Can the universe be in a mixed state? or did God have a choice in creating the universe?

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

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Abstract

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently. Moreover, it has been conjectured that the universe is not in a pure state but in a mixed state in W-QM. In this paper, I argue that this conjecture has several potential problems that quantum mechanics does not have.

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently (Dürr et al, 2005; Chen, 2019, 2021). It replaces the wave function in quantum mechanics (QM) with the density matrix and correspondingly the Schrödinger equation with the von Neumann equation. Since quantum dynamics can be formulated directly in terms of the density matrix, it seems reasonable to assume that the ontic state of the universe is represented not by a wave function in QM but by a density matrix in W-QM, which may be a mixed state. In other words, the universe may be not in a pure state but in a mixed state in W-QM (Chen, 2021). In this paper, I will present a critical analysis of this interesting conjecture.

First of all, when the universe is in a general mixed state in W-QM, we cannot find the effective density matrix for a subsystem even in the case of decoherence, since the subsystems of the universe are strongly entangled. This is unlike the situation in QM, in which the effective wave function can be defined for a decohered subsystem. Thus, we don't have a workable W-QM theory for a subsystem of the universe such as a hydrogen atom in a laboratory to give predictions which can be tested by experiments.[1]

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[1] Note that impure density matrices are indeed needed for understanding the time evolution of subsystems with spin in a universe governed by Bohmian mechanics (see Dürr et al. 2005). But this is not relevant to the workable W-QM theory here.

By contrast, we have QM for subsystems of the universe, such as Bohmian mechanics with effective wave functions (Goldstein, 2021), and it agrees with experiments.

Next, in QM we have several ψ-ontology theorems such as the PBR theorem to prove that the effective wave function of a subsystem is real, representing the relevant ontic state of the subsystem (Pusey, Barrett and Rudolph, 2012). However, in W-QM we don't have W-ontology theorems to prove that the effective density matrix of a subsystem is real, representing the relevant ontic state of the subsystem. One reason is that we don't have a workable W-QM theory for subsystems based on which we can give a similar proof. Another reason is that even a decohered subsystem does not have a well-defined ontic state (see below). Then, even though we may extend the reality of the effective wave function to the universal wave function in QM, we cannot do this in W-QM.

Third, when the universe is initially in a general mixed state in W-QM such as according to Chen's (2021) initial projection hypothesis, all subsystems of the universe will be strongly entangled, no matter whether they have interactions with each other or not. This seems to pose an issue for non-interacting subsystems. Suppose there are two non-interacting subsystems or very-weakly-interacting subsystems in the universe. Then, how can their ontic states be strongly entangled at the initial instant?[2] Note that the entanglement is reflected not only in the ontic state but also in the change of the ontic state during a measurement. For example, in W-collapse theories, when one subsystem is measured, the ontic state of the other non-interacting subsystem will be also changed, no matter how far away they are in space. Usually the entangled state of two subsystems is formed by their interaction. If two subsystems do not interact with each other or they have only a very weak interaction, then it will be natural to assume that their ontic states are independent or very weakly entangled. By contrast, QM does not have this issue of entanglement for non-interacting subsystems.

Lastly, since we don't have a workable W-QM theory for subsystems when assuming that the universe is in a mixed state, it seems that we cannot test the mixed state conjecture and confirm that it agrees with experiments. The proponents of W-QM resort to the empirical equivalence of W-QM with QM to avoid the issue. Let's see if this is possible.

According to Dürr et al (2005) and Chen (2019), W-QM and QM are empirically equivalent when assuming that in QM a random wave function is assigned to the universe such that the associated statistical density matrix equals the fundamental density matrix assigned to the universe by W-QM. Suppose the fundamental density matrix of the universe at a given instant

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[2] Note that it has been widely argued that the space where the ontic state of an N-body system exists is three-dimensional, not $3N$-dimensional, and thus taking the mixed state of an N-body system directly as a whole physical field in a $3N$-dimensional space is not plausible (Gao, 2017).

t_0 is

$$W_0 = \sum_{i=1}^{N} p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \tag{1}$$

where N is the dimension of the Hilbert space, $p_i \in (0,1)$ satisfies the nomalization relation $\sum_i p_i = 1$, and $|\psi_i\rangle$ is a set of orthogonal states in the Hilbert space. The equivalence between W-QM and QM means that one can assign a random wave function $|\psi_i\rangle$ with probability p_i or a mixed state W_0 to the universe and use either QM or W-QM for the same empirical predictions.

Then, one can test the mixed state conjecture by using QM for subsystems of the universe. However, it is obvious that there is underdeterminism here. Since W-QM and QM give the same empirical predictions, no experiments can determine whether the universe is in a mixed state or in a pure state. Thus, the mixed state conjecture is still untestable by experiments when resorting to the empirical equivalence of W-QM with QM. Certainly, the assumption that the universe is in a random pure state in QM is untestable by experiments either. I will turn to this issue of underdeterminism in QM later.

Furthermore, it can be argued that the empirical success of QM for subsystems of the universe does not provide empirical support for the mixed state conjecture in a deterministic W-QM. The reason is that the empirical equivalence of W-QM with QM requires an additional condition which is arguably not satisfied if QM is deterministic. The equivalence requires that in QM the wave function of the universe at the initial instant is "chosen" at random from a statistical ensemble represented by the fundamental density matrix in W-QM. Here the choosing probability is not subjective or epistemic, related to our knowledge about the initial state of the universe, but objective and ontic, inherent to the initial state of the universe itself. However, the universe is single and unique and there does not exist a statistical ensemble of universes. Moreover, there is neither an external system outside the universe nor an internal physical process which "chooses" the wave function of the universe randomly with a particular probability at the initial instant in a deterministic QM.[3] If there is no such a random choice required by the empirical equivalence of W-QM with QM, then there will be no empirical evidence supporting the mixed state conjecture. In this case, since the quantum state of the universe is a pure state in W-QM, the mixed state conjecture is also wrong.

Note that we can prepare a statistical ensemble of subsystems of the universe in which each subsystem has a random effective wave function with

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[3] For example, in Bohmian mechanics, the dynamics for the wave function is continuous and deterministic, and there is no stochastic process to make the wave function of the universe random.

a particular probability. In a deterministic QM such as Bohmian mechanics, the effective wave function of each subsystem is determined by the initial positions of the Bohmian particles of the subsystem, as well as the setting of the external preparation apparatus. Different initial positions of the Bohmian particles lead to different effective wave functions.

Anyway, it would be strange to assume that the initial state of the universe must have a certain random feature in a deterministic theory.[4] A more natural assumption is that the initial state of the universe in a theory with deterministic dynamics is also determined by a deterministic law such as a definite boundary condition, one example of which is the no-boundary condition suggested by Hartle and Hawking (1983). A theory with a separate mixture of random initial state and deterministic dynamics seems to be neither simple nor unified. Indeed, it is usually regarded as an advantage of W-QM over QM that the initial universal quantum state in the theory is assumed to be a definite mixed state, not a random mixed state (Chen, 2021). By the same reasoning, QM with a definite initial universal wave function is better than QM with a random initial universal wave function.[5]

Here I have a conjecture on how to solve the puzzle of the arrow of time. It is usually thought that the past hypothesis and the statistical postulate are needed to solve the puzzle (see, e.g. Loewer, 2020). It is the statistical postulate in QM that motivated the mixed state conjecture in W-QM (Chen, 2021). Now QM with a definite initial universal wave function can solve the puzzle of the arrow of time in a simpler way. The statistical postulate is dropped and not needed. The past hypothesis can be naturally derived, since the universe has only one possible initial wave function, the initial entropy of the universe is zero, which means that the universe begins with the minimum entropy. Penrose's (1994) Weyl curvature hypothesis is arguably one example of such a theory.

To sum up, I have argued that the mixed state conjecture in quantum mechanics with a fundamental density matrix (W-QM), which says that the universe is not in a pure state but in a mixed state in W-QM, has several potential issues that quantum mechanics (QM) does not have. One issue is that the ontic states of non-interacting subsystems of the universe are strongly entangled. Another issue is that the empirical success of QM for subsystems of the universe does not provide empirical support for the mixed

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[4] Even in a stochastic theory such as collapse theories, since the initial state cannot be randomly collapsed from an earlier state by definition, it is arguable that the initial state of the universe cannot be randomly choosen either.

[5] QM with a definite initial universal wave function will also avoid the issue of underdeterminism plagued by W-QM with the mixed state conjecture and QM with a random initial universal wave function. If there were indeed two fundamental theories that are completely empirical equivalent, then God would have another type of freedom to choose the theory for the universe (besides choosing the initial state of the universe), and we would be never able to find the choosen theory even if we obtain all empirical evidence. In this case, to paraphrase Einstein, God would be not only subtle, but also malicious.

state conjecture, since the empirical equivalence of W-QM with QM requires an additional condition which arguably cannot be satisfied. It remains to be seen if W-QM can be revised to solve these issues.

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