Relationism Rehabilitated?
II: Relativity

Oliver Pooley*
Exeter College, University of Oxford

28 March, 2001

Abstract

In a companion paper (Pooley & Brown 2001) it is argued that Julian Barbour's Machian approach to dynamics provides a genuinely relational interpretation of Newtonian dynamics and that it is more explanatory than the conventional, substantival interpretation. In this paper the extension of the approach to relativistic physics is considered. General relativity, it turns out, can be reinterpreted as a perfectly Machian theory. However, there are difficulties with viewing the Machian interpretation as more fundamental than the conventional, spacetime interpretation. Moreover, this state of affairs provides little solace for the relationist for, even when interpreted along Machian lines, general relativity is a substantival theory although the basic entity is space, not spacetime.

1 Introduction

When a position finally starts to be described as orthodoxy, it is normally a sure sign that the consensus has started to fracture. The current state of the substantivalist–relationist debate proves no exception. The belief that substantivalism has the upper hand is still, surely, the orthodox opinion. But there is also a growing awareness that the relationist can respond to apparently strong substantivalist arguments.

Some of the most important recent insights into what a relationist position might involve have been provided by Julian Barbour who, together with various collaborators, has pursued an approach to the foundations of dynamics inspired by Ernst Mach. The significance of this work has yet to be fully appreciated by philosophers with an interest in interpretative questions concerning spacetime physics. In a companion paper (Pooley & Brown 2001), the implications of Barbour's approach for pre-relativistic physics are explored. There it is argued that Barbour's Machian programme provides an as yet *unique* relational alternative

*e-mail: oliver.pooley@philosophy.oxford.ac.uk

to standard Newtonian mechanics, namely Barbour and Bertotti's intrinsic particle dynamics (first described in Barbour & Bertotti 1982). This theory exactly matches the zero angular momentum fragment of standard Newtonian theory. Although alternative formulations of this fragment of Newtonian theory are possible (see, especially, Belot 1999, Belot 2000), only Barbour's notion of "best matching" provides a relational understanding of the inertial structure that must be a component of any such formulation.[1]

This paper is ultimately concerned with the various surprises that are in store when one adopts the Machian approach in the context of the variable geometry of general relativity (GR). If cast in its 3+1, dynamical form, it turn out that GR can already be viewed as a perfectly Machian theory.[2] But even from the Machian perspective, it is a theory about substantival *space* (rather than space*time*). Further, although such a Machian interpretation of GR naturally suggests a "sophisticated" substantivalist attitude to space, it nevertheless involves an indeterminism more pernicious than that familiar from discussions of the Hole Argument.

Before turning to Barbour's interpretation of GR in Section 3, I first consider the extension of the Machian programme to special relativistic theories. This will serve as a useful introduction to some of the interpretative issues.

2 Special Relativity

The most influential anti-relationist arguments concern the scientific treatment of motion. In a nutshell, the arguments are that dynamics requires the use of inertial structure and that this structure is *prima facie* non-relational. The arguments are summarized in Pooley & Brown (2001, §4) where Barbour and Bertotti's relational *reduction* of inertial structure in the context of Newtonian mechanics—effectively a Machian refutation of the substantivalist arguments—is described in some detail.

Now the relationist must confront two further issues, both concerning the viability of his point of view in the specific context of relativistic physics. One concerns the apparent tension between relativity's denial of a preferred foliation of spacetime, and the basic ontological ingredients of Barbour and Bertotti's relational alternative to standard physics, namely *instantaneous* relative configurations of the entire universe. DiSalle (1994, 275), for example, suggests that this tension strips relationism of the philosophical attractiveness it might have enjoyed prior to relativity. Its true significance is explored below (Sections 2.2 and 3.2). The

[1] As described in, e.g., Earman (1989, Ch. 5), Pooley & Brown (2001, §6) and Barbour (1999*a*), intrinsic particle dynamics is not the only genuinely relational non-relativistic theory. The problem is that these alternatives, in one way or another, fail to be empirically adequate. Of course, in failing to be a relativistic theory, intrinsic particle dynamics itself is not, ultimately, empirically adequate. This paper is primarily concerned with the interpretation of the theories one obtains when one extends the basic principles of intrinsic particle dynamics to the relativistic domain.

[2] The claim that GR can be viewed as a fully Machian theory is controversial. In endorsing it I am not claiming that the geometry of spacetime is fully determined by the matter content of the universe. *That* claim is generally acknowledged to be false (see Barbour & Pfister 1995). The sense in which GR *is* perfectly Machian is explained in Section 3.

other issue, which I address first, concerns a popular substantivalist conception of *fields.*

2.1 Fields

The question of whether an ontology of fields, rather than of Newtonian point particles, has any implications for the substantivalist–relationist debate is one that commands little consensus. In his *World Enough and Space-time*, Earman endorses the following argument of Hartry Field:

> From the platonistic point of view, a field is usually described as an assignment of some property, or some number or tensor, to each point of space-time; obviously this assumes that there are space-time points, so a relationist is going to have to either avoid postulating fields (a hard road to take in modern physics, I believe) or else come up with some very different way of describing them. (Field 1980, 35)

Earman claims that "in postrelativity theory, it seems that the electromagnetic field, and indeed all physical fields, must be construed as states of M [the spacetime manifold]" (1989, 155). The standard characterization of a field in spacetime involves assigning, for each coordinate system, a set of numbers to each point p of M. These are the components (in each coordinate system) of the field at each point p. Earman claims that while "the antisubstantivalist can, of course, attempt to dispense with some or all of this apparatus in favor of another means of specifying a relationally pure state of affairs…the burden of proof rests with the antisubstantivalist" (1989, 159).

How persuasive is the argument? Certainly a number of philosophers have felt that an alternative conception of fields is readily to hand. Belot urges that one can think of fields as being "extended non-material objects possessing infinitely many degrees of freedom" rather than as assignments of properties to space or spacetime: "neither option appears mandated by physical considerations" (1999, 45).[3] Substantivalists have responded that "dispensing with space-time regions in favor of "parts of a field" is possible… However… this "saves" relationism only by trivializing it" (Field 1980, 41). And Rynasiewicz, observing the debate, concludes that this is all grist to his sceptical mill: it merely emphasizes that what one counts as "space" and what one counts as "physical object" is really no more than a matter of linguistic choice (Rynasiewicz 1996, 300–1).

To note these moves is only to begin to engage with the real issues. Earman points out that the standard specification of fields invokes spacetime points. Even if it is claimed that fields are extended objects *in* spacetime rather than *properties of* spacetime, it might seem that an alternative way of specifying them is needed if a commitment to the existence of spacetime points is to be avoided. Field's charge of trivialisation is off-target too. Suppose an alternative to a substantival

[3]Brown (1997) makes a similar point. I endorse Belot's intuition but since ordinary matter receives a field-theoretic treatment in modern physics (in the context of GR and quantum field theory, one standardly talks of "matter fields") I would question the qualification "non-material."

conception of fields is viable. For relationism to be vindicated, an empirically adequate dynamical theory of such entities which does not involve primitive inertial or temporal structure is still needed. Otherwise the anti-relationist arguments mentioned above are effective. Point particle theories which eschew such structure are possible. But are there field analogues? And if there are, are they empirically adequate?

As already anticipated, such theories do in fact exist. One of Barbour and Bertotti's motivations for constructing intrinsic particle dynamics (for a description of the theory, see Pooley & Brown 2001, §7) was to find a framework that could also deal with fields. To get a flavour of such a field theory, consider the simplest case, that of a scalar field ϕ. Imagine a configuration of such a field that is non-zero only in some finite region. According to the relationist, ϕ does not represent an assignment of properties to space; it is an extended, material thing. But equally, since space itself is supposed not to exist, this extended object should not be characterised in terms of the spatial locations of the various field intensities. Rather it is to be characterised by the relative dispositions of the field intensities: the infinite number of facts about the relative distances and angles between particular values of ϕ that fully capture the pattern of field intensities. Facts, for example, of the form "an instance of $\phi = a$ is b metres from an instance of $\phi = c$". Together, these specify the *relative field configuration.* As in the case of a relative configuration of point particles, a very economical way of capturing these relational facts is to refer the field to a Cartesian coordinate system, $\phi = \phi(x)$. But to stress a point made in Pooley & Brown (2001, §7), these coordinates are not to be understood as 'names' for the points of space. Rather one should imagine laying a coordinate grid onto the field itself.

There is one important difference between the field case and the particle case. It is a difference that surely is in part responsible for Earman's belief that the way in which fields are standardly specified lends *prima facie* support to substantivalism. In the particle case, while one can choose to capture the relative distances in terms of a coordinate system, one does not have to do so. The alternative, of course, is to state explicitly a sufficient number of the relative distances r_{ij}. In the field case there are an infinite number of such distances to catalogue—fields have infinitely many degrees of freedom.

The correct relationist response to this observation is to note that one nonetheless can have a clear understanding of what a relational specification of the field would consist in, even if such a specification is not possible in practice. After all, specification of a substantivalist field configuration will also, in general, not be possible in practice. *In principle*, Earman's challenge to the relationist to provide a "direct characterisation" of the reality underlying the substantivalist's description of a field is easily met, at least in the case of fields 'in' flat space(time) (cf. Earman 1989, 171).

The route to a relational field theory parallels exactly the route to a relational particle theory. Consider two patterns of field intensities ϕ and ϕ' which differ intrinsically. One can describe each with the help of Cartesian coordinate systems. In doing so, how the coordinate systems are 'placed' relative to the field configurations is entirely arbitrary. This allows the construction of a trial measure, $\mathrm{d}s$, of the difference between them, for example via $\mathrm{d}s^2 = \int d^3x(\phi'(x) - \phi(x))^2$. The coordinatization of ϕ' can now be varied: coordinate systems related to each other by Euclidean transformations capture the same relative dispositions of field intensities. This variation will affect our trial value of $\mathrm{d}s$ and the minimum value resulting from such variation will constitute a measure of the *intrinsic* difference between ϕ and ϕ'. Barbour calls this variation "best matching" for rather than describing the process in terms of the recoordinatisation of one of the field configurations, one could imagine the two 3-dimensional field configurations being rigidly shifted with respect to each other to obtain the 'best fit' relative placement. The measure of intrinsic difference between relative field configurations it provides gives us a metric on the field's relative configuration space, $\mathcal{Q}_0$. This in turn can be used to construct a geodesic variational principle on the relative configuration space, yielding a preferred class of curves as representative of physically possible histories: relational histories will be sequences of relative field configurations.[4]

2.2 Simultaneity

The problem is that this notion of a possible history embodies absolute simultaneity, something which the abundant empirical evidence supporting special relativity militates against. Nonetheless, there are two senses in which relational field theories of the type just described can be said to be Lorentz-covariant.

In Barbour and Bertotti's intrinsic particle dynamics one has a uniquely favoured time parameter and best matching provides an equilocality relation between successive configurations. These surrogates for Newton's absolute time and space allow one to transform a sequence of 3-dimensional relative particle configurations into a 4-dimensional configuration of particle trajectories in Newtonian spacetime. Similarly, for a given relational field theory, the best-matching minimization of relative field configurations and the simplifying time parameter associated with the action principle (if it is of Jacobi type) will allow the construction of an effective spacetime; i.e. it will transform physically possible sequences of 3-dimensional relative field configurations into a 4-dimensional field configuration.

We can now ask a number of questions. First, is this 4-dimensional field configuration a solution of a special relativistic field theory formulated in Minkowski spacetime? The answer will be yes if the relational theory bears a certain relationship to some Lorentz-covariant field theory.[5] Such theories can be said to Lorentz-covariant in a weak sense.

So far this 4-dimensional field configuration is described with respect to a particular inertial frame (that obtained from the best matching of the 3-dimensional relative configurations). To set the scene for the second question, imagine per-

[4] For further details of how best matching provides a metric on the relative configuration space of point particles and for why a geodesic principle on this configuration space constitutes a genuinely relational theory, see Pooley & Brown (2001, §§6–7). A complementary discussion can also be found in Butterfield (2001, XXX) which contains a very clear description of best matching in the point particle case employing rather different notation.

[5] Details are given in the appendix.

forming a Lorentz transformation. The result will be a new description of the same 4-dimensional configuration which can be reinterpreted as a sequence of instantaneous 3-dimensional relative configurations. In general this sequence will be different from the original one—it will be represented by a different curve in the field's $\mathcal{Q}_0$—for by describing things from the perspective of a different inertial frame, we have sliced spacetime by different hyperplanes of simultaneity. The second question is: is this new sequence also a solution of a relational field theory? Finally, if it is, one can ask: is it a solution of the same relational field theory?

As explained in the appendix, the two parts of the second question must receive the same answer: either yes or no. The answer is yes only if the field's energy-momentum 4-vector vanishes.[6] In this case, the relational theory is Lorentz-covariant in a strong sense: the dynamical law describing the universe (the geodesic principle on the universe's relative configuration space) takes the same form no matter from which inertial frame one chooses to obtain a sequence of 3-dimensional relative configurations.

One might be tempted to see the need for a restriction on the value of energy-momentum in order to achieve strong Lorentz-covariance as a bonus. If, for some reason, one believes the theory *should* be Lorentz-covariant in this sense, then the value of the energy is fixed to be identically zero. It is arguably a weakness of the point particle theory that, although E is a fundamental constant, it is *arbitrary.* Although the particle theory is particularly elegant when $E = 0$, nothing mandates this value.

However, strong Lorentz-covariance has some rather unwelcome consequences for the relationist. From the relationist perspective, 3-dimensional relative configurations are ontologically primary. This is certainly Barbour's view:

> The world is to be understood, not in the dualistic terms of atoms (things of one kind) that move in a framework and container of space and time (another quite different kind of thing), but it terms of more fundamental entities that fuse space and matter into a single notion of a possible arrangement, or configuration, of the entire universe. Such configurations... are the ultimate things. There are infinitely many of them; they are all different instances of a common principle of construction; and they are all, in my view, the different *instants of time*... The world is made of Nows. (Barbour 1999*b*, 16)

The 4-dimensional spatio-temporal framework of an inertial frame, and everything properly described with respect to it, are to be understood as emerging from the dynamics of these truly fundamental "ultimate things". This itself makes the very formulation of the invariance requirement—that the laws of nature take the same form in every inertial frame—rather odd. That aside, the real problem comes when one asks, of any particular possible universe, *which* sequence of 3-dimensional

[6]The 4-dimensional angular momentum tensor about every point will also vanish. I put aside the question of whether there exist physically interesting field theories which have non-trivial solutions of this form.

configurations represents the genuine ontology; for in general, the sequence of relative configurations associated with each inertial frame will be different. They will correspond to different curves in the appropriate relative configuration space, even though each curve is a geodesic of the dynamical variational principle.

Suppose the actual universe could be modelled as the solution of such a theory. Each member of an infinite family of curves in $\mathcal{Q}_0$ would be equally adequate to the observable phenomena. But surely, if one wishes to adopt the relationist perspective, one must hold that our universe is one particular sequence of relative configurations. All the others represent physically distinct yet observationally indistinguishable worlds.[7] The alternative—to admit that each curve is a different representation of the same reality—reintroduces a 4-dimensional perspective and with it, arguably, the fundamental reality of spacetime. The situation is ironically close to that faced by the naive Newtonian substantivalist who insists that one particular set of inertial trajectories are the world lines of the points of absolute space, but who has to admit that no empirical evidence can tell him which set it is.

If, on the other hand, the world could be modelled by a relational field theory which was Lorentz-covariant only in the weak sense then the problem does not arise. The *apparent* absence of a standard of absolute simultaneity is taken care of by the fact that the theory is weakly Lorentz-covariant. But it is *only* apparent; the genuine simultaneity surfaces are those of the inertial frame for which $\mathbf{P} = 0$. Moreover, the relational theory *predicts* that in this frame the 3-dimensional angular momentum with respect to the centre of mass coordinate will be zero.

As we are about to see, this story comes close to being repeated in the context of GR, but in rather an unexpected fashion.

3 Geometrodynamics

So far the spatial relations between the material parts of possible instantaneous configurations of the universe have been assumed to be Euclidean. Can the framework be further generalized to arbitrary Riemannian spatial relations? What are the analogues, in the context of GR, of the instantaneous relative configurations of the universe so far considered? An answer is only forthcoming for general relativistic spacetimes that admit a global foliation into a family of 3-dimensional spacelike hypersurfaces. (For simplicity, suppose such hypersurfaces are compact and of fixed topology.) Each such hypersurface has a certain matter content (fields with various patterns of intensities) with definite spatial relations (no longer necessarily Euclidean) holding between the various field values. Such data can be specified in terms of a 3-dimensional differentiable manifold, Σ, with a Riemannian metric h_{ij} and matter fields defined on it. Consider shifting the matter and

[7] Each sequence of relative configurations corresponds to the same spacetime and they are thus "observationally indistinguishable worlds" in the following sense: in principle, inhabitants of such worlds could discover through observations which spacetime allowed by the theory they were part of. But no observation could reveal to them which particular sequence of relative configurations constituted the building blocks of their spacetime.

metric fields with respect to the base manifold Σ by an active diffeomorphism. The result involves exactly the same pattern of field intensities with the same spatial relations holding between them. The relationist will thus identify them. The space of all possible 3-dimensional entities of this type appears to be the appropriate relative configuration space for a relational analogue of GR.

For the moment, overlook the fact that, as traditionally conceived, the relationist–substantivalist debate is about the *reality* of space or spacetime and consider the matter-free case. The space of all Riemannian metrics defined on Σ is normally denoted $\text{Riem}(\Sigma)$. The relationist will wish to identify as physical spaces points of $\text{Riem}(\Sigma)$ that differ solely in how the metric field is 'placed' on Σ. He will identify points of $\text{Riem}(\Sigma)$ that are related by a diffeomorphism of Σ. The resulting relative configuration space is the space of all possible intrinsic 3-*geometries* (of fixed topology), denoted $\text{Geom}(\Sigma) = \text{Riem}(\Sigma)/\text{Diff}(\Sigma)$, where $\text{Diff}(\Sigma)$ is the group of all diffeomorphisms of Σ. Following John Wheeler, $\text{Geom}(\Sigma)$ is known as *superspace.*

Once the details of the relative configuration space have been fixed, the broad outlines of a Machian dynamical theory are easy to state. One wants to define a best-matching based metric on the relative configuration space and then use this to construct a geodesic principle to single out a preferred class of curves as representative of physically possible histories. The generalization from Euclidean to Riemannian geometry requires a generalization of the best-matching procedure. In effect *all possible* coordinatizations of two 3-geometries which differ intrinsically must now be considered in the minimization of some quantity chosen to represent a trial difference between them. It was Barbour and Bertotti's hope to construct a Machian alternative to GR along these lines. It came as quite a surprise to them (and an initially unwelcome one!) when, following discussions with Karel Kuchař, they realised that the *"geometrodynamical"* formulation of GR was already a theory of exactly this type.[8]

To get an idea of how this can be, it is instructive to compare Barbour and Bertotti's action principle for Machian non-relativistic particle dynamics (equation 3.1) with the "BSW" action (equation 3.2): the action principle for GR found by Baierlein, Sharp and Wheeler (1962):[9]

$$\begin{aligned} \delta S_{\text{BB2}} = 0, \qquad S_{\text{BB2}} &= \int \mathrm{d}\lambda \sqrt{F_E T_{\text{BB2}}}\ , \\ T_{\text{BB2}} &= \frac{1}{2}\sum_{i=1}^{n} m_i \left(\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda} - \textstyle\sum_\alpha a_\alpha(\lambda)\, O_\alpha \mathbf{x}_i\right) \cdot \left(\frac{\mathrm{d}\mathbf{x}_i}{\mathrm{d}\lambda} - \textstyle\sum_\alpha a_\alpha(\lambda)\, O_\alpha \mathbf{x}_i\right); \end{aligned} \tag{3.1}$$

$$\begin{aligned} \delta I_{\text{BSW}} = 0, \qquad I_{\text{BSW}} &= \int d\lambda \int d^3x \sqrt{R T_{\text{BSW}}}\ , \\ T_{\text{BSW}} &= G^{ijkl}\left[\frac{dh_{ij}}{d\lambda} - 2N_{(i;j)}\right]\left[\frac{dh_{kl}}{d\lambda} - 2N_{(k;l)}\right]. \end{aligned} \tag{3.2}$$

[8] For more details see (Barbour 1999*b*, 167–177).

[9] (3.2) is taken from Barbour (1994, 2868, equation 35). It is not given in exactly this form in Baierlein, Sharp & Wheeler (1962). For an extensive discussion of the interpretation of (3.1), see Pooley & Brown (2001, §7).

In (3.2), the *3-dimensional* metric h_{ij} (describing the intrinsic geometry of space at an instant with respect to some *arbitrary* coordinate system) is the dynamical variable, analogous to the various particle coordinates, $\mathbf{x}_i$, in (3.1). The *shift vector* N^i is the generator of 3-dimensional diffeomorphisms and, according to the Machian point of view being expounded, appears in the action to effect the generalized best-matching mentioned in the paragraph above.[10] The 3-dimensional curvature scalar R is a conformal factor analogous to F_E. G^{ijkl}, a functional of h_{ij}, is the *DeWitt supermetric.* It is needed to define a metric on Riem(Σ) which in turn defines a metric on superspace after minimization with respect to the N^i.

The principal conceptual difference between the action principles (3.1) and (3.2) is the positioning of the integration over 3-space outside the square root in (3.2): the sum over particle coordinates appears within the square root in (3.1). In both cases time does not appear in the kinematics: λ is an arbitrary parameter labelling consecutive 3-geometries. But in GR, the time defined by the dynamics is defined *locally.* By setting the conformal factor (R) equal to the kinetic term (T_{BSW}) *at each point of space* one obtains a *different* time parameter corresponding to the local proper time along the time-like trajectories joining best-matched points of successive 3-geometries.[11]

At this point it might be helpful to mention the Hamiltonian formulation of GR, an alternative but related formulation of geometrodynamics more familiar in the canonical quantum gravity literature. When cast in Hamiltonian form, GR turns out to be a *gauge theory* in a technical sense; it is a *constrained Hamiltonian system.*[12] Central to this formulation is a subspace, often denoted Γ, of the cotangent bundle $\mathcal{T}^*$Riem(Σ). It is a *constraint surface* defined by a set of four constraint functions, the three *momentum* or *vector* constraints and the *Hamiltonian* or *scalar* constraint.

Every point of Γ is a possible spacelike hypersurface of some general relativistic spacetime. Points of $\mathcal{T}^*$Riem(Σ) that do not lie on Γ are 3-manifolds equipped with a combination of metric and extrinsic curvature tensors that is incompatible with that 3-space being embeddable in any general relativistic spacetime. Associated with the constraint functions are a set of transformations $\Gamma \to \Gamma$ which partition the constraint surface itself into subspaces. These are the so-called *gauge transformations* and the subspaces are known as *gauge orbits.* It turns out that the gauge orbits are in one-one correspondence with equivalence classes of isometric general relativistic spacetime models.

Most philosophical discussion of canonical quantum gravity takes the Hamil-

[10] Under an infinitesimal diffeomorphism generated by N^i, $h_{ij} \to h_{ij} - (N_{i;j} + N_{j;i}) = h_{ij} - 2N_{(i;j)}$. The raising and lowering of indices and covariant differentiation are defined with respect to the 3-metric h_{ij}. The O_α in (3.1) are the infinitesimal generators of the Euclidean group and occur in the action to effect a 'rigid' best matching of relative particle configurations analogous to that described for fields in the previous section (see page 4).

[11] The position of the square root in (3.2) may be extremely significant. Barbour, Foster & Ó Murchadha (2000) claim that the BSW action principle (i.e., GR) is the only consistent theory of this type and, further, that when matter is coupled to the theory one can derive local Lorentz-covariance, Maxwellian electrodynamics and the gauge principle.

[12] For an excellent discussion of GR as a gauge theory in this sense, see Belot & Earman (2001).

tonian formulation of GR, rather than a Lagrangian formulation, as its starting point. When interpreting the Hamiltonian formalism, the basic decision one faces is whether to regard the transformations generated by the constraints as 'genuine' gauge transformations indicating that the formalism contains non-physical degrees of freedom. If one does interpret them in this way, points of Γ related by the gauge transformations are held to represent the same state of affairs.

If one takes the BSW Lagrangian formulation as fundamental, however, then the momentum constraints on the one hand and the Hamiltonian constraint on the other are seen to be quite different in nature. The momentum constraints arise as a result of the generalized best matching and hence Barbour views the transformations generated by them as arising from a genuine redundancy in the formalism: one can represent one and the same 3-geometry by way of many nominally different metric tensors defined on Σ. The Hamiltonian constraint, however, is analogous to identities that hold between the canonical momenta of the standard Newtonian Jacobi principle action (discussed in Pooley & Brown 2001, §6) and those of its Machian analogue (3.1). The identities arise in these cases case because the momenta are not independent; for example, there are $3N$ canonical momenta associated with the standard Jacobi principle action for N gravitating particles yet one needs to specify only $3N - 1$ numbers to define a direction (as opposed to a direction and a speed) in the $3N$-dimensional configuration space $\mathcal{Q}$. The identities correspond to the reparameterization invariance of the actions; they do not arise due to a redundancy in the way the *configuration* data are represented.[13]

To summarize, it is clear why Barbour concludes that GR is Machian: only global relative 3-dimensional configurations count in the dynamics. The local inertial frames and local proper time do not feature in the kinematical foundations of the theory. Rather they are determined dynamically via a generalized best-matching and a localized version of the dynamical temporal metric.[14]

3.1 Sophisticated substantivalism

The immediate question one now faces is: what is the theory just described a theory of? After all, the relative configurations are intrinsic 3-geometries: they are 'relational' specifications of the geometrical properties of *space*. Even when cast as a dynamical theory concerning the evolution of 3-dimensional entities, GR turns out to be a theory involving substantival space if not substantival spacetime.

Two rather desperate strategies of retreat for the relationist should be considered.[15] In the first, which can be disposed of rather quickly, one simply stipulates that the vacuum solutions of pure geometrodynamics are unphysical, insisting that there must be matter fields present. The 3-metric field is then to be seen merely as

[13]For a very clear discussion of the analogy between the constraints of Hamiltonian GR and those of Barbour and Bertotti's 1982 theory, see Barbour (1994), especially §§6 and 12. For the relation between the BSW action for GR and the Hamiltonian formulation, see Barbour et al. (2000, sec. III).

[14]For discussion of these aspects of Barbour and Bertotti's non-relativistic relational theory, see Pooley & Brown (2001, §§7–8).

[15]Note that neither is advocated by Barbour.

encoding the distances between the various values of these other fields. This seems a hopeless position for two reasons. First, the precise form of the geometry of any Einstein-style "holes" in the matter fields of an initial data hypersurface will affect the solution. It seems contrived at best to say that the geometrical features of such empty spaces—which can be as convoluted as one likes—are manifestations of the spatial relations between parts of the surrounding matter fields. Second, and far more damaging, the specific way in which the metric field occurs in the action principle makes it appear a player very much on equal footing with any matter fields present.[16]

The second strategy is well illustrated in the following quote from Carlo Rovelli:

> Einstein's identification between gravitational field and geometry can be read in two alternative ways:
>
> i. as the discovery that the gravitational field is nothing but a local distortion of spacetime geometry; or
>
> ii. as the discovery that *spacetime geometry is nothing but a manifestation of a particular physical field*, the gravitational field.
>
> The choice between these two points of view is a matter of taste, at least as long as we remain within the realm of nonquantistic and nonthermal general relativity. I believe, however, that the first view, which is perhaps more traditional, tends to obscure, rather than enlighten, the profound shift in the view of spacetime produced by general relativity. (Rovelli 1997, 193–194)

Exact analogues for views (i) and (ii) (now reading "spatial" for "spacetime") exist for the 3-metric field of geometrodynamics. The claim that it is a matter of taste which perspective one adopts is reminiscent of Rynasiewicz's view mentioned in Section 2.1. Rovelli goes on to suggest that in light of view (ii) "it is perhaps more appropriate to reserve the expression *spacetime* ["space" if we are considering the 3-dimensional Σ] for the differential manifold, and to use the expression *matter* for everything dynamical... *including the gravitational field.*" Once the identification of diffeomorphic entities (whether 3- or 4-dimensional) has convinced us to jettison the bare manifold, "physical reality is now described as a complex interacting ensemble of entities (fields), the location of which is only meaningful with respect to one another" (1997, 194).

At this stage one risks getting mired in terminological niceties, but it is possible to be clear about what is at stake. At the beginning of this subsection I claimed that geometrodynamics is naturally interpreted as a dynamical theory *about space* (coupled to matter, if any is present). I then considered and rejected the relationist strategy which denies the primitive reality of the 3-metric field. What is now under consideration is whether one can admit the concrete reality of the 3-metric field *and yet deny the reality of space.* I maintain that to call the 3-metric "matter" is

[16] For similar problems, in a context other than geometrodynamics, confronting a reductive understanding of the 4-dimensional metric of GR, see Brown & Pooley (2001).

strained and misleading. The same point in the context of the 4-metric is forcibly put by Hoefer (1998, 459–460) who stresses the differences between the metric field and other fields, in particular that one can imagine space without matter fields but that there can be no space without the metric field. Rovelli himself admits that the 'gravitational field' is unlike any other field, in particular in the way that it couples to every other field. In addition to these features I would also stress the *indispensability* of the metric field: it is not just another field which one may or may not include in a model of a dynamically possible world as one chooses. One cannot formulate a theory of the other physical fields without the metric field.

Note, also, that one cannot apply the relationist conception of fields discussed in Section 2.1 directly to the metric field. There one used the brute spatial relations between the various intensities of matter fields to specify a relative field configuration. Here the field itself defines such distances. Although one can talk, for example, about varying values of curvature and the distances between them, a catalogue of all the shortest distance relations between points suffices to define the values of the curvature at them and anything else there is to define. In a sense there really is nothing other than the distance relations and their relata. Relata that are most simply and naturally interpreted as the the points of substantival space.

One of Rovelli's primary concerns is to stress that determination of location becomes 'relational': all that matters is contiguity between fields. But this one can readily admit while maintaining that one of these fields represents nothing other than space or spacetime itself. For although geometrodynamics is naturally interpreted as a theory about the evolution of the geometrical properties of substantival space, it equally suggests a *sophisticated* form of substantivalism; i.e., that isometric 3-spaces should be *identified*.[17] This is because the role of the shift vector in (3.2) precisely parallels that of the generators of the Euclidean group in (3.1).

Geometrodynamics can be regarded as a degenerate[18] geodesic principle on *superspace*: putatively distinct 3-spaces with the same geometry but which differ solely in terms of which points of the manifold instantiate which geometric properties are stipulated to represent the same physical possibility. Just as the Cartesian coordinate system of intrinsic particle dynamics is a convenient way to represent the relative distances between particles and does not name the points of an inertial frame, so the coordinate systems of geometrodynamics are not to be thought of as naming the points of a bare 3-manifold, points that possess primitive thisness.[19] In

[17] *Sophisticated substantivalism* can be defined as the combination of two doctrines: (1) that isometric spaces (whether they be 3- 4- or n-dimensional) should be identified (hence the "sophisticated") and, (2), that such spaces are to be thought of as primitive entities (as 'substances', hence "substantivalism")—in particular, they are not ontologically reducible to the network of actual or possible relations holding between their material contents. The phrase is due to Belot & Earman (2001, 228) who, as we are about to see, use "sophisticated" in a rather a pejorative sense.

[18] I return to the significance of this degeneracy in the next section

[19] The terminology is due to Adams (1979). Very roughly, an individual possess primitive thisness if its being the very thing it is does not supervene on purely qualitative facts. The claim

the generic, asymmetrical case, the points of space can be individuated directly in terms of their geometric properties.[20] The coordinate charts of the differentiable manifold are simply a vastly more convenient way of doing so.

Belot and Earman reject this form of substantivalism. Their reasons for doing so concern the links (as they see them) between the different interpretations of classical GR and different approaches to quantizing the theory. I briefly mentioned the Hamiltonian formulation of GR and its status as a gauge theory (see above, page 9). A *gauge-invariant* interpretation of this formalism asserts that only quantities that are invariant under the transformations generated by the constraints (the gauge transformations) are physically real.

Belot and Earman equate gauge-invariant interpretations of the classical theory with relationist interpretations and thus see relationism as underwriting gauge-invariant approaches to quantizing the theory. They also claim that "straightforward substantivalism" can be understood as underwriting particular non gauge-invariant approaches.[21] Sophisticated substantivalism, on the other hand, is in trouble:

> there is one sort of response to the hole argument which *is* clearly undesirable: the sort of sophisticated substantivalism which mimics relationism's denial of the Leibniz-Clarke counterfactuals. It would require considerable ingenuity to construct an (intrinsic) gauge-invariant substantivalist interpretation of general relativity. And if one were to accomplish this, one's reward would be to occupy a conceptual space already occupied by relationism. Meanwhile, one would forego the most exciting aspect of substantivalism: its link to approaches to quantum gravity... To the extent that such links depend on the traditional substantivalists' commitment to the existence of physically real quantities which do not commute with the constraints, such approaches are clearly unavailable to the relationist. Seen in this light, sophisticated substantivalism, far from being the savior of substantivalism, is in fact a pallid imitation of relationism, fit only for those substantivalists who are unwilling to let their beliefs about the existence of space and time face the challenges posed by contemporary physics. (Belot & Earman 2001, 248–9)

that spacetime points possess primitive thisness is very closely related to the claim that the identity relation between spacetime points of different possible worlds is primitive. The route to sophisticated substantivalism by way of a denial of such a primitive identity relation is advocated by Hoefer (1996). Hoefer himself does not endorse substantivalism (he is a relationist); he merely argues that the most defensible form of substantivalism is a sophisticated one.

[20] For issues relating to the symmetrical cases, see Butterfield (1989, 27) and Saunders (forthcoming).

[21] 'Gauge-invariant' approaches to quantizing the theory require that all observables commute with quantum-operator versions of the constraint functions. *Straightforward substantivalism* is the obvious alternative to sophisticated substantivalism. The straightforward substantivalist believes in qualitatively identical yet distinct physically possible worlds, differing solely in terms of which points of space or spacetime instantiate which geometric properties.

Their claim that the sophisticated substantivalist is required to provide an *intrinsic* gauge-invariant interpretation of GR needs to be motivated. Consider, for a moment, the spacetime formulation of the theory. The sophisticated substantivalist urges that a single physical possibility corresponds to an equivalence class of isometric spacetime models. At a mathematical level he has not provided an intrinsic description of the possibility; there is redundancy in his description corresponding to the freedom one has in choosing how to paint the particular metric and other fields onto the base manifold. But despite this, surely one has a very clear idea of the reality underlying the equivalence class, viz. a spacetime of a particular geometry whose points' identities depend on their particular geometric properties and relations. There is no obscurity either in how each member of the equivalence class can represent this single entity or in why the model–possibility representation relation is many to one. To demand an intrinsic gauge-invariant interpretation is to demand that one devise a mathematical formalism in which the relation becomes one to one. But Belot and Earman do not give any reason why the sophisticated substantivalist (or the relationist) *needs* to provide such a formalism or why he should be troubled by the prospect that providing such a formalism might prove to be an intractable problem.

Returning to geometrodynamics, although 3-space is described in terms of a coordinate-dependent metric tensor, the machinery of best-matching, and the analogy with non-relativistic Machian theories, strongly support *identifying* isometric 3-spaces. Again, there is no difficulty in grasping the nature of the entities underlying such equivalence classes; they are 3-spaces of a particular geometry. Note too that if one does not equate isometric 3-spaces in the way suggested, then consistency would seem to require that one regard Barbour and Bertotti's intrinsic particle dynamics as an indeterministic theory formulated in absolute space, surely an absurd position. The description of geometrodynamics expounded in this section is thus both substantivalist and partially gauge-invariant (the demand for an intrinsic description of a 3-geometry is simply to be rejected). The gauge-invariance is only partial because of the foliation invariance of the theory, something addressed in the next section.

What would be much harder to provide—both because of the existence of non-trivial vacuum solutions and because of the non-reducibility of metrical relations to structural properties of matter fields—is a *relationist* gauge-invariant description of GR in either its spacetime or its geometrodynamical formulation. Initially Belot and Earman characterize the opposition between relationism and substantivalism in terms of the ontological status of space and spacetime:

> substantivalists understand the existence of spacetime in terms of the existence of its pointlike parts, and gloss spatiotemporal relations between material events in terms of the spatiotemporal relations between points at which the events occur. Relationists will deny that spacetime points enjoy this robust sort of existence, and will accept spatiotemporal relations between events as primitive. (Belot & Earman 2001, 227)

But in pure geometrodynamics without matter fields what are the material events meant to be? It seems that there are only the points of space and their spatial relations. What is to be decided is how the physics treats these: whether it is in what might be called *absolutist* (or haecceitist) terms, by assigning primitive thisness to the points of the bare spatial manifold, or whether it is in 'relational' terms in that the individuality of a point of space is held to be settled by the sum total of its geometrical properties and relations. The appearance of the shift vector in the BSW action (equivalently the momentum or vector constraint of the Hamiltonian formulation of geometrodynamics) strongly suggests the latter position. After all, a geodesic principle on Riem(Σ) which did not implement generalized best-matching is conceptually possible. Such a theory would recognise a *physical* distinction between two spaces with the same geometry that differed solely in terms of which points instantiated which properties.[22]

Thus a preferable point of view to that of Belot and Earman is to see the question of whether there exist physically real quantities which do not commute with all the constraints of the Hamiltonian formulation of GR as being decisive in deciding between straightforward substantivalism on the one hand and the disjunctive set of sophisticated substantivalism and antisubstantivalist relationism on the other. If the second of these options is vindicated (say, for example, by all the observables of an empirically successful, but as yet unknown, quantum theory of gravity commuting with the constraints) then there will remain a further interpretative task. One will need to decide whether the resulting theory is a quantum theory about (intrinsically described) regions of space or spacetime, or whether quantum spacetime is reducible to something else, as the traditional antisubstantivalist relationist (such as Hoefer) desires.[23]

This shows that one needs to distinguish carefully between two uses of the term "relationism". One brand of relationism (antisubstantivalism) involves denying the fundamental existence of space or spacetime. But anti-haecceitism—the claim that a possibility is fully specified by a complete description of the qualitative properties of, and the relations holding between, its parts—is also called "relationism" by some. Saunders (forthcoming) is an example of a relationist ("sophisticated sub-

[22]Elsewhere Belot reports favouring a reading of the substantivalist–relationist debate "according to which relationism and its denial (substantivalism) are theses concerning the ontological instantiation of a given physical geometry" (Belot 2000, 575). However he goes on to "stipulate" that the substantivalist should side with Clarke in his dispute with Leibniz and regard, e.g., two possible worlds differing solely in terms of where qualitatively identical matter distributions are situated in space as genuinely distinct possibilities. Failure to grant relationism the sole right to identify such situations is meant to risk restricting it to "some other, typically quite barren, demesne" (Belot 2000, 576–7). I hope to have made it clear both why I see no principled reason for such a restriction on how the substantivalist can count possibilities and why I see no such threat to the vitality of relationism lurking on the horizon.

[23]Belot and Earman claim that Kuchař's internal time approach to quantum gravity is allied to a 'straightforward' substantivalist interpretation of GR. But while Kuchař believes that the observables of GR need not commute with the Hamiltonian constraint, he does believe that they should commute with the vector constraint. In classical terms this amounts to regarding all 3-spaces which instantiate the same 3-geometry as physically indentical but regarding different 3-geometries which nonetheless can form (different) spacelike hypersurfaces of the same general relativistic spacetime as physically distinct. Such a position seems perfectly compatible with sophisticated substantivalism (about space).

stantivalist") in this latter sense. He reserves the phrase "reductive relationism" for antisubstantivalism. I have already quoted Carlo Rovelli, who is taken by Belot and Earman as a paradigm relationist in the field of canonical quantum gravity. His prime concern as a relationist is that it is only relations of contiguity between the various dynamical fields that are physically meaningful (though this itself is a problematic claim when there is only one field) and elsewhere his descriptions of loop quantum gravity suggest a point of view which sounds suspiciously like sophisticated substantivalism:

> Loop quantum gravity is a rather straightforward application of quantum mechanics to Hamiltonian general relativity. . . In conventional QFT, states are quantum excitations of a field over Minkowski (or over a curved) spacetime. In loop quantum gravity, the quantum states turn out to be represented by (suitable linear combinations of) spin networks. A spin network is an abstract graph with links labeled by half-integers. . .
>
> Intuitively, we can view each node of the graph as an elementary 'quantum chunk of *space*'; the links represent (transverse) surfaces separating quanta of space. . . The spin network[s] represent relational quantum states: they are not located in a space. Localization must be defined in relation to them. (Rovelli 2001, 110, my emphasis)

3.2 Indeterminism and conformal superspace

One reason why Belot believes that the substantivalist should resist the temptation to regard diffeomorphically related spacetime models as physically equivalent is that the indeterminism he is thereby committed to is of only a "strangely mild variety" (1999, 47). And certainly if, in the context of geometrodynamics, one insists on regarding diffeomorphically related 3-spaces as physically distinct, then the theory is beset by a mild form of indeterminism: for a given 3-geometry, the dynamical equations determine everything except which points of 3-space will instantiate which geometrical properties. Adopting such an interpretative stance, one can perhaps regard the points of Σ as heirs to the persisting points of Newton's absolute space, though now stripped of their inertial significance (the local inertial frames are determined by the best-matching dynamics).

Ironically, however, taking the geometrodynamical formulation of GR seriously—regarding the 3-geometries (Barbour's "Nows") as the "ultimate things"—implies a thoroughly pernicious indeterminism, even when a sophisticated substantivalist interpretation of them is adopted. The situation is vividly captured by DeWitt's characterization of general relativistic spacetimes as corresponding to *sheaves* of geodesics in superspace. Recall the interpretational problems associated with the $E = 0$ relativistic field theory discussed in Section 2.2. In that case, the sequences of relative field configurations obtained from different inertial frames corresponded to observationally indistinguishable yet distinct possible worlds. In GR the uncountable number of possible foliations between two given space-like hypersurfaces will (in general) correspond to distinct sequences of 3-geometries, *all of which satisfy the action principle of geometrodynamics*. If the "Machian" geometrodynamical ontology of 3-geometries is taken at face value, each sequence corresponds to a physically distinct, though observationally indistinguishable, history. (As collections of individuals, the 3-geometries of two such sequences are qualitatively distinguishable from each other. However, each corresponds to the same spacetime and hence they constitute observationally indistinguishable *histories*, cf. footnote 7.)

Moreover—and here is the real sting—two foliations of a given spacetime can match up to a given hypersurface and diverge thereafter. The specification of an initial sequence of 3-geometries is not sufficient to allow us to predict which continuation of the sequence will be actualized—a blatant case of indeterminism. And this is not merely the indeterminism of the Hole Argument which only concerns which objects (spacetime points) play which roles in two possible worlds, the set of objects and the roles to be played being identical in the two cases (cf. Melia 1999). The indeterminism afflicting the Machian interpretation of geometrodynamics concerns which sequence of qualitatively *distinguishable* entities—3-geometries—will exist. Distinct sequences of 3-geometries involve different *roles* being played by the points of space at different times; thus, if one accepts the anti-haecceitist claim that there is no individuation except by reference to the properties of the individuals in question, they even involve different space points.

The easy moral to draw from this situation is that the spacetime formulation of GR is more fundamental than geometrodynamics. An initial point and direction in superspace *does* suffice to determine a unique space*time* geometry. If we regard the 3-geometries as arbitrary slicings through a fundamental spacetime, the indeterminism is only apparent. In the spacetime context, the sophisticated substantivalist urges us to identify models related by *4-dimensional* spacetime diffeomorphisms. By only identifying hypersurfaces related by *3-dimensional* diffeomorphisms, we have failed to take into account one redundant degree of freedom per spacetime point. No wonder indeterminism resulted.

There is perhaps one alternative that should be mentioned. One could attempt to regard a spacetime as genuinely constructed from *all possible* compatible sequences of 3-geometries. Note that these 'basic entities' would share parts. It is not at all clear whether this rather subtle and nebulous form of Machianism could offer a genuine reduction of spacetime to 3-dimensional entities and avoid collapsing into a thoroughly 4-dimensional perspective. Consider, for example, Barbour's admission:

> Machian relationships are manifestly part of the deep structure of general relativity. But are they the essential part? If the world were purely classical, I think we would have to say no, and that the unity Minkowski proclaimed so confidently is the deepest truth of space-time. (Barbour 1999*b*, 180)

He believes, however, that once quantum mechanics is taken into account, 3-dimensional entities are finally seen as ultimate.[24]

[24]For reasons that lie far beyond the scope of this paper; they are to be found in, e.g., Barbour

Before proclaiming the ascendancy of the spacetime representation of GR, at the classical level at least, there is one further discovery of Barbour's to consider. What if the redundant degree of freedom was not ultimately connected to spacetime diffeomorphism invariance? A *conformal* 3-geometry has only two true degrees of freedom per space point. Any two metrics related to each other by a space-dependent positive multiplicative factor correspond to the same conformal geometry. In representing such a geometry by a Riemannian 3-metric h_{ij} defined on a 3-dimensional differentiable manifold (six numbers per space point) there are *four* redundant degrees of freedom: three associated with the arbitrary coordinate system with respect to which h_{ij} is written, and one associated with the particular choice of h_{ij} from the equivalence class $\{h'_{ij} : h'_{ij} = \lambda(x)h_{ij}\}$.

We could thus imagine a theory defined on *conformal superspace*, the space obtained by identifying points of superspace that correspond to the same conformal geometry. Technically, to define a metric on this new configuration space, best-matching would need to be further generalized. In comparing two 3-metrics corresponding to two intrinsically different conformal geometries one would need not only to extremize with respect to 3-dimensional diffeomorphisms (in order to take into account the arbitrariness of the coordinate system), one would also need to extremize with respect to the conformal "coordinate" $\lambda(x)$ at each space point.

Recently, Barbour and Ó Murchadha have pursued this idea (Barbour & Ó Murchadha 1999). They show that one of a family of possible theories yields sequences of conformal 3-geometries corresponding to the constant mean curvature hypersurfaces of a restricted class of general relativistic spacetimes. As yet this line of work is in its early stages; but if it proved fruitful, especially in the context of quantum gravity research, it would clearly have a significant moral for the interpretative enterprise. For now a *unique* curve in the relative configuration space (which is now conformal superspace) corresponds to each general relativistic spacetime. Thus absolute simultaneity is regained: the 'real' Nows are the constant mean curvature spacelike hypersurfaces. Foliating spacetime by other sequences of hypersurfaces leads to different curves in conformal superspace, but not curves which satisfy the dynamical action principle. Regarding 3-dimensional entities (described relationally, now even with respect to local scale) as fundamental becomes a viable option once more. But these entities are nothing other than instantaneous states of substantival space.

4 Some conclusions

I wish to conclude by summarizing the principal claims of the last few sections. Barbour has argued that, when cast in the geometrodynamical form given by Baierlein, Sharp and Wheeler, GR is naturally interpreted as a Machian theory. I endorse this point of view but it does not then follow that GR can also be viewed as a relational theory. On the contrary, the most natural interpretation of geometrodynamics is as a theory about substantival *space*. (Space*time* is no

(1999*b*, Parts 4 and 5).

longer seen as a fundamental entity and herein lies the Machian nature of the interpretation: inertial structure is given a reductive, dynamical explanation.)

The natural interpretation of geometrodynamics is also a *sophisticated* substantivalism: putatively distinct spaces which differ solely in terms of which points of space instantiate which geometrical properties are stipulated to be numerically identical. Only if one takes this interpretative stance does one do justice both to the close analogy between the BSW action and Barbour and Bertotti's prerelativistic intrinsic particle dynamics and to the machinery of best-matching as it occurs in the BSW action. Belot and Earman's qualms about sophisticated substantivalism seem ill-founded. In particular when two senses of "relationism" are distinguished (anti-substantivalism and anti-haecceitism) it is possible to view many self-proclaimed relationists in the field of canonical quantum gravity as sophisticated (i.e., anti-haecceitist) substantivalists.

However, the interpretation of geometrodynamics that I have advocated is only sustainable as an interpretation of GR if the geometrodynamical formulation of the theory can be taken to be more fundamental than the spacetime formulation. In the final section I point to a severe problem with viewing it in this way: the theory is radically indeterministic. One could view the indeterminism as merely apparent if one was able to view different foliations of a given spacetime as corresponding to a single reality. My claim is that to do so is to concede that the spacetime viewpoint as fundamental.

If one wishes both to assert the primacy of 3-spaces over spacetime and to avoid indeterminism, one's theory cannot treat all foliations of spacetime on an equal footing. Barbour and Ó Murchadha's recent investigations into the possibility of formulating geometrodynamics on conformal superspace suggest that such theories are possible and that GR can be reinterpreted as just such a theory.

There are still many unsettled philosophical questions concerning the nature and status of spacetime and the interpretation of GR. The most obvious relate to the range of different formulations of the theory that are available, the connections these have to different approaches to quantum gravity and the different interpretative stances they suggest. But despite the range of live options, two tentative conclusions can be reached. The most natural interpretations of the theory are substantivalist, whether the basic entity is taken to be space or spacetime. Moreover, the most promising type of substantivalism would appear to be a sophisticated one.

Acknowledgements

I am very grateful to Julian Barbour, Harvey Brown and Simon Saunders for many helpful comments on, and discussions of, drafts of this paper. Particular thanks are due to Jeremy Butterfield for a very extensive set of comments on an earlier draft that led to numerous significant improvements.

A Appendix: relativistic relational field theory

In this appendix, I principally follow Barbour (forthcoming, chapter 7). Consider the following Lorentz-invariant action principle for a scalar field ϕ:

$$\delta S = 0, \qquad S = \int d^4x \mathcal{L}\left(\phi, \phi_{,\mu}\right) = \int d^4x \left(\phi_{,\nu}\phi^{,\nu} - U\left(\phi, \phi_{,\mu}\right)\right). \tag{A.1}$$

where U is a Lorentz scalar. A field will satisfy this action principle in some region of spacetime if the value of S for this region is stationary under variations of the field variables which vanish on the boundary of that region. Specializing to a specific inertial frame and effecting a "$3 + 1$ decomposition" of (A.1), one writes:

$$S = \int dt \int d^3x \left(\left(\phi_{,t}\right)^2 - (\nabla\phi)^2 - U\left(\phi, \phi_{,\mu}\right)\right). \tag{A.2}$$

t is now to be regarded as the sole independent variable and the field values at each point of space are to be regarded as the dynamical degrees of freedom. Treating the spatial and time coordinates differently in this manner means that one really has chosen a specific inertial frame with respect to which the dynamics is formulated. Clearly, however, the same expression is obtained no matter which inertial frame is chosen.

One can now pass to a Jacobi form of (A.2). We are considering sequences of 3-dimensional field configurations (defined with respect to some inertial frame) labelled by t, the time of that inertial frame. One can relabel these curves by an arbitrary monotonic parameter λ and regard $t = t(\lambda)$ as a dependent variable. Assuming $U = U(\phi)$, one obtains the following parameterized form (with respect to time) of (A.2):

$$S = \int d\lambda \int d^3x \left(\frac{\dot{\phi}^2}{\dot{t}} - \dot{t}\left((\nabla\phi)^2 - U(\phi)\right)\right), \tag{A.3}$$

where the dots denote differentiation with respect to λ. Since only the derivative of t occurs in (A.3), time can be completely eliminated to give the Jacobi type action:

$$S_{\phi\text{Jac}} = \int d\lambda \sqrt{\int d^3x \left(E - (\nabla\phi)^2 - U(\phi)\right)\left(\frac{d\phi}{d\lambda}\right)^2}, \tag{A.4}$$

where E is the total energy (see Lanczos 1970, 125–129, 132–135).

(A.4) is formulated with respect to an inertial frame, but one obtains a related relational field theory by replacing the kinetic term $\int d^3x\,(d\phi/d\lambda)^2$ by the "best-matching" relational analogue to obtain:

$$S_{\phi\text{Mach}} = \int d\lambda \sqrt{\int d^3x \left(E - (\nabla\phi)^2 - U(\phi)\right)\left(\frac{d\phi}{d\lambda} - a_\alpha(\lambda) O_\alpha \phi\right)^2} \tag{A.5}$$

(A.5) yields a string of relative field configurations as solutions. One can stack these according to the equilocality relation found by the best-matching minimization and then 'space' them according to the simplifying time parameter to obtain a 4-dimensional field configuration. Just as in the particle case, this will be a solution of the action principle (A.2) and hence represent a solution to a Lorentz-invariant field theory. However it will be a field configuration of energy $\int d^3x T^{00} = E$ for which the linear momentum and the intrinsic angular momentum vector vanish, $\mathbf{P} = 0 = \mathbf{S}$. $P^i = \int d^3x T^{i0}$ and S^i is given by $\epsilon^{ijk} S^k = S^{ij} = \int d^3x \left(\left(x^i - x^i_{\text{CM}} \right) T^{0j} - \left(x^j - x^j_{\text{CM}} \right) T^{0i} \right)$. That these are conserved quantities follows from the invariance of (A.1) under spacetime translations and spatial rotations respectively. The centre of mass coordinate is given by $x^i_{\text{CM}} = (1/E) \int d^3x T^{00} x^i$ and the energy-momentum tensor is given by:

$$T^\mu_\nu = \frac{\partial \mathcal{L}}{\partial (\phi_{,\mu})} \phi_{,\nu} - \delta^\mu_\nu \mathcal{L} \tag{A.6}$$

P^μ transforms as a Lorentz 4-vector under Lorentz transformations. Hence if $P^0 = E \neq 0$ and $P^i = 0$ in some inertial frame F, then $P^0 \neq E$ and $P^i \neq 0$ in inertial frames moving relatively to F.

Consider the field's total intrinsic angular momentum,

$$S^{\mu\nu} = \int d^3x \left(\left(x^\mu - x^\mu_{\text{CM}} \right) T^{0\nu} - \left(x^\nu - x^\nu_{\text{CM}} \right) T^{0\mu} \right), \tag{A.7}$$

where the point x^μ_{CM} can be anywhere on the centre of mass's worldline. In the rest frame ($P^i = 0$), $S^{0j} = 0$. Hence in the best-matched frame for which $P^i = 0 = S^{ij}$, $S^{\mu\nu} = 0$.

The total angular momentum on an arbitrary hypersurface Σ about spacetime point p, coordinates $\{a^\mu\}$ is given by

$$J^{\mu\nu}(\text{about } p) = \int_\Sigma d^3\Sigma_\alpha \left(\left(x^\mu - a^\mu \right) T^{\nu\alpha} - \left(x^\nu - a^\nu \right) T^{\mu\alpha} \right) \tag{A.8}$$

and is independent of the hypersurface Σ on which it is calculated (Misner, Thorne & Wheeler 1973, Box 5.6). The relation between the total angular momentum calculated about two points p and q, coordinates $\{a^\mu\}$ and $\{b^\mu\}$, is given by

$$J^{\mu\nu}(\text{about } p) - J^{\mu\nu}(\text{about } q) = -c^\mu P^\nu + c^\nu P^\mu, \tag{A.9}$$

where $c^\mu = a^\mu - b^\mu$ is the 4-vector from q to p.

Now if $E = 0$ in the best-matched inertial frame then $P^\mu = 0$, clearly a Lorentz-invariant condition. Note that for the action principle we are considering, E can vanish only if U can take negative values, for $(\nabla\phi)^2$ is always positive unless ϕ vanishes everywhere. In this special case in which $P^\mu = 0$ it follows from (A.9) that $J^{\mu\nu}$ takes the same value about every point of spacetime. Although the expression for $S^{\mu\nu}$ given above ceases to be well defined when $E = 0$, we may suppose that, by continuity, $J^{\mu\nu} = 0$. It follows that with respect to every inertial frame, $E = 0$, $P^i = 0$ and $S^i = 0$. Therefore the sequences of relative field configurations obtained from every inertial frame satisfy the relational variational principle (A.5).

References

Adams, R. M. (1979), 'Primitive thisness and primitive identity', *Journal of Philosophy* **76**, 5–26.

Baierlein, R. F., Sharp, D. H. & Wheeler, J. A. (1962), 'Three-dimensional geometry as carrier of information about time', *Physical Review* **126**(5), 1864–1865.

Barbour, J. B. (1994), 'The timelessness of quantum gravity: I. the evidence from the classical theory', *Classical and Quantum Gravity* **11**, 2853–2873.

Barbour, J. B. (1999*a*), The development of Machian themes in the twentieth century, *in* J. Butterfield, ed., 'The Arguments of Time', Oxford University Press, Oxford, pp. 83–109.

Barbour, J. B. (1999*b*), *The End of Time: The Next Revolution in Our Understanding of the Universe*, Weidenfeld & Nicholson, London.

Barbour, J. B. (forthcoming), *Absolute or Relative Motion? The Deep Structure of General Relativity*, Vol. 2, Oxford University Press, New York.

Barbour, J. B. & Bertotti, B. (1982), 'Mach's principle and the structure of dynamical theories', *Proceedings of the Royal Society, London* **A 382**, 295–306.

Barbour, J. B., Foster, B. Z. & Ó Murchadha, N. (2000), 'Relativity without relativity', `http://arxiv.org/abs/gr-qc/0012089`.

Barbour, J. B. & Ó Murchadha, N. (1999), 'Classical and quantum gravity on conformal superspace', `http://arxiv.org/abs/gr-qc/9911071`.

Barbour, J. B. & Pfister, H., eds (1995), *Mach's Principle: From Newton's Bucket to Quantum Gravity*, Vol. 6 of *Einstein Studies*, Birkhäuser, Boston.

Belot, G. (1999), 'Rehabilitating relationism', *International Studies in the Philosophy of Science* **13**(1), 35–52.

Belot, G. (2000), 'Geometry and motion', *British Journal for the Philosophy of Science* **51**, 561–595.

Belot, G. & Earman, J. (2001), Pre-Socratic quantum gravity, *in* C. Callander & N. Huggett, eds, 'Physics Meets Philosophy at the Planck Scale', Cambridge University Press, Cambridge, pp. 213–255.

Brown, H. R. (1997), 'On the role of special relativity in general relativity', *International Studies in the Philosophy of Science* **11**(1), 67–81.

Brown, H. R. & Pooley, O. (2001), The origins of the spacetime metric: Bell's Lorentzian pedagogy and its significance in general relativity, *in* C. Callender & N. Huggett, eds, 'Physics Meets Philosophy at the Plank Scale', Cambridge University Press, Cambridge, pp. 256–272.

Butterfield, J. N. (1989), 'The hole truth', *British Journal for the Philosophy of Science* **40**, 1–28.

Butterfield, J. N. (2001), 'The end of time?', *British Journal for the Philosophy of Science* **XXX**, XXX.

DiSalle, R. (1994), 'On dynamics, indiscernibility, and spacetime ontology', *British Journal for the Philosophy of Science* **45**, 265–287.

Earman, J. (1989), *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time*, MIT Press, Cambridge MA.

Field, H. (1980), *Science Without Numbers: A Defence of Nominalism*, Princeton University Press, Princeton.

Hoefer, C. (1996), 'The metaphysics of space-time substantivalism', *Journal of Philosophy* **93**(1), 5–27.

Hoefer, C. (1998), 'Absolute versus relational spacetime: For better or worse, the debate goes on', *British Journal for the Philosophy of Science* **49**, 451–467.

Lanczos, C. (1970), *The Variational Principles of Mechanics*, fourth edn, Dover Publications, Inc., New York.

Melia, J. (1999), 'Holes, haecceitism and two conceptions of determinism', *British Journal for the Philosophy of Science* **50**, 639–664.

Misner, C. W., Thorne, K. S. & Wheeler, J. A. (1973), *Gravitation*, W. H. Freeman and Company, San Francisco.

Pooley, O. & Brown, H. R. (2001), 'Relationism rehabilitated? I: Classic mechanics.', *British Journal for the Philosophy of Science* **XXX**, XXX.

Rovelli, C. (1997), Halfway through the woods: Contemporary research on space and time, *in* J. Earman & J. Norton, eds, 'The Cosmos of Science', University of Pittsburgh Press, Pittsburgh, pp. 180–223.

Rovelli, C. (2001), Quantum spacetime: What do we know?, *in* C. Callender & N. Huggett, eds, 'Physics Meets Philosophy at the Planck Scale', Cambridge University Press, Cambridge, pp. 101–122.

Rynasiewicz, R. A. (1996), 'Absolute versus relational space-time: An outmoded debate?', *Journal of Philosophy* **93**, 279–306.

Saunders, S. W. (forthcoming), Indiscernibles, general covariance, and other symmetries, *in* J. Renn, ed., 'Revisiting the Foundations of Relativistic Physics. Festschrift in Honour of John Stachel', Kluwer, Dordrecht.