theorem and confirmed that the standard version, interpreting the values of elementary spin quantities as scalars, ends in the familiar contradiction between locality and QM. But in the course of this reconsideration, we have learned that a version interpreting the values as vectors creates no contradiction. Such a version can reproduce the QM bounds while respecting the locality assumptions used in Bell's theorem. Moreover, it can be motivated from the physics of spin. Hence, recognizing the choice between scalar and vector values of spin quantities suggests a reappraisal of the theorem.

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[13] See [2], p.150.

[14] [5], p.32.