Mandersian Relationism: Space, Modality and Equivalence

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Abstract

Modal relationism is the view that our best physical theories can dispense with substantival space or spacetime in favor of possible configurations of particles. Kenneth Manders argued that the substantivalist conception is equivalent to this Leibnizian conception of space. To do so, Manders provides a translation f from the Newtonian theory T_N into the Leibnizian modal relationist account T_L. In this paper, we show that the translation does not establish equivalence, since there is no translation $g : T_L \mapsto T_N$ that preserves theoremhood. This seems to show that the modal relationist theory T_L is less parsimonious than its substantivalist rival.

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> Thus it appears that any tenable relationalism must incorporate a powerful, primitive notion of geometric modality. Manders (1982) provides the most promising route to such a relationalism.
>
> *(Belot 2000, 578)*

1 Introduction

In this journal, Kenneth Manders has argued that the Newtonian conception of space, which postulates spatial points and treats them as fundamental, is theoretically equivalent to the Leibnizian modal conception, which treats spatial points as logically constructed from actual particles and possible configurations thereof (Manders 1982). To demonstrate the equivalence between the substantival and the modal relationist metaphysics, Manders provides a translation from the language of the Newtonian theory of space T_N into the language of Leibnizian modal relationism.[1] He suggests that an adequate relationist theory T_L can be found, namely the translation of the Newtonian theory T_N (Manders 1982, 589). However, the existence of a translation does not amount to a proof of equivalence, under reasonable standards of formal equivalence. While Manders defines a translation $f : T_N \mapsto T_L$, he does not show that there is a translation g: $T_L \mapsto T_N$ such that $T_N \models \phi \leftrightarrow g(f(\phi))$ for all formulae ϕ of T_N (see (Halvorson 2019,

[1]The notion of translation at play here is the notion of a generalized translation between first-order theories that one can find in (van Benthem and Pearce 1984) and (Halvorson 2019). The notion of Morita equivalence, defined in (Barrett and Halvorson 2016), is equivalent to the notion of intertranslatability presupposed here (see (Halvorson 2019)).

27)). In fact, we show that there is no interpretation from T_L to T_N (a translation that sends theorems into theorems).

This result seems to show that the modal relationist introduces redundant structure, absent in the substantivalist theory. We substantiate this claim by using a criterion for comparing amounts of structure. The one we employ is inspired by a criterion for the equality of structure due to (Barrett 2022).

The philosophical conclusion of the paper is that modal relationism ought to be abandoned on parsimony grounds. We base this recommendation on another principle of parsimony, whose most precise formulation is again due to (Barrett 2022), namely that *all else being equal*, we should favor theories that posit less structure. We think that this principle is an improved form of *Ockham's Razor*, and is both intrinsically plausible and exemplified by famous cases of theory choice. This combination of our structure counting criterion and the parsimony principle of (Barrett 2022) is in tension with the requirement of (Belot 2011) that a relationist theory should state truth conditions for all statements about space. If a systematic paraphrase is a translation function f, then theories of the form $f(T_{subst})$ cannot be superior on parsimony grounds.

2 The Theories T_N and T_L

Let us begin by describing the two systems T_N and T_L. We follow Manders by identifying T_N with Tarski's formalization of Euclidean geometry (see (Tarski 1959)). The ontology of T_N consists of spatial points.[2] The primitive extralogical predicates of the substantivalist language L_N are the following:

[2]By ontology we mean the range of the variables in unabbreviated notation.

(a) A three place predicate 'Between $v_1v_2v_3$' that holds of the points v_1, v_2 and v_3 when v_2 stands between v_1 and v_3 on the segment connecting them.

(b) A four place predicate 'Congruence $v_1v_2v_3v_4$' that holds of four points v_1, v_2, v_3 and v_4 when the segment $\overline{v_1v_2}$ is as long as $\overline{v_3v_4}$.

The language L_L of the relationist theory T_L contains two types of variables. The variables $x_1, x_2, ..., x_n$ range over point particles. The variables $c_1, c_2, ..., c_n$ range over a new type of entity: possible configurations of point particles. Manders (1982) does not clarify whether the ontology of T_L includes only *actual* particles or both actual and *merely possible* particles. But, as (Field 1984, 57) has pointed out, a 'possible point particle' appears to behave in the relationist theory exactly like a space or a spacetime point. Since there is no distinction between particles and points, we cannot distinguish the merely possible particles from spatial points in terms of location. These 'possible particles' seem to stand in exactly the same relations, and satisfy exactly the same principles as points of space.[3] If quantification over merely possible point particles is admitted, modal relationism appears to collapse immediately into substantivalism. Even if one could distinguish possible particles from spatial points, on philosophical grounds, merely possible point particles raise the same epistemological and parsimony objections

[3]One might hold that the spatial points have their relations of betweenness and congruence necessarily (Maudlin 1988), while those of the merely possible point particles can be freely recombined, inside different possible configurations. While the former view strikes us as arbitrary, it does distinguish the two types of entities. But then, the point is that a theory of merely possible point particles is as parsimonious as a substantivalist theory.

as space or spacetime points. For this reason, we suppose that the particles that T_L quantifies over are only the actually existing ones. Possible configurations of particles will be described better below, when we come to the predicates that apply to them. But they correspond roughly to possible worlds, or scenarios in which the particles are arranged in some fashion.

Language of T_L: For every geometrical predicate R^n of the Newtonian theory T_N, the language L_L has a predicate R^{n+1} for point particles with an extra place for configurations. Since L_N contains two geometrical predicates of betweenness and congruence, T_L contains the following two predicates:

(1) A four-place predicate 'Between$'$ $c_1x_1x_2x_3$', which holds of a configuration and three particles when the particle x_2 is between the particles x_1 and x_3 in the configuration c_1.

(2) A five-place predicate 'Congruence$'$ $c_1x_1x_2x_3x_4$', holding of a configuration c_1 and four particles $x_1x_2x_3x_4$ when the distance between x_1 and x_2 is the same as the distance between x_3 and x_4 in c_1.

Moreover, the theory T_L contains two other predicates:

(3) A four place predicate '$c_1x_1 \sim c_2x_2$' holding of two point particles and two configurations when the point particle x_1 as it occurs in c_1 is in the same place of x_2 as it occurs in c_2. We call this the 'same-place-as relation'.

(4) A two place predicate of membership '$x_1 \in c_1$' that holds of a point particle and a configuration when the former is an element of the latter.

The same-place-as relation is crucial for the claim that T_L is adequate to physics, and is also the most unfamiliar. It requires, therefore, a more detailed explanation than the others. Manders (1982, 579-582) introduces it by drawing upon intuitions about absolute space. Two particles in two respective configurations satisfy this relation if they stand in the same point of *absolute space*. However, this reference to absolute space is syncategorematic, and does not require quantification over space, regions, or points. Configurations, in this sense of the word, are like possible assignments of the particles to absolute locations, rather than simply networks of relative distances. We can contrast this approach with another natural primitive: transworld congruence (Field 1989, 72). Four particles in two configurations are transworld congruent if they stand at the same distance in their respective configurations. In terms of transworld congruence, it is possible to define a weaker same-place-relation: two particles y and y' stand in the same place in two configurations c and c', if they have the same number of particles, and there is a one-to-one map that preserves congruence and maps y to y' (fig. 1).

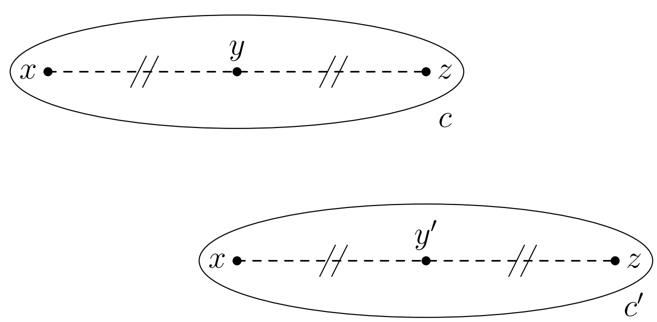

Figure 1: c, y, c' and y' need not stand in the same-place-as relation.

In T_L particles stand in the same place independently of what other particles in the configurations are doing. The theory T_L is consistent with the claim that there exist two isomorphic configurations, in the previous sense, with the particles in different places of absolute space. For example, there could be a configuration c with three collinear particles, x, y, z, equally spaced, and also a configuration c' where the three particles are shifted by five centimeters to the left.[4]

This same-place relation will pay its dividends when we come to motion and inertia. For an instant, let us add variables ranging over times $t_1, ...t_n$ and a binary relation $R(t, c)$ between times and the configuration realized at time t. The theory with transworld congruence does not seem to be able to define what it means for a symmetrical object to rotate around an axis. The contemplated extension of T_L can. It can state, for example, that x remains in the same place at t_2 as it was at t_1, but that y and z have moved (fig. 2). It can also quantify the angle of rotation by contemplating a merely possible and unactualized configuration c in which some particles y' and z' are located at t_2 where y and z were in t_1. Therein lies the promise of physical adequacy.

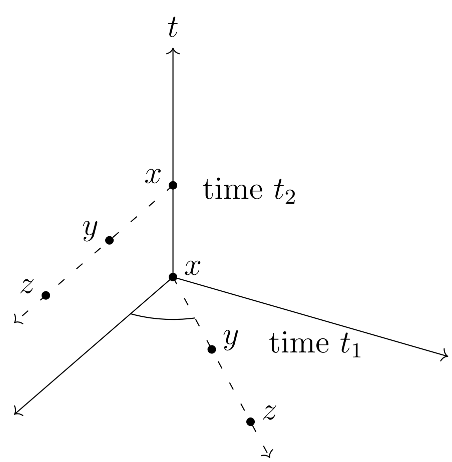

Figure 2: Rotation

[4]In the figure, a circle represents a configuration. A point represents a particle. Two particles are part of the same configuration if they are part of the same circle.

Ultimately, it is the axioms that fix the interpretation of the same-place relation. Manders states in the appendix that the postulates of T_L are simply the translation of the axioms of T_N, together with three axioms stating that the same-place-as relation is an equivalence relation on configurations (Manders 1982, 589). Therefore, in order to specify the axioms, or postulates, of T_L, we need to specify the Mandersian translation first. In general, we can specify the axioms of a relationist theory by giving a translation, and then taking the translation of the axioms of some substantivalist physical theory. Let us state explicitly the three equivalence relation axioms for the same-place-as relation:

- ψ_1 is the formula $\forall c_1 \forall x_1 (x_1 \in c_1 \rightarrow x_1 c_1 \sim c_1 x_1)$
- ψ_2 is the formula $\forall x_1 \forall c_1 \forall x_2 \forall c_2 \; (c_1 x_1 \sim c_2 x_2 \rightarrow c_2 x_2 \sim c_1 x_1)$
- ψ_3 is $\forall x_1 \forall c_1 \forall x_2 \forall c_2 \forall x_3 \forall c_3 \; (c_1 x_1 \sim c_2 x_2 \wedge c_2 x_2 \sim c_3 x_3 \rightarrow c_1 x_1 \sim c_3 x_3)$

Then, T_L is the following theory:

$$T_L = \{f(\phi); T_N \vdash \phi\} \cup \{\psi_1, \psi_2, \psi_3\}$$

The intuitive interpretation of the same-place-as relation may already raise suspicions that the resulting theory is 'substantivalist' in some sense. However, it is important to note that the theory T_L satisfies the classical definition of a relationist theory, as laid out in Field (1984, 33): the theory does not postulate space or spacetime points, except as logical constructions out of aggregates of matter. If the theory is to be ruled substantivalist rather than relationist, the debate must be redefined. We will return to this point in the conclusion.

3 The Translation from T_N to T_L

Manders sets out a translation from the Newtonian, or geometric, language L_N into the Leibnizian language L_L of point particles and their possible configurations. A translation or interpretation f of T_N into T_L is entirely determined by a specification of the translation of the atomic formulae of L_N. Let us begin with the identity relation. Since the same-place-as relation is an equivalence relation, we can identify spatial points with the equivalence classes of pairs (x, c) of a particle and a configuration under the same-place-as relation. After all, a particle *in a configuration* does indicate a position in absolute space. This suggests that the translation f should map the identity between points (that is formulae such as $v_i = v_j$) into the same-place-as relation $\sim$:[5]

Definition of f for the identity between points.

$$v_i = v_j \mapsto_f c_i x_i \sim c_j x_j$$

Equivalently, in a Morita extension L'_N of L_N, new variables for spatial points may be introduced as equivalence classes of pairs of point particles and configurations under the same-place-as relation $\sim$.

[5]We will use the symbol $\mapsto_f$ to indicate the translation function f. The notion of a generalized translation at play here allows us to map a formula of one language and the sequence of its free variables to a formula of the other language and a sequence of sequences containing all the free variables in that formula. For example, the identity relation $=$ and the sequence of its two free variables $< v_i, v_j >$ is mapped to the same-place-as relation $\sim$ and a sequence of pairs of a point particle variable and a configuration variable $< [c_i, x_i], [c_j, x_j] >$.

There are two other atomic formulae of the substantivalist theory T_N: the two geometrical primitives of betweenness and congruence. If $\phi(v_{j_1}, v_{j_2}, v_{j_3})$ is 'Between $v_{j_1}v_{j_2}v_{j_3}$', then its translation $f(\phi)$ states that there is a configuration c' in which the three points $v_{j_1}, v_{j_2}, v_{j_3}$ are occupied by particles, and the three particles located at these points satisfy the betweenness predicate in c'.

Definition of f for the predicate of Betweenness.

$$\text{Between } v_{j_1}v_{j_2}v_{j_3} \mapsto_f \exists c' \exists x'_1 \exists x'_2 \exists x'_3 (c_{j_1}x_{j_1} \sim c'x'_1 \wedge c_{j_2}x_{j_2} \sim c'x'_2$$
$$\wedge\, c_{j_3}x_{j_3} \sim c'x'_3 \wedge \text{Between}'\, c'x'_1x'_2x'_3)$$

Explanation: The formula says that a point v_{j_2} is between two points v_{j_1} and v_{j_3} if and only if there is a configuration c' and three particles x'_1, x'_2 and x'_3, such that (i) x'_2 is between x'_1 and x'_3 in c' and (ii) the pairs of particles and configurations that correspond to the points v_{j_1}, v_{j_2} and v_{j3} are the same points as, that is, they stand respectively in the same-place relation to, the pairs x'_1c', x'_2c' and x'_3c' (see Figure 3 below).

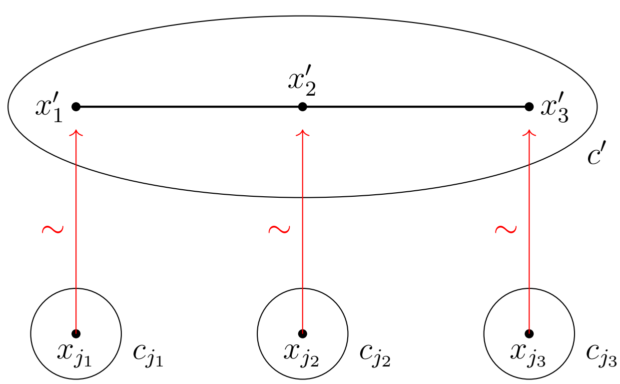

Figure 3: Betweenness in T_L

Congruence: The translation f is defined in a similar way when $\phi(v_{j_1}, v_{j_2}, v_{j_3}, v_{j_4})$ is 'Congruence $v_{j_1}v_{j_2}v_{j_3}v_{j_4}$'. There is a configuration c' in which the four points $v_{j_1}, v_{j_2}, v_{j_3}, v_{j_4}$ are occupied, and the four particles located at these points satisfy the congruence predicate in c'.

Definition of f for the predicate of Congruence.

$$\begin{aligned}\text{Congruence } v_{j_1}v_{j_2}v_{j_3}v_{j_4} \mapsto_f \exists c' \exists x'_1 \exists x'_2 \exists x'_3 \exists x'_4 (c_{j_1}x_{j_1} &\sim c'x'_1 \\ \wedge\, c_{j_2}x_{j_2} \sim c'x'_2 \wedge c_{j_3}x_{j_3} &\sim c'x'_3 \\ \wedge\, c_{j_4}x_{j_4} \sim c'x'_4 \wedge \text{Congruence}'c'x'_1x'_2x'_3x'_4)\end{aligned}$$

The translation is extended, in the usual way, to complex formulae:

Definition of f for complex formulae.

$$\neg\phi \mapsto_f \neg f(\phi)$$

$$\phi \wedge \psi \mapsto_f f(\phi) \wedge f(\psi)$$

$$\exists v_j \phi \mapsto_f \exists c_j \exists x_j (x_j \in c_j \wedge f(\phi))$$

Manders has given us a conservative translation f from T_N into T_L. Trivially, for any formula ϕ of the language of T_N, if ϕ is a theorem of T_N then $f(\phi)$ is a theorem of T_L. However, he has not shown how to translate the formulae of L_L back into L_N. In other words, he has not shown that T_N and T_L are in fact theoretically equivalent. It is not difficult to see that there is in fact no reverse translation f^{-1} from the language L_L to the language L_N that maps theorems into theorems.[6] The reason is that the function f is not *essentially surjective*:

Definition 1. *Let $f\colon T \to T'$ be a translation between theories. f is essentially surjective iff for each sentence ψ of $L_{T'}$, there is a sentence ϕ of L_T such that $T' \vdash \psi \leftrightarrow f(\phi)$ (Halvorson 2019, 120; Barrett and Halvorson 2022, 7).*

T_L talks about point particles and configurations, whereas T_N has no resources to talk about either point particles or their configurations. T_L is designed to describe relative distances between physical bodies, while T_N is a purely geometrical theory and lacks any physical content. We can sketch a rigorous argument to see that f is not essentially surjective, and therefore that there cannot be such a reverse translation from T_L into T_N. This follows from the fact that T_N is complete but T_L is incomplete.[7] We will from now on omit the essentially identical arguments for extensions of these theories.

Lemma 1. *The Leibnizian theory T_L is incomplete.*

[6]Technically, the definitions of conservativity and essential surjectivity in [Halvorson 2019) apply to ordinary translations, also in the case of formulae with free variables. The definitions are more complicated for generalized translations, when free variables are present, since a generalized translation can send formulae to formulae with a different number of variables. For now, we restrict them to closed formulae or sentences. This suffices for present purposes.

[7]A theory T is incomplete if and only if for some formula ϕ of the language of T, some model of T verifies ϕ and some model of T falsifies ϕ. It is possible, of course, to interpret an incomplete theory T in a complete theory T'. However, a complete and an incomplete theory cannot be *equivalent*.

Proof. We show that there is a formula ϕ of the language of T_L and two models M and M' of T_L such that ϕ is true in M' and $\neg\phi$ is true in M''. The particles of M consists of the set $\{1\}$. The particles of M'' consist of the set $\{1, 2\}$. We take the configurations to be the functions from particles to $\mathbb{R}^3$. The interpretation of the geometrical and same-place relation of T_L are the obvious ones. We easily verify that the axioms of T_L hold in both M and M'. Let ϕ be the formula that says 'there is exactly one particle'. ϕ is true in M but it is false in M'. $\square$

Proposition 1. *The translation f is not essentially surjective.*

Proof. T_N is complete (see (Schwabhauser, Szmielew and Tarski, 1983, 218 ff.)). Since T_L is incomplete, take a sentence ψ that T_L does not decide. Assume that f is essentially surjective, so that there is a formula ϕ of L_T such that $T_L \vdash \psi \leftrightarrow f(\phi)$. By completeness, either $T_N \vdash \phi$ or $T_N \vdash \neg\phi$. Therefore, by definition of T_L and of f, either $T_L \vdash f(\phi)$ or $T' \vdash \neg f(\phi)$. By logic and the assumption, either $T' \vdash \psi$ or $T' \vdash \neg\psi$. Contradiction.

$\square$

Let us now state what we mean when we say that g is an inverse to f:

Definition 2. *If $f : T \to T'$ is a translation and $g : T' \to T$ is a translation, then g is an inverse of f iff for every closed formula ϕ of L_T and for every closed formula of $L_{T'}$ we have that $T \vdash \phi \leftrightarrow g(f(\phi))$ and $T' \vdash \psi \leftrightarrow f(g(\psi))$.*

Corollary 1. *There is no translation f^{-1} that is an inverse to f from the language of T_L to the language of T_N which preserves logical form, and such that theorems of T_L are mapped to theorems of T_N.*

Proof. Assume an inverse f^{-1} exists. Let ψ be a formula. Then $T_L \vdash f(f^{-1}(f(\psi)) \leftrightarrow \psi$. Therefore f is essentially surjective. Contradiction. $\square$

4 Conclusion

In this last part of the paper, we will argue that the theory T_L is a worse theory than T_N, and therefore ought to have been rejected in favour of substantivalism, even if classical mechanics had turned out to be empirically adequate.

Since the only other modal relationist approach that is known to us, namely the theory based on transworld congruence, is incapable of defining rotation and acceleration, this result casts doubt on modal relationism in general.

We will argue that T_L is inferior on grounds of parsimony. We have in mind a principle of theory choice like the following:

Structural Parsimony. *All other things equal, we should prefer theories that posit less structure (Barrett 202, 296).*

This principle explains, for instance, the fact that relativistic electrodynamics is better than Lorentz's electrodynamics that posits the luminiferous aether, or an undetectable foliation of spacetime (Bradley 2019, 1055). Intuitively, a theory is more parsimonious than another if everything in the first theory is definable in the less parsimonious theory, but not the other way around (Barrett 2022, 307). The counting of structure at play has been rationally reconstructed in terms of category theory (Barrett 2022, 305), but this is equivalent to a condition in terms of translations (Barrett 2022, 310):

Equal structure. *A theory T posits the same amount of structure as a theory T' if and only if there is an essentially surjective translation $f : T \mapsto T'$.*

This principle deals only with the case in which the two theories posit the same amount of structure. What we need is a similar criterion for when the structure posited by one theory is a proper part of the structure posited by the other. In such a situation, it is natural to assume that there is a translation $f : T \mapsto T'$, but there is no *essentially surjective* translation. This is equivalent to the following condition (by Lemma 1 in (Barrett and Halvorson 2022, 8)) :

Less Structure. *A theory T posits less structure than a theory T' if and only if there is an interpretation $f : T \mapsto T'$ but there is no interpretation $g : T' \mapsto T$.*[8]

We will assume this syntactical structure counting criterion also for many-sorted theories, and employ generalized translations as well.

Barrett (2023) has applied his parsimony principle to choose between nominalistic and platonistic physical theories. He analyzes the requirement of (Putnam 1971) that a nominalistic physical theory interpret standard platonistic theories. He rejects it as unreasonable since it commits the nominalist to just as much structure as the platonist. We will return to this point shortly.

The principle similarly indicts the theory T_L of (Manders 1982). T_L posits more structure without any compensating explanatory gain, and therefore there is no reason to prefer it to the substantivalist theory T_N.

The relationist may insist that "*not all else is equal*", but it is unclear what compensating advantages T_L in (Manders 1982) could have. The syntactical complexity of its axioms is clearly similar to those of the theory T_N. The translation f preserves

[8] An interpretation is just a conservative translation that maps theorems into theorems. We should add the usual proviso that the translation should preserve observational vocabulary and that the two theories agree on prediction reports.

logical structure. (Manders 1982, 576) says that modal theories have a *prima facie* epistemological advantage. But it is unclear why because in order to code the physics we need to add a functor $a(t)$ for the actual configuration at a time. We can then translate statements such as "the sun is at absolute rest" and "the sun is moving at 17'000 km/h" in the relationist language, and the translations will be similarly underdetermined by the evidence. Even if we adopt a Galilean invariant primitive of collinearity on an inertial line, for example, we will be left with what Dasgupta (2015, 620-21) calls self-locational uncertainty and inexpressible ignorance about which of the infinitely many isometric configurations is the actual one. Finally, the complexity introduced by the possible ways of locating particles in space (Belot 2000, 576) is matched by the redundancy in distinct configurations with the same particles and same relative distances (see fig. 1).

The requirement of (Putnam 1971) that a nominalistic theory translate a platonistic theory has its relationists counterpart in the methodology of Belot (2011, 35-37), which requires a relationist theory to give truth conditions to all the geometric statements that its substantivalist counterpart can express. Is this requirement of interpreting the substantivalist theory reasonable or not?

The matter is partly a verbal dispute about the definition of relationism. It seems to us that there are two varieties of relationism. The first can be called eliminativism, and simply denies the existence of space and time. On this view, the only entities that exist are material objects, and spacetime is banished from the ontology together with the aether, quintessence, caloric, ghosts, and the like.

The second variety can be called reductive relationism, and admits the existence of a substantial spacetime, but treats it as ontologically derivative from matter. On this view, facts about spacetime regions are grounded on facts about material objects. Hartry Field

(1984) suggests that the relationist may view spacetime regions as logical constructions out of matter. A materialist might think in the same way that mental states are grounded on physical brain states. If one has this second view, then it makes sense to ask for a relational truthmaker for all or at least many basic facts about space or spacetime.

Gordon Belot (2011, appendix A, 2012, 81) calls the first variety of relationism ‘antirealism’ and reserves the term ‘relationism’ for what we call reductive relationism. In this context, it makes sense to ask for a translation function f from the standard formulation of mechanics into a reformulation that only quantifies over material objects. The goal is not to provide a rival theory, but a more metaphysically perspicuous formulation of the same theory.

However, none of these considerations save the theory of (Manders 1982) or modal relationism. If one wants to pursue the second strategy, one has to provide a theory that is genuinely equivalent to the standard version. The novel presentation should be a description of the same facts, but show how some of them reduce to facts of a simpler sort. It should not introduce additional structure, or posit *additional facts*. In other words, there should be a reverse translation f^{-1} from modal relationist facts to substantival facts. This is precisely what we have shown to fail in the case of the theory T_L of Manders (1982). Therefore, it does not matter whether one is an eliminativist or a reductionist relationist. Modal relationism is not the way to go.

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