Can the ontology of Bohmian mechanics include only particles?

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Abstract

It has been suggested that the wave function of the universe is not ontic but nomological, and there are only particles in the ontology of Bohmian mechanics. In this paper, I argue that this view will lead to certain impossible situations, such as that two free Bohmian particles, which have exactly the same properties and the same state of motion initially, have different states of motion later.

Key words: Bohmian mechanics; particles; wave function; nomological view

1 Introduction

Bohmian mechanics or the pilot-wave theory of de Broglie and Bohm provides an ontology of quantum mechanics in terms of particles and their trajectories in space and time (de Broglie, 1928; Bohm, 1952). However, it has been debated if the ontology of Bohmian mechanics includes only particles. According to some authors, the universal wave function is not ontic, representing a concrete physical entity, but nomological, like a law of nature (Dürr et al, 1992; Allori et al, 2008; Goldstein and Zanghì, 2013; Esfeld et al, 2014; Goldstein, 2021). On this view, there are only particles in the ontology of Bohmian mechanics.[1] While according to others (Bohm and Hiley, 1993;

[1]Note that unlike Humeanism and dispositionalism, primitivism about laws as suggested by Maudlin (2007) attributes a fundamental ontic role to the universal wave function. Thus, on primitivism one may also say that the ontology of Bohmian mechanics includes both particles and the wave function even when assuming the nomological view of the wave function (see Dorato and Esfeld, 2015; Dorato, 2015 for a different view). In

Holland, 1993; Gao, 2017; Hubert and Romano, 2018; Valentini, 2020), the ontology of Bohmian mechanics includes both particles and the wave function. In this paper, I will present a new result which may help examine the ontology of Bohmian mechanics. In particular, I will argue that if the ontology of Bohmian mechanics includes only particles, then there will exist certain impossible situations, such as that two free Bohmian particles, which have exactly the same properties and the same state of motion initially, have different states of motion later.

2 Bohmian mechanics

In Bohmian mechanics, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The law of motion is expressed by two equations: a guiding equation for the configuration of particles and the Schrödinger equation, describing the time evolution of the wave function which enters the guiding equation. It can be formulated as follows:

$$\frac{dX(t)}{dt} = v^{\Psi(t)}(X(t)), \tag{1}$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t), \tag{2}$$

where $X(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function, and v equals to the velocity of probability density in standard quantum mechanics.

The status of the above equations is different, depending on whether one considers the physical description of the universe as a whole or of a subsystem thereof. Bohmian mechanics starts from the concept of a universal wave function (i.e. the wave function of the universe), figuring in the fundamental law of motion for all the particles in the universe. That is, $X(t)$ describes the configuration of all the particles in the universe at time t, and $\Psi(t)$ is the wave function of the universe at time t, guiding the motion of all particles taken together. To describe subsystems of the universe, the appropriate concept is the effective wave function.

The effective wave function is the Bohmian analogue of the usual wave function in standard quantum mechanics. It is not primitive, but derived from the universal wave function and the actual spatial configuration of all the particles ignored in the description of the respective subsystem (Dürr, Goldstein and Zanghì, 1992). The effective wave function of a subsystem can be defined as follows. Let A be a subsystem of the universe including N

this sense, the result of this paper is still consistent with the nomological view of the wave function when assuming primitivism about laws.

particles with position variables $x = (x_1, x_2, ..., x_N)$. Let $y = (y_1, y_2, ..., y_M)$ be the position variables of all other environmental particles not belonging to A. Then the subsystem A's conditional wave function at time t is defined as the universal wave function $\Psi(x, y, t)$ evaluated at $y = Y(t)$:

$$\psi_A(x,t) = \Psi(x,y,t)|_{y=Y(t)}, \tag{3}$$

where $Y(t)$ is the positions of the environmental particles at time t. If the universal wave function can be decomposed in the following form:

$$\Psi(x,y,t) = \varphi(x,t)\phi(y,t) + \Theta_t(x,y,t), \tag{4}$$

where $\phi(y,t)$ and $\Theta(x,y,t)$ are functions with macroscopically disjoint supports, and $Y(t)$ lies within the support of $\phi(y,t)$, then $\psi_A(x,t) = \varphi(x,t)$ (up to a multiplicative constant) is A's effective wave function at t. It can be demonstrated that the temporal evolution of A's particles is given in terms of A's conditional wave function in the usual Bohmian way, and when the conditional wave function is A's effective wave function, it also obeys a Schrödinger dynamics of its own.

3 A special case

Now I will analyze the view that there are only particles in the ontology of Bohmian mechanics.

First, I will present a result useful for later analysis. Suppose there are two free (uncorrelated) particles that have the same properties. Moreover, they have the same state of motion at an initial instant, and the law of motion is deterministic for them. The question is: will they have the same state at later instants? If the laws of motion are the same for the two particles, then they will have the same state at later instants. If the laws of motion are different for the two particles, then they may not have the same state at later instants. But this is an impossible situation; since the two particles have exactly the same properties, the law of motion cannot distinguish them, and thus it must be the same for the two particles.

Next, I will argue that this impossible situation also appears in a special Bohmian universe when there are only particles in the ontology of Bohmian mechanics. Suppose there are two free electrons A and B being in a product state $\psi(x_A, t_0)\varphi(x_B, t_0)$ at an initial instant t_0, where $\psi(x_A, t_0)$ and $\varphi(x_B, t_0)$ are two spatially separated wavepackets. Moreover, the interactions between these two electrons and the interactions between each of them and the environmental particles are so weak that they can be ignored, and thus the two electrons will keep being in a product state for a long time. The universe whose wave function is a product state is a very special universe.

If there are only particles in the ontology of Bohmian mechanics, then we have two Bohmian particles A and B (besides the Bohmian particles in the environment) in ontology, and the state of motion of each particle at each instant is represented by its position and velocity at the instant. The velocity of each Bohmian particle is determined by the guiding equation: $v(x,t) = \frac{1}{m}\nabla S(x,t)$, where m is the mass of electron, and $S(x,t)$ is the phase of the wave function of the corresponding electron. Suppose the velocities of the two Bohmian particles at the initial instant are the same, namely $v_A(x_A(t_0),t_0) = v_B(x_B(t_0),t_0)$, where $x_A(t_0)$ and $x_B(t_0)$ are the initial positions of the two Bohmian particles, respectively. Then we will have two Bohmian particles which have the same state of motion at an initial instant (by space translation invariance).[2] According to the guiding equation, when $\nabla S_A(x_A(t),t) \neq \nabla S_B(x_B(t),t)$ at a later instant t, which is permitted when the two electrons have different initial wave functions, the velocities of the two Bohmian particles will be different at the instant.[3] This means that the Bohmian particles of two free electrons initially have the same state of motion, but laterly have different states of motion.

This is an impossible situation. Since the two Bohmian particles have exactly the same properties, the law of motion cannot distinguish them, and thus it must be the same for them, which means that when they have the same state of motion initially, they must have the same state of motion laterly. Note that the two free electrons and the environment are initially in a product state and their interactions can be ignored, and thus the Bohmian particles in the environment have no influences on the Bohmian particles of the two electrons, and the Bohmian particle of each electron has no influences on the Bohmian particle of the other electron either.[4]

4 A general case

The general universal wave function is not a product state but an entangled state. In this case, we need to analyze the effective wave functions of subsystems of the universe.

Suppose there are two free electrons A and B whose effective wave function is a product state $\psi(x_A,t_0)\varphi(x_B,t_0)$ at an initial instant t_0, where $\psi(x_A,t_0)$ and $\varphi(x_B,t_0)$ are two spatially separated wavepackets. Moreover, the interactions between these two electrons and the interactions between each of them and the environmental particles are so weak that they can be ignored, and thus the two electrons will keep being in an effective product

[2] If the two Bohmian particles are in the same initial position, which is permitted by Bohmian mechanics, then space translation invariance is not needed. In this case, the wavepackets of the two electrons can still be spatially well-separated.

[3] Even when the two electrons have the same initial wavepacket centered at different positions, the velocities of their Bohmian particles may be also different at a certain instant if the wavepacket assumes a particular form and the two Bohmian particles are in different positions within the support of their wavepackets at the instant.

[4] I will analyze the influences of the interactions in more detail later.

state for a long time. In this case, the universal wave function at t_0 can be written as

$$\Psi(x_A, x_B, y, t_0) = \psi(x_A, t_0)\varphi(x_B, t_0)\phi(y, t_0) + \Theta(x_A, x_B, y, t_0), \qquad (5)$$

where y is the position variables of all other environmental particles not belonging to A and B, $\phi(y, t_0)$ and $\Theta(x_A, x_B, y, t_0)$ are functions with macroscopically disjoint supports, and the positions of the environmental particles at time t_0, $Y(t_0)$, lies within the support of $\phi(y, t_0)$.

As argued before, there are situations in which the Bohmian particles of the two free electrons initially have the same velocity, but laterly have different velocities. The key for the general case is to argue that the later difference of the velocities of the Bohmian particles of the two free electrons does not completely result from the influences of the environmental particles. This can be done with two steps. First, when the positions of the environmental particles, $Y(t)$, keeps being within the support of $\phi(y, t)$, the change of the effective wave function of the two free electrons results not from the influences of the environmental particles, but from its own free Schrödinger evolution. Next, the change of the effective wave function of the two free electrons can result in the difference of the velocities of their Bohmian particles.

In fact, one can even argue that the difference of the velocities of the Bohmian particles of the two free electrons completely results from the change of their effective wave function (at least in one inertial frame). In order to see this, consider an inertial frame in which the Bohmian particles of the two free electrons have the same zero velocity initially. In this case, after the initial instant t_0, if the effective wave function of the two free electrons did not change, then their Bohmian particles would still have the same zero velocity, no matter how the environmental particles move. Only the effective wave function of the two free electrons changes (due to its free Schrödinger evolution), can their Bohmian particles have different velocities.

Certainly, after the initial instant t_0, even if the effective wave function of the two free electrons does not change in an inertial frame, their Bohmian particles may also have different velocities lately when these particles have the same nonzero velocity initially in this inertial frame. The difference of the velocities of these Bohmian particles may result from the spatial difference of the effective wave functions of the two free electrons. Moreover, it can be argued that the difference of the velocities of these Bohmian particles does not result from the influences of the environmental particles by considering an extreme case. Suppose the function $\phi(y, t)$ has nodes and the environmental particles are in one of these nodes before the initial instant t_0. Then, the difference of the velocities of these Bohmian particles cannot result from the influences of the environmental particles, since these environmental particles already disappear and no longer exist after the initial instant t_0.[5]

Therefore, we also have the impossible situation in the general case. If there are only particles in the ontology of Bohmian mechanics, then it is impossible to explain why the Bohmian particles of two free electrons initially have the same state of motion but laterly have different states of motion when the influences of the environmental particles can be excluded. Since the two free Bohmian particles have exactly the same properties and their motion is not influenced by the environmental particles, the law of motion must be the same for them, which means that when they have the same state of motion initially, they must have the same state of motion laterly.

5 An analysis of subsystems with interactions

The above analysis assumes that the interactions between the two electrons and the interactions between each electron and the environmental particles are so weak that they can be ignored. In this section, I will consider the influences of the interactions and clarify in what sense they can be ignored in deriving the impossibility result.

As argued above, if there are only particles in the ontology of Bohmian mechanics, then for two free electrons, the situation that their Bohmian particles initially have the same velocity but laterly have different velocities will be an impossible situation. However, for two interacting electrons, the situation that their Bohmian particles initially have the same velocity and laterly have different velocities may be not an impossible situation, since the later difference of the velocities of the two Bohmian particles may result from the interactions between the two electrons. Thus we need to analyze how the interactions between the two electrons influence the difference of the velocities of their Bohmian particles.

According to the Schrödinger equation, there are two evolution terms that determine the time evolution of the wave function: one is the free Hamiltonian, and the other is the interactive Hamiltonian. When the interactions between the two electrons and the interactions between each electron and the environmental particles are very weak, the interactive Hamiltonian can be ignored when compared with the free Hamiltonian for the time evolution of the wave function of the two electrons. Then, the change of the

[5] In the final analysis, although the effective wave function of a subsystem is determined by both the universal wave function and the positions of all other Bohmian particles not belonging to this subsystem, the role played by these Bohmian particles is only selecting which function the effective wave function of the subsystem is, while each selected function is independent of these Bohmian particles and completely determined by the universal wave function. This conclusion can be reached by a careful analysis of the definition of the effective wave function, namely Eq.(4).

wave function of the two electrons over time mainly results from its free evolution, not from the influences of the interactions.

Furthermore, according to the guiding equation, the change of the wave function of the two electrons over time results in the difference of the velocities of their Bohmian particles. Then, it is the free evolution of the wave function of the two electrons, not the interactions between the two electrons or the interactions between each electron and the environmental particles, that results in the most of the difference of the velocities of the Bohmian particles of the two electrons. In other words, the interactions alone cannot explain the later difference of the velocities of these Bohmian particles. Therefore, for two interacting electrons, the situation that their Bohmian particles initially have the same velocity and laterly have different velocities is still an impossible situation.

6 Can a stochastic law of motion avoid the result?

The above analysis is based on the *deterministic* law of motion of Bohmian mechanics. An interesting question is: can a stochastic law of motion avoid the impossibility result? It can be expected that the answer depends on the specific stochastic law of motion. As we know, there are stochastic variants of Bohmian mechanics, a typical one of which is the Bohm-Bell-Vink dynamics or Vink's dynamics (Bell, 1984; Vink, 1993; Barrett, 1999, p.203). In this section, I will analyze this stochastic theory.

The continuity equation in the discrete position representation $|x_n\rangle$ for a one-particle system is:

$$\hbar\partial P_n(t)/\partial t = \sum_m J_{nm}(t), \tag{6}$$

where

$$P_n(t) = |\langle x_n\,|\psi(t)\rangle\,|^2, \tag{7}$$

$$J_{nm}(t) = 2Im(\langle\psi(t)\,|x_n\rangle\,\langle x_n|H\,|x_m\rangle\,\langle x_m\,|\psi(t)\rangle), \tag{8}$$

where $|\psi(t)\rangle$ is the wave function of the system, and H is the Hamiltonian of the system.

In Vink's dynamics, the position jumps of the Bohmian particle of the system are governed by a transition probability $T_{mn}dt$ which gives the probability to go from position x_n to x_m. The transition matrix T gives rise to a time-dependent probability distribution x_n (for an ensemble of identically prepared systems), $P_n(t)$, which has to satisfy the master equation:

$$\partial P_n(t)/\partial t = \sum_m (T_{nm}P_m - T_{mn}P_n). \tag{9}$$

Then when the transition matrix T satisfies the following equation:

$$J_{nm}/\hbar = \sum_m (T_{nm}P_m - T_{mn}P_n). \tag{10}$$

the above continuity equation can be satisfied.

Vink (1993) showed that when choosing Bell's simple solution where for $n \neq m$[6]

$$T_{nm} = \begin{cases} J_{nm}/\hbar P_m, & J_{nm} \geq 0 \\ 0, & J_{nm} < 0, \end{cases} \tag{11}$$

the dynamics reduces to the guiding equation of Bohmian mechanics in the continuum limit.

In this stochastic theory, the velocity of a Bohmian particle in Bohmian mechanics is replaced by the transition probability of a Bohmian particle, which gives the probability for the Bohmian particle to go from its current position to another future position. Thus, the state of motion of a Bohmian particle includes both its position and its transition probability. Since the law of motion for the transition probability is deterministic and different wave functions will lead to different evolution of the transition probability, we will have the similar impossible situations as in Bohmian mechanics (when assuming that the ontology of the theory includes only particles); two free Bohmian particles, which have exactly the same properties and the same transition probability initially, have different transition probabilities later.

7 Conclusion

In this paper, I have argued that the view that the ontology of Bohmian mechanics includes only particles will lead to certain impossible situations, such as that two free Bohmian particles, which have exactly the same properties and the same state of motion initially, have different states of motion later. There are two possible ways to avoid this impossibility result. One way is to find a stochastic variant of Bohmian mechanics. Although the Vink dynamics fails, maybe another stochastic theory may succeed. The other way is to include the wave function in the ontology of Bohmian mechanics. If the wave function is in the ontology, then why the Bohmian particles of two free electrons, which initially have the same state of motion, have different states of motion later is because they are not really free but affected by different wave functions. It remains to be seen which way is a better way to avoid the impossibility result.

[6]The probability $T_{nn}dt$ follows from the normalization relation $\sum_m T_{nm}dt = 1$.

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