Measuring up to the Measurement Problem

Decoherence and Bohr's ideas through the lens of the measurement problem and quantum erasers

Master's Thesis
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Abstract

In this thesis, interpretations of the formalism of quantum mechanics are investigated in terms of their address to the classic measurement problem as well as the more modern quantum erasers. The main focus is on the interpretational insight provided by Niels Bohr and the concept of decoherence, but with an overview of other important interpretations as well. The measurement problem is described and strategies for its solution is divided into two main categories: solutions and dissolutions, which are associated with collapse and no-collapse interpretations respectively. Decoherence is found to require an interpretational basis in order to properly address the measurement problem, while Bohr's interpretation has some unresolved points, mainly relating to the understanding of Bohr's notion of context, which is central to his idea of quantum mechanics. By comparing Bohr's ideas and decoherence, I argue that each can be of use to the other; decoherence can formalise some of Bohr's concepts, while Bohr's ideas provides a constructive interpretational basis for decoherence. Lastly, I argue that quantum erasers provides a ground for discussions on interpretational questions, as the insight into the nature of quantum mechanics challenges several aspects of the aforementioned different interpretations, the understanding of the Bohrian context among them.

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Jeg hører godt, du kalder mig for ingen,
men det er mig, der svøbt i kejserkåbe
ser dig an fra sommerfuglevingen.

Inger Christensen, Sommerfugledalen, 1991

Author's Note

This is an edited version of the original thesis. Expect for correcting minor errors, this edition differs from the original thesis by quoting Niels Bohr in English, rather than (mainly) in Danish. I have not translated the quotes myself, but have found English versions of the same Danish texts, which were used in the original thesis. This edition of the text was finished in May 2023.

Contents

1 Introduction

Since the development of quantum mechanics the question of how to interpret the formalism has been a difficult topic. To this day, no interpretation is generally agreed upon, and discussions of the understanding of a formalism, which has been so successful in physics during the last century, remain open. The advancement in the field of quantum information in recent years can be said to have kindled these discussions. Quantum information is information theory which rather than being based on the use of classical bits, uses two-level quantum systems (qubits). This field of research has uncovered many new features and uses of quantum mechanics, with which the established interpretations can be confronted. This can turn out both as a clarification and as a challenge to the different interpretations, and can in either case serve as a fresh foundation on which to conduct interpretational discussions, and thus give rise to new insight.

At the heart of discussions on the understanding of quantum mechanics lies an issue known under the name of the measurement problem. This problem was present in the early days of quantum theory and is to this day far from being settled to general satisfaction. Many different angles, views and conclusions have been drawn from it and, like the new discoveries of quantum information, it serves as a ground on which different interpretations can be presented and discussed.

In general terms, the measurement problem is concerned with the discrepancy between the quantum theoretical description, where a system can be in e.g. indefinite states of superposition and which has produced very accurate predictions upon varied behaviours of atomic and subatomic systems, and the definite, separable world around us, which has given rise to the laws of classical physics. If, when and how or how not this change from quantum to (apparently) classical descriptions takes place is the questions that the many attempts to solve the measurement problem strives to tackle. It is a problem that any theory will have to directly address, and as different interpretations will do so differently, it can serve as an illustration of an interpretation's idea of the world.

This thesis is concerned with the interpretation of quantum mechanics. I will examine interpretations in the light of the classic measurement problem as well as from the perspective of a new piece of quantum physics: quantum erasers. This is a protocol, which by erasing information stored in a qubit allows one to observe interference in the data from measurements where interference has not been seen due to the storing of information in the qubit in the first place.

I will mainly be concerned with the ideas of Niels Bohr on how to understand quantum mechanics and with the feature of the quantum formalism known as decoherence. Decoherence was, with questionable success, put forward as a solution to the measurement problem from the quantum formalism itself, while Niels Bohr, who was among the founders of quantum mechanics, has written and said much on interpretational questions. Bohr's ideas will be set up against other interpretations, the Everett interpretation, Bohmian mechanics and Ghirardi–Rimini–Weber (GRW) theory, whose addresses to the measurement problem and accounts of quantum erasers will also be discussed. The thesis will serve the purpose of giving an idea of the current state of affairs in regards to the measurement problem, as well as a discussion of how a Bohrian understanding of quantum mechanics can be useful in decoherence and vice versa and how such an understanding fare against the insight quantum erasers can give into the nature of quantum mechanics.

Section 2 will be a characterisation of the measurement problem. This includes an account of different precise definitions or formulations of the problem, which has been presented elsewhere and which represents different angles from which one can view the problem. Furthermore I will discuss the terms "classical" and "measurement", which will occur frequently throughout the thesis, and therefore require more precise definitions than those provided by ordinary language. Section 3 is dedicated to an overview of the many different attempts to solve or dissolve the measurement problem by employing different interpretations of the quantum formalism. Here I will discuss Bohmian mechanics, the Everett interpretation and GRW Theory, and how each of these hold up against the different variations of the measurement problem. In section

4, I will give an account of decoherence as well its role as a possible (partial) solution to the measurement problem and a discussion of some of the issues it faces from being a feature of the quantum formalism rather than an interpretation. Niels Bohr's ideas on quantum mechanics and how these relates to the measurement problem will be the topic of section 5, and in section 6 I will compare these with the characteristics of decoherence. Here, I will argue that many similarities exist between the two, and thus motive the idea of using each to give insight into the understanding of the other. Section 7 will consist of an account of quantum erasers and a discussion of the consequences this piece of physics has for the different interpretations. Lastly, in section 8, I will summarise the main points of the thesis and give an idea of the open questions worth of further investigation.

2 Characterisation of the measurement problem

Despite its central place in many discussions on the foundational issues of quantum mechanics, it is not always perfectly clarified what the measurement problem actually is. Fundamentally, it concerns the question of the relationship between the quantum description of the (sub)atomic world and the (at least apparently) classical experiences we have of macroscopic objects. However, an exact definition depends on what approach one is taking to the problem, and formulations vary between different accounts, according to the different focuses. Here, I will first give a general presentation of the measurement problem. Then, I will discuss the term "measurement" and its importance in the problem, followed by an attempt at defining the terms "quantum" and "classical", which are frequently used in connection with the measurement problem as well as discussions on the interpretation of quantum mechanics in general. In the last sections, I will introduce different specific formulations of the problem, which each focus on different aspects of the general problem.

2.1 Schrödinger's famous cat

The measurement problem is famously illustrated by the so called Schrödinger's cat paradox, which was published in 1935 in an article, *Die gegenwartige Situation in der Quantenmechan* by Erwin Schrödinger [Schrödinger, 1980]. In this article, Schrödinger sought to make an account of the state of affairs in quantum mechanics and to point out the difficulties, which it faced. By his own account, he was inspired by the likewise famous Einstein-Podolsky-Rosen paper [Einstein et al., 1935], which brought to light the concept of entanglement in seeking to illustrate the incompleteness of quantum mechanics. [Schrödinger, 1980, p. 323] Schrödinger put forward a thought experiment, which has since been known as "Schrödinger's cat" and has become an emblem of quantum mechanics.

The cat paradox shows that applying only the formalism of quantum mechanics leads to the existence of cats in undefined states of aliveness; a concept which is unfamiliar to our experiences. It consists of an example of how "an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy" [Schrödinger, 1980, p. 328]. The idea is that a radioactive atom is in a quantum state, $|\Psi\rangle$, which is an equal superposition of being decayed and not decayed. In Schrödinger's original account, it is a small amount of radioactive material, which is, after a certain amount of time, as likely to have one decayed atom as none [Schrödinger, 1980, p. 328]. However, for simplicity one can restrict oneself to a single, radioactive atom, the state of which, after a period of time corresponding to its half life, will have evolved (in accordance with the Schrödinger equation) into a superposition state yielding equal probability of having decayed as of not having decayed. In Dirac notation [Sakurai and Napolitano, 2017, p. 10-23], this state is written as,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\text{not decayed}\rangle + |\text{decayed}\rangle\right), \tag{2.1}$$

or, in matrix representation [Sakurai and Napolitano, 2017, p. 20-23] in the basis of the two states: not decayed and decayed,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}. \tag{2.2}$$

In case the of the atom decaying, a Geiger counter is triggered, which in turn triggers the release of a poisonous gas, thus killing a cat which has been placed in a closed box with the container of the poison. The state of the atom and the cat becomes entangled through the interaction with the Geiger counter and the poison, as the composite state of the atom and the cat evolves as $|\text{not decayed}\rangle|\text{alive}\rangle \rightarrow |\text{not decayed}\rangle|\text{alive}\rangle$ and $|\text{decayed}\rangle|\text{alive}\rangle \rightarrow |\text{decayed}\rangle|\text{dead}\rangle$, due to the chain of reactions triggered by the decaying atom. This type of interaction corresponds to what is known as a *control not* (CNOT) gate in quantum information, where the state of one system flips or does not flip according to the state of another system [Nielsen and Chuang, 2010, p. 20-22]. The matrix representation of the CNOT gate is given by

$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} . \tag{2.3}$$

In the same basis as before, the state of the initially alive cat is

$$|\mathrm{alive}\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} , \tag{2.4}$$

making the composite state evolve as

$$\begin{aligned} \mathrm{CNOT}\,|\Psi\rangle \otimes |\mathrm{alive}\rangle &= \mathrm{CNOT}\,\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} && (2.5) \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} && (2.6) \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} && (2.7) \\ &= \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) && (2.8) \end{aligned}$$

Or, in Dirac notation,

$$|\Psi\rangle_{\mathrm{atom}}|\mathrm{alive}\rangle_{\mathrm{cat}} \to \frac{1}{\sqrt{2}} \left(|\mathrm{not\ decayed}\rangle_{\mathrm{atom}}|\mathrm{alive}\rangle_{\mathrm{cat}} + |\mathrm{decayed}\rangle_{\mathrm{atom}}|\mathrm{dead}\rangle_{\mathrm{cat}}\right) . \tag{2.9}$$

This is a state of entanglement between the atom and the cat, as it cannot be written as a product state of the state of the cat and the state of the atom respectively. It is also clear that the cat is in no definite state of being either dead or alive, but in some sort of superposition state - a result that clearly does not correspond with our usual experiences with cats. Schrödinger describes it as having "the living and the dead cat (pardon the expression) mixed or smeared out in equal parts" [Schrödinger, 1980, p. 328]. This, therefore, gives rise to the paradox: if one applies only the quantum formalism, macroscopic objects will not always have definite properties (such as being either definitely dead or definitely alive). The measurement problem is concerned with what attitude one is to take towards this paradox. This can entail an explanation of how to account for the fact, that the quantum formalism seems to be not always applicable, or how to otherwise explain the appearance of definite properties in the macroscopic world or, conversely, the appearance of indefinite properties in the (sub)atomic world.

This account of the measurement problem follows the spirit of a point made by David Wallace [Wallace, 2012], in that the problem is given as, not a problem of any interpretation of quantum mechanics, but as a problem that arises out of the quantum formalism, and which exists independently the interpretation of that formalism [Wallace, 2012, p. 4579]. That is to say, the problem has been presented with the aim of relying on as few assumptions about how to understand or interpret the quantum formalism as possible, thus allowing for different interpretations to provide a range of different solutions to the problem. Wallace characterises the measurement problem as the problem of accounting for the fact that the quantum state space is sometimes understood as a phase space and sometimes as a distribution space. A phase space is here understood as a space where different points represent systems with different physical properties, whereas a distribution space is a space of probability distributions over different physical states. The latter corresponds to having the so-called "ignorance interpretation" of the quantum state; i.e. the indefiniteness is an expression of our lack of definite knowledge. Treating the quantum state space as a distribution space explains the use of quantum states to give probabilities of different basis states as the absolute square of the corresponding coefficient, such as a 0.5 probability of Schrödinger's cat being dead or alive respectively, or the 0.5 probability of an atom having decayed after a period of its half life. This understanding does not apply in all cases, as it for instance makes no account (at least not straightforwardly) of the relative phases of the coefficients, which gives rise to e.g. observable interference effects in situations such as the double slit experiment and interferometers in quantum optics. In quantum mechanics, therefore, both understandings are, according to Wallace, in use, and the measurement problem can therefore be stated as the problem of accounting for the transition between them [Wallace, 2012, p. 4579-4580]. In the Schrödinger's cat paradox, this amounts to the question of when one is to view the quantum state as a state in phase space and when one is to view it as a state in distribution space, i.e. apply the ignorance interpretation.

2.2 The role of measurement

Given the account of the measurement problem given in this work, it may seem unclear why this issue has been given the name "measurement problem". Indeed, the reference to measurements has been purposefully avoided. An account of the problem can be given as relating to the unknown nature of measurement processes, where quantum systems go from being in superpositions to entering into definite states of measurement outcomes. This version of the problem could be considered as misleading as the focus on the effect upon systems by measurement processes hints at the importance of the intentions of conscious beings in subjecting the systems to these processes, as the word "measurement" implies some intent behind the physical process. It would imply that the cat would only be expected to be either definitely dead or definitely alive upon a conscious being opening the box with the view of ascertaining the fact. On the contrary, the account given here speaks only of interactions with objects considered as "classical" (see the following section for a discussion of this term), a more general notion which does indeed contain interactions with instruments of measurement. Though, as will be looked into later, some sort of address must be made to the measurement problem by any interpretation of the quantum formalism, it is, arguably, not necessary to include the role of human intentions implied by the reference to only measurement interactions, and therefore the general account is more appropriate.

That being said, accounts of measurement processes play an important role in quantum mechanics and are central in the development of different interpretations. Any interaction we have with the rest of the world, which allows us to gain information, can after all be said to take the form of a measurement. Furthermore, the analysis of the measurement process gives general insight into the mathematical background of the measurement problem. Attempting a formulation of a formal definition of what is meant by "measurement" will therefore be to advantage.

In a classical sense, a measurement can be said to describe the process of ascertaining, to some degree of accuracy, the value of some particular property possessed by some particular system. This corresponds to what is understood by "measurement" in ordinary language; one can for example find the following definition in *The Oxford Paperback Dictionary*:

> **measure** *v.* **1.** to find the size, quantity, or extent of something by comparing it with a fixed unit or with an object of known size [Hawkins, 1988]

By this definition, to measure is to determine the value of some independently existing property by translating it into known units; i.e. to "find" something, implies that that something is already there to be found.

But if the descriptions given by quantum mechanics are to be considered in any other way than as a measure of our knowledge of the state of a system, that is if we want to apply any other interpretation than the ignorance interpretation to the quantum state, "measurement" must take on another meaning. For there to be a measurement, a measurement apparatus must become entangled with the measured system, which is to be measured, such that certain states of the apparatus occur together with certain states of the system. For instance, the different positions of the pointer of some meter occur with different states of the system. If a set of a system's eigenstates for some observable are $|\alpha_i\rangle$, then the interaction with the measurement apparatus, which is to measure this observable (whose initial state is $|\beta_0\rangle$) is such that $|\alpha_i\rangle|\beta_0\rangle \rightarrow |\alpha_i\rangle|\beta_i\rangle$, where the states $|\beta_i\rangle$ are orthogonal states of the apparatus in the ideal case. This is of course a case of a perfect measurement; in real measurement interaction this will only occur with some probability, i.e. it will be possible for the measurement to fail. But in the ideal case, the interaction of the measurement apparatus with the system in a general state, leads to the following entangled state, where the state of the system has been written down in terms of the eigenstates of the observable being measured,

$$\left(\sum_i \alpha_i|\alpha_i\rangle\right)|\beta_0\rangle \to \sum_i \alpha_i|\alpha_i\rangle|\beta_i\rangle\,. \tag{2.10}$$

To relate this to the dictionary definition, one can say, that this type of correlation between system and apparatus states corresponds to the "comparing with a fixed unit".

The probabilities of the different outcomes of a measurement of an observable are given by the Born rule. An observable is represented by a hermitian operator, $\hat{A} = \sum_i \alpha_i \Lambda_{\alpha_i}$, where $\Lambda_{\alpha_i} = |\alpha_i\rangle\langle\alpha_i|$ is the projection operator. In a measurement of this observable on a system in a state $|\Psi\rangle$, the probability of obtaining a certain eigenvalue, α_j, is $|\langle\alpha_j|\Psi\rangle|^2$, which in the case of the measured system in equation 2.10 equals $|\alpha_j|^2$ [Sakurai and Napolitano, 2017, p. 19, 23-25].

As a criterion for something to be meaningfully considered as a measurement, Schrödinger suggests that repeated measurements must yield the same results [Schrödinger, 1980, p. 329]. A way of capturing this in the quantum formalism, is for the system, after the measurement, to be in an eigenstate corresponding to that eigenvalue, which is the measured outcome. If this is the case, then any repetition of the same measurement will give the same result. This effect of a measurement is captured in John von Neumann's introduction of "process 1" [Howard, 2021, p. 2], by the projection postulate, which states, that if the outcome of a measurement of $\hat{A}$ turned out to be α_k, then the system has been projected into the corresponding eigenstate:

$$\Lambda_k \sum_i a_i|\alpha_i\rangle|\beta_i\rangle = a_k|\alpha_k\rangle|\beta_k\rangle\,. \tag{2.11}$$

If the state of the measurement apparatus were to put back to $|\beta_0\rangle$ (e.g. the pointer moved back to it's original position) and the measurement repeated, only one outcome is possible: α_k. In other words, after the measurement, the value of the observed property has a definite value, which can be (once again) ascertained by further measurements on the system [Janssen, 2021, p. 8-9]. This view, that an definite eigenvalue, α_k is obtained only when the system is in the corresponding eigenstate, $|\alpha_k\rangle$ is known as the eigenvalue-eigenvector link (or the e-e link) [Wallace, 2012, p. 4580].

Using these definitions, the measurement problem can be formulated mathematically, as arising from the fact that the way quantum systems evolve in time (in the quantum formalism) is linear. In the matrix representation, the time evolution is always represented by multiplying the state vector with unitary matrices, $U = \mathrm{e}^{i\hat{\mathrm{H}}t}$, where H is the Hamiltonian of the system and $U\,U^\dagger = \mathbb{1}$ (as it is unitary) [Sakurai and Napolitano, 2017, p. 66-71]. This was for instance the case in the account of Schrödinger's cat, where the evolution was represented by multiplication of the CNOT-matrix, which is indeed unitary. For two-level systems, this unitary and linear evolution is usually described as rotations on the surface of a sphere in configuration space, known as the Bloch or Poincaré sphere [Sakurai and Napolitano, 2017, p. 15] (the name depends on whether the system described is a photon or not), and is characterised by being reversible, as suggested by the fact that multiplying with the complex conjugate equals identity. If the general quantum state is to evolve into one with definite properties, one must have some sort of non-unitary evolution, as represented by the projection operator. The system will subsequently be found to be in this exact eigenstate, i.e. the value of the relevant property will have the corresponding eigenvalue.

As mentioned, what has been considered here is a case of a perfect measurement. This is obviously not a reasonable description of real measurement situations, where errors of different sorts are unavoidable. However, the simple case presented here, easily extends to more complicated situations. For the purpose of this work, consideration of this idealised measurement situation suffices.

To return to the point of stating the measurement problem more generally, i.e. not only relating to definite measurement outcomes, but rather to definite states in general, it can be observed that the entanglement in equation 2.10 corresponds exactly to that which occurred in the case of Schrödinger's cat, equation 2.9, where there was no mention of a measurement occurring. Likewise, the projection postulate is (as mentioned) essentially seen as a formalisation of the requirement of definite values of observables.

This illustrates that the defining feature of the account of the measurement problem given here, has nothing to do with whether the system, with which the quantum system interacts, is a measurement apparatus or not (in the sense of a conscious being intending it to be so or not).

This is further exemplified by a point made by Niels Bohr [Bohr, 1935], about the relativism of to what part of the physical system the term “measurement apparatus” is applied. He argues, that the physical apparatus in the experimental setup of a measurement is sometimes treated as an object of investigation and sometimes as a measurement instrument. Without going into much detail about Bohr’s view (yet), Bohr is of the opinion that measurement apparatuses are to be described with classical physics, while the objects of measurement are subjected to quantum mechanical laws. According to Bohr, different parts of a system employed as a measurement apparatus, will, according to what property is being measured, need to be treated quantum mechanically, and therefore not as a measurement apparatus, in Bohr’s sense, but as an object of investigation itself. [Bohr, 1935, p. 698] This implies, that (at least in some interpretations of quantum mechanics) the term “measurement apparatus” applies rather to the role some part of the joint system can play in the equations describing an interaction, than to an actual, physical apparatus employed by conscious beings. Thus, also “measurement” can be said to cover types of interactions in general, rather than the act of employing some apparatus to some system in order to gain knowledge.

The measurement problem therefore extends to other types of interactions than those of conscious beings gaining knowledge. A similar viewpoint are expressed by Wallace, who stresses the physical nature of measurement devices, by arguing that they are carefully designed objects, the construction of which (at least in modern day experimental physical) itself relies on quantum physics [Wallace, 2012, p. 4578]; and also by Don Howard, who writes that there is nothing special about measurements, and that they therefore cannot hold special place in theory [Howard, 2021, p. 2]. In this light, the measurement problem is not concerned with measurements in a functional sense, i.e. as the means by which humans, or other beings, gain knowledge, but in a physical sense, which extends to systems and interactions beyond measurement situations.

In order not to loose generality, therefore, the measurement problem, will be considered as relating to any interaction which leads to the transferring of indeterminacy from quantum systems to systems, which one would not expect to be indeterminate. It therefore introduces the need to either explain why one's expectations are wrong, or explain why and how something, which is represented by the projection operator, is at work. One returns to the general account of the measurement problem reached at the end of section 2.1. Hence, whenever measurements will be mentioned later in this thesis, it must be observed, that what is referred to is the more general notion presented here, and therefore not measurement in the sense of conscious beings intending to gain knowledge, except when it is stated otherwise.

2.3 Defining classicality

In the previous section, the terms "quantum mechanical" and "classical" have been used without further specification. They are usually used in relation to the measurement problem in literature in general; the macroscopic objects being described as being classical as opposed to the indefinite (sub)atomic systems. Though, there is some general understanding of what these terms cover, to use these them more clearly and unambiguously, proper definitions are needed.

A system being described as quantum mechanical, is simply understood as a system whose state and evolution in time is given by the quantum formalism. A classical system is then understood as something which is in contrast with this; something in a state different from that given by the quantum formalism. Exactly what this entails must be further specified.

In his PhD thesis, Howard [Howard, 1979] identifies the key difference between quantum mechanics and what he calls "classical scientific realism" (CSR) as being the concept of separability. This concept is a defining feature of CSR, whereas its general absence is characteristic of quantum mechanics.

CSR is described as a view held by Max Planck and others, which fits well with classical physics, but is severely challenged by quantum mechanics. Howard quotes Planck as stating that the task of physics is "die reale Aussenwelt zu erkennen" [Howard, 1979, p. 16] and that this Aussenwelt is independent of the scientists, the "knowing subjects" [Howard, 1979, p. 29]. The "realism-part" of CSR expressed here is not what concerns the characterisation of classicality, which is the topic of this section. However, the "classicality-part", is relevant here and is, according to Howard, related to the point of independence from the knowing subjects. Howard argues that it is essential in CSR to have kinetic independence of the observer and the observed object in order to assure objectivity. That is to say, the observer can only influence the object through physical interactions, that can be compensated for; thus making the two systems in principle separable [Howard, 1979, p. vii, 327].

The interaction between the observer and observed is in general treated as a physical process, and in the classical view, this process is deterministic and governed by physical laws of the behaviour of systems with definite properties. Therefore the disturbance of an observer can be compensated for, and there is independence between the systems. Howard argues, that this independence is therefore a consequence of viewing the systems as in principle separable [Howard, 1979, p. 313, 322-327]. In this view, spatial separation of two objects is a sufficient condition for separability [Howard, 1979, p. 330]. Howard identifies this as being the crux of the debate concerning the aforementioned EPR-paper [Einstein et al., 1935], as an assumption in the thought experiment presented by Einstein, Podolsky and Rosen was that separability, the independent reality, was assured by the spatial separation of two systems. Bohr, on the other hand, denied this and considered the systems as nonseparable despite the distance between them [Howard, 1979, p. 257-259]. That it is the assumption of separability which is the source of the different conclusions dawn from the EPR thought experiments by EPR themselves and Bohr respectively, is underlined by the combined work of W. H. Furry and Barnard d'Espagnat, which are also mentioned in [Howard, 1979]. Furry identifies an assumption, which d'Espagnat shows can be derived from the separability principle, about the interaction between the two systems in the thought experiment. This is shown to give rise to different empirical predictions made by the original version and by the quantum formalism, the latter of which makes another account of the interaction between the two systems; one that does not lead to the two systems being in definite, independent states when the interaction ceases [Howard, 1979, p. 265-269].

In Howard's thesis, he sets CSR up against the ideas of Bohr. For this work, however, this is one step further than what is intended at this point. Rather than Bohr's denial of separability, one can compare CSR with the general nonseparability in the quantum formalism, where entanglement between systems, such as in the state in equation 2.10, persists regardless of the termination of any interaction between the subsystems.

As was touched upon in the previous section, Bohr himself mentions, in the article constituting his response to the Einstein-Podolsky-Rosen paradox [Bohr, 1935], the need to separate different parts of a system, specifically to make a distinction between what parts act as measurement instruments and what parts are measured, as being the essential difference between classical and quantum mechanics.

> This necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical description of physical phenomena.* [Bohr, 1935] [original italics]

He continues to argue, that this need for distinction between measurement apparatus and measured object is of no consequence to the nature of the descriptions given in classical physics, though it is highly important for the way he understands quantum mechanics [Bohr, 1935]. This could be taken to be a consequence of the fact that in classical physics it is assumed that objects and apparatuses of measurements and are in principle separable, and therefore making the distinction straightforward, whereas in quantum mechanics it is not. Bohr concludes that classicallity is an idealisation, where all phenomena are considered to be infinitely divisible and where any interaction can be accounted and compensated for [Bohr, 1961b, p. 115]. This clearly corresponds to an assumption of separability, which becomes an idealisation as quantum mechanics is inherently nonseparable. This nonseparability is the source of the different role of measurements in quantum mechanics, indicated by e.g. considerations of the measurement problem, where the effect of the measurement seems not to follow deterministic evolution, and therefore cannot be compensated for as in CSR. The measurements and instruments mentioned here, can be generalised to situations other than that of measurements and systems other than measurement instruments, as indicated in section 2.2.

Howard goes on to formalise the notion of separability versus nonseparability by pointing out the correspondence with mixtures as opposed to pure states:

> [...] a separable ensemble can be described as a mixture, while a nonseparable ensemble cannot, and an ensamble that can be described as a mixture is separable, while an ensemble that has to be described as a pure case is nonseparable. [Howard, 1979, p. 360-361]

Furthermore, Howard identifies a classical description of something, as a description that treats what it describes *as if* it is separable, i.e. describe it as a mixture [Howard, 1979, p. 360]. This is (partly) motivated by separability being the assumption made by Einstein, Podolsky and Rosen in their thought experiment, which does not hold in the quantum formalism, where quantum systems are entangled even when there is no longer any interaction between them, as well as by the observer-observed independence of classical descriptions of measurement. Thus, classical states are associated with mixtures, as opposed to pure quantum states, which are generally nonseperable [Howard, 1979, p.360-361].

If classical states are mixtures, it means that they can always be given the ignorance interpretation without leading to problems, such as those which occur for pure states, where phase-dependent interference terms cannot be accounted for in the ignorance interpretation. But from the fact that the ignorance interpretation *can* be applied, it does not follow that it so to speak *has been* applied, i.e. that the system has gone into a definite eigenstate. This view of classicality, therefore, does not necessarily assume that the classical world is definite, in the sense of classical objects being in definite eigenstates, only that there is separability and no quantum interference.

This account of classical descriptions is also coupled to Bohr's doctrine of classical concepts [Howard, 1979, p. 135-147]. This is the statement, that one must always come back to a description "expressed in common language supplemented with the terminology of classical physics" [Howard, 1979, p. 139] where the terminology of classical physics means a (mathematical) precision of everyday language. This is, according to Bohr, the only way one is able to make sensible, unambiguous communication about one's observations to other people; this being at the heart of scientific practise [Howard, 1979, p. 139-140]. The idea at present is not to give an account of Bohr's views on quantum mechanics or scientific practise, but to try to identify what he understood by "classical". It appears from his doctrine of classical concepts, that a classical description is something that makes sense to us; something that can be expressed in everyday language and thus communicated in a sensible way. It is not that some objects/systems are classical and some are not, but rather that one must treat them *as if* they were classical. Therefore a classical description is one which ignores these aspects, that does not allow for a description in classical physics, and could therefore be said to be an idealisation (as implied earlier and as Bohr actually does sometimes, e.g. in [Bohr, 1949b, p. 45]). This ties in with Howard's notion of separability, as he argues that in order to have unambiguous communication, one must assume observer-observed independence, and that this assumption is inherent in the everyday language and classic terminology [Howard, 1979, p. 141-146].

Howard points out that one cannot view certain concepts, such as momentum and position, as being classical, while other concepts which have no place in classical mechanics, such as spin, are not. Position and momentum enter into quantum mechanics in the same way as e.g. spin, and are therefore not marked out as possesing some sort of classical feature, while, on the other hand, spin states can be mixtures, and thus have a similar "classical" nature to that of position or momentum states [Howard, 1979, p. 365-366]. Classicallity is thus not associated with specific types of observables, and consequently all observables can be quantum mechanical.

This seems to have been pointed out by Schrödinger [Schrödinger, 1980], who argued, that position, momentum, etc. refer to something new in quantum mechanics. Here, the terms take on a different meaning than in classical physics, where they refer to definite, independent properties of objects. In his 1935 paper, he identifies the defining feature of classical models as "absolute determinacy" [Schrödinger, 1980, p. 323], i.e. that from the complete state of a system, as well as its state at any past or future point in time, can be determined by exact knowledge of its properties, or "determining parts":

> The representation in its absolute determinacy resembles a mathematical concept or a geometric figure which can be completely calculated from a number of determining parts [...] Yet the representation differs intrinsically from a geometric figure in this important respect, that also in time as fourth dimension it is just as sharply determined as the figure is in the three space dimensions. [Schrödinger, 1980, p. 232]

In contrast, he sees the central point of quantum mechanics as being exactly the negation of this premise of determinacy. In quantum mechanics, he writes,

> [...] models with determining parts that uniquely determine each other, as do the classical ones, cannot do justice to nature. [Schrödinger, 1980, p. 324]

This account of Schrödinger's about determinacy might be connected to Howard's separability, as the indeterminacy, Schrödinger mentions, comes from the fact, that observables do not have independent reality, where they "uniquely determine each other". In other words, one cannot view a system as a well-defined "geometrical figure", the determining parts of which can be found out by measurements.

In two articles by J. Baez [Baez, 1987] and Guido A. Raggio [Raggio, 1988] respectively, the separability of a state is related to the commutation of the corresponding operators. This is done by looking at Bell's inequality, which is derived from an assumption of separability and is violated by entangled quantum states. States, therefore, which satisfies the inequality are separable. Baez proves two formulations of the Bell's inequality for C*-algebras; the algebra to which operators on the Hilbert space belongs. In his first version, he assumes that the states on a composite system $A \otimes B$ (where A and B are

C*-algebras) are *decomposable* in order to derive the inequality [Baez, 1987]. That a state is decomposable means that it is a product state, i.e. is not entangled. This is a special case of the more general term *separable state*, which is a mixture (a weighted sum) of product states. For pure states, these terms coincide, as there will be only one term is the weighted sum. Baez writes that,

> If A and B are the C*-algebras corresponding to two physical systems, the product system has C*-algebra $A \otimes B$, and admits 'local hidden variables' in Bell's sense when all its states are decomposable. This happens if at least one of the two systems is classical. [Baez, 1987]

In the last sentence Baez relates the requirement of decomposable states to one of the systems being "classical", by which he refers to an algebra where the operations commute. This relationship between decomposability and commutability is stated formally as "If either A or B is Abelian, all states on $A \otimes B$ are decomposable" [Baez, 1987]. That is to say, that when one treats something as if it commutes (an Abelian group is a commutative group), the states on the composite system become separable. From this Baez proves his second version of Bell's theorem, which states that if A and B are commuting and one of them is Abelian, then Bell's theorem will be satisfied [Baez, 1987]. This is an argument for commutation being a condition for classicality, as quantum systems are known to violate the Bell inequality, which is satisfied by all classical systems. Baez' previous point then connects this to decomposability.

In his article, Raggio cites Baez' theorem in a proof of the equivalence of different characteristics of W*-algebras (the special type of C*-algebra, which corresponds to the operators of the Hilbert space). He states that for two W*-algebras, A and B, the condition of satisfying Bell's inequality, of A and B commuting and of every normal state on $A \otimes B$ being decomposable are equivalent. This further underlines the mathematical fact of the close connection between decomposable states and commuting algebras (operators in the quantum formalism) and the fact that under these conditions Bell's inequality will also be satisfied. Classicality is associated with Bell's inequality being satisfied, with separability and with commutation. Raggio shows that all these criteria correspond to one another. This, I think, is in favour of the idea of using separability as *the* criterion.

There are of course other characteristics of CSR, which might be challenged in quantum mechanics. In addition to the kinetic independence, which leads to the separability of the observer and the observed, Howard mentions that a theory in CSR are causal/deterministic, pictorial and uses space-time coordination and dynamical conservation laws in unrestricted combination [Howard, 1979, p. 99-112]. The latter of these characteristics are necessary for determinism, as this requires both the location (in space and time) and the dynamical properties of all particles to be well-defined and abide by the dynamical laws in order for all future configurations to be predictable from precise knowledge of the initial conditions. It is also necessary for the pictorial nature of classical theories, as one would visualise particle as objects possessing these properties [Howard, 1979, p. 111-112]. Howard argues, that this characteristic is actually a consequence of the assumption of separability, as he writes,

> And the unrestricted combination of the concepts of position and momentum turns out to presuppose the principle of system-instrument independence, for, as we shall see, the primary consequence of the denial of the latter in quantum mechanics is just the impossibility of simultaneously specifying precise values of any two conjugate variables. [Howard, 1979, p. 124]

Similarly, Howard argues that also the deterministic and pictorial nature of CSR are connected to separability. In the latter case, this follows from the fact that the ability to visualise a system is based on the idea of measured variables being properties of the system independently of the measurement taking place [Howard, 1979, p. 123-124]. Determinism is also mentioned on several occasions by Bohr in connection with classical physics. In one article, he mentions determinism as characteristic for the type of descriptions given in Newton's mechanics and in electrodynamics [Bohr, 1958a, p.11] and in a lecture he adds that it is compatible with thermodynamics [Bohr, 1955a, p. 102-103]. Howard again argues, that this characteristic can be connected to separability as it relies on the unrestricted definition of initial conditions, which is challenged by the lack of object-instrument separability [Howard, 1979, p.122-123].

Howard also mentions a further consequence of the kinetic independence of CSR: that unlimited divisibility is assumed, i.e. that all processes can be divided into as small steps as desired [Howard, 1979, p. 115-117]. The infinite divisibility was mentioned previosly in connection with Bohr's idea of classicality, and is obviously violated in quantum mechanics, whose very name implies that it builds upon the postulate that processes (such as jumps between energy levels in atoms) are quantised and thus not divisible. As this nondivisibility is derived from nonseparability, this is another argument for using an assumption of separability as the defining characteristic of classicality.

Howard clearly argues that quantum mechanics is inconsistent with CSR, and that this inconsistency boils down to the fact that observers and observed objects cannot be assumed to be (in principle) separable in the quantum formalism. One could argue that a theory cannot be either realist or antirealist; only people can have these views. But this is a view that separates theories from people by looking at a theory as something in itself to which one can relate. If one, in special relativity, says that the theory is equal to the Minkowski metric, then it makes sense to say that one can be either realist or antirealist about it. But the theory can be said to be more than the metric; it could include e.g. conclusions about the equal status of reference frames and the resulting reference frame dependence of all statements. CSR has ideas of what a good theory is like: a description of reality independent of "us". But quantum mechanics, in e.g. Bohr's view, show that these ideas cannot be right, as they are conceptually impossible due to the object-observer inseparability. If the separability of quantum mechanics is a statement about the world, a part of the theory, then it looks as though quantum mechanics must at least deny the kind of independent realism that was supposed in classical physics.

The point here is not to say that any interpretation must necessarily give up on all the mentioned characteristics of CSR. It is only to motivate the choice of separability as characterising classicality in this thesis, by showing how different concepts, which one would connect with classical theories, can be tied to an assumption of separability (or specifically the separability of measured systems and measurement instruments) and how the quantum formalism does not allow for such a general assumption of separability, due to entanglement.

2.4 Specific formulations

Although the measurement problem has been characterised in general terms, an exact definition of what the problem is, and thus what needs to be solved, is still lacking. It turns out to be possible to make several such definitions according to what aspect of the problem one chooses to focus on and what makes sense to ask and what does not according to one's interpretational standpoint. In an article [Maudlin, 1995], Tim Maudlin gives an account of three versions of the measurement problem, which are summarised briefly here.

> **The problem of outcomes:** The wave function being complete, evolution being due to a linear dynamical equation (the Schrödinger equation) and having definite measurement outcomes are mutually inconsistent [Maudlin, 1995, p. 7-10].
>
> **The problem of statistics:** The wavefunction being complete, evolution being due to a deterministic dynamical equation and measurements that are described by the same initial wave function sometimes having different outcomes (with probabilities given by the Born rule) are mutually inconsistent [Maudlin, 1995, p. 10-13].
>
> **The problem of effect:** A measurement has an influence on the future development of the system, which has been measured [Maudlin, 1995, p. 13-14].

Maudlin, himself, describes the aim of his article as being to "distinguish and analyze several difficulties confronting attempts to reconcile the fundamental quantum mechanical dynamics with Born's rule" [Maudlin, 1995, p. 7]. To reconcile the quantum formalism and Born's rule, can be taken as a brief summary of the account of the measurement problem given earlier, and Maudlin then seeks to analyse different aspects of this problem. The first problem, the problem of outcomes, concerns the question of accounting for the reconciliation of the (seeming) indefiniteness of quantum systems and the (seeming) definiteness of many other systems, such as measurement outcomes or cats. The proof of the inconsistency consists of showing that by assuming completeness and linearity, one obtains a contradiction of definite outcomes. This corresponds to what is shown in the Schrödinger's cat paradox (section 2.1). If the wavefunction constitutes a complete description, and if it behaves according to the dynamical laws of the quantum formalism, i.e. the Schrödinger equation, then how can one account for the definite measurement outcomes, which are inconsistent with the indefinitness of this description? If not, what is lacking?

The problem of statistics is also addressed by David Albert [Albert, 1992a], where he accounts for the measurement problem as the inconsistency of quantum dynamics, which is linear and deterministic, and the wave collapse postulate, the essential feature of which is the stochastic nature of assigning definite measurement outcomes (with probabilities given by the Born rule). These two types of evolution contradict each other [Albert, 1992a]. If the dynamical laws of the evolution of quantum states are deterministic and the quantum states are a complete description of a system, then two identical systems will be identical still after having been subjected to the same interactions. This constitutes a contradiction of the final assumption in the problem, thus proving the inconsistency [Maudlin, 1995, p. 11]. The difference to the problem of outcomes is, that rather than focusing on the definiteness of measurement outcomes, the attention is here drawn to the question of the (seeming) indeterminacy involved in obtaining these definite outcomes, captured by the fact that the same initial quantum state may give rise to different measurement outcomes. To this problem belongs the problem of how to account for the probabilities that are at play in this process.

Lastly, the problem of effect points out, that a solution to the measurement problem must also account for the fact, that repeated measurements yield the same results, i.e. the future state of the system seems to somehow depend on the measurement being made, and a measurement outcome enables predictions about the future state [Maudlin, 1995, p. 13-14].

A point has been made by Alexander Meehan [Meehan, 2019], concerning an additional background assumption present in these formulations of the measurement problem.

This assumption is that of competent measurement, which includes two things: a) the person making a measurement always experiences a definite observed outcome and b) measuring devices are to some degree reliable and informative, so as to e.g. give an output α_i if the input is determinatly $|\alpha_i\rangle$ (using the notation from equation 2.10) [Meehan, 2019, p. 3]. This assumption has also been made in the general account of the measurement problem given earlier. It could be argued, that the process of measurement, or indeed inquiring into nature at all, is somewhat meaningless without it, which is also the reason why it is presented as a background assumption by Meehan, and not included explicitly in the formulations of the measurement problem given by Maudlin.

Similarly to Maudlin, Hanneke Janssen [Janssen, 2021] presents several definitions of the measurement problem in her master's thesis. She begins with,

> **The minimalist measurement problem:** How can one make sense of expressions such as the one in equation 2.9 and 2.10? [Janssen, 2021, p. 12]

$$|\Psi\rangle_{\text{atom}}|\text{alive}\rangle_{\text{cat}} \rightarrow \frac{1}{\sqrt{2}}\left(|\text{not decayed}\rangle_{\text{atom}}|\text{alive}\rangle_{\text{cat}} + |\text{decayed}\rangle_{\text{atom}}|\text{dead}\rangle_{\text{cat}}\right) \tag{2.9}$$

$$\left(\sum_i \alpha_i|\alpha_i\rangle\right)|\beta_0\rangle \rightarrow \sum_i \alpha_i|\alpha_i\rangle|\beta_i\rangle \tag{2.10}$$

This version of the problem amounts to the same as the general question one is left with by the Schrödinger's cat paradox, as accounted for in section 2.1. As such, it is more general than e.g. the problem of outcomes, as it assumes nothing about the status of the quantum state or the expectations of measurement outcomes, but only asks the wide question of what attitude one takes regarding both these issues. Janssen proceeds to present different specifications of this minimalist or general problem, corresponding to different ways in which the problem can be viewed. She mentions the problem of outcomes, like Maudlin, as well as the following,

> **The problem of interpretation:** Is it possible to formulate an interpretation, that gives epistemological or ontological meaning to the state in equation 2.9 and

> 2.10? [Janssen, 2021, p. 12]
>
> **The problem of collapse:** What kind of process causes the jump from the superposition state to a definite state, i.e. what is the process that causes the collapse of the wavefunction? [Janssen, 2021, p. 16]
>
> **The problem interference:** Why is there no interference present, i.e. why are the off-diagonal terms in the density matrix zero, at the end of a measurement? [Janssen, 2021, p. 20]

All these three versions of the measurement problem approaches the general question from different angles. The problem of interpretation specifically addresses the question of how one is to understand the superposition state of the cat. It asks for an explanation of what the quantum state means; to what it refers. It is clearly related to the measurement problem, as a solution to the problem of interpretation, i.e. an interpretation of the superposition state, would shed light on the issue of the measurement problem, which concerns the explanation of why characteristics of the superposition state, such as indefinitness, does not appear in e.g. the macroscopical domain [Janssen, 2021, p. 12-13]. The problem of collapse is a more direct formulation of the measurement problem, as it concerns how to account for a process which takes one from such a superposition state to one of the basis states, i.e. an either a dead or an alive cat. As will become clear later, in section 3, solutions to the measurement problem can be categorised according to whether this is a meaningful question to ask or not, i.e. whether a collapse of the wavefunction forms part of the solution or not. Janssen argues that the problem only occurs for interpretations, which include the e-e link and “a one-to-one correspondence between the formalism and our experiences” [Janssen, 2021, p. 16]. This requires a collapse, as the experience of a definite outcome in this case must correspond to the system definitely being in the corresponding eigenstate.

The problem of interference concerns the fact that quantum systems in superposition states can exhibit interference phenomena, such as is seen in for instance double slit experiments or in interferometers used in quantum informational setups. The measurement problem formulated as the problem of explaining the lack of such interference effects after measurements, can be related to the version of the measurement problem given by Wallace [Wallace, 2012] (accounted for in section 2.1), as it concerns the fact that the distribution space understanding of the quantum state space, where no interference effects are present, is sometimes appropriate, whereas effects of phases (e.g. interference) are observable in other cases. A solution to the problem of interference would give an account of the reason why these interference effects (apparently) disappear after certain interactions of the quantum system, such as a measurement process, or in certain situations. Mathematically, this is equivalent of an account of how the off-diagonal terms in the density matrix of a system, $|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$ (here the simple case of a system with two orthogonal states),

$$\rho = |\Psi\rangle\langle\Psi| = \begin{bmatrix} |c_0|^2 & c_0 c_1^* \\ c_0^* c_1 & |c_1|^2 \end{bmatrix} \tag{2.12}$$

becomes zero. In a density matrix, the diagonal elements gives the probabilities for the corresponding basis states of the system, whereas the off-diagonal terms account for interference effects, and have no meaning in Wallace's distribution space and are not present after a measurement. That is to say, they are not observed directly, though they affect the measurement outcomes, thus making the simple distribution space understanding in general impossible. This issue is different from the problem of outcomes, as it requires less; the problem of outcomes, would require an account of why it is suddenly possible to apply the ignorance interpretation to the quantum state (when before, it was unfeasible) in addition to the explanation of why the reasons that it was unfeasible are no longer present, which is what is required by the problem of interference. In other words, even though the interference terms disappear, thus allowing an ignorance interpretation, that does not explain why something which was not a probability distribution becomes a probability distribution of definite measurement outcomes.

A quantum state can be represented as a vector in a configuration space known as the Hilbert space. This state can be written down in terms of several different basis states in the same way, as a vector in ordinary space can be written down in terms of different basis vectors. The same spin state, $|\Psi\rangle$, can e.g. be expressed in terms of spin in the z-as well as in x-directions (i.e. two orthogonal directions), as the spin in the x-direction is given by $|\rightleftarrows\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$, where the arrows in the vertical direction represents the spin in the z-direction, and arrows in the horizontal direction represents spin in the x-direction. One could for instance reformulate a spin state defined in the z-direction in terms of x-direction basis states:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) = \frac{1}{\sqrt{2}}(|\rightarrow\rangle + |\leftarrow\rangle) \tag{2.13}$$

Thus, without the specification of an observable, which can provide a set of eigenstates as basis for the state vector of the system, it is not given to which eigenstates the Born rule is to assign probabilities, as probabilities cannot be assigned to all observables simultaneously [Janssen, 2021, p. 17-22]. Janssen writes:

> [...] the pure quantum state is just a unit vector in Hilbert space that does not carry any empirical content unless a particular observable has been specified. [Janssen, 2021, p. 21]

Janssen presents two additional versions of the measurement problem concerning this issue. The first relates to the statistical nature of measurements, as it concerns the problem of the arbitrariness in the formalism as to the basis in which probabilities are to be assigned.

> **The problem of preferred basis (general):** What determines which orthogonal basis states the Born rule assigns probabilities to for a given quantum state? [Janssen, 2021, p. 17-19]

This issue is also touched upon by Wallace, who, in addition to an account of when the transition between a phase space and a distribution space understanding of the quantum state space takes place (as mentioned in section 2.1), includes a question of “the basis with respect to which the probabilistic interpretation is to be specified” [Wallace, 2012, p. 7] in the measurement problem.

The second of Janssens problems of preferred basis, stems from an analysis of the measurement interaction, where the same joint state of the system and measurement apparatus can be decomposed in terms of different basis states, all equally valid. In a measurement, states of the system and states of the apparatus are correlated, as described in equation 2.10, where this state is defined on a product Hilbert space $\mathcal{H}_\alpha \otimes \mathcal{H}_\beta$, where $\mathcal{H}_\alpha$ and $\mathcal{H}_\beta$ are the Hilbert spaces of the system and of the apparatus respectively. Janssen calls a specific decomposition of the joint state, such as equatuion 2.10, a bi-decomposition and, in cases where both apparatus- and system states are orthogonal, bi-orthogonal decomposition. Janssen points out that no bi-decomposition is unique - not even in the case of a bi-orthogonal decomposition if coefficients $\alpha_i = \alpha_j$ for any two different i and j [Janssen, 2021, p. 21-22]. The problem of preferred basis can then be expressed as a problem of how to determine which bi-decomposition rightly corresponds to measurement situation at hand.

> **The problem of preferred basis (decomposition):** Of all the possible bi-decompositions of a quantum state what determines which rightly corresponds to the measured states? [Janssen, 2021, p. 22]

Another issue related to the measurement problem is also discussed in Janssens thesis: the emergence of classicality. This is the idea that the classical world is to somehow be extrapolated from the quantum theoretical description, in the same manner as the classical equations of motion appear from special relativity in the limiting case of velocities which are low relative to the speed of light. In the case of classicality, or rather approximate classicality, emerging from a quantum description, however, it is usually not by taking some limit (e.g. $\hbar \to 0$) that the classical laws are to be found; it is rather through a dynamical process. Though this issue is related to the measurement problem, it goes beyond what is required for solving the problem. Whereas the measurement problem concerns the reconciliation of the quantum formalism and the (apparent) definiteness of the classical world, the concept of emergence of classicality aims to explain the very occurrence of the classical world. It might be considered as overly ambitious to require quantum mechanics to give an explanation of why and how classical physics is the way it is, and, in any case, it is an additional requirement to that of explaining the seemingly inconsistency in the quantum mechanical formalism, that is at the heart of the measurement problem. This text is mainly concerned with the measurement problem in itself and emergence of classicality will not be considered thoroughly. For further discussion upon this subject, see for example [Janssen, 2021], [Crull, 2013] and [Tanona, 2013].

Rather than choosing one specific formulation of the measurement problem, this thesis seeks to investigate how different approaches to proposed solutions to the measurement problem answers all the different versions of the problem. One formulation of problem might be completely meaningless for one particular interpretation, whereas it is of central importance for another. Applying the different interpretations to the measurement problem will serve as an additional specification of those interpretations as well as answering the question of the extent to which the measurement problem has been solved in the view of the interpretations.

I have used, and will continue to use, the terms *unitarity* and *linearity* interchangebly in relation to the evolution of the wavefunction in the quantum formalism. Both are characteristics of quantum mechanics, and the measurement problem can be derived from either.

2.5 The no-cloning theorem and the control problem

A problem, known as the control problem, has been presented by Alexander Meehan [Meehan, 2019], which can be considered as yet another formulation of the measurement problem. Connected with it is a central theorem in quantum mechanics: the no-cloning theorem. Independently of its connection to the control problem, the no-cloning theorem can be considered relevant in discussions of the measurement problem, as it bears many similarities to the problem. The no-cloning theorem states that according to the quantum formalism, it is not possible to make copies of a quantum state. As we have no experience of such copy-making, this does not become a problem, in the style of the measurement problem, but rather, as the name suggests, a theorem of quantum mechanics. When considering different interpretations of quantum mechanics, it could be fruitful to examine their attitude towards the no-cloning theorem, in a similar manner to how they treat different formulations of the measurement problem. In addition, there is a connection to the control problem, in its consequences in limiting our ability of state preparation and measurement, as will be seen. Because of these considerations, the no-cloning theorem will here be accounted for along with Meehan's control problem.

In short, the no-cloning theorem states that it is impossible to prepare a system in a state, which is identical to the state of another system. If some unknown state of one system (for simplicity, a two-level system), $a|0\rangle_1 + b|1\rangle_1$ is to be copied to another system, which (again for simplicity) is initially in the state $|0\rangle_2$, then one would require some operation, which can take,

$$(a|0\rangle_1 + b|1\rangle_1) \otimes |0\rangle_2 \rightarrow (a|0\rangle_1 + b|1\rangle_1) \otimes (a|0\rangle_2 + b|1\rangle_2) \tag{2.14}$$

The no-cloning theorem states that this is not possible in the quantum formalism, as it cannot be achieved by any unitary and linear operator. This can be seen by writing out the product states as vectors in the basis of $|0\rangle$ and $|1\rangle$,

$$(a|0\rangle_1 + b|1\rangle_1) \otimes |0\rangle_2 = \begin{bmatrix} a \\ b \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad (a|0\rangle_1 + b|1\rangle_1) \otimes (a|0\rangle_2 + b|1\rangle_2) = \begin{bmatrix} a^2 \\ ab \\ ab \\ b^2 \end{bmatrix} . \tag{2.15}$$

Thus, the operator, U, required for the cloning of the state from one system onto another, must fulfill,

$$\begin{bmatrix} a^2 \\ ab \\ ab \\ b^2 \end{bmatrix} = U \begin{bmatrix} a \\ b \\ 0 \\ 0 \end{bmatrix} . \tag{2.16}$$

As the state of the first system is unknown, the matrix U cannot contain a or b, and as the equation is linear, it is therefore not possible to satisfy equation 2.16, since it would require operations like $a \rightarrow a^2$. Therefore the linearity of quantum mechanics results in the impossibility of cloning an unknown state of one system onto another system [Nielsen and Chuang, 2010, p. 2-3, 24-25]. In addition, equation 2.16 cannot be solved if U is unitary [Meehan, 2019, p. 32].

The no-cloning theorem shows, that linearity and unitarity are incompatible with a specific type of state preparation: the preparation of one system in the same, unknown state as another system. There are however general cases of state preparation to which some limitations follow from the problem presented by Meehan [Meehan, 2019]: the control problem. Meehan states, that this new problem constitutes an aspect of the measurement problem, when the latter is viewed a general way [Meehan, 2019, p. 5], as has been the case in this thesis. The control problem concerns itself with what can be done with quantum states [Meehan, 2019, p. 4] and, in short, it consists of the following,

> **The control problem:** Evolution being due to a unitary, dynamical equation (the Schrödinger equation), having determinate inputs and being able to successfully prepare a quantum state are jointly incompatible [Meehan, 2019, p. 3].

Meehan also draws attention to the need of another background assumption being that of competent measurement, which was discussed in section 2.4 in connection with the formulations of the measurement problem given by Maudlin. By having determinate inputs, Meehan understands that it is always determinate what a system has or has not been given as an input to a device used for measurement or state preparation [Meehan, 2019, p. 3]. Meehans definition of succesful state preparation is rather weakly formulated, so as to exclude as little as possible:

> [...] at least some of our preparation devices are such that, if determinately fed many inputs, they output a non-trivial fraction of those inputs in some specified range of quantum states. [Meehan, 2019, p. 3]

Meehan argues that state preparation in general is consistent with both linearity and unitarity. The difference from the situation in the no-cloning theorem is here, that the state of the "device" used to prepare the state of the system (corresponding to system that is to be cloned), need not remain in the same state after the preparation, i.e. it is not a case of cloning. The final state may e.g. depend on the initial state of the system, that has been prepared [Meehan, 2019, p. 10-11]. An example is quantum teleportation, where one system is prepared in the same state of another (distant) system. In this protocol the state of the system, from which the initial state is copied, is destroyed

[Nielsen and Chuang, 2010, p. 26-28]. However, there are cases of state preparation, which seem to be at odds with linearity. These are types of preparation where one part of a superposition state is filtered out, to obtain a system in the state of the other part. In the quantum formalism, the final state, i.e. the state after the filtering, cannot be equal to that part of the superposition (this would require an evolution formally corresponding to a collapse of the superposition state into one eigenstate) [Meehan, 2019, p. 1]. An example could be a polarization beam splitter (PBS), which splits light into vertically and horizontally polarized components. If a photon was sent through a PBS, where one path, corresponding to a horizontal polarization, is blocked, and the other, corresponding to vertical polarization, is used in further experiements, the photon cannot be said to have been prepared in a state of vertical polarization. This issue is known as the preparation problem, but is not as fundamental like the measurement problem, as state preparation is not in general incompatible with linearity and unitarity [Meehan, 2019, p. 11]. However, it can be said to be connected to the problem of effect, as repeated measurements on the photon, would be consistent with it being in a state of vertical polarisation. Thus, the problem of how to account for repeated measurement outcomes when the quantum formalism does not allow for the system to be in the corresponding eigenstate is the issue here, and therefore the preparation problem can be said to be a special case of the problem of effect [Meehan, 2019, p. 19].

Meehan points out, however, that state preparation is in conflict with unitarity, if one also assumes determinate inputs (and the background assumption of competent measurement), which is what the control problem states [Meehan, 2019, p. 11]. Meehan shows this via an example. In case someone measures some system (e.g. an electron) as having spin $\uparrow_z$, when that person will feed a bunch of other systems (denoted by $i = 1, 2, 3, ..$) to one preparation device, D, leading to (many of) theses systems being in associated D-states. If however, spin $\downarrow_z$ is measured, then the systems will be the input of another preparation device, D', which will result in (many of) them being in D'-states, which are supposed to be distinct from the D-states.

$$|\uparrow_z\rangle_{\text{electron}} \otimes |\text{initial}\rangle_{\text{rest of laboratory}} \rightarrow |\text{final}\rangle_{\text{rest of laboratory}} \otimes |D\rangle_1 \otimes |D\rangle_2 \otimes ... \tag{2.17}$$

$$|\downarrow_z\rangle_{\text{electron}} \otimes |\text{initial}\rangle_{\text{rest of laboratory}} \rightarrow |\text{final}\rangle_{\text{rest of laboratory}} \otimes |D'\rangle_1 \otimes |D'\rangle_2 \otimes ... \tag{2.18}$$

If the measured system is in a state, $|{\rightarrow}\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_z}\rangle + |{\downarrow_z}\rangle)$, a problem arises, as, by determinate inputs, it must be determinate which of the preparation devices are given the inputs, but this cannot be the case without a type of evolution which would be a violation of unitarity [Meehan, 2019, p. 13-16]. That this is a kind of measurement problem becomes clear from the fact that it involves a measurement [Meehan, 2019, p. 18].

Meehan connects his control problem to the no-cloning theorem, by arguing that the no-cloning theorem marks out a conflict between our ability to prepare and our ability to determine a quantum state [Meehan, 2019, p. 30-31]. Meehan points out that the no-cloning theorem is the formalisation of a feature of quantum mechanics, which arises from the fact of its unitarity and determinate inputs. That is to say, if one holds on to the assumptions of unitarity and determinate inputs, there must, according to the control problem, be an incompatibility of the assumption of state preparation with the background assumption of competent measurement. Or, Meehan writes, with the assumption of competent state determination, which is also shown to be in tension with the other assumptions in the control problem: unitarity, preparation and determinate inputs. [Meehan, 2019, p. 30] State determination in quantum mechanics is different from that of classical mechanics, as only eigenstates, and not states in general, can be measured directly (see section 2.2). It is therefore not in general possible to simply determine what state a quantum system is in by a measurement. In quantum tomography, a state is determined by rather performing measurements on an ensamble of identical states, thus giving information that converges towards the unknown quantum state.

There is therefore a conflict between our ability to know about and our ability to control quantum states [Meehan, 2019, p. 31-32]. The connection to the no-cloning theorem is captured in the observation that, if cloning was possible, the possibility of state preparation and determination, would be unlimited - but as cloning is not allowed in the quantum formalism, these abilities are limited [Meehan, 2019, p. 30-31]. If both of these concepts were possible, a gate (to use quantum informational terminology) could be made, which first determines the state of a quantum system and then prepares another system in that state, thus cloning it. The no-cloning theorem states, that this would not be possible (when unitarity is assumed), and thus limits these abilities.

As seen in the definition of the control problem, no assumption is made of the completeness of the wavefunction, and therefore interpretaions of quantum mechanics must give some account of the limitations to state preparation (as well as determination) regardless of their views on the completeness of the wavefunction [Meehan, 2019, p. 31-32].

3 Solutions, dissolutions and illusions

The solution to the measurement problem is strongly connected to the interpretation of quantum mechanics. On one hand, the measurement problem serves as an offset for interpretations, as it specifies what a possible interpretation must contain, in addition to e.g. fitting experimental results, making sense of the formalism and accounting for violations of Bell's inequality. On the other, the measurement problem is a problem of interpretation, as its solution calls for an interpretational basis. This is reflected in the vast variety in the different solutions given in different interpretations.

An interpretation can therefore be analysed in terms of its solutions to the different formulations of the measurement problem. If the measurement problem is the Gordian knot (though not, arguably, impossible to untie), its solution can be found by either untying or by cutting the knot. The untying of the knot is what I will call a solution to the problem, as it is done by giving some account of the transition from quantum mechanical to classical descriptions. Interpretations that solve the measurement problem in this manner are generally collapse-interpretations, i.e. interpretations which contain some sort of process corresponding to a collapse of the wavefunction into a classical eigenstate. These interpretations sacrifice the idea that the wavefunction always evolves unitarily and linearly as described by the Schrödinger equation, and includes another process, which takes the wavefunction into one of its eigenstates, i.e. a wave collapse [Maudlin, 1995]. This collapse then can be employed to explain why no superpositions and no quantum interference effects are found on the macroscopic level. The second strategy for solving the measurement problem, that of cutting the knot, I will call a dissolution, as rather than describing a quantum-to-classical transition, the problem is addressed by giving some account of how this transition does not actually take place. This is generally associated with non-collapse interpretations, where there is no concept of a non-unitary type evolution, and thus no transition from quantum to classical. In these interpretations the world is either always described by quantum mechanics, though we fail to notice it, or the quantum state is not actually a correct deception of the (sub)atomic systems, that are in fact classical, though we cannot (yet) access this knowledge.

There remains one last strategy for dealing with the Gordian knot; that of passing it by. This would be achieved by an instrumentalist understanding of quantum mechanics, and perhaps science in general. As long as the formalism can give useful predictions, which can be tested experimentally, the role of scientific research is fulfilled. As this text is concerned with the measurement problem and is conducted under the assumption that there is relevance in the discussions of it, this standpoint will not be explored further. In this section I will instead give a short overview of different interpretations of quantum mechanics, and how each of them are able to address the different formulations of the measurement problem. The focus is on three interpretations; the two non-collapse theories Bohmian mechanics and the Everettian many-world interpretations, as well as the collapse-theory known as GRW Theory.

3.1 Non-collapse interpretations

Non-collapse interpretations do not strive to explain where and how a change from the quantum formalism to the classical definite properties takes place, but rather maintain that the proper description of the world remains the same throughout any process. Thus the world is either always classical (in the sense of separability) or always non-classical (the non-separability remains). In non-collapse theories, the Gordian knot is cut rather than untied, as the measurement problem no longer exists, and the solution such a theory could provide is actually a dissolution. Obviously, such interpretations will have no issue with the problem of collapse, as there is no collapse to explain the process of. However, the other formulations of the measurement problem are addressed differently by the different interpretations within this category.

3.1.1 Bohmian Mechanics

One group of non-collapse theories is known as the *hidden-variable theories*. In this group reality is considered to be always classical, in the sense of states always being separable and definite. The quantum state (as described in the formalism of quantum mechanics) is actually not a complete description of the state of a system, as the systems described do in fact posses some additional definite property or properties. These are called "hidden variables", since they have traditionally been considered to be new kinds of variables, which were inaccessible to us, as they did not have empirical consequences. This is not always the case, however, and in some literature (e.g. [Maudlin, 1995]) hidden-variable theories are rather referred to as "additional-variable" theories. One example of this is Bohmian mechanics, which adds a definite position to the quantum formalism. In a book by Maudlin [Maudlin, 2011], three main principles of Bohmian mechanics are identified:

1. The wavefunction is incomplete as, in addition to the wavefunction, the state of a system is characterised by its definite position. This is the case also when the wavefunction is not in an eigenstate in the position basis. Thus two systems with identical wavefunctions might still have different states.
2. The is no collapse, and the wavefunction always evolves according to the Schrödinger equation.
3. There is determinism, as the wavefunction evoloves only in accordance with the Schrödinger equation and the additional variable, the definite position, also evolves deterministically.

The idea in Bohmian mechanics is that the definite position of a system is guided by the system's wavefunction; as the wavefunction evolves it determines how the particle will move. As the position is always definite, particles have well-defined trajectories. The definite position is linked to the wavefunction via an equation known as Bohm's equation, which gives the "velocity" of the particle as the graditent of the phase of the wavefunction. Together with the Schrödinger equation, Bohm's equation gives the

Bohmian dynamics. In some formulations the so-called *quantum potential* is added to the formalism. However, Maudlin argues that this is in principle unnecessary. Without it, the dynamics at any future point is given by Bohm's and Schrödinger's equations as well as the two fundamental variables of Bohmian mechanics: the position and *the wavefunction*. The quantum potential only serves to recover Newtonian mechanics, where the fundamental variables are position and *velocity*. [Maudlin, 2011, p. 106-111] Thus, though Bohmian mechanics recovers a kind of classicality through the definite trajectories of particles, it does not (in itself) constitute a return to Newtonian mechanics and (all of) the classical definite variables.

Bohmian mechanics is a deterministic theory, where definite initial states evolve deterministically into definite final states, and therefore the usual probabilities of quantum mechanics comes to be an expression of our lack of knowledge of the initial state [Maudlin, 2011, p. 108]. For Bohmian mechanics to be observationally equivalent to quantum mechanics, the probabilities of the two must be the same: the absolute square of the wavefunction, Ψ. This criteria is fulfilled if the probability distribution describing the initial uncertainty about the position of a particle is equal to $|\Psi(0)|^2$, as the uncertainty at any later point, t, will then be given as $|\Psi(t)|^2$ in accordance with quantum mechanics [Maudlin, 2011, p. 109]. This distribution is known as the *quantum equilibrium* [Daumer et al., 1996, p. 384]. Maudlin writes, that it is a rather unclear why the initial uncertainty should be given as the absolute square of the wavefunction and thus ensure the equivalence with the probabilistic predictions of quantum mechanics. It is not due to a fundamental relationship between the wavefunction, which guides the motion, and the uncertainty about the position that is captured in the probabilities, and therefore it must be due to some other factor [Maudlin, 2011, p. 109]. What this can be is a subject of discussion in literature on Bohmian mechanics.

Previously, in section 2.1, the interference of quantum phenomena was mentioned as standing in the way of interpreting the indefiniteness of quantum states as an expression of our lack of knowledge (the ignorance interpretation). Bohmian mechanics gets around this problem by giving an explanation of how the interference occurs. As a particle's position is guided by the wavefunction, it will be affected by the interference patterns of the wavefunction, though the particle itself follows a determinate trajectory. Maudlin writes regarding the double slit experiment: “The particle itself only goes through one slit, but the parts of the wave-function associated with the both slits come to overlap in configuration space, resulting in the famous interference fringes.” [Maudlin, 2011, p. 110]. The wavefunction in Bohmian mechanics therefore seems to serve a dual purpose; it both guides the particle’s trajectory (it is sometimes called a *pilot wave*) and give the statistical predictions. The latter of these purposes indicates that the wavefunction, or at least its absolute square, is an expression of our ignorance of the real, definite position of the particle. This epistemological understanding, however, cannot explain the role of guiding the particle position and the significance of the off-diagonal terms. The answer to this is that the wavefunction is understood ontologically in Bohmian mechanics, and he fact that it can also give the statistical predictions is seen as an *accidental* feature. There are, however, problems with viewing the wavefunction as a physical wave. As mentioned previously, the wavefunction is defined in configuration space (with a number of dimensions corresponding the the degrees of freedom) rather than physical space and has a complex phase, and in addition to this the wavefunction does not obey Newton’s 3$^{\text{rd}}$ law; it influences the particle, but not vice versa. [Romano, 2020, p. 10606] Because of these problems, the wavefunction is not seen as physical in the sense of classical waves, but rather along the lines of a law of nature. Here Bohmian mechanics entails a radical departure from the laws of nature in classical physics, which are uniform and thus does not depend on the initial state of the systems to which they apply. In Bohmian mechanics the wavefunction is used to determine how the particle motion will change; to give the “velocity” in some sense. Thus, the laws of motion is dependent on the state of the particle.

If the position in Bohmian mechanics is definite, one might ask whether other quantum operators, such as momentum and spin, have definite values or not. The Bohmian answer would be that these other operators are not real; only the position is real and definite and the theory must explain why one can apparently measure other quantities and why these seems to fit the statistical predictions of quantum mechanics. In general, one does not really measure e.g. momentum, but actually the position of a pointer, which is not correlated with any sort of momentum-property of the measured system.

In an article on the meaning of observables in Bohmian mechnaics it is written that,

> For an N-particle universe, these two equations [the Schrödinger equation and Bohm's equation] form a complete specification of the theory. There is no need, and indeed no room, for any further axioms, describing either the behavior of "other observables" or the effects of "measurement". [Daumer et al., 1996, p. 383]

They argue that experiments can be perfectly accounted for in Bohmian mechanics, as the outcome, which corresponds to a position of a pointer, is determined by the initial configuration and the Bohmian dynamics. This does *not* constitute a measurement of any property of the system [Daumer et al., 1996, p. 385-387]. A spin-measurement involving a Stern-Gerlach apparatus can e.g. be accounted for without any need for a spin-property or spin-degrees of freedom, and "merely reflect the way spinor wave functions are incorporated into a description of the motion of configurations" [Daumer et al., 1996, p. 389]. An account of Stern-Gerlach-like measurements made by Albert [Albert, 1992b], shows how the output, i.e. which way the particle goes after passing through the Stern-Gerlach apparatur, will depend on in which part of the wavefunction (in the space-basis) the particle is actually located upon entering the apparatus relative to the orientation of that apparatus [Albert, 1992b, p. 147]. Outcomes of measurements of such experiments will depend on the exact experimental context [Albert, 1992b, p. 153] but this is not in conflict with CSR, where science describes a observation-independent reality (see section 2.3), as they are not considered to be a reflection of any real property of the measured system.

Bohmian mechanics is necessarily a non-local theory. This follows from the observed violations of Bell's inequality, which show that a hidden-variable theory cannot be local [Maudlin, 2011]. In Bohmian mechanics the motion of a particle can change instantaneously at-a-distance due to changes in the wavefunction and thus things that occur at one place can instantaneously affect the behaviour of systems at another arbitrarily far away [Maudlin, 2011, p. 110-111]. Maudlin argues that the world must in any case be non-local, but I consider the non-locality of Bohmian mechanics to be different from what is generally necessary, as it must be a more "physical" kind of non-locality, where a definite state actually change instantaneously at-a-distance and the wavefunction becomes a transmitter of signals that determine this change.[1]

In regards to the measurement problem, the general answer given by Bohmian mechanics is, as mentioned, one of dissolution, since there is to quantum-to-classical transition to account for; the world is always classical. As Bohmian mechanics is a non-collapse theory, the problem of collapse is obsolete and need not be discussed further. The minimalist measurement problem seeks an explanation of how to understand a state such as the atom-cat-state of the Schrödinger cat paradox (equation 2.9). Maudlin addresses this issue in saying that though the wavefunction may be in a superposition, the cat will be in a definite state, as it is made up of particles, which all have definite positions and which must be in a configuration corresponding to *either* a dead or an alive cat. Similarly, in measurement processes, the measurement instruments will be in definite states determined by the likewise definite states of the measured systems [Maudlin, 2011, p. 107-108]. The quantum state in 2.9 is therefore just part of the complete state of the system, which follows a definite "trajectory" from the atom, through the Geiger counter and the poison, to the cat. This also answers the problem of interpretation, as the state is interpreted as the real, but incomplete wavefunction of the system, the complete state of which is (also) given by its definite positional configuration. For hidden-variable theories in general, the answer to these problems will depend on the specific theory, i.e. what the additional variables and their dynamics are.

The problem of outcomes and the problem of statistics both concern the mutual inconsistency of the completeness of the wavefunction, the evolution being given by the Schrödinger equation and having determinate outcomes and different outcomes (with probabilities given by the Born rule) for the same initial conditions, respectively. Both these are in Bohmian mechanics solved by abandoning the idea of the wavefunction being complete; in hidden-variable theories, it indeed is not, as the system is not fully described by it but has some definite variables in addition [Maudlin, 1995]. The definite outcome will be determined by the definite initial state, while the "identical" initial

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[1] This is a topic of discussion in [Albert, 1992b], and an in-depth analysis of this issue and how it relates to relativity theory is made in [Maudlin, 2011].

conditions in the problem of statistics will *not* be identical, in the Bohmian picture. The additional variable, the position, must be different in order to lead to different outcomes. The physics governing the hidden variables must (at least approximately) replicate the Born statistics, and, as mentioned, this is secured in Bohmian mechanics by assuming an uncertainty of the initial position corresponding to the absolute square of the wavefunction at the initial time.

Regarding the problem of effect, Maudlin writes,

> A theory without wave collapse can only solve the problem of effect if the dynamics of the additional variables force the additional variables to carry information about the results of measurements through time. This will be an intrinsically more difficult task for a theory in which those dynamics are stochastic. [Maudlin, 1995, p. 14]

Maudlin is saying, that the fact that a second measurement will have the same outcome as the first must have an explanation in the physics of the hidden variables. He further argues, that this is achieved by Bohmian mechanics, whose additional variable is not stochastic but deterministic. Though the wavefunction is not affected by the measurement in a way which secures that repeated measurement yields the same result by undergoing a collapse, this is not a problem, as the actual result of the first measurement depend on the definite additional value, and the Bohmian dynamics secures the sameness of the repeated measurements if nothing is done to change the state of the system [Maudlin, 1995]. If e.g., at the output-path corresponding to $\uparrow_z$ of a Stern-Gerlach apparatus, a second Stern-Gerlach apparatus is placed, then this second apparatus will give an output consistent with the wavefunction being in the eigenstate $|\uparrow_z\rangle$, since the $|\downarrow_z\rangle$-part of the wavefunction (in the position basis) will be zero in the region of the second apparatus, and thus have no effect on the direction which the particle takes at that point [Albert, 1992b, p. 148-151].

The problem of intereference, i.e. why there is no interference at the end of a measurement, is solved in a similar manner. The interference terms of the density matrix only serves to guide the particle, they do not constitute the actual state. A particle follows a well-defined trajectory, and a measurement gives a well-defined outcome accordingly. Though these outcomes will be distributed according to the wavefunction, including the interference fringes, the actual state of a particle is never in an indefinite state which can interfere and the measurement outcome is a reflection of this.

As only the position is a real observable in Bohmian mechanics, the other bases of the wavefunction does not represent other observables in terms of which the state of the system can be expressed or of whose value the Born rule can give statistical predictions. Therefore the problem of preferred bases becomes the problem of explaining how other observables seem to be measured, and the outcomes of these seem to fit the statistical predictions of the other bases of the wavefunction. Bohmian mechanics, as shown in [Daumer et al., 1996] and [Albert, 1992b], succeeds in this through accounts of experiments, that does not include the measurement of different properties corresponding to different operators, but explain why outcomes, corresponding to pointer positions etc., occur in accordance with the probabilistic predictions of quantum mechanics. Albert wirtes, that as long as a measurement of a quantum mechanical observable is recorded in a position of something, Bohmian mechanics will reproduce the outcomes of quantum mechanics [Albert, 1992b, p. 151-153].

The control problem does not have completeness of the wavefunction as one of its assumptions, but only unitary evolution, determinate inputs, preparation and competent measurement (see section 2.5). Since the response of Bohmian mechanics to the measurement problem is in general to deny the completeness of the wavefunction, it would seem like it does not provide much towards a solution of the control problem. Bohmian mechanics would maintain unitary evolution, determinate inputs and competent measurements, and thus would have a problem with state preparation, which would perhaps makes sense, as we definitely cannot prepare the positional configuration, and perhaps not the wavefunction either. On the other hand, Bohmian mechanics might deny the assumption of competent measurement in all other bases than position, as measurement outcomes in these cases depend on the context and does not reflect the value of any property of the measured system. I will leave this here as an open question, in need of further discussion.

3.1.2 Everett's many-world interpretation

Another example of a non-collapse interpretation are variations of Everettian, or many-world interpretations. As opposed to Bohmian mechanics, where the world is in some sense always classical, the correct description is in Everett interpretations always that of quantum theory and the classical realm we experience is an illusion. The Everett interpretation is often regarded as coming directly from the quantum formalism. Wallace e.g. writes that the Everett interpretation adds no new physics and corresponds to an interpretation of the formalism in a "straightforward realist way" [Wallace, 2013, p. 464]. This is of course under the assumption that the interpretation of the formalism does not constitute "new physics". To do this, according to Wallace, is to have the superposition state describe *more than one thing.* Rather than an object being in an indefinite state of superposition of e.g. two eigenstates, this superposition state describes a situation where *both* eigenstates are definitely realised. That is to say, the superposition state does not describe a superposition of states of an object, but rather a superposition of different "worlds" (the precise meaning of this term varies) in which the different eigenstates are realised [Wallace, 2013, p. 464-465]. Janssen writes that the Everett interpretation concerns "relative states", where definite outcomes occurs relative to an eigenstate. The definite outcome of spin up is thus measured relative to the eigenstate $|\uparrow\rangle$ etc. [Janssen, 2021, p. 122-123].[2] Similarly, Wallace argues that only one outcome is experienced, as people's experiences are bound to one "world", and that the "worlds" do not interact strongly enough for the other(s) to be detectable [Wallace, 2013, p. 465]. Janssen formulates it in the following manner: there is definiteness on the single-world level, but the universe includes all the different worlds, and thus all the different outcomes are realised [Janssen, 2021, p. 123-124].[3]

The vocabulary varies between different versions and different litterature; the different "worlds" are often called different "branches" and the universe, consisting of all the branches or worlds, are sometimes called the "multiverse".

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[2] Janssen argues that this is as far as Everett himself went, but that the many-world interpretation takes this further in arguing that the world splits up into several "worlds" [Janssen, 2021, p. 124].

[3] Janssen here talks of the many-world interpretation, not Everett's relative states.

In modern Everett interpretations it would be more appropriate to use the term "branch", as "world" is a bit too simple an well-defined to really work when going in depth with quantum theory. It suggests that the world splits into several worlds, which each are classical, i.e. have well-defined, definite quantities. But this is not possible, as within one world, or branch, all variables cannot be definite. If e.g. the world splits into different, definite spin-z values, $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$, the spin in the x-direction will not be well-defined in either. "Branch" carries less meaning than "world", and can thus more easily accommodate the complexities and abstractness of the notion. In addition to this, the quantum state might e.g. not be a superposition of two well-defined outcomes, which each can be realised in one world. A wavefunction might e.g. look like figure 1, where it is localised in two distinct peaks, but with a non-zero area in between. Here one could perhaps talk of two branches, but they would not be as distinct as "splitting into two worlds" would suggest. In many ways Bohmian and Everett interpretations have the same goal: to be realist about quantum mechanics. But to achieve this they go in different directions when confronted with subtleties and difficulties of quantum theory; Bohmian mechanics comes to require more detailed accounts to e.g. make sense of the statistical predictions in bases other than position, whereas the Everett interpretation must replace the initial, intuitive and well-defined "worlds" with the much more abstract "branches".

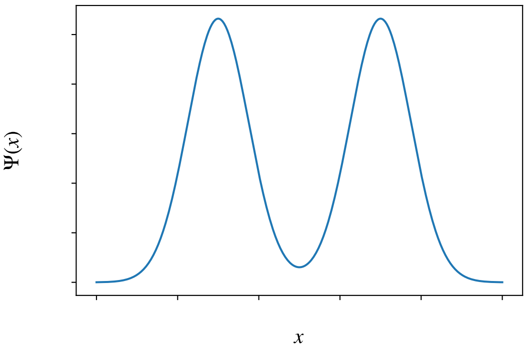

Figure 1: An example of a wavefunction in one-dimensional space.

As mentioned, there are different variations of the Everett interpretation. The many-mind theory associates the different relative states, with different mental states of the observer and the many-exact-worlds theory, add an ensamble of worlds to formalism [Wallace, 2013, p. 467]. I will here not go into these different versions, but will keep to the acocunt of the Everett interpretation which have been sketched above.

The Everett interpretation solve the problem of outcomes by denying the existence of definite measurement outcomes, while maintaining both that evolution is always due to the Schrödinger equation and that quantum mehcanics is complete. As argued earlier, one needs only account for why there *appears to be* definite outcomes, and not necessarily why there *are* definite outcomes, and Janssen argues, that this is what the Everett interpretation does [Janssen, 2021, p. 122]. The theory succeeds in this by having all possible outcomes of a measurement occur in different branches, the effect of which is that quantum state is still in a superposition (of different branches rather than different states), where different measurement outcomes occur and are registered by humans, who cannot appreciate the existence of more than one branch, and therefore perceive the outcome as being definite [Maudlin, 1995]. Thus the experience of a definite outcome comes to be relative to such a branch. Wallace argues that in this picture, objects such as measurement instruments are *not* in indefinite states, as it is rather the case that several definite states all occur [Wallace, 2013, p.464]. But the definiteness is branch-relative and the overall state of the universe (or multiverse), is still given by the superposition. This means that even though the outcomes might in this sense be called definite, the Everett interpretation is still a negation of the type of definite outcomes, which are assumed in the problem of outcomes, and which require some sort of breaking of the symmetry of the superposition - either from the beginning (corresponding to incompleteness of the wave function) or upon some event (corresponding to a collapse taking place, i.e. the evolution being not always given by the Schrödinger equation). Rather than breaking this symmetry, the Everett interpretation breaks up reality into different branches. Thus the assumption of definite outcomes in the problem of outcomes is broken in a way which allows for a solution to the problem.

These considerations also clarify how to understand the entangled superposition states, which occur upon e.g. measurements or the poisoning of cats, is to be understood. The states in equation 2.9 and 2.10 correspond to the right description of reality, where the different terms represent different branches, that all take part in forming this reality; the cat is alive in one branch and dead in the other. This solves the minimalist measurement problem and the problem of interpretation. These type of states are ontological and describe the state of a universe, that consists of several branches relative to which definite states are experienced. This ties in very well with the claim made by e.g.

Wallace about the Everett interpretation being the result of interpreting the quantum formalism in a realist way.

As there is no wave collapse, the problem of collapse become meaningless. The problem of effect is also solved by the observation that a second measurement will be done relative to each of the branches, and thus will be influenced by the definite outcome of that branch. In other words, the entanglement continues in adding another component to the total system: $|\uparrow\rangle_{sys}|M_{\uparrow}\rangle_{M1}|M_{\uparrow}\rangle_{M2}... + |\downarrow\rangle_{sys}|M_{\downarrow}\rangle_{M1}|M_{\downarrow}\rangle_{M2}....$

The problem of statistics, which concerns the circumstance of the same initial conditions resulting in different outcomes following probabilities given by the Born rule, is rather troublesome in the Everett picture. Maudlin [Maudlin, 1995] actually argues, that the Everett interpretation cannot solve it, as the probabilities can have no meaning in a theory where every outcome occurs. If the world evolves into several branches in which all the possible measurement outcomes respectively are realised, assigning probabilities to the different outcomes makes no sense, since all do occur with certainty, and there therefore is nothing for the probabilities to be probabilities of. It is not clear what the difference between the superposition $\frac{1}{\sqrt{2}}|\alpha\rangle + \frac{1}{\sqrt{2}}|\beta\rangle$ and $\sqrt{0.9}|\alpha\rangle + \sqrt{0.1}|\beta\rangle$ would be, as in both cases there will be a branch in which $|\alpha\rangle$ is realised and a branch in which $|\beta\rangle$ is realised (or two collections of branches in which either occurs). The well confirmed Born statistics therefore seems to become meaningless [Maudlin, 1995]. Wallace identifies such a type of critique, as arguing that the probabilities does not make sense, due to the fact that the theory contains no uncertainty (we know all outcomes will occur) and no alternative possibilities (there is only one scenario: all outcomes occur) [Wallace, 2013, p. 475]. Wallace disagrees with Maudlin that these concepts are necessary requirements for assigning probabilities and thus that the probabilities given by the Born rule can in fact be meaningfully assigned to the different branches [Wallace, 2013, p. 477]. He further argues that defining the meaning of probabilities is generally a problem, and that it is no greater problem in the Everett interpretation than in other interpretations [Wallace, 2013, p. 477-479]. I am not really convinced by Wallace's defence of the use of probabilities in the Everett picture, as I find that there is not much left in the term "probability" without uncertainty and alternative possibilities. However, this is a subject on which a lot of literature exist. The meaning of probabilities in the Everett picture is e.g. discussed by Lev Vaidman and Kelvin J. McQueen [McQueen and Vaidman, 2019] who argue that the probabilities can be understood in terms of "self-location".

The problem of preferred basis is likewise an obstacle in the Everett picture. This is pointed out by Janssen, who argues that "it is not clear what determines the interpretation basis" [Janssen, 2021, p. 124] if one does not simply refer to e.g. observables. The physical phenomena of decoherence, which will be gone through later in this thesis, is generally used as a solution to this problem, as decoherence leads to suppression of interference with respect to a specific basis. This allows for the different eigenstates, which form that basis, to be considered as different things, since they no longer interfere. Therefore, decoherence can serve as the definition of the "branching". In itself, the Everett interpretation does not really provide an answer to when and how the branching occurs, but since decoherence was formulated the custom among Everettians has been to use decoherence; the branching is said to occur when a state decoheres and the basis in which is occurs is the basis selected by decoherence, i.e. in the basis in which decoherence diagonalises the density matrix. There are however many remaining issues, and Janssen argues that decoherence is not sufficient to solve the Everett interpretation's issues with the problem of preferred basis [Janssen, 2021]. Wallace also mentions that the approximate nature of decoherence is the cause of scepticism of its success in solving the preferred basis problem [Wallace, 2013, p. 470], but argues that one can accept the approximate nature, and regard the branching as an emergent phenomenon [Wallace, 2013, p. 470-471, 474].

Regarding a solution to the problem of interference, the Everett interpretation does not in itself give any answer to why the interference disappears. This is another matter where the Everett interpretation usually draws on decoherence, which, as we shall see, can be used to explain why the interference disappears upon interaction with e.g. a measurement apparatus or the environment. Decoherence can e.g. show how an interference pattern is replaced by something like figure 1, where one can talk of two distinct branches which do not interfere.

In the control problem, the inputs would in the Everett interpretation not really be determinate, as all inputs would be given relative to the different branches. Relative to a branch a determinate input would be given, corresponding to the outcome of the measurement in that branch, and a definite state would be prepared. But in the overall state of the universe (or multiverse) the prepared state would be a superposition following from the superposition of the different inputs. The no-cloning theorem would however remain as a feature of quantum mechanics, as it is derived from the unitary and linear evolution of quantum states, something which is maintained in the Everett interpretation.

3.2 Collapse Interpretations

Interpretations that include a wave collapse acknowledge the quantum and the classical realm as equally real, and strive to give an account of the process where a systems goes from being in an indefinite quantum state to being in a definite classical state (i.e. the wave collapse) and set up some parameters for where the division between classical and quantum should be.

The "traditional", or "orthodox", view of quantum mechanics is usually taken to be as described by John von Neumann. It is often viewed as being closely linked with Niels Bohr, and the rather ill-defined Copenhagen interpretation, which will be discussed later. In this thesis, however, I will argue that this is a misunderstanding. In von Neumann's account, a collapse is postulated to take place upon *measurement*. Thus the superposition state in equation 2.9 would, upon measurement, collapse into either $|\text{not decayed}\rangle|\text{alive}\rangle$ or $|\text{decayed}\rangle|\text{dead}\rangle$, with probabilities given by the Born rule. Similarly, the state of any system collapses stochastically into one of its eigenstates upon measurement [Howard, 2021, p. 2].

The problem of outcomes is thus solved, as there are definite measurement outcomes, which are obtained at the cost of adding a second type of evolution, the collapse, to the evolution which is due to the Schrödinger equation. Similarly, the problem of statistics is solved by the addition of this second type of evolution, which is stochastic and follows the Born rule; a state can collapse into any eigenstate with probabilities of each given by the Born rule. The problem of interference is solved, as no interference can take place in the collapsed eigenstate which is the post-measurement state of the system. Repeated measurements will, as mentioned in section 2.2, be measurements of this eigenstate, and thus yield the same result as the first measurement, thus solving the problem of effect.

The basis in which the collapse occurs is in von Neumann's account determined by the measured observable, as it results in the system being in an eigenstate of that observable. Regarding the problem of preferred basis, therefore, the Born rule applies to the eigenvalues of the measured observable and the measured states are those expressed in this basis. There is however no more detailed, physical explanation of how the choice to measure something results in such a collapse, as the terms "measurement" and "observables" are not defined in any detail.

The Control problem is in the von Neumann picture solved by denying the assumption that the evolution is always unitary as expressed by the Schörodinger equation. A collapse will occur during the preparation as some measurement takes place in the determination of input, and thus the inputs and the prepared states are definite states. In this picture one does not arrive at the no-cloning theorem, which makes sense as this theorem is derived from an assumption of unitarity; something which do not hold in von Neumann's account.

Even though many of the measurement problems seems to be solved by the von Neumann approach, the reliance on the ill-defined term "measurement" means that the interpretation is in general not taken seriously outside its instrumentalist use. This issue is captured by the failure of the von Neumann approach to properly address the problem of collapse. As it is by no means clear what constitutes a measurement and indeed why this kind of physical interaction should hold such a special place in the theory of quantum mechanics, it is not clear how, when and why a collapse takes place. In the case of Schrödinger's cat, does the collapse take place when the Geiger counter clicks or doesn't click? When the poison is released or not released? When the cat is poisoned or not poisoned? Or not until a human being (or some other being who is aware that they are making a measurement) opens the box, looks at the cat and registers, in their consciousness, whether it is dead or not? The ad hoc use of the concept of measurement and the possibly significant role of the human intellect, which it could be taken to imply, renders the bare von Neumann interpretation unsatisfactory to most people.

The entangled superposition state of a measurement instrument and a measured system would, in von Neumann's account, be collapsed into an eigenstate, with the coefficients in the superposition giving the probabilities of the different eigenstates. But in the case of entanglement with other macroscopical objects it is not equally clear what would be the right description, as there is no physical definition of when a measurement, and thus a collapse, takes place. Therefore, the minimalist measurement problem (how to make sense of states of entanglement with macroscopical objects) and the problem of interpretation (how to interpret such states meaningfully) cannot really be answered satisfactorily. As no macroscopical objects in states of superposition are observed, it might be argued that in some way or another a "measurement" always takes place upon interactions with macroscopical objects. The problem with this orthodox interpretation of quantum mechanics is therefore that the simple use of the word "measurement" is not well-defined enough to satisfactorily account for when or how a wave collapse occurs.

3.2.1 GRW Theory

This issue is dealt with rather innovatively by another collapse interpretation known as the Ghirardi–Rimini–Weber (GRW) theory. In this theory the question of what circumstances causes a wave collapse is avoided, as the collapse is stochastic, and can occur at any time regardless of what interactions a system might or might not have with other systems. The process of the collapse is given by a multiplication of the wavefunction (in the position basis) with a narrow Gaussian function, which thus localises the particle in space. The position of the Gaussian function is also given by a stochastic process, which, if GRW theory is to give the right predictions, must depend on the wavefunction in such a way as to make the localisation more likely to occur where the absolute square of the wavefunction is higher [Maudlin, 2011, p. 225-229].

In this matter, GRW theory is similar to Bohmian mechanics, where the laws of nature also comes to depend on the state of the system to which they apply. Here the position of the Gaussian depends on the wavefunction and thus the state of the system. In GRW theory, a collapse always occurs in the position basis. When the wavefunction is localised in position space, it will, according to the Fourier transformation, flatten out in momentum space. Thus, no collapse can be said to make the momentum of a system definite, nor any other quantum operator. Thus, again like Bohm, GRW must account for why one does not really measure these other variables, but actually measure the spacial configuration of a measurement apparatus. The probability of a collapse occurring is fixed at a very small number. The exact value is considered to be a constant of nature and is unknown, however, a mean time of 10^8 years between collapses has been shown to be compatible with all observations [Maudlin, 2011, p. 228]. [4]

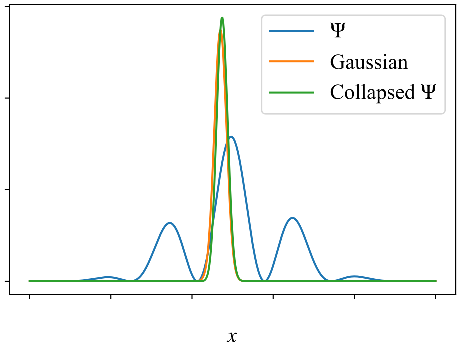

Figure 2: An example of a GRW-collapse of a wavefunction, Ψ.

So far only a single particle has been considered. If several particles are entangled,

$$|\Psi\rangle = \sum_i |\alpha_i\rangle_\alpha |\beta_i\rangle_\beta |\gamma_i\rangle_\gamma \dots , \tag{3.1}$$

and one particle (particle α) undergoes a collapse, and becomes localised in the eigenstate $|\alpha_j\rangle$, all the other particles will likewise collapse into the state given by the entanglement as the corresponding state, i.e. $|\beta_j\rangle$, $|\gamma_j\rangle$ etc.. Thus, with an increasing number of particles in the entanglement, the probability of a collapse likewise increases, since

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[4] The book by Maudlin in from 2011, but seems to be consistent with the more resent developments of GRW Theory. In an account of the theory made in an article from 2021 [Lorenzetti, 2021], the collapse is said to occur at a mean rate of $10^{-16}\,\mathrm{s}^{-1}$, corresponding to a single particle system undergoing a collapse every 10^8-10^9 years.

only one of the particles need to collapse to ensure the collapse of all the particles. For systems of only one or a few particles (such as those where quantum effects are observed), it is highly unlikely that the system collapses within a reasonable time frame. However, when such a system interacts with a macroscopic item, such as a cat or a measurement device, it becomes entangled with all the particles of that item. Macroscopical systems consist of an incredibly large number of particles and therefore one would expect a collapse within a very short time frame, and such objects will therefore remain in the entangled superposition state for only minuscule amount of time. Due to the entanglement, quantum systems will collapse upon e.g. measurement interactions involving a measurement instrument containing a large amount of particles [Maudlin, 2011, p. 226-229].

The localisation of the wavefunction is only approximate. The degree of precision is given by the width of the Gaussian function, which is another constant of nature in the GRW theory. The measurement device will therefore never exactly point at one result and a the cat will never be exactly in the alive-state [Maudlin, 2011, p. 228]. The width of the Gaussian is considered to be small enough to make particles seem like point particles from a human perspective. The fact that it is a Gaussian, however, means that the collapsed wavefunction will always have tails, and thus is never completely localised.

In the Schrödinger cat paradox, the state quickly collapses into (approximatly) either of the two eigenstates. This would happen already upon the entanglement of the atom with the Geiger counter, as the latter would contain a huge amount of particles, thus significantly increasing the probability of a collapse. The minimalist measurement problem and the problem of interpretation are therefore solved, as superposition states of entangled, macroscopical bodies can only last a very short time and thus are never observed. That such states are entangled is obviously essential in GRW Theory as the probability of collapse is otherwise as low as for single particles. States such as 2.9 and 2.10 therefore physically exists but quickly collapses into the definite eigenstates, as chances are high that one of the many particles, that enters into the entanglement undergoes a collapse. The problem of outcomes and the problem of statistics are likewise solved, as the supposition that the evolution of a system always happens according to the Schrödinger equation is denied; in addition to this type of evolution, there are the stochastic collapses. This accounts for the definite outcomes of measurements, and also for the fact that the same initial conditions can result in different outcomes, as the localisation of the wavefunction occurs randomly (i.e. this additional type of evolution is not deterministic) though still in accordance with the Born rule.

The problem of interference is solved by reference to the large number of entangled particles in macroscopic measurement devices, which makes the unlikely wave collapse highly likely. The wavefunction collapses shortly after the interaction of a quantum system with an instrument of measurement, and after the collapse the total state is in an eigenstate and there is no interference. Similarly, the problem of effect is managed easily, as the collapse brings the system into an eigenstate, thus securing that additional measurement will give the same outcomes as the first. This is also pointed out by Maudlin, who writes that "The dynamics of the wave-function [...] propagates the effect into the future" [Maudlin, 1995, p. 13], and argues that this marks a clear distinction from Bohmian mechanics, where the effect is mediated by the dynamics of the additional variables and *not* the wavefunction.

The problem of collapse concerns the question of what process is the cause of the collapse of the wavefunction. The unique answer made by GRW to this problem is the key idea in the formulation of GRW theory: nothing causes the collapse. The theory tells us that there is always some possibility of a collapse happening and the only reason it seems to occur e.g. upon measurement is due to the large number of particles involved in these interactions. GRW theory is therefore fundamentally stochastic in two ways; the spacial position of the localisation of the wavefunction is random and given by a probability distribution consistent with Born's rule, but the very occurrence of the collapse is stochastic as well and thus there is no determinate process from which it is a result.

The localisation of the wavefunction always occur in the position basis, and this means that the problem of preferred basis becomes similar in GRW theory to how it is in Bohmian mechanics, where only the position is considered to be a real property of the system. The theory must give an explanation of why statistical predictions of measurement outcomes in other bases than that of position seem to fit observations. One can argue, like one does in Bohmian mechanics, that what one really measures when doing experiments are positions of pointers. To say that Schrödinger's cat is alive means that all the particles constituting the cat are in a positional configuration corresponding to an alive cat as opposed to the configuration of a dead cat. A spin-measurement done with a Stern-Gerlach apparatus, will send the particle in different directions according to its spin, and thus a localisation in space (upon e.g. entanglement with macroscopical detectors) will correspond to a collapse in one spin-eigenstate. One could perhaps in this case argue that the spin is a real property of the measured system, which becomes correlated with the position and thus indirectly measured. This picture is however, becomes problematic when one considers measurements of momentum. If the particle is localised in space it is, as mentioned, "smeared out" in momentum space. A collapse in the position basis therefore cannot correspond to a collapse into a momentum eigenstate. It must be concluded than one does not really measure other observables in the GRW view. As in Bohmian mechanics, it is considered enough to be able to explain measurements of definite positions, as this is what we always observe: positional configurations of measurement instruments.

The control problem is solved by the fact that GRW theory denies that evolution is always due to Schrödinger equation. If the inputs are determined by anything involving a macroscopical object (a person, a meaasuring device), then wavefunction will have collapsed and will be in an eigenstate. Therefore the inputs will be definite and a definite state will be prepared. If the determination of inputs and the preparation is done somehow by a system of only a few particles, then there only a miniscule chance of a collapse taking place during the time of the preparation, and the prepared state can be an indefinite one. This is however not in conflict with the control problem, as the inputs are not determinate in the cases where the evolution is unitary. As in the von Neumann account, where the assumption of unitarity is also abandoned with the inclusion of a collapse, the no-cloning theorem is not derived.

3.3 An overview of the different solutions

It can be seen from this account, that the different interpretations of quantum mechanics, which I have presented here, address the measurement problem in widely different ways. Therefore it varies between them which formulations of the problem are easily solved and which becomes problematic. In this manner, the measurement problem draws attention to the different strengths and weaknesses of the interpretations, which I will quickly summarise here.

Bohmian mechanics avoids many of the departures from classical physics, which quantum mechanics otherwise seems to imply. The world consists of particles, the state of which remain well-defined and definite. The evolution of these states is always deterministic. Bohmian mechanics does not include a collapse, which would require further explanation, and it manages to solve the problem of statistics, without giving up on the determinism, by arguing that systems of identical wavefunctions need not be in identical states, as their positions may be different. But though Bohmian mechanics in many ways succeeds in creating a theory of quantum phenomena which (in these aspects) do not depart from the classical world view, this comes at a cost. In Bohmian mechanics the position of a particle becomes its only real property. All measurements of momentum, spin etc., that appears to be made in physics, are actually no such things. They are only correlations of particle and pointer positions. The wavefunction also seems to become a novel type of object in the Bohmian picture. It is not a physical wave, yet it changes the physical behaviour of particles; it can be viewed along the lines of a law of nature, but one which depends on the state of the system to which it applies. Bohmian mechanics must also endorse non-locality in a very concrete way, as change of behaviour is instantaneously mediated by the wavefunction. Lastly, it is unclear why the uncertainty of the particle position should be in the quantum equilibrium.

The Everett interpretation also has no concept of collapse that requires explaining, but contrary to Bohmian mechanics, this is due to the fact that quantum mechanics never seizes to be the right description of the world. The Everett interpretations need not divide the world into classical and quantum mechanical, and need not explain any transition between different kinds of descriptions. But by introducing a concept of "branches", another term in need of explanation is created and for this, the Everett interpretation relies heavily on decoherence. The use of decoherence in the Everett interpretation includes the choice of basis in which the "branching" occurs, an enterprise the success of which e.g. Janssen [Janssen, 2021] is sceptical about. I also find the term "branch" to be a rather abstract notion. At least, the Everett interpretations turns out to be far less clear and simple than what it appears to be at a first glance. The idea of a splitting the world into different worlds each with definite, classical properties does not suffice for the subtleties of quantum mechanics. In the Everett interpretation the meaning of the probabilities given by quantum states also become more complicated, since all possible outcomes, to which the probabilities refer, will occur.

GRW theory, in contrast to Bohmian mechanics and the Everett interpretation, introduces a collapse which constitute a transition from a quantum mechanical to a classical description of a system. By doing so, GRW theory is able to solve many of the formulations of the measurement problem, as they rely on an assumption that the evolution of quantum systems is always unitary. This includes the problem of outcomes, the problem of statistics, the problem of interference and the problem of effect. Since the inclusion of a collapse is done, not by letting some process be the cause of it, but by making it occur randomly, the problem of explaining the circumstances of the collapse is mostly avoided. This however, means that one must add a stochastic law of the occurrence of the collapses to the theory in addition to the collapses themselves being stochastic. The latter of these stochastic features of the theory must depend on the state of the system which undergoes the collapse in order to replicate the Born statistics; i.e. the localisation must be subjected to a probability distribution corresponding to the wavefunction squared. Like in Bohmian mechanics, measurements of variables other than position becomes effects of the correlation between the wavefunction and the pointer position rather than actual measurements of these variables.

4 Decoherence: a dead end or a way forward?

Decoherence is not an interpretation of quantum mechanics but a quantum theoretical process. This fact, however, has been in favour of the idea of decoherence as a possible solution to the measurement problem, as it would be a solution from within the quantum formalism itself, not requiring new variables, new ontologies or new types of evolution of the quantum state. Scott Tanona writes,

> The decoherence approach explicitly assumes that instruments and environment have a complete quantum description, and systems are determined to be classical not by fiat (and not via interaction with other systems declared to be classical by fiat), but rather *in virtue of the quantum description itself* [Tanona, 2013, p.3629] [my italics]

The basic idea of decoherence is that when a quantum system interacts with its environment or a measurement apparatus, it becomes entangled with it. When looking at just the system by itself, one no longer has a pure state, but a mixture, that does not have any quantum interference. Therefore, the inability to isolate a system completely from its surroundings, is what causes the quantum effects to disappear. In this capacity, decoherence has been expected to give a physical explanation of the apparent wavecollapse upon e.g. measurement, and of the apparent definiteness of macroscopical objects, as the larger a system is, the less possible it is for it to be without contact with the environment and avoid decoherence. Thus, a cat would never be found in a pure quantum state. These expectations may be said to have been partially fulfilled and partially disappointed. This will be investigated in section 4.2 where I will discuss decoherence as a solution to the different measurement problems, after first giving an account of decoherence the following section. This account will deviate from the type of decoherence, which is commonly discussed in literature upon the subject of the measurement problem. Usually, the so-called environment-induced decoherence is employed in relation to the measurement problem. This is a process by which the combined object-instrument state is shown to approach a mixture, as it becomes entangled with many environmental degrees of freedom. Here I will make a more general account, using only the entanglement between two systems. Lastly, some general points in discussions on what to understand by decoherence are touched upon.

4.1 An account of decoherence

To illustrate a case of decoherence, a system in a state $a\,|0\rangle + b\,|1\rangle$, becomes entangled with some other system, $|\phi\rangle$,

$$(a\,|0\rangle_A + b\,|1\rangle_A)\,|\phi\rangle_B \rightarrow a\,|0\rangle_A|\phi\rangle_B + b\,|1\rangle_A|\phi_\perp\rangle_B = |\Psi\rangle_{AB} \tag{4.1}$$

Here, it is assumed that the states of both systems are orthogonal, and that there is an interaction between the them, which causes $|\phi\rangle_B \rightarrow |\phi\rangle_B$ if system A is in the state $|0\rangle$ and $|\phi\rangle_B \rightarrow |\phi_\perp\rangle_B$ if system A is in the state $|1\rangle$. This is the same type of correlation of eigenstates leading to entanglement, as that described for measurement processes in equation 2.10, i.e. the CNOT-gate. This could therefore be the entanglement between a quantum system and a measurement device, though it need not be.

If one is only concerned with system A, then one can trace out system B by calculating the reduced density matrix, ρ_A, which gives the statistics of system A taken on its own.

$$\rho_A = \mathrm{Tr}_B(|\Psi\rangle_{AB\;AB}\langle\Psi|) = |a|^2\,|0\rangle\langle 0| + |b|^2\,|1\rangle\langle 1| = \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}, \tag{4.2}$$

where the last matrix is written in the basis of $|0\rangle$ and $|1\rangle$. This can be compared with the initial density matrix of system A before the entanglement with system B:

$$\rho_{\text{initial}} = \mathrm{Tr}_B((a\,|0\rangle + b\,|1\rangle)(a^*\,\langle 0| + b^*\,\langle 1|)) = \begin{pmatrix} |a|^2 & a\,b^* \\ a^*\,b & |b|^2 \end{pmatrix}. \tag{4.3}$$

The off-diagonal terms in equation 4.3 are clearly not present in equation 4.2. As stated previously, the diagonal terms of a density matrix gives the probabilities (according to the Born rule) of finding the system in either of the two basis states respectively, whereas the off-diagonal terms characterises the interference between the two basis states. The interference is thus not present when considering only one part of an entangled pair. As in general, $a = |a|\,\mathrm{e}^{i\phi_a}$ and $b = |b|\,\mathrm{e}^{i\phi_b}$, the off-diagonal terms becomes $a^*\,b = |a|\,|b|\,\mathrm{e}^{i(\phi_b - \phi_b)}$ and the complex conjugate. The interference is therefore an effect of the relative phase of the coefficients, and this effect vanishes in the reduced density matrix, where only the diagonal terms $|a|^2$ and $|b|^2$ remain. If system A, a particle, becomes entangled with system B, the environment or a measurement device, the particle on its own will have gone from being in a pure state with interference to being i a mixture with no interference. In section 2.3, classicality was associated with mixtures, following a point made by Howard. Thus, the state in the reduced density matrix can be said to be a classical state, as opposed to the quantum state prior to the interaction with the other system.

The account of decoherence presented here has been of a more general nature of what is usually the case. In most literature, decoherence refers to environment-induced decoherence, where entanglement exists between an object, an instrument *and* the environment, and the reduced density matrix of the combined object-instrument state becomes a mixture due to the tracing out of the environmental degrees of freedom (e.g. [Janssen, 2021] and [Adler, 2003]). The more general approach employed here is in agreement with the use of the word "decoherence" by Tanona, who writes,

> For the purposes of this paper, decoherence will be considered a general term signifying the loss of coherence in individual subsystems represented by reduced density matrices, regardless of the type of system involved in the entanglement. [...] observers measuring a system can be thought of as decoherence-inducing environment. But also, measured systems can act as the environment of a measuring apparatus. [Tanona, 2013, p. 3632]

In agreement with the more general account, Wallace also uses a Hilbert space decomposed into only two subspaces, even though he refers to one as the "system" and the other as the "environment" [Wallace, 2012, p. 4584-4586]. As implied by this, there is some ambiguity as to what exactly is understood by environment-induced decoherence. Sometimes, the importance lies in the inclusion of a third system (the environment) in the entangled state, while at others the main characteristic is that the "environment" has many degrees of freedom with which the measured system becomes entangled. In this case, the "environment" might actually be (parts of) the measurement apparatus. However, I will mostly be concerned with decoherence in the general sense, accounted for here, and thus avoid such problems with the exact definition.

The account of decoherence given here is also an idealised case of entanglement, where the states of the environment or measurement apparatus are perfectly orthogonal and perfectly correlated. In the case of environment-induced decoherence (in either sense of the term), the environmental states cannot be supposed to be orthogonal. However, the interference terms can be shown to very rapidly approach zero as the states of the environment approaches orthogonality [Adler, 2003, p. 6-8]. Therefore, though the interference terms are not exactly zero in the reduced density matrix, they will become zero for all practical purposes almost immediately. In the general, idealised case presented here, the interference terms disappear, as the inner product $\langle\phi|\phi_\perp\rangle = 0$. In a case where the states of system B are not orthogonal (such as when the system is entangled with the environment, which is not designed in such a way as to have perfect correlation with orthogonal state, as in an ideal measurement situation) one can replace $|\phi_\perp\rangle$ with the more general $|\phi'\rangle$. In this case of environmental entanglement, decoherence will have the effect of causing $\langle\phi|\phi'\rangle \to 0$ as as time $\to \infty$. How exactly this is accounted for varies between different models of the environment (the system B) and its interaction with system A. [Adler, 2003, p. 6-8] [Janssen, 2021, p. 42-45]

Janssen also considers the case of imperfect measurements. These are measurement interactions either with errors, i.e. where the wrong states are coupled, such as having

$$(a\,|0\rangle_A + b\,|1\rangle_A)\,|\phi\rangle_B \to a'\,|0\rangle_A|\phi\rangle_B + a''\,|0\rangle_A|\phi_\perp\rangle_B + b'\,|1\rangle_A|\phi_\perp\rangle_B + b''\,|1\rangle_A|\phi\rangle_B \quad (4.4)$$

instead of equation 4.1, or *2nd kind measurements*, where the states of the measured object are changed by the measurement interaction,

$$(a\,|0\rangle_A + b\,|1\rangle_A)\,|\phi\rangle_B \rightarrow a'\,|\alpha\rangle_A|\phi\rangle_B + b'\,|\beta\rangle_A|\phi_\perp\rangle_B\,. \tag{4.5}$$

In these cases, the off-diagonal terms does in general not converge towards zero [Janssen, 2021, p. 89-90]. Janssen are here talking about the system-apparatus reduced density matrix, ρ_{AB}, in environment-induced decoherence. In the case of 2^{nd} kind measurements, ρ_A would be diagonal in the basis of $|\alpha\rangle$ and $|\beta\rangle$ where the correlations with the instrument states are. In the case of the measurement with error, the degree of diagonalisation depends on the relative sizes of a'' and b''. If they are small compared to a' and b', the off-diagonal terms will be likewise small.

The relevance of decoherence is not restricted to discussions upon interpretational issues; rather it is something that every experimentalist working with quantum physics must deal with. Here, decoherence due to entanglement with the environment is something that destroys the quantum state, which one wants to work with, or something which makes one loose control over the states. If the quantum system is not perfectly isolated from the environment, it becomes entangled with environmental degrees of freedom. This causes decoherence in the (now necessarily reduced) density matrix for the system and the measurement apparatus, prior to any possible measurement or other types of interactions one wishes to do with the quantum system, and which would have required a pure quantum state. In other words, as stated in an article by Elise Crull, “the phase relations of that system become smeared out, or decohered, into the new degrees of freedom provided by the environment” [Crull, 2013]. Thus, the state of the quantum system is essentially destroyed in a way not planned for or controllable. As such, decoherence is related to the “lifetime” of the quantum states in experimental physics. In quantum informational research one seeks to construct qubits (i.e. two state quantum systems, that can interact and evolve corresponding to different gates). To be able to work with these in the running of algorithms, they must interact very weakly with the environment so as to avoid decoherence and thus “live longer”.

As apparent in this account of decoherence, the essential point, which allows for a quantum mechanical evolution from a pure state to a mixture, is that the unitary quantum formalism applies to the total system, i.e. the whole universe or some perfectly isolated part [Meehan, 2019, p. 6]. It is therefore possible to have a *local* time evolution which is not unitary, such as in the case of decoherence, where a system becomes entangled with another system, thereby making the total system larger. The mixture is thus obtained when one does not take the total system into account, i.e. when restricting oneself to only a part of it, where unitarity therefore does not apply [Crull, 2013] [Janssen, 2021, p. 69].

4.2 Decoherence as a solution to the different measurement problems

The role of decoherence in a solution to the measurement problem has been widely discussed. The overall idea is, that decoherence, as a process in which a quantum system (viewed by it self) goes from a superposition state with quantum effects such as interference, to a mixture of definite outcomes, explains the shift from quantum to classical descriptions of the world. There are however, several factors to consider with regards to the measurement problem, some of which can indeed be said to be solved by decoherence, and some of which are more problematic. In the following, I will discuss decoherence as a (partial) solution to the different formulations of the measurement problem, which were mentioned in section 2.

It is clear why decoherence can be seen as a solution to the problem of interference, as it explains how the off-diagonal interference terms of the (reduced) density matrix becomes (or approaches) zero in a measurement situation, where the quantum system is entangled with another system, or due to interactions with the environment. Decoherence makes a quantum mechanical account of why the off-diagonal terms in the density matrix of a system, the state of which has been measured, are zero. It springs from the fact that upon measurement the system necessarily becomes entangled with the measurement apparatus, meaning that the reduced density matrix of the system become a mixture with only diagonal terms. However, the interference terms of the density matrix disappearing is not alone enough to account for why there is no interference at the end of a physical measurement; something linking the density matrix in general, and the reduced density matrix in particular, to the physical world i missing. Here, decoherence suffers from being purely a feature of the formalism and thus manifestly *not* an interpretation of that formalism. Though decoherence can in the case of the disappearing interference be a usefull tool or give an important clue towards a way of thinking about this issue, an understanding of that the density matrix represents is needed prior to decoherence being applied. As we shall see, this type of conclusion regarding the usefulness of decoherence will reappear throughout this discussion.

The minimalist measurement problem perhaps cannot be answered by the interpretation neutral decoherence, as it concerns the understanding of what the meaning of (entangled) superposition states are; something that will depend on one's interpretation of quantum mechanics. However, decoherence can be said to shed some light on the matter, and thus having significance for the ways in which these states *can* be understood. The states in 2.9 and 2.10,

$$|\Psi\rangle_{\text{atom}}|\text{alive}\rangle_{\text{cat}} \to \frac{1}{\sqrt{2}}\left(|\text{not decayed}\rangle_{\text{atom}}|\text{alive}\rangle_{\text{cat}} + |\text{decayed}\rangle_{\text{atom}}|\text{dead}\rangle_{\text{cat}}\right) \tag{2.9}$$

$$\left(\sum_i \alpha_i|\alpha_i\rangle\right)|\beta_0\rangle \to \sum_i \alpha_i|\alpha_i\rangle|\beta_i\rangle\,, \tag{2.10}$$

are the states of the total system; of the cat *and* the atom or of α- and β-systems, the latter of which could for instance be an object and an instrument of measurement. It would be wrong to conclude from this state, that e.g. the cat or the α-system is in a state of superposition. An entangled state is defined by not being a product state of the states of the subsystems, and this means that one cannot simply look at one subsystem separately. This is exactly the main feature in decoherence. As shown in the above account, there is a difference in the density matrix of the total system, which is indeed described by a pure quantum state such as 2.9, and the reduced density matrix of a subsystem of the total system, such as the cat. The reduced density matrix of the cat would be a mixture, with only diagonal elements giving the probabilities of it being either dead or alive. This does not constitute an answer to the minimalist measurement problem, but is does direct a possible answer: one does not need to explain what can be meant by a cat being in a superposition state.

Similarly, decoherence cannot be made to address the problem of interpretation, as it itself is a feature of the formalism rather than an interpretation. This problem could be said to form exactly the part of the minimalist measurement problem, that cannot be answered by decoherence. But as above, decoherence has saved one the trouble of giving meaning, epistemological or ontological, to a cat in a superposition state, as one can distinguish between the state of the total system and the state of the subsystem. Decoherence will have to be combined with a proper interpretation, however, in order to direct the understanding of these states any further. In section 6, such an address will be made, as decoherence will be viewed in the light of the writings of Niels Bohr - and vice versa.

Tanona regards it as generally acknowledged, that decoherence is not able to solve the measurement problem, partly due to the fact that having a diagonal density matrix does not equal having definite outcomes, as the ignorance interpretation does not apply [Tanona, 2013, p. 3632]. Here, he draws attention to a serious issue with decoherence: its difficulties with the problem of outcomes. Though decoherence leads to a state, which is a mixture of different definite outcomes, what one would need in order to maintain the idea of definite outcomes of a measurement is a probabilistic interpretation of this mixture; that is, one would need to apply the ignorance interpretation to the mixture. For this to be at all possible, there must be no interference, as interference is what stands in the way of applying a probabilistic interpretation to any quantum state, as it is an effect of the phases, which are irrelevant for the probabilities. Decoherence does take care of this matter by the suppression of the off-diagonal terms in the reduced density matrix [Wallace, 2012, p. 4584]. But as pointed out by Stephen L. Adler [Adler, 2003], the problem of outcomes is not eliminated by producing a state, that could possibly be subjected to the ignorance interpretation. Decoherence does not directly account for how one definite orthogonal measurement outcome is obtained, only for how the interference between different eigenstates disappear. The contradiction in formalism between the exclusively unitary and linear evolution of the quantum state and the outcome of a measurement being in one of several orthogonal states still remain. The need of a stochastic collapse to go from the mixture without interference to one definite state has not been eliminated or, alternatively, some other interpretational background to explain away the apparent need of a wavecollapse [Adler, 2003].

Janssen likewise concludes that the problem of outcomes is not solved by decoherence, since definite outcomes are only implied, not accounted for [Janssen, 2021, p. 35]. She differentiates between interference and indefinitness, in arguing that though decoherence might remove the interference, this does not mean that it has removed the indefinitness of quantum states. She does not consider it to be possible for decoherence to achieve this single-handedly, as she finds the argument that because a quantum state (a reduced density matrix) looks like a classical, statistical mixture it must mean that it is ontologically the same, to be flawed [Janssen, 2021, p. 24-30]. A mixture, which is obtained by taking a partial trace of the density matrix of an entangled system, is known as an improper mixture, whereas a classical, statistical mixture, is known as a proper mixture. Decoherence thus enable a reformulation of the problem of outcomes, as the problem of accounting for why one can interpret an improper mixture as a proper mixture, and thus apply the ignorance interpretation [Janssen, 2021, p. 23]. Though this problem is not solved by decoherence, Janssen does however, consider the possibility that in unison with an interpretation, decoherence might be able to account for the lack of indefiniteness, as she argues that "The point at which the need for an additional interpretation becomes manifest is when one needs to go from the diagonal reduced density matrix to the occurrence of definite and unique measurement outcomes." [Janssen, 2021, p. 63-64]

Janssen argues that applying the igonrance interpretation to the decohered density matrix is problematic. This, she argues, is due to the fact that the term ignorance referred to in decoherence is different from that of classical physics (and the ignorance interpretation). A classical mixture of different outcomes, to which the ignorance interpretation applies, represents the ignorance about which outcome has been obtained, which is the result of unknown conditions. Also in decoherence, a mixture is obtained through *not knowing* the conditions, i.e. the tracing out of entangled subsystems. But this type of ignorance is fundamentally different, as in decoherence, more knowledge would result in the disappearance of the classical behaviour, i.e. in the state not being a mixture, whereas more information in the classical mixture, would lead to more precise knowledge, represented by a change in the diagonal elements. Janssen further points to a problem in reconciling the ignorance interpretation with the e-e link, as the real eigenstates are those of the total system, not those of the subsystem. Thus the real eigenstates are not in a mixture, and the ignorance interpretation cannot be applied. Janssen argues, that this is a problem for the ignorance interpretation of the mixture, as this application is motivated by the e-e link; i.e. by the idea of the basis states of the reduced density matrix corresponding the the states of the (sub)system. [Janssen, 2021, p.82-83] As will be seen in section 6, I do not consider these issues to be insurmountable, when seen in the light of certain interpretations; in Bohr's view these matters will be addressed by the concepts of contextuality and fundamental statistics. But they are something, that one must make sure to consider when using decoherence as a basis in interpretational considerations.

Decoherence as a solution to the problem of statistics has much the same issues as for the problem of outcomes, since it, in itself, offers no explanation of how one of the possible outcomes is obtained, and thus gives no account of the fact that the same initial conditions can lead to different outcomes. It has been suggested that the specific measurement outcome might be determined by the initial state of the environment with which the measurement object is entangled. This possibility of obtaining a deterministic process of going from a superposition to an eigenstate is reviewed by Adler, who mentions that it has been shown not to work [Adler, 2003, p. 10]. Decoherence has no problem, however, with accounting for the statistics of measurement outcomes, as the decohered mixture will have the same diagonal elements as the initial pure state, i.e. the same probabilities as those given by the Born rule when applied to the pure state. The part of the problem of statistics relating to the fact of the measurement outcomes being subjected to probabilities given by the Born rule, therefore fits naturally with decoherence.

Decoherence is in general regarded as useful in addressing the problem of preferred basis. The idea is that decoherence selects a preferred basis, thus solving the problem of in what basis the density matrix is diagonalised [Janssen, 2021, p. 42-45]. This happens, as decoherence on causes diagonalisation in a specific basis: that in which the states of the two entangled systems are correlated.

If the (orthogonal) states of one system, such as a measurement instrument, are correlated with the spin states of another system in the z-basis, so that the entangled state looks like

$$|\Psi\rangle_{\alpha\beta} = a\,|\uparrow\rangle_{\alpha}|\psi\rangle_{\beta} + b\,|\downarrow\rangle_{\alpha}|\psi_{\perp}\rangle_{\beta}\,, \tag{4.6}$$

the same entangled state with a basis change of the spin states (to an x-basis), will look like,

$$|\Psi\rangle_{\alpha\beta} = \frac{a}{\sqrt{2}}|\rightarrow\rangle_{\alpha}|\psi\rangle_{\beta} + \frac{a}{\sqrt{2}}|\leftarrow\rangle_{\alpha}|\psi\rangle_{\beta} + \frac{b}{\sqrt{2}}|\rightarrow\rangle_{\alpha}|\psi_{\perp}\rangle_{\beta} + \frac{b}{\sqrt{2}}|\leftarrow\rangle_{\alpha}|\psi_{\perp}\rangle_{\beta}\,. \tag{4.7}$$

The reduced density matrix of the system α will in this basis become,

$$\begin{aligned}\rho_{\alpha} =& \mathrm{Tr}_{\beta}(|\Psi\rangle_{\alpha\beta\;\alpha\beta}\langle\Psi|) \qquad (4.8)\\ =& \frac{|a|^2{+}|b|^2}{2}|\rightarrow\rangle\langle\rightarrow| + \frac{|a|^2{-}|b|^2}{2}|\rightarrow\rangle\langle\leftarrow| \\ &+ \frac{|a|^2{-}|b|^2}{2}|\leftarrow\rangle\langle\rightarrow| + \frac{|a|^2{+}|b|^2}{2}|\leftarrow\rangle\langle\leftarrow| \qquad (4.9)\end{aligned}$$

In this bases, the reduced density matrix clearly has non-zero off-diagonal elements. It is not surprising as a matrix can always be diagonalised in *some* basis, while it will not be diagonal in others. The main thing here, is that the reduced density matrix becomes diagonal in the basis in which the states are correlated with the states of the other system. In what basis the density matrix is diagonalised thus depends on the interaction with the other system, e.g. the measurement interaction. There is however a problem with systems that are correlated in all possible bases. This type of entangled state is known as a *maximally entangled state*, and is defined by having entanglement in all bases. An example can be two particles the total spin of which is zero, i.e. their spin will be opposite when measured in any direction. In these cases, the reduced density matrix will correspond the the (normalised) identity, and will be diagonalised in all bases. This can be seen from equation 4.9, as it is the case of $a = b = \frac{1}{\sqrt{2}}$.

Decoherence's solution to the problem of preferred basis will in these cases not apply.

Decoherence's response to the problem of preferred basis is thoroughly discussed by Janssen. Janssen concerns herself with environment-induced decoherence, and argues that a third system (the environment) is needed to remove the ambiguity about what base the state is decomposed in. She writes,

> So by introducing a third system, the basis ambiguity is removed: rewriting the state of one of the subsystems in a different basis comes at the expense of a more complicated expression for the total state on $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ which does not represent a one-to-one correlation between, for instance, the quantum system and the measurement apparatus. [Janssen, 2021, p. 47]

This might seem strange in the light of the previous account of how the basis of the diagonalisation corresponds to the basis in which the eigenstates are correlated with the states of the measurement instrument. I will return to this issue, but first I will follow Janssen's argument through. She argues that, as the density matrix is diagonalisable in the basis in which the system states are correlated with the instrument states, one must look at where these correlations can exist. She argues, that only when the observable commutes with the Hamiltonian of the interaction between the instrument and the environment can the correlations be stable, as they are not perturbed by the interaction with environment. Here the observable is the operator, the eigenstates of which are to be correlated with the instrument states [Janssen, 2021, p. 47]. These eigenstates constitute what is called the "pointer basis". A case of perfect commutation is an idealised case, as the observable and the interaction-Hamiltonian will in general not commute. In this more general case, something known as a "predictability sieve" selects the basis states that are *least affected* by the interaction with the environment, in the sense of having the smallest increase in entropy [Janssen, 2021, p. 48]. Janssen thus concludes that "only eigenstates of the pointer observable [...] will not be perturbed by the 'continuous measurement' by the environment." [Janssen, 2021, p. 52] This process of selecting the basis in which the decoherence is stable, and which can therefore be used for measurements, is known as *einselection* (environment-induced selection rules) [Janssen, 2021, p. 49-50]. The einselected basis corresponds to the pointer basis, and is thus the basis in which the density matrix is diagnonalised, and where this diagonalisation is robust to pertubations from the environment.

Jansses stresses the point about the einselection being in general only approximate: "What decoherence arguments pursue, therefore, is a dynamical theory of 'approximate' superselection rules." [Janssen, 2021, p. 51] She argues that because of this the uniqueness of the pointer basis-decomposition of the density matrix is not guaranteed [Janssen, 2021, p. 53]. But issue relates to the "imperfect" cases. For the idealised two-system interaction, there is nothing approximate about the selection of basis.

We can now return to the meaning of including the environment. Janssen herself argues that the correlations between two systems is enough for the basis selection (the environment is said to be superflous in this regard). She writes the following, in a discussion of what observables are in the decoherence picture:

> [...] only the decomposition in terms of the [apparatus state] basis that commutes with the [apparatus-environment] interaction Hamiltonian can be stable under the interaction with the environment. So far, this argument is correct, but in the present context it seems rather superfluous to me. Zurek claims to solve the problem of accidental degeneracy by pointing out that it is in fact a matter of dynamics. But if we are to take recourse to the dynamics, we *could just as well refer to the dynamics of the measurement interaction directly*, i.e. invoke the [system-apparatus] hamiltonian, which selects the measured observable in a completely unambiguous way. [Janssen, 2021, p. 77] [my italics]

This latter suggestion is exactly what was previously shown; that the correlations between the measured system and the measurement instrument determines the basis of diagonalisation. What Janssen argues is the role of the third system, the environment, is in showing why the observables are what they are. She writes,

> [...] why is there only a limited set of "classical" quantities that appear to be definite for "ordinary" objects? The answer decoherence offers to this ques-

> tion is that any open system is also in a measurement-like interaction with its environment, which similarly determines a preferred basis of "effectively classical" states. [Janssen, 2021, p.78]

These are questions which lie in the field of *emergence of classicality* (which Janssen herself points out) as they concern the explanation of why the classical world appear in the way it does. This is therefore, in my opinion, beyond a mere solution of the measurement problem. I will therefore maintain that the general two-system decoherence is sufficient, and that the addition of a thrid subsystem does not bring the problem of the preferred basis closer to a solution.

Returning to the formulation of the two problems of preferred basis, the general version is stated as the problem of accounting for which basis the Born rule applies to. One can quite easily argue, that the Born rule applies to the basis in which the density matrix is diagonalised, i.e. the basis in which the states of the two systems are correlated. The decomposition version is more specific in seeking an answer to the question of which decomposition of the system-instrument state corresponds to the measured states. Here one can again appeal to the basis in which the correlations are, or take a further step back and appeal to einselection. However, one might still argue that something is needed to connect the preferred basis to an observable. In other words, one needs to argue that the preferred basis is that in which the statistical content of the quantum state has empirical meaning. This is what Janssen calls the *interpretational basis* [Janssen, 2021, p.18]. In quantum physics a basis is usually said to correspond to an observable, and the state in that basis is used to give predictions of the outcome of a measurement of the corresponding observable; that of which the basis states are eigenstates. But, as was shown in section 3, observables take on different meanings in different interpretations, and the question of the interpretational basis might not be a simple matter. It is in any case another example of how little decoherence can do without an interpretational background to give the mathematical formalism physical meaning.

It is not easy to identify decoherence as describing either a collapse or non-collapse situation. Depending on the perspective it is both. Looking at just one part of the entangled pair, decoherence is a dynamical explanation of the "collapse", in the sense of going from a pure state to a mixture and loosing the quantum effects that are due to the phase relations (here disregarding the problem of obtaining definite outcomes). It is a process of the quantum formalism which makes the quantum interference etc. of the system disappear upon interaction with e.g. a measurement device or with the environment, the latter of which explains why macroscopical systems, which are never isolated, appear classical. This is exactly where a collapse of the wavefunction is postulated to take place and what it is postulated to do (again excepting the issues with the problem of outcomes). But in another sense, the wavefunction never collapses, as no non-unitary, non-linear, physical evolution takes place. Looking at both parts of the entangled pair as a whole, the quantum states at all points evolve just as predicted by the Schrödinger equation. The presence or absence of the collapse therefore depends on what one is looking at (what can be understood by "looking at" will be discussed later). Tanona uses the world "locally effective collapse" about this type of situation where no physical collapse takes place, yet a process leading from a pure state to a mixture can be said to occur in parts of a system [Tanona, 2013, p. 3641]. The sort of answer, which decoherence gives to the problem of collapse, will follow these lines.

The problem of effect concerns how to account for the effect of a measurement on later measurements, i.e. why a measured system behaves as though it is in an eigenstate. Tanona is of the opinion, that this problem is solved by decoherence, as he writes that the effect of entanglement on future interactions is *robust* in the local system where the "effective collapse" has taken place [Tanona, 2013, p. p. 3641]. He writes,

> Once entanglement has created local "decoherence" in even a simple two-system (e.g., singlet state), quantum effects may be seen in correlations, but no reemergence of coherence locally at the subsystem alone is possible unless the entire entangled system is acted upon in a way that will eliminate the entanglement, e.g., by reversal of the entangling interaction. This means that for a particular interaction depending on a certain subsystem's observables alone, if that system has decohered in those observables, then the effect of that decoherence on the interaction is effectively complete, unless the decohering entanglement is reversed before the interaction. [Tanona,

2013, p. 3641]

That is to say, for other interactions (e.g. later measurements) performed only on the decohered subsystem, the interactions will be with the decohered state, i.e. will be influenced by the effective collapse of that system. The coherence cannot be regained by any future interactions of the subsystem, but only by globally counteracting the entanglement. Thus, future interaction of a decohered subsystem, will "proceed as if the system had collapsed into the correlation basis" [Tanona, 2013, p. 3642], in accordance with what is required by the problem of effect. There is however a problem. Even if a second measurement is a measurement of the decohered state, this will be a mixture and not a definite state. This therefore cannot explain why the same eigenvalue would be measured twice. Bas van Fraassen [van Fraassen, 1997] has given an account of repeated measurements, which is useful for many non-collapse theories with regards to the problem of effect. He argues, that two measurements of a system must give the same result due to the entanglement of the total system. A second measurement brings in a third system (C) in the entangled state, rather than being a repetition of the measurement process. The entanglement of the system (A) with the first measurement apparatus (B) therefore still holds, and the state becomes

$$\Big(a\,|0\rangle_A|\phi\rangle_B + b\,|1\rangle_A|\phi_\perp\rangle_B\Big)\,|\phi\rangle_C \rightarrow a\,|0\rangle_A|\phi\rangle_B|\phi\rangle_C + b\,|1\rangle_A|\phi_\perp\rangle_B|\phi_\perp\rangle_C\,, \tag{4.10}$$

i.e. a superposition of measuring one outcome twice and the other outcome twice. This is very much like the answer to the problem of effect in the Everett interpretation (section 2), but without the terminology of the second measurements being branch-relative. Rather this state in general shows, that a second measurement must necessarily give the same result as the first.

Tanona also writes, that future entanglements with the environment will serve to stabilise a prior decoherence, as they are "proceeding as if the system had collapsed" [Tanona, 2013, p. 3646]. He thus argues, that a measurement alone will result in decoherence, which is then reinforced by further entanglement with the environment.

In regards to the control problem, decoherence maintains the condition of definite inputs, as the state of what input is being given (possibly by a person) to a preparation device would undoubtedly be a decohered state due to the impossibility of isolating such a system from the environment. Likewise, decoherence maintains that any evolution is due to the unitary Schrödinger equation. With these two conditions, the control problem becomes the no-cloning theorem, i.e. the limitation of simultaneous state preparation and determination. This theorem is in quantum mechanical research (such as quantum information) regarded as a settled feature of quantum mechanics rather than an issue that needs to be solved. In my opinion decoherence cannot shed further light upon whether the no-cloning theorem is a problem or not, as it is not an interpretation but merely a quantum mechanical process. I would however, consider it an asset of decoherence, that it enables this reading of the control problem. The fact that decoherence is (often) approximate, however, might be considered to mean that the no-cloning theorem is only approximate as well. I will discuss this further in the following section.

The control problem relates to cases where one would expect determinate inputs, such as when a person feeds the input to a macroscopic device, and these are the cases where environment-induced decoherence will definitely have taken place. But what if the whole process was performed by perfectly isolated interacting quantum systems? It might be considered possible to avoid the decoherence of input states if microscopical systems were used and the input was somehow determined by a quantum system. In this case one would have no indefinite inputs and end up with a superposition of prepared states. But this is only if one considers the total system, the full density matrix. Even if there is no environment, decoherence would still have taken place, several systems are still entangled, meaning that decoherence could occur if one of these were traced out. The problem is that decoherence in itself does not provide any rules for when some subsystem is to be traced out and the reduced density matrix is the appropriate description. That depends on what the traced out system corresponds to and when it happens, and therefore the conclusion again becomes that decoherence needs an interpretational basis.

To conclude, decoherence seems to be a useful step towards solving the measurement problem. There are however several remaining difficulties, especially concerning how to understand different aspects of the formalism. Decoherence must be paired with an interpretation to fully solve interpretational difficulties.

4.3 Further discussions on decoherence

Decoherence entails several different claims about what the world is like. As there is no physical collapse, the world is in decoherence perceived as indefinite, though this is not experienced due to indistinguishably of e.g. an object-instrument state (in the case of environment-induced decoherence) from a statistical mixture [Howard, 2021]. It is not exactly a statistical mixture, though, as the off-diagonal terms which distinguish the pure state from that of a mixture are not in general exactly zero in the decohered density matrix. Crull thus points out that it is in principle possible to measure superposition states, e.g. by observing interference patterns in a single measurement. She argues, that the apparent definiteness of measurement outcomes, that is to say observations of eigenstates, is a consequence of the inability to measure these damped superpositions [Crull, 2013]. This has consequences for how the no-cloning theorem can be viewed in a decoherence-account of measurements. If it is in principle possible to measure a superposition, it would be possible to directly measure the full quantum state by a single measurement. This would in turn allow one to make a protocol corresponding to a cloning. State determination and preparation are no longer fundamentally limited, and the no-cloning theorem becomes approximate. But is decoherence generally approximate? In a perfect measurement the off-diagonal terms are zero, and therefore one could argue that the approximate nature is practical rather than fundamental. But in this case a less perfect measurement gives one greater ability to know things, which seems at least to be slightly odd. This might be a point, where experimental advancements could in principle help settle these questions, by attempting to get around the no-cloning theorem in an experiment with a low degree of decoherence. The possibility of measuring superpositions is also investigated by Janssen, who however argues that it is not likely that the off-diagonal terms can ever be perceived due to the speed at which they decay in the usual decoherence models

[Janssen, 2021, p. 66].

Janssen also discusses the grounds on which the interference terms can be said to vanish due to how small they are. They are ignored due to the empirical indistinguishability. But though the smallness can be said to entail statistical equivalence with mixtures, the definiteness also needs the definite selection of basis, which decoherence claims to give a dynamical explanation for. This, Janssen argues, becomes problematic under considerations of the fact that the off-diagonal terms are not exactly zero [Janssen, 2021, p. 85-90]. She writes, “the decoherence theorist cannot appeal to an orthodox application of the statistical algorithm to argue for near statistical equivalence, for this argument implicitly assumes that one can meaningfully assign probabilities to observables that are not measured” [Janssen, 2021, p. 87]. The approximate nature of decoherence thus becomes problematic in decoherence’s role in basis selection. How does one get from something being almost in the position basis, to stating that one can use statistical predictions for the classical concept of position?

The fact that decoherence is about restricting what parts of a total system are considered in our description of the world, it would seem like human intentions are inherent in the process. Words like “view”, “look at” and “choose”, which are laden with a notion of human intent, keep creeping up in accounts of decoherence. It appears that to write down a reduced density matrix corresponds to choosing not to consider some things. Jansen seems to be of this opinion, as she writes that that decoherence is based on “non-observation”:

> The theory of decoherence, however, introduces an anthropocentric notion of “observation” (or rather, non-observation) in the formulation of (part of) the condition that reduces classical physics to quantum mechanics itself. One could therefore say that for environment-induced decoherence, classical reality is a good enough approximation (for all practical purposes) only as a consequence of our way of looking at things. [Janssen, 2021, p. 66]

From this point, she argues that one can characterise (environment-induced) decoherence by 1) allowing for classical physics being a good approximation in some cases and

2) having an observer that plays a hand in the appearance of the classical behaviour [Janssen, 2021, p. 66-67]. Contrary to the second of these points, I would argue (as does Janssen) that different interpretations might take different stances towards the role of conscious beings in decoherence. As with the rest of quantum mechanics, it concerns to what physical scenarios different interpretations relates the formalism to. Interpretations might consider the reduced density matrix as representing the state relative to a physical subsystem rather than "what one is looking at". As will be seen later, Bohr's understanding of quantum mechanics might give some insight into how decoherence can be understood in a way, that is relative to, but not dependent on an observer.

Janssen concludes that in decoherence "the classical world loses its fundamental status and instead becomes a relative, approximate and *anthropocentric* concept" [Janssen, 2021, p. 63] [my italics], as the appearance of classicality in decoherence relies on the "non-observation" of parts of the total system, as well as a notion of practical indistinguishability, which she also calls anthropocentric [Janssen, 2021, p. 66]. The characterisation of classicality, which is given here, seems to fit the definition of an idealisation, and this term is often used regarding the classical world in decoherence theory. Using the word "idealisation" about the classical world in the decoherence-picture, directs the thoughts towards decoherence being about ones lack of knowledge, or "non-observation". Classical idealisations are epistemological as they occur when one (purposely) disregards certain aspects of a physical description, such as ignoring air resistance in an oblique throw. However, the idealisation in the case of decoherence is fundamentally different from these types of classical idealisations, as it is not about ignoring some aspect which gives a small correction to one's calculations, but about changing the whole behaviour of the system. An analogue is found in the nature of the statistical predictions of outcomes in a decohered state.

Janssen argues, that the statistical predictions that result from decoherence differ fundamentally from predictions of classical probability distributions, as more information in the latter case would allow one to predict the outcome with (more) certainty, whereas more information, i.e. including more degrees of freedom in the density matrix rather than tracing them out, would mean that the outcomes for which one had probabilities no longer exists [Janssen, 2021, p. 82-83].

Similarly, including more things in the density matrix does not give small corrections to the idealised case, but makes the classical idealisation disappear (or at least change completely), since these effects are a consequence of treating a local system as global, i.e. using the reduced density matrix. As this is the case, the use of the word "idealisation" in connection with decoherence, is manifestly different from its use in classical physics, and does not necessitate an epistemological nature of its account of classicality. Again this issue falls back on what an interpretation has to say about what the reduced density matrix is representative of (what we look at or a physical subsystem) and what kind of process causes it to be a good description (a mental process or something independent of consiousness). Decoherence does not in itself provide such answers. I will therefore argue that whether the idealisation of the classical world is really anthropocentric, and relies on non-observation, or not cannot be settled by decoherence alone. Depending on the interpretation with which it is coupled, the answer can swing either way.

In the cases, where the states of the system with which the measured system is entangled are not (initially) orthogonal, decoherence is an approximate, emergent process. Wallace draws attention to this point in his article, where he mentions the fact that the off-diagonal terms are not exactly zero and that the choice of basis has "more a pragmatic than a fundamental character" [Wallace, 2012, p. 4587]. He argues that, due to this approximate and emergent character, decoherence "cannot have a place in the axioms of fundamental physics, precisely because they emerge from those axioms themselves" [Wallace, 2012, p. 4588]. Wallace describes decoherence as a dynamical processes coming from the axioms of quantum mechanics, which therefore cannot form part of those axioms. Tanona comes to a similar conclusion. He argues, that decoherence cannot be a part of what constitutes quantum mechanics, as it itself relies on a classical assumption of what is considered an instrument, what is considered an object etc., as these divisions of the world are not given by nature. He conducts an investigation on whether decoherence can live up to the goal of accounting for the appearance of classicality by only relying on the quantum description itself. Tanoa argues, that decoherence relies on a quantum-classical cut, as decoherence accounts for clasicality via the omission of parts of an entangled pair, and this omission can, according to Tanona, only be justified by making something like a Copenhagen cut, which he argues can only be motivated by a classical assumption [Tanona, 2013, p. 3634-3638]. Tanona thus concludes that decoherence must necessarily make some sort of classical assumption in order to be able to account for classicality. Then the questions arises of when it is appropriate to make these assumptions:

> This justification is what is missing from typical claims of the explanatory power of decoherence: given the relativity of decoherence claims, to treat a system as decohered in some basis requires an implicit or explicit cut, and we should have justification for the pretermission cut that enables that description. [Tanona, 2013, p. 3643]

As mentioned, Tanona argues, that this cut is necessary for choosing a subsystem, to which decoherence can apply, and further argues that the justification for that cut can only come from classical assumptions [Tanona, 2013, p. 3638]. Tanona therefore argues, that decoherence fails in accounting for classicallity from within the quantum formalism itself.[5]

Thus both Tanona and Wallace argue that decoherence cannot have a fundamental place in solutions to the measurement problem, but they do so differently, as Wallace argues that it is due to the approximate nature of decoherence, while Tanona bases the conclusion of the fact that decoherence must itself rely on the classicality it seeks to account for. They are thus addressing different issues; Wallace talks of what kind of result decoherence gives, while Tanona talks of how one gets to this result. These different angles relflects their different views. Wallace, who is an advocate of an Everettian point of view, sees the wavefunction as being fundamental. However, we cannot experience this, but only an approximation. In this decoherence is central, but i cannot be a part of the of the fundamental quantum mechanics. Wallace rather views decoherence as an emergent process, and compares with e.g. zoology and similar fields of study, which are

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[5] When Tanona talks of accounting for classicality, he talks of ensuring seperability between the object and instrument of measurements. In section 2.3, I argued, that seperability can be connected with classicality, as shown in e.g. Raggio's theorem.

both real and practical, but not part of a fundamental reality [Wallace, 2013, p. 470-474]. Tanona, on the other hand, is more inclined towards a Bohrian view, though he has an opinion which I will argue differs from Bohr's. Tanona seems to have an idea of a fundamental reality, that one imposes a cut on. As decoherence necessitates such a cut, it cannot be part of the fundamental reality. As will be discussed in section 6, there might be another, more truly Bohrian way to view this.

Wallace also argues, that decoherence, in resulting in a suppression of the off-diagonal terms, secures the fulfilment of the conditions for a probabilistic interpretation of the density matrix. This he uses in concluding that the measurement problem, due to decoherence, becomes a philosophical, rather than a physical or practical problem, as one does not get predictions, which contradict the experimental results. On the contrary, the predictions correspond with those given by the Born rule and observered in experiments [Wallace, 2012, p.4586]. Wallace here uses the same kind of terminology as when saying that the Everett interpretation does not require new physics (see section 3.1.2); that the interpretation of the formalism is separate from "the physics". A physical problem is thus when something gives wrong predictions, while a philosophical problem occurs when there is nothing to show that something is false, but it is nevertheless puzzling. I am sceptical towards this sharp distinction between "physics" and "interpretation". One might e.g. argue that decoherence gives no predictions at all without some understanding of what the reduced density matrix represents.

Janssen calls decoherence a piece of "good physics" [Janssen, 2021, p. 35] on par with other quantum mechanical calculations. She argues, that it has a physical, but not a conceptual relevance. This is more in line with my own thoughts on the subject, as she means that decoherence alone does not provide answers to questions such as those which are brought forward in the measurement problem.

Decoherence does have definite consequences for how the world is viewed in a decoherence picture, such as the fundamentality of quantum mechanics. However, it falls short in explanatory power in many questions; what the role of conscious observers are, how classicality can be understood, what the role of decoherence in the axioms of quantum mechanics is etc.. This is mostly due to the fact that it is a feature of the quantum formalism and not an interpretation. Though it seems to imply different things it cannot, without the understanding of what the reduced density matrix is, provide any real answers. In this regard, Bohr's view can be put forward as a suggestion for a natural partner of decoherence. In the next section, I will give an account of Bohr's thoughts concerning quantum mechanics, with the idea that his account of the quantum formalism and his general way of thinking will be helpful in understanding decoherence in a fruitful way, which can give a meaningful address to the measurement problem. Similarly decoherence can help clarify and concretisise Bohr's different concepts.

5 The ideas of Niels Bohr

In addition to being involved in the development of quantum mechanics during the first half of the 20th century, Niels Bohr has written and said several things about how to understand this new field of physics in his view. Bohr's views are often equated with some idea of *the Copenhagen Interpretaion*; a muddled term coined much later, which mixes the views of several people associated with Bohr and the Institute of Theoretical Physics in Copenhagen. Specifically, Bohr is often regarded as an instrumentalist or anti-realist, probably due to his epistemological argumentation and his departure from many of the attributes of *classical scientific realism*. In this section, I seek to make an account of how Bohr can be understood, based on his own writings as well as accounts made by others. First, I will give a rough sketch of the different features of CSR which, according to Bohr, must be given up in the light of the developments in physics that lead to quantum mechanics. Then an account of Bohr's understanding of the right way to interpret quantum mechanics will follow, through an examination of concepts such as complementarity, object-instrument nonseparability and the doctrine of classical concepts. This will eventually give a picture of Bohr as a "contextual realist". Lastly, I will construct answers to the different versions of the measurement problem from Bohr's ideas on quantum mechanics. This will serve both to clarify Bohr's views as well as providing insight into how the measurement problem might be advantageously addressed.

5.1 The background: a break with classical scientific realism

The background of the development of quantum mechanics was the opening of a new field of study; that of atomic physics. According to Bohr, this new research resulted in new experiences, that were at odds with the views held in Classical Scientific realism (an account of which was given in section 2.3), and thus necessitated a change in the requirements for and ideals of a scientific theory. Several aspects of early quantum mechanics were problematic when viewed from a classical standpoint. The proposed stationary states of the atoms and quantum leaps between them, does not subject to classical mechanical descriptions; the contradictory behaviour of the same system observed under different experimental setups, such as particle-wave duality demonstrated in the double-slit experiment, is a paradox if one views measurements classically; the relation between different spectral lines were found to be linear, and thus incompatible with different mechanical atomic models, which would give these relations as either quadratic or continuous [Bohr, 1961a, p. 51].

That an epistemological change is not merely inspired, but is actually necessitated, by these new issues in physics is expressed by Bohr in a lecture given in 1939 at an antrhopological congress [Bohr, 1939a], where he talks of "[...] a general epistemological attitude which we have been *forced to* adopt in [...] the analysis of simple physical experiments." [Bohr, 1939b, p. 31] [my italics]. Thus, the physical evidence of experimental results, forces a new epistemological standpoint on us. The deviations from the ideals of classical physics, that characterises Bohr's understanding of quantum mechanics, and which will be presented in this section, is therefore an unavoidable consequence of the new developments in physics in Bohr's eyes. The new physics which lead to quantum mechanics, was therefore not only the means of formulating new laws for a new area of investigation (the atoms and later the particles), but also brought forward discussions on epistemological questions [Bohr, 1955a, p. 101]. It is considerations of the role of the observer that gives rise to these questions, as it is (apparently) widely different in atomic physics, from how it is considered in CSR [Bohr, 1949c, p. 115]. As discussed in section 2.3, the observer is no longer separable from what is observed, as it is assumed in CSR, due to the general non-separability of quantum states.

This new status of the observer marks a limitation of the mechanical descriptions of classical mechanics, as observed by Bohr in a later lecture,

> [...] modern development of atomic physics, at the same time as it has augmented our knowledge about atoms and their constitution of more elementary parts, has revealed the limitation in principle of the so-called mechanical conception of nature and thereby created a new background for

> [the pertinent] problem, as to what we can understand by and demand of a scientific explanation. [Bohr, 1949d, p. 97]

By the "mechanical conception of nature", Bohr refers to the classical view where all behaviour is the result of causal chains of events, defined by the dynamical properties and space-time coordinates of all particles, i.e. a deterministic description. Bohr is thus of the opinion that the new type of experimental results represents a break with this deterministic, mechanical world view and in that process, he also changes the notions of what can be required of a theory, as a theory can no longer be expected to give such a description if it is to account for quantum phenomena.

The limitation of the mechanical description and of the separability of the observer is captured in Heisenberg's uncertainty relation (which Bohr invariably calls "ubestemthedsrelation", i.e. *indeterminancy* relation, as we shall see later);

> Heisenberg had made a most significant contribution to the elucidation of the physical content of quantum mechanics by the formulation of the so-called indeterminacy principle, expressing the reciprocal limitation of the fixation of canonically conjugate variables. This limitation appears not only as an immediate consequence of the commutation relations between such variables, but also directly reflects the interaction between the system under observation and the tools of measurement. The full recognition of the last crucial point involves, however, the question of the scope of unambiguous application of classical physical concepts in accounting for atomic phenomena. [Bohr, 1961e, p. 91]

This paragraph, which originates in a lecture given at the 12th Solvay conference in 1961, includes many of the deviations from CSR considered necessary by Bohr. The object-instrument nonsperability means that the interaction between the measured system and the measurement instrument cannot be ignored or compensated for as assumed in classical physics ("the interaction between the system under observation and the tools of measurement"). The limitations to conservation of energy and momentum, which shows itself in effects such as quantum tunnelling, is a further testimony of the limitations of classical laws and their foundations in determinism and pictorial descriptions [Bohr, 1961a, p. 80].

Similarly, the uncertainty relation is a limitation of the ability to simultaneously ascribe values of space-time coordinates and dynamical properties to a system. In classical physics it is assumed that all these concepts can always be ascribed to a system, but, at least in Bohr's opinion, Heisenberg's uncertainty relation raises doubts about the foundation for the use of these classical concepts. Bohr argues that this calls for a revision of the basis on which these concepts can be ascribed unambiguously to a system [Bohr, 1949b, p. 77] and in a sense this is what his account of quantum mechanics amounts to.

Since an area of research where the fundamental principles of CSR no longer works has been reached, Bohr is of the opinion that the range of the frame in which classical physics can be applied has thus been found [Bohr, 1961a, p. 47-48]. This might seem worrying, but does not appear to have surprised Bohr much, as he considers that the field of atomic physics lies far beyond the experiences which classical physics was developed to describe; i.e. the macroscopic events directly apparent to our senses [Bohr, 1955a, p. 103]. As our area of research expands, it is perhaps not surprising that our epistemological notions much change also. This is the case elsewhere in physics. For example, the classical notion of absolute time, which fits everyday experiences of speeds much slower than the speed of light, had to be given up, with the field of research leading to the formulation of special relativity.

To conclude, Bohr views the new developments in physics as having exposed classical physics as an idealisation [Bohr, 1949b, p. 45] and that the deviations therefrom are not accidental or temporary, but necessary characteristics of areas of physics which, he argues, lies beyond the range of classical descriptions [Bohr, 1958a, p. 17].

5.2 An account of Bohr's understanding of quantum mechanics

The quantum formalism was made to account for the, to classical physics unfamiliar, phenomena observed in atomic physics. The crucial step was to replace the classical concepts, i.e. space-time coordinates and dynamical properties such as energy and momentum, with new symbols, or operators, which abide by the new features of quantum mechanics. These are summarised in the Heisenberg uncertainty relation, which is in turn captured by the non-commutative nature of the operators. Bohr writes,

> Thus, in the quantal formalism, the quantities by which the state of a physical system is ordinarily defined are replaced by symbolic operators subjected to a non-commutative algorism involving Planck's constant. [Bohr, 1958b, p. 2]

This then gives the formalism, the interpretation of which continues to give rise to much discussion. Bohr does not seek to understand quantum mechanics by expanding this formalism, but rather investigates its features and builds up his views on the theory from these.

As mentioned, the non-commutation of the operators gives rise to Heisenberg's uncertainty relations. The commutation relation of the canonical operators are for instance $[\hat{p}, \hat{q}] = i\,\hbar$ (in one dimension), from which it follows that the uncertainty (the dispersion, or variance) of these variables must satisfy the inequality, $\langle(\Delta\hat{p})^2\rangle\langle(\Delta\hat{q})^2\rangle \geq \frac{1}{4}\hbar^2$, or, more loosely, $\Delta p\,\Delta q \sim \hbar$ [6] showing how the precise determination of both variables is limited by $\hbar$ [Sakurai and Napolitano, 2017, p. 48, 33-35]. These type of relations can be understood as a limit to how well it is possible to *know* the properties of a system, i.e. as a limitation of the precision of measurements. But Bohr was of another opinion. He writes,

> These circumstances find quantitative expression in Heisenberg's indeterminacy relations which specify the reciprocal latitude for the fixation, in

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[6] By using $\Delta A = \sqrt{\langle(\Delta A)^2\rangle}$ as a variable (the "uncertainty") rather than as an operator.

> quantum mechanics, of kinematical and dynamical variables required for the definition of the state of a system in classical mechanics. In fact, the limited commutability of the symbols by which such variables are represented in the quantal formalism corresponds to the mutual exclusion of the experimental arrangements required for their unambiguous definition. In this context, we are of course not concerned with a restriction as to the accuracy of measurements, but with a limitation of the well-defined application of space-time concepts and dynamical conservation laws, entailed by the necessary distinction between measuring instruments and atomic objects. [Bohr, 1958b, p. 5]

It is clear from this quote, that Bohr is of the opinion, that it is nonsensical to talk of, or even picture a system as possessing values of e.g. position and momentum to a degree of mutual definition beyond that expressed in the inequality which follows from the commutation relation. This is also argued by Howard, who emphasises that Bohr does not view the indeterminacy, which is captured by the uncertainty relation, in the manner of a Heisenberg-microscope, where the properties of the measured system is disturbed by the interactions with the measurement apparatus, thus resulting in an indeterminacy in the *knowledge* of that property. Bohr does not want to presuppose the independent existence of such properties, and therefore talks of the uncertainty relations as providing a limit of how well two properties can be *defined* [Howard, 2021, p. 12].

To understand what exactly is meant by this is the key to understanding Bohr's thoughts on quantum mechanics as a whole. Concepts such as complementary and the doctrine of classical concepts are crucial in this understanding and in understanding what is meant by the "necessary distinction between measuring instruments and atomic objects" mentioned in the quote above. These concepts will be explored in the remainder of this section as well as their consequences. All these concepts are interconnected and the order in which they are presented is therefore somewhat arbitrary. Some aspects of a concept might only be understood properly in the light of another. In the cases where these other concepts are yet to be accounted for, this will be pointed out, and the point will be returned to when the full meaning can be given.

5.2.1 Non-separability, Complementarity and the use of classical variables

Bohr's perhaps most famous concept is that of complementary. The key to understanding complementarity, lies in Bohr's view of the uncertainty relation as the formal counterpart of the fact that an experimental setup allowing for the determination of one variable exclude the possibility of also measuring another variable, if the operators corresponding to the two variables do not commute. To appreciate his full meaning of the term, one must first consider the radical revision of the meaning of the classical variables that enter into these complementary relationships.

For Bohr, the development of quantum mechanics drew attention to the caution with which one must regard the use of different concepts outside the scope of everyday experiences [Bohr, 1939a, p. 37]. The background of his considerations is the idea that the physics for which quantum mechanics was developed lies outside the frame of classical mechanics, and in this capacity, has changed the very foundation of scientific explanations, by requiring that conditions for the use of the concepts are met with, which were not considered in classical physics. Bohr writes,

> [...] we must realize that the discovery of the quantum of action has thrown new light on the very foundation of the description of nature and revealed hitherto unnoticed presuppostitions to the rational use of the concepts on which the communication of experience rests. [Bohr, 1955b, p. 91]

He thus states, that the conditions for using the concepts, were previously not noticed, but that quantum mechanics has brought them forward. This phrasing implies that the conditions were always there, and that the world is therefore properly described by quantum mechanics, whereas the classical description is an idealisation, where we can get away with ignoring these aspects. The reason for the previously unheeded conditions is the non-separability of quantum mechanics, which is foreign to classical physics and which necessitates a redefinition of the meaning of scientific inquiry as well as the classical concepts. Bohr here relates the noncommuting and the nonseparable nature of quantum mechanics to one antoher, since it is the discovery of Plancks constant which results in the uncovering of the new relationship between observed objects and the instruments of observation. This ties in with Raggio's theorem, which were accounted for in section 2.3, and which shows the mathematical equivalence of the commutation of operators A and B, and the separability of states on $A \otimes B$.

As was discussed in section 2.3, Bohr frequently mentions the lack object-instrument separability as *the* distinction between quantum mechanics and classical physics [Bohr, 1955c, p. 88]. He argues that quantum mechanics, with its nonseparability, gives rise to epistemological issues, in the special case of the nonseparability of measured objects and measurement instruments. This new epistemological position, which will be investigated presently, is paralleled with ideas from old eastern philosophy, as Bohr compares the problems that arose in connection with the development of quantum mechanics with

> [...] that kind of epistemological problems with which already thinkers like Buddha and Lao Tse have been confronted, when trying to harmonize our position as spectators and actors in the great drama of existence. [Bohr, 1937, p. 20]

By alluding to the complicated role of being both an actor and a spectator in the world, Bohr draws attention to the fact that as we seek to observe some part of the world, we are ourselves a physical system, which will interact with that part, and are subjected to the same laws which applies to all other physical systems. The non-separable nature of quantum states makes this interaction impossible to account and compensate for.

Importantly for Bohr's understanding of quantum mechanics, this nonseparability limits the ability to observe the independent behaviour of systems [Bohr, 1939a, p. 37]. When talking about the double slit experiment, where interference is either observed or not depending on the setup of the experiment, he argues that the wave- and particle behaviour, that gives rise to interference and no interference respectively, are complementary, but also that we in this experiment

> [...] are just faced with the impossibility, in the analysis of quantum effects, of drawing any sharp separation between an independent behaviour of atomic objects and their interaction with the measuring instruments which serve to define the conditions under which the phenomena occur. [Bohr, 1949a, p. 47]

What is meant here, is that it is impossible to speak of the behaviour of the particle while passing through the slits independently of the interaction with the setup of the experiment, i.e. independently of whether which-way information is measured or not. As the system becomes entangled with the instrument, it's state cannot be separated from that of the instrument (i.e. the experimental setup).

This means that the interaction between an object and its the experimental context is inextricably bound to the observed phenomenon, i.e. the behaviour of the object [Bohr, 1958a, p. 14]. This leads Bohr to reserve the term *phenomenon* for accounts that include all relevant parts of the experimental context. This is the only way, according to Bohr, to give an unambiguous account of a quantum system [Bohr, 1958a, p. 14, 16]. A classical variable, can only be attributed to a system, i.e. is only a real property of that system, within a phenomenon in which the experimental context fulfils the conditions for the use of that variable. This corresponds to a context suitable for the measurement of the variable.

It can be unclear what exactly are the relevant parts of the experiemental context. In another article, Bohr writes, "As regards the specification of the conditions for any well-defined application of the formalism, it is moreover essential that the *whole experimental arrangement* be taken into account" [Bohr, 1949a, p. 50] [original italics]. In Bohr's response to the EPR paper, he writes, that performing a measurement on one of the entangled pair has an effect upon the other as there is "*an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system*" [Bohr, 1935] [original italics]. Being entangled, the two particles are one phenomenon, together with any instrument used to perform measurements on either, since these also become entangled with the system. Performing a measurement on one of the pair, thus changes the experimental context, that forms part of the phenomenon, thereby changing which state of both particles can be well-defined. It is clear that systems with which a measured system is entangled cannot be disregarded in any discription. It can therefore be argued that what Bohr means by all of the experimental arrangement is the systems with which the measured system in some way becomes entangled. This also makes sense as a measurement process is necessarily one in which the measured system becomes entangled with the measurement instrument (see section 2.2), and as entanglement corresponds to nonseparability. The statistical laws of quantum mehcanics, applies to observables under such specified experimental contexts [Bohr, 1958a, p.13]. A spin state $|\Psi\rangle = a_x|{\rightarrow}\rangle + b_x|{\leftarrow}\rangle = a_z|{\uparrow}\rangle + b_z|{\downarrow}\rangle$, can give meaningful statistical content of the probabilities for spin up or down in the x-direction (as the squared coefficients in the x-basis, $|a_x|^2$ and $|b_x|^2$) in a phenomenon where the spin is measured in the x-direction, or, in another phenomenon in which the spin in the z-direction is measured, the probabilities $|a_z|^2$ and $|b_z|^2$ will be meaningful.

In summary, Howard states that according to Bohr, phenomena is what quantum mechanics applies to. In the phenomenon, not only the system, but the entire (experimental) context is included, and only in such a context, can different properties of the system be meaningful and quantum mechanics give its physical predictions in the shape of probability distributions for these properties [Howard, 1979].

In returning to complementarity, one can identify two aspects. The first is the inability to unite the values of different variables in one picture of a state, i.e. to simultaneously ascribe them to a system. The other is the fact that only together can these variables give a complete description of said system. Bohr writes,

> Consequently, evidence obtained under different experimental conditions cannot be comprehended witin a single picture, but must be regarded as *complementary* in the sense that only the totality of the phenomena exhausts the possible information about the objects. [Bohr, 1949a, p. 40] [original italics]

The latter of these aspects comes from the fact that it is in classical mechanics enough to know the position *and* the momentum of a system in order to specify anything else about it. Bohr argues that such knowledge is not possible in quantum mechanics, but that the two quantities will complement each other, in that both are necessary for the description of the system to be complete. What is meant by this can be formalised in the quantum counterpart of the role of position and momentum in classical physics. From the statistics of these quantities it is possible to deduce the statistical predictions of all other quantities.[7] Thus the two complementary quantities can still serve as a kind of basis of all knowledge of the system.

In classical physics the basis for use of these variables are always assumed, and their values for some system could thus (in principle) be determined by a single experimental setup, and together constitute a complete picture of the state of the system [Bohr, 1958a, p. 15]. But the first mentioned aspect of complementarity constitutes a departure from this view, as it states that the non-commutation of the operators in the formalism of quantum mechanics, leads to the inability to unite different variables in a definition of the state of the system [Bohr, 1958a, p. 15-16]. To get to this point, one must consider two things: a) that the context in which a variable can be measured, is necessary for the very definition of that variable, or, in other words, for the ability to ascribe its value as a property of a system, and b) that the non-commutation of the operators is considered as the formal representation of the mutual exclusion of the different contexts in which the variables corresponding to the operators could be measured (and thus defined according to a)) [Bohr, 1961a, p.79]. Complementarity is therefore the mutual exclusion of the conditions for the use of different variables, e.g. for the use of dynamical variables and space-time coordination [Bohr, 1955c, p. 89] or, correspondingly, a limitation of the ability to ascribe their value to a system [Bohr, 1955c, p. 90-91].

The term "position" refers to a classical picture of an object being located at some point, which can be quantised by reference to a coordinate system. Similarly, other variables refer to other classical pictures. In classical physics these pictures can be united, to form

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[7] A sketch of the proof of this: 1) Let $B(\mathcal{H})$ be all (bounded) linear operators on the Hilbert space $\mathcal{H}$. Let S be a set of linear operators on $\mathcal{H}$, i.e. a subset of $B(\mathcal{H})$, and let S' be the set of all operators that commute with every operator in S. Then the algebra generated by S is $B(\mathcal{H})$ if and only if S' consists of only scalar multiples of the identity operator $\mathbb{I}$. 2) Let S be the set of all (bounded) functions of the position and momentum operators. Then S' consists only of scalar multiples of $\mathbb{I}$. This sketch has been written by my supervisor Hans Halvorson.

a single picture of e.g. an object being located somewhere with a momentum of a certain size pointing in a certain direction, but, as mentioned, the different variables cannot be joint in a single picture of the state of a system in quantum mechanics. Complementarity means, that the variables necessary for a complete, classical description of the system, can only be applied relative to different, mutually exclusive contexts (experimental setups); in one context the system will have a definite position, in another a definite momentum. In measurement situations the experimental setup will dictate for which variable's application to the system the conditions are satisfied. This explains apparent contradictions in the behaviour of systems in different experimental situations; the contradictions arise because one tries to unite the description of the different contexts, i.e. different phenomena, in a single picture of the system - something which is simply not possible, according to Bohr [Bohr, 1949b, p. 50-53].

In an article by Howard [Howard, 2021], complementarity is stated to be the logical consequence of entanglement. Howard argues, as was done here in section 2.2, that upon measurement, the joint object-instrument state must necessarily be entangled [Howard, 2021, p. 13]. This leads to complementarity, as an entangled state is one where, though the composite system is real, its parts (the object and the instrument respectively) does not have *independent* reality [Howard, 2021, p. 11-13]. Howard argues, that the lack of independent reality, is essential in complementarity. He writes:

> Bohr is saying that we cannot ascribe definite positions and times without performing measurements, which requires a physical interaction between the object and the instrument, but, since the object will then no longer have a well-defined, independent state, thanks to entanglement, there can be no talk of energy and momentum conservation, not because energy and momentum conservation are violated, but because they can no longer be clearly formulated, at least not in the classical sense, since that requires the ascription of well-defined dynamical states. [Howard, 2021, p. 15]

Thus, the experimental setups of complementary variables exclude one another due to the nonseparability of the system and the measurement apparatus. If the setup is set to measure the position of some particle, the measurement process will result in an in principle uncontrallable momentum transfer to the particle, thus making a measurement of its momentum nonsensical. Conversly, if the experiement is set to measure momentum, a description in spacial coordinates is impossible. [Bohr, 1960a, p.22-23] The need for the replacement of the concepts from classical physics with non-commuting operators, which gives rise to the Heisenberg uncertainty relation, can thus be said to come from the general nonseparability of the quantum states. Or, as Bohr writes, while speaking more specifically of measurement situations, "This limitation [Heisenberg's uncertainty relation] also directly reflects the interaction between the system under observation and the tools of measurement." [Bohr, 1961e, p. 91].[8]

5.2.2 The doctrine of classical concepts

Apart from the experimental setup being included in any quantum phenomenon, the setup and the results of any measurements must also be described in classical terms in order for the experiment to serve its purpose. This is Bohr's *doctrine of classical concepts*, which he formulates it in the following way:

> [...] even when the phenomena transcend the scope of classical physical theories, the account of the experimental arrangement and the recording of the observations must be given in plain language, suitably supplemented by technical physical terminology. This is a clear logical demand, since the word "experiment" refers to a situation where we can tell others what we have done and what we have learned. [Bohr, 1955d, p. 72]

It is clear from the last sentence in this quote, that Bohr builds the doctrine upon epistemological considerations. For something to be an experiment, it cannot just occur; it must be possible to communicate to others what the results are and how they were obtained, in order for it to be possible for others to replicate the experiment. Bohr argues that this cannot be achieved by reference to an abstract quantum state, but that one must use everyday words and concepts such as the terms from classical

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[8] Here, the connection between commutation and seperability is again mentioned. A formal version of this connection can be found in Raggio's theorem (see section 2.3)

physics, whose meanings are well-defined and precise. In other words, one must use the language of classical physics, which is developed to describe our surroundings as they appear to our senses and our investigations of their causal connections [Bohr, 1958a, p. 11]. In this, mathematics is viewed as a refinement of everyday language enabling more precise communications [Bohr, 1955c, p. 84]. In regards to experimental outcomes, one must talk of properties such as position and momentum in their classical sense and not as non-commuting abstract operators of the quantum formalism, in order for the experiment to make sense.

The doctrine of classical concepts, according to Howard, boils down to the necessity of being able to treat measurement outcomes *as if* there was object-observer separability. Since the physical world, as described by quantum mechanics, is nonserarable, it is not possible with a system in totality, only in some respect [Howard, 1979, p. 143]. Using separability as the definition of classicallity was discussed in section 2.3. And if a separable state corresponds to a classical one, it fits in very well to translate Bohr's doctrine of classical concepts into a doctrine of as-if separability. This point is of course somewhat circular, as Bohr's ideas of separability from the observer formed part of the motivation for characterising classicality by separability.

The doctrine of classical concepts serves as Bohr's way of securing a kind of objectivity of science, and in his PhD-thesis [Howard, 1979], Howard motivates the doctrine of classical concepts with a review of objectivity. As mentioned in section 2.3, classical scientific realism is primarily characterised by the seperability of the observer and the observed, and is, according to Howard, underlying all of the classical physical understanding. Separability serves as the foundation of objectivity in science by allowing for an instrument to be used in the determination of independent properties of the objects under investigation. The general nonseparability of composite states in the quantum formalism (if complete) therefore leaves no space for classical understanding of objectivity [Howard, 1979, p. 142-143]. Bohr, however has another conception of objectivity, which relies on unambiguous communication. Objectivity is the liberation from the restrictions of experiences to a subject, and is achieved by communicating the contents of science to others, who can in turn replicate it for themselves [Howard, 1979, p. 139-142].

Bohr therefore understands objectivity in science as the lack of dependence on or influence from a specific subject; a kind of inter-subjectivity. This is secured by the doctrine of classical concepts [Bohr, 1960b, p. 37]. The circumstance that the doctrine of classical concepts is necessary due to the general nonseparability of quantum mechanics, which stands in the way of the classical objectivity through observer-independence of described systems, strengthen Howard's point about it corresponding to the emplyment of as-if separability.

Previously, Bohr's conditions for the use of classical concepts were discussed. These can only be defined for a system in a specified context. The doctrine of classical concepts builds upon this in arguing that to have objective science, the results of measurements must be in terms of these classical concepts, and in order to have this, the context must be described classically with respect to that variable. This is what Howard translates into as-if separability in the basis of the measured variable.

A classical example is position and momentum measurements of a particle passing through a screen with a single slit. This is e.g. used as en example by Bohr in [Bohr, 1949b, p. 62-63]. If the screen is bolted down, this allows for a measurement of the particle position as it passes through the slit, as the position in this case becomes well-defined. Momentum on the other hand meaningless to talk about, as the position of the bolted down screen is considered as absolutely fixed. For the measurement of the momentum, the screen is mounted on springs, allowing a determination of momentum transfer while the position at the time of the particle passing through the slit, on the other hand cannot be defined. Thus, to treat one variable as classical, it appears that one must ignore some other aspect of the classical description; here momentum conservation or well-defined position respectively.

Henrik Zinkernagel [Zinkernagel, 2016] expresses an opinion that differes significantly from Howard's concerning the entanglement between a measured system and the measurement apparatus. He derives this point from the doctrine of classical concepts, as he argues that the system cannot be entangled with the measurement apparatus in totality, as this must be classically described, and must therefore encompass classical parts, which cannot be in a state of entanglement [Zinkernagel, 2016, p. 7]. It might be difficult to compare Howard and Zinkernagel on this point, as they speak about Bohr differently. As I understand him, Zinkernagel talks of the quantum mechanical description as being relative to the experimental context [Zinkernagel, 2016, p. 7], and thus Bohr's doctrine, which states that some part of this context must be described classically, comes to mean that it cannot be in an entangled state. Howard, on the other hand, rather talks of the wavefunction as translating between contexts, and the entanglement with the instrument, to be the cause of the need to employ *as-if* separability. Personally, I'm inclined to use Howard's way of speaking, as I think it suits Bohr's own terminology best. Bohr talks of the observer-observed nonseparability, which I think translates quite naturally into entanglement in the basis of the observed quantity; i.e. entanglement with the part of the instrument, which must be described classically. The momentum of the particle passing through the one-slit screen on springs, becomes entangled with the momentum of the screen - the exact thing which is treated classically in the momentum measurement. The nonseparability is what causes the observed phenomena to depend on the classically described observational context for its very definition.

5.2.3 The Copenhagen Cut

In continuation of the doctrine of classical concepts, Bohr writes that, "Thus, the description of quantum phenomena requires a distinction in principle between the objects under investigation and the measuring apparatus by means of which the experimental conditions are defined." [Bohr, 1961c, p. 78] The line between the object and the instrument is known as *the Copenhagen cut*. Bohr here argues, that this cut is necessary in order to fulfil the doctrine of classical concepts, as it corresponds to the choice of what to describe classically. Thus it follows from the doctrine of classical concepts, that in any description, one must make such as cut between what is observed, and what is the instrument [Bohr, 1958a, p. 14]. Making the cut corresponds to defining the measurement context. If one in a one-slit experiment wants to measure the position of a particle, the one-slit apparatus must be bolted down, i.e. the position must be fixed. As mentioned, the momentum of the apparatus becomes undefined in this case, and bolting it down thus amounts to considering the concept of momentum as not applying to the apparatus. In this context, the position of the particle can be well-defined, whereas the momentum can not. The cut is therefore made with regards to some *aspect of* the apparatus; in the context where the apparatus is bolted down, the entangled position of the apparatus and of the particle if treated as if they were separable and a position can be defined for the particle, while momentum is undefined.

Understanding exactly what Bohr is saying regarding the measurement context is a difficult matter. On one hand, the context serves as a condition for applying classical concepts, and on the other, the context is to be described with classical concepts. This question is a reoccurring topic throughout this thesis. Bohr considers the context to be a necessary reference for assigning classical properties to a system. If a context is to work as a reference for the position of a system, it must itself have a well-defined position. If, on the other hand the context is to measure momentum, it cannot have a fixed position, as its momentum could in that case not be defined.

Kristian Camilleri and Maximillian Schlosshauer [Camilleri and Schlosshauer, 2015] consideres the cut to be *functional*. In general they argue for the need of a functional, rather than a structural language for discussions of quantum mechanics. Here, one views a measurement instrument not as a physical system, but as a "technological artifact", and the structural understanding of the world depends on a functional understand of the instrument [Camilleri and Schlosshauer, 2015, p. 5]. This means that an instrument is to be understood in terms of what role it is to play. That does not mean, that its structural description is irrelevant, but rather that it is relevant in respect to how the functional role is achieved: what must an experimental physicist consider when setting up an experiment? It is possible to treat an instrument as purely a structural object (this would mean describing it quantum mechanically), but if it is to work as an instrument, this will not do, as implied by Bohr's doctrine of classical concepts [Camilleri and Schlosshauer, 2015, p. 5]. It must be described in terms in which it can *function* as a measuring device. Thus there is no fundamental, ontological difference between the quantum object and the experimental apparatus. Rather, what is treated as what is due to functional, as opposed to dynamical, considerations. This, however, does not mean that the cut between object and instrument is arbitrary, or due to a mental process. It is on the other hand a fixed by the experimental setup which reflects the functional considerations:

> [...] the cut corresponds to something "objective" in the sense that the object–instrument distinction was essentially fixed by the functional-epistemological considerations dictated by the choice of the particular experimental arrangement. [Camilleri and Schlosshauer, 2015, p. 8]

Camilleri and Schlosshauer argue that this cut differs from a cut between the quantum realm and the classical realm, as both the measured system and the measurement instrument must be described in classical terms [Camilleri and Schlosshauer, 2015, p. 6].

In classical physics there is no principal difference in the kind of descriptions one would give of the measured systems and the instruments of measurement; concepts such as position, momentum etc. can be used unambiguously to give the state of both or either [Bohr, 1955a, p. 107-108]. As have been shown, this is not possible in quantum mechanics due to complementarity, where there is a need to have a cut between object and instrument. In order to use a variable in describing a system, one must be clear about what constitutes the system, and what constitutes the instrument, which gives the condition for application of the variable. Bohr writes, that it is necessary "of distinguishing, in the study of atomic phenomena, between the proper measuring instruments which serve to define the reference frame and those parts which are to be regarded as objects under investigation and in the account of which quantum effects cannot be disregarded." [Bohr, 1949a, p.55-56] Bohr argues, that if an instrument is used as an instrument to measure momentum, then it is treated like an object of investigation with regards to position, meaning that Heisenberg's uncertainty relation must apply. Thus, the cut determines which classical variables can be well-defined [Bohr, 1935].

Earlier, when the object-instrument separability has been discussed, it has been stated, that the interaction between the object and the instrument is impossible to compensate for due to the lack of separability. It is here, that the explanation of how this can be is found. It is due to the fact that if instruments are to serve their purpose as instruments, they cannot simultaneously be subjected to investigation themselves [Bohr, 1933b, p. 16]. If something is to give the conditions for the application of a variable, it cannot also be an object of investigation. Bohr writes,

> [...] such [object-instrument] interaction represents in quantum physics an integral part of the phenomena, for which no separate account can be given if the instruments shall serve the purpose of defining the conditions under which the observations are obtained. [Bohr, 1961e, p. 91-92]

This is what Camilleri and Schlosshauer also argue when they, as mentioned, write that, according to Bohr, it is always possible to describe e.g. a measurement instrument quantum mechanically, but in doing so one looses functional role as acquiring empirical knowledge.

Zinkernagel argues, that to apply the quantum mechanical formalism to a system, one must disregard, in principle not in practice (such as air resistance can be disregarded in the description of the path a thrown ball takes), the quantum of action (Planck's constant) of some other system. This corresponds to the formulation of the doctrine of classical concepts, i.e. that one must treat something else classically. [Zinkernagel, 2016, p. 13-17] But this further points to a kind of infinite regression of reference frames; to describe a system quantum mechanically, one needs another system described classically, and to describe that system quantum mechanically one needs yet another system described classically, etc. etc.. All systems can thus be treated quantum mechanically, but not necessarily simultaneously, and the quantum-classical divide is therefore flexible [Zinkernagel, 2016].

5.2.4 Contextual reality

All these considerations of different concepts of quantum mechanics, comes together to form Bohr's interpretation of the quantum formalism. The main idea is that reality is relative to a context and that all scientific statements must therefore be made with a reference to the (type of) context, since they are not statements about the independent state of a system, but of its state in its interaction with the experimental setup used to gain the information:

> [...] no result of an experiment concerning a phenomenonwhich, in principle, lives outside the range of classical physics can be interpreted as giving information about independent properties of the objects, but is inherently connected with a definite situation in the description of which the measuring instruments interacting with the objects also enter essentially. [Bohr, 1939b, p. 26]

The physical reality of concepts such as position and momentum, can seem challenged by assertions that they are not independent properties of a system, knowledge of which can be gained by measurement. Bohr, however, does not dispute their physical reality, but states that it can only be talked of with reference to the conditions for their unambiguous use; i.e. to a context in with the eigenstates of either variable are well-defined. These contexts exclude one another in the case of non-commuting variables, thus limiting the degree to which both can be real [Bohr, 1949b, p. 54].

Howard constructs an alternative to classical scientific realism based on Bohr's concept of complementarity: Maximal Bohrian realism, where reality is context-dependent [Howard, 1979, p. 333-382]. Since complementarity shows the mutual exclusion of experimental contexts and that these contexts are necessary for the definition of classical variables, Howard argues, that complementarity can be though of the mutual exclusion of these variables. As these varibale cannot be simultanously defined, he further argues that they cannot have independent reality. Camilleri og Schlosshauer also mentions this, and argue that Howard (by choice) goes beyond Bohr here [Camilleri and Schlosshauer, 2015, p. 9].

The contextuality of quantum mechanics is also a key point of Carlo Rovelli's understanding of quantum mechanics. I do not whish to equate Rovelli's ideas with those of Bohr, as I (and Rovelli himself) will argue that this would not be appropiate. However, several of his points resemble those of Bohr and can serve as a further motivation and clarification of the ideas presented here. Rovelli argues, that quantum mechanics is a theory of different systems' information about one another and therefore reconstructs quantum mechanics in terms of information theory. In doing this he avoids using the term *state*, as he is of the opinion, that quantum mechanics does not describe states, only information about different systems from the point of view of another system [Rovelli, 1996, p. 6-30]. Here he differs from Bohr, who talks of contextual states of systems, and thus, while rejecting the idea that one can give an account of the *independent* state of a system, is still talking about states (albeit contextual, or relative states) and not about information. Rovelli also writes about how his understanding relates to Bohr's interpretation. He writes, that Bohr assumes the classical world; i.e. gives certain systems (conscious observers) special status. This, he argues, is consistent with his own ideas, but misses out on the generality. In my opinion this is not the way to understand Bohr. I do not think, he gives the conscious observer a special status in the physical theory. Rather, the contextuality of the observed quantities means that observers become inseparable from what they observe.

Rovelli arrives at his understanding through considerations of the context-dependence of quantum mechanics, which, as mentioned, can be relevant to consider in connection with Bohr. He argues that the source of what he calls "uneasiness" and paradoxes about quantum mechanics comes from a (mistaken) assumption of observer-independent states of a system [Rovelli, 1996, p. 3]. I think this captures the idea of the uneasiness about Bohr's ideas quite well. CSR builds upon an assumption of independent states of objects, which can be found out by measurements. Bohr breaks with this by arguing that measured quantities are only real relative to the contexts in which they are measured due to the nonseparability of quantum mechanics. This does not necessarily mean that objects do not exist independently. As I read Bohr, he means that the *classical* states do not exist independently of the observer, and these are the only things we can describe. But towards a description of the state of a system, to use a Kantian phrase, *an sich*, one cannot get any closer than the wavefunction, which, as we shall see, is symbolic and so to speak, translates between different phenomena.

Bohr often describes quantum mechanics as being a non-pictorial theory. For instance, he writes,

> Owing to the very character of such mathematical abstractions [the non-commuting operators of the quantum formalism] the formalism does not allow pictorial interpretation on accustomed lines, but aims directly at establishing relations between observations obtained under well-defined conditions. [Bohr, 1955d, p. 71]

This alludes to a previously mentioned point made by Bohr; that the complementary properties of a system, necessarily observed under different experimental contexts, cannot be united in a single picture of the state of that system, due to the non-separability of entangled quantum systems. This means that it is not possible, through measurement to gain a pictorial description of a system *an sich*, but only to gain partial pictures of the state entwined in specific contexts. One could imagine that the wavefunction could be a candidate for such a description, but Bohr argues that the wavefunction is merely symbolic, due to the fact that it is not defined in space-time, but in a configuration space with as many dimensions as the system's degrees of freedom [Bohr, 1961b, p. 113]. The wavefunction, which can perhaps be said to constitute a description of the state of a system *an sich*, does not provide a picture of what that state is. Bohr consideres the wavefunction as a mathematical abstraction, which only has real physical content with reference to specific contexts, where it gives statistical predictions of the values of different variables [Bohr, 1958a, p. 16]. Zinkernagel also uses the word "symbolic" about the wavefunction. He argues, that the wavefunction cannot be understood as a real wave, as it is imaginary and defined in configuration-space rather than in space-time. However, it is not epistemological in the sense of being an expression of our knowledge either, as it is a representation of the system [Zinkernagel, 2016, p. 5-7]. But even though the wavefunction represents the system, it cannot be interpreted as a picture of that system, or rather Zinkernagel argues, that it can only be pictured in such a way in specific situations, such as e.g. the double slit experiment, where Zinkernagel points out that Bohr himself drew the wavefunction as travelling wavefronts [Zinkernagel, 2016, p. 6]. I consider Bohr's main point in saying that quantum mechanics is not pictorial to be that complementary phenomena does not come together to form a consistent picture of the observed system. It is therefore a continuation of the contextuality of the theory. By calling the wavefunction pictorial in certain contexts, Zinkernagel uses a terminology which varies from mine, though I agree with what is meant. This is also apparent in him describing the wavefunction as context-dependent, in that it "is a representation of a quantum system in a particular, classically described, experimental context." [Zinkernagel, 2016, p. 7] As hinted in section 5.2.2, my reading of Bohr is rather that the *content* of the wavefunction, the statistical predictions, are context-dependent, and the symbolic wavefuction allows one to make these predictions in many different contexts, i.e. bases representing contexts in which the corresponding observable can be defined.

With this non-pictorial nature in mind, many of the classical "paradoxes" of quantum mechanics are solved, as these, according to Bohr, arise when one tries to ascribe physical pictures to quantum effects [Bohr, 1949b, p. 52]. The particle-wave duality for instance, looses its apparent contradiction, when one restricts the picture of a particle as either a wave or a physical particle to the contexts in which the respective conditions of the use of such concepts are fulfilled, and does not try to unite these in a single picture of what the particle is like outside of any or across different contexts. That this appears to be Bohr's attitude is shown by his writing, in continuation of a short account of complementarity and the need to include a reference to the experimental context in any attribution of properties to a system,

> Such considerations not only have clarified the above-mentioned dilemma with respect to the propagation of light, but have also completely solved the corresponding paradoxes confronting pictorial representation of the behaviour of material particles. [Bohr, 1955b, p. 90]

The inability to picture quantum phenomena is connected to the inability to subdivide the phenomena. Bohr mentions that a single photon passing through an interferometer, will behave as though it has passed through both paths, as the probability distribution will exhibit an interference pattern, but a photon sent through a beamsplitter will be registered as being either reflected or transmitted. Combining these two aspects in a picture of how the photon travels is not feasible. To Bohr, therefore it makes no sense to talk about following step-by-step the history of a phenomenon [Bohr, 1949b, p. 65].

In a sense, quantum mechanics can be viewed as a relativistic theory as it describes reference frame dependent phenomena; i.e. the contexts can be viewed along the lines of reference frames. To understand what kind of objects the reference frames in Bohr's interpretation are, whether ontological or epistemological, a comparison with the theory of relativity can be beneficial. Bohr himself compares quantum mechanics to relativity several times. In a lecture from 1933, he writes,

> Just as the general concept of relativity expresses the essential dependence of any phenomenon on the frame of reference used for its coordination in space and time, the notion of complementarity serves to symbolize the fundamental limitation, met with in atomic physics, of the objective existence of phenomena independent of the means of their observation. [Bohr, 1933a, p. 7]

Here Bohr compares the reference frame dependence that arises from special relativity, with the dependence on the systems with which a system interacts ("the means of their observation") arising from complementarity in quantum mechanics. In both cases there is a question of the necessity of referencing a context when ascribing properties, rather than systems possessing these properties in themselves (in the case of relativity, this would mean relative to an ether). In relativity an object cannot be said to be travelling at some velocity; only to be travelling at some velocity relative to a specific frame of reference. In quantum mechanics, objects cannot be said to posses definite properties except in specified contexts where these properties can be defined.

Relativity also gave rise to epistemological insight in the so-called subjective nature of human description, as all descriptions are necessarily relative to the the describers point of origin [Bohr, 1939a, p. 36]. This is not subjectivity, in the sense of being in conflict with objectivity, but in the sense of necessitating a communication of a reference frame as well as the observations in order for the observations to be understood by others. It does not make sense to talk of the velocity of some obejct without specifying relative to what reference frame this velocity is defined, and similarly, does it not, according to

Bohr, make sense to talk of some quantity of a quantum system except in relation to a context in which that quantity can be defined. Without these references the statements become subjective. Likewise, relativity prompted a revision of the foundations for the use of the concepts space and time, proving them to be reference frame dependent, and thus only meaningful relative to a certain observer/frame. However, relativity also provided laws that transcend different observers and serve as tools for translation of the description given in different frames [Bohr, 1960a, p. 21]. All this can be considered to be analogue to the revisions to classical physics made by quantum mechanics. In both cases, observations as well as concepts previously considered absolute were shown to be relative to the reference frame, and in both cases, the theory gives laws for how the properties in different reference frames relate to one another.

This latter point is related to another similarity, pointed out by Bohr: that neither of their formal content can be considered pictorial like classical physics [Bohr, 1958a, p. 17]. This seems to imply a comparison in how the formalism in relativity and in quantum mechanics can be understood. As mentioned, Bohr does not consider the wave function to be a picture of reality, but as a symbolic mathematical abstraction, that gives physical content relative to specific contexts and combines what can be observed in the different contexts together. The Minkowski metric in special relativity can be understood as a similar abstract symbolic objects, that translate between different reference frames.

Rovelli also compares the two theories in arguing that the uneasiness resulting from assuming observer-independent (classical) states of systems in quantum mechanics, correspond to the uneasiness resulting from assuming absolute time in relativity [Rovelli, 1996, p. 3-5]. He thus points to a similarity in the development of the theories, as both involved changing previous conceptions, resulting in the appreciation of the reference frame dependence of things previously considered absolute.

The reference frame dependence of the two theories can thus in many ways be considered analogue to one another. But there are fundamental differences in the types of reference frames. In relativity, the reference frames are spacio-temporal; coordinate systems in space-time, which can quite easily be pictured. The reference frames of quantum phenomena are different, as they relate to the interaction and following nonseparability of physical systems, and thus express how different properties emerges in the interaction with specific systems. But the analogue can perhaps give an idea of the answer to the question of whether these reference frames are ontological or epistemological. What can one answer if the same question is put to relativity? The frames are neither physical objects, nor do they exist only in the mind. The need to refer to one in order to use different concepts reflects the complex and relativistic nature of things, and the lack of an absolute background. Similarly, the contextuality of quantum mechanics can be viewed as the symptom of the failure of absoluteness of descriptions in terms of classical concepts.

In Bohr's account of the observer-dependence of relativity, he mentions that in relativity the requirement of a deterministic descriptions is fulfilled [Bohr, 1955c, p. 85-86]. This, however, he does not seem to be possible in quantum mechanics. Bohr writes, "we meet regularities which are completely foreign to the mechanical conception of nature and which defy pictorial deterministic desciption" [Bohr, 1955b, p. 85]. It appears that Bohr is of the opinion that the non-classical behaviour of quantum systems are not compatible with deterministic descriptions.

In the very formulation of quantum mechanics there already seems to be a conflict with determinism in the indivisibility of the leaps from one energy level to another in an atom [Bohr, 1955a, p. 105]. In the deterministic view every process can be divided in infinitely small steps of cause and effect, but this is not the case in quantum mechanics where the jumps between energy levels happen without any intermediate steps, and where it is generally not possible to follow processes in detail, as seen previously. Dividing a phenomena in smaller steps would require modifications to the experimental setup, and the smaller steps would therefore constitute different phenomena, meaning that they cannot be unified as parts of the picture of the first phenomena [Bohr, 1958a, p. 14-15]. This is also seen in the need for an object-instrument cut, which Bohr argues is instrumental in the limitations of a deterministic understanding of quantum mechnics,

> The very fact that quantum regularities exclude analysis on classical lines necessitates, as we have seen, in the account of experience a logical distinction between measuring instruments and atomic objects, which in principle

> prevents comprehensive deterministic description. [Bohr, 1958b, p. 6]

This is because a division of the phenomena would require a new object-instrument cut, as processes that were part of the instrument would have to be subjected to investigation themselves. As new cuts means new phenomena, this points to the impossibility in quantum mechanics of fulfilling the demand of pictorial description of cause and effect [Bohr, 1958a, p. 17].

Furthermore, determinism (at least in classical physics) requires that all properties of all systems are precisely defined, though they may not be precisely known, in order for them to precisely determine all future configurations. Bohr's understanding of Heisenberg's uncertainty principle does not allow this, as it is a limit of how well-defined two canonical variables can be [Bohr, 1958a, p. 13]. The non-communitation of the operators corresponding to different classical variables, therefore, is in conflict with the unlimited compatibility of these variables, which constitutes the foundation of classical, deterministic descriptions [Bohr, 1955a, p. 108].

Bohr uses the words "causal" and "deterministic" interchangeable. Howard, however, uses definitions proposed by Northrop to distinguish between these two concepts, and conclude that while classical theories are both causal and deterministic, quantum mechanics (in the Bohr-version) is only causal. Northrop identifies two conditions for determinism: a) that from the initial state of a system it is possible to predict the state at any later time and b) the state must be specified with precise, classical variables and not e.g. statistics. He then defines *causality* as fulfilling condition a), while the word *determinism* is only used when both conditions are fulfilled. Howard argues, that Bohr, when talking of both causality and determinism, refers to theories which fulfil both conditions, i.e. are deterministic, and that quantum mechanics in Bohr's view is causal in the Northrop-sense [Howard, 1979, p. 102-103]. The Schrödinger evolution is causal, and therefore one can always, if given the initial state, get the state at any later point, so long as one, by "future state" does not understand the values of measurement outcomes, but only the statistical predictions for these. The causal connection is not applicable across different phenomena (e.g. between different complementary variables), as different phenomena cannot be connected in one picture. The states which are causally connected are therefore not those given in the definite, classical variables as the use of these is limited to within different phenomena and the second condition is not fulfilled.

If one is using the Northrop-definitions therefore, Bohr actually means determinism, when he e.g. writes "[...] any question of a return to a mode of description consistent with the principle of causality [was] excluded by unambiguous experience of the most varied kind" [Bohr, 1937, p. 18]. This is seen in that he, after this quote, contrasts a causal theory with quantum mechanics qua a *fundamentally statistical theory*, which is clearly incompatible with a classical, deterministic theory, but does violate Northrop's condition a).

The failure of determinism is therefore also present in Bohr's view on the statistical prediction of quantum mechanics. In classical physics, statistical theories are epistemological, in the sense of being descriptions of what we know of the state of something else; it is in principle possible to determine whether a coin toss will end up heads or tails from the initial conditions, and thus the 50/50 chance is just a reflection of our lack of knowledge about these conditions. But in Bohr's view, the statistics of quantum mechanics differs from this kind of statistics, by being *fundamentally statistical*. The statistical content of quantum mechanics is the probability of different eigenvalues of an observable given as the absolute squares of the coefficients in the corresponding basis. To Bohr the basis represents the reference to the context in which this variable is defined, and he writes "In conformity with the non-pictorial character of the formalism, its physical interpretation finds expression in laws, of an essentially statistical type, pertaining to observations obtained under given experimental conditions." [Bohr, 1958b, p. 3], that is, the formalism of quantum mechanics is symbolic (as previously discussed), and gives the actually physical content in the shape of context-dependent probabilities, which are fundamentally statistical, and therefore not an expression of our lack of knowledge of an underlying deterministic chain of events. This is also apparent by Bohr writing in another article [Bohr, 1949b], that the statistical laws of quantum mechanics differs from those of classical mechanics by *not* being a practical tool for describing complicated systems, but rather being inevitable in the description of quantum phenomena [Bohr, 1949b, p. 47]. The statistical description is thus not a temporary solution in lack of a deeper deterministic description, which is to come, but a complete discription in itself [Bohr, 1955a, p. 106].

Bohr comes to this conclusion from the fact that the same initial conditions in the same experiment can result in different outcomes. He writes, “Corresponding to the circumstance that different individual quantum processes may take place in a given experimental arrangement, these relations [the content of the quantum formalism] are of an inherently statistical character.” [Bohr, 1955d, p. 71]. If indeed the same initial conditions give different outcomes, it is clear that the probabilities of these outcomes do not reflect ignorance of a deterministic description. Other interpretations, such as hidden-variable theories, will argue that the initial conditions are in fact not the same, but Bohr does not seek to introduce more things into the theory rather than accepting the failure of notions from classical physics. He therefore regards the statistical predictions of quantum mechanics as fundamentally different from those of classical statistical theories.

5.2.5 A rational generalisation

Though Bohr’s view of quantum mechanics have many deviations from what is classically considered to be marks of a good theory (determinism, pictures of an independent reality), Bohr considered the theory as being both comprehensive and free from contradictions.

There are no contradictions in the quantum formalism, and according to Bohr this excludes the possibility of any contradictions in the theory [Bohr, 1962b, p. 38]. As mentioned, apparent contradictions comes from trying to extend pictorial descriptions outside of the contexts in which they can be defined and unite different phenomena in a single picture of an independent state of the system. For Bohr, this is really all one can demand of a theory for a new area of investigation; “Here, of course, we cannot seek a physical explanation in the customary sense, but all we can demand in a new field of experience is the removal of any apparent contradiction.” [Bohr, 1955b, p. 90] In

Bohr's opinion, one cannot assume, that one's previous notions of what a good theory is can be carried across to these new types of observations.

If the formalism is without contradictions, Bohr argues that the only thing that can make the theory insufficient is if it leads to predictions which are at conflict with experimental observations, or if it does not provide predictions for all possible observations [Bohr, 1949b, p. 72], none of which he would argue is the case with quantum mechanics. Bohr is of the opinion that, as the quantum formalism can be applied to any imaginable context, the different complementary properties of an object obtained in the different situations exhausts all the information of the object which can be defined. Though it is not possible to get *every* variable with measurements done on one system, Bohr considers it to be sufficient that one can get *any* variable. The departure from the ideals of classical theories is necessary in a field where concepts can only be used contextually [Bohr, 1949c, p. 120-121]. Under these epistemological circumstances, objectivity is secured by explicit reference to the object-instrument cut in each case [Bohr, 1955a, p. 110].

Regarding the measurement problem, it can be argued, that there must either be a contradiction in the formalism (between unitarity and definite measurement outcomes) or the formalism is at odds with observed behaviour (definite outcomes are not observed). If one of these things were the case, quantum mechanics would, by Bohr's own account, not be free from contradictions. Bohr, however, is of neither of these opinions. As mentioned, he considers the quantum formalism to be free from contradictions and to explain every observed phenomena. The key to this is of course the contextual and the fundamentally statistical nature of the theory.

In the light of these considerations, Bohr does not view quantum mechanics as merely a limitation of classical mechanical framework, but sees it as a generalisation thereof. He e.g. writes that quantum mechanics can be regarded as "a rational generalization of classical physics" [Bohr, 1955b, p.90]. My understanding of this view, is that it relates both to Bohr's idea of classical mechanics as a special case or idealisation of quantum mechanics, and to how he views the development of quantum mechanics: that it was the "rational" way to expand classical physics so as to incorporate the new observations without any contradictions. Zinkernagel points to the nature of the relationship between Bohr's quantum mechanics and classical physics as being one of mutual dependence. He cites Bohr on the subject of quantum mechanics as a generalisation of classical physics, while classical physics, on the other hand, is required in the formulation of quantum mechanics, as quantum mechanics, in order to be applied to a system, needs a reference to another, classical system [Zinkernagel, 2016, p. 17].

Because many of Bohr's ideas are based on epistemological considerations, there is much talk of "observers" and "measurements", which can incline one to think that consciousness plays an important role in the theory. However, as pointed out by Howard, "He [Bohr] is always careful to speak not of a conscious, human observer but of the physical measuring apparatus." [Howard, 2021, p. 13] When talking about the interactions that results in the impossibility of observing independent behaviour of the phenomenon, it is always interactions with measurement instruments or experimental configurations. This implies, that it is the interaction with another physical system and not (necessarily) a person, that is central in complementary and constitute the conditions for definition of variables. This point was also touched upon in section 2.2, where it was argued that as far as the formalism concerns it does not matter whether a system is designed to be a measurement instrument or not.

The freedom of choice in what complementary property to "look at" or where to make the object-instrument cut is not (merely) a mental process. The choice in these cases corresponds to the choice of setting up different experimental arrangements, and it is the interaction with these arrangements that creates the entanglement with the measured system [Bohr, 1955c, p.90]. As mentioned, the inability to describe a quantum process in smaller steps comes from the fact that to do so would require adjustments to the experimental setup, which would render this investigation a new phenomenon, not to be united in one picture with the old [Bohr, 1955a, p. 108-109]. Here it is again the physical setup of the new experiment, that marks it out as a new phenomena, and not the decision of looking at something else. These points emphasise that Bohr considers the experiments as physical systems, and these rather than cognitive processes are relevant in the theory. That this is the case is supported by a remark made by Bohr, in a comparison of quantum mechanics with relativity:

> [...] in niether case [relativity theory and quantum mechanics] does the appropriate widening of our conceptual framework [that comes with both relativity and quantum mechanics] imply any appeal to the observing subject, which would hinder unambiguous communication of experience. In relativistic argumentation, such objectivity is secured by due regard to the dependence of the phenomena on the reference frame of the observer, while in complementary description all subjectivity is avoided by proper attention to the circumstances required for the well-defined use of elementary physical concepts. [Bohr, 1958b, p. 7]

Thus, it is exactly the necessity of referring to the experimental circumstances in all descriptions of quantum phenomena, that saves one from the dependence on an "observing subject". Consciousness does enter into the theory, as human being are also physical systems, and are to be subjected to the same laws of physics as other systems. But as consciousness plays no special role, it is not a necessary element in the formulation of the theory.[9]

Camilleri and Schlosshauer, however, empathises that Bohr's ideas are epistemological in nature. In an article [Camilleri and Schlosshauer, 2015], they argue that the doctrine of classical physics is epistemological and that Bohr was in general concerned with the epistemology of experiments rather than ontology. They argue that Bohr uses epistemological considerations of the meaning of doing experiments to motivate the doctrine of classical concepts: in order to perform an experiment, one must necessarily read out the result in a way which require a spacio-temporal description (e.g. the position of a pointer), and thus give a classical description of parts of the apparatus [Camilleri and Schlosshauer, 2015].

Again it appears to be difficult to make a final decision about the nature of Bohr's notion of context. The bolted down one-slit apparatus is the reference frame for position,

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[9] E.g. Rovelli [Rovelli, 1996] gives a contextual account of quantum mechanics, with physical systems as reference frames without any inclusion of consciousness.

due to the fact that it stays still. But physically it cannot stay really still, meaning that Bohr must mean that one need to use the context as if it is something it is not; one must *assume* that the concept of momentum does not apply. Thus, there seems to be a functional dimension to the reference frames as well. However, contexts really correspond to physical contexts. The definition of observables depend on the experimental setup and to subdivide a phenomenon one needs to change the physical system constituting this context. But though this question is not entirely settled here, it is at least completely clear that the context does not correspond to a state of mind, and is not defined by someone's decisions about what to measure - not before this decision is reflected in the physical setup of an experiment.

5.3 Bohr's ideas as a solution to the different measurement problems

In the previous section, Bohr's ideas, and his own assessment of them, were explored and on this basis, I will see how Bohr's interpretation can address the different versions of the measurement problem. Bohr himself wrote an article titled *Om Maalingsproblemet i Atomfysikken* [On the Measurement Problem in Atomic Physics] [Bohr, 1946], where he discusses the new situation in physics brought forward by the development of quantum mechanics. He mentions the central place in "the exact sciences" that measurements has had since the time of Galileo Galilei, and argues for the insight in the failure of many of the classical notions of measurement and scientific theories, which came with the new field of study, and which was mentioned in the previous account of his understanding of quantum mechanics. He further stresses that quantum mechanics is no less diverse in its content than classical physics. In this article, Bohr seems to be talking about something other than the measurement problem, as it was described in this thesis. He argues, that measurement in quantum mechanics has a different and less straightforward role than in classical mechanics, and has therefore been brought to the foreground of discussions in this field. He mentions in another article, that quantum mechanics brought attention to "iagttagelsesproblemet" [the problem of observation] and thus to the role of measurements in scientific theories [Bohr, 1961a, p. 79]. In the article on the so-called measurement problem, he argues that looking into the physics of the atomic world has lead to a new epistemology. As mentioned, he views Planck's constant as a lower limit for the inevitable interaction between object and instrument, and thus makes it impossible to distinguish between the independent behaviour of the object and the object-instrument interaction. Measurements, therefore, has a completely new status in science compared to classical physics. This appears to be what Bohr understands as *the measurement problem* in this article; the problem of understanding the new role of measurements in quantum mechanics. It is of course related to "our" measurement problem, as this is exactly an example of how the role of measurement has become the basis for discussions. Though Bohr's article seems to rather concern coming to terms with a new epistemological situation, whereas the measurement problem in general concerns explaining apparent contradictions in the theory, Bohr's ideas concerning both are highly interlinked, as the basis on which Bohr argues for the new epistemological situation is his understanding of the quantum formalism and his ideas on how to address different apparent issues of the theory.

Bohr also mentions a "measurement problem" in a lecture from 1958:

> Still, the recognition of the reciprocal latitude for the fixation of any two canonically conjugate variables, expressed in the principle of indeterminacy formulated by Heisenberg in 1927, was a decisive step towards the elucidation of *the measuring problem in quantum mechanics.* Indeed, it became evident that the formal representation of physical quantities by non-commuting operators directly reflects the relationship of mutual exclusion between the operations by which the respective physical quantities are defined and measured. [Bohr, 1961d, p. 61] [my italics]

Again, one gets the impression, that the here mentioned "measurement problem" is not the measurement problem discussed in this thesis. Bohr is of the opinion that this problem is (partly) solved by considering complementarity; that is, that different variables represented by non-commuting operators, can only be "defined and measured" in different mutually exclusive contexts. This implies that the problem, Bohr talks of here, is a problem of reconciling *different* measurements, i.e. the problem of how different measurements relates to one another and the apparent contradictions, which appears when a unification of different measurements is attempted. The measurement problem discussed in this thesis rather concerns understanding the process *within* one measurement. An idea of Bohr's opinion of this issue can be constructed from his general ideas, though it does not seem to have been directly addressed by himself. This is pointed bout by both Zinkernagel, who writes that "In his published writings Bohr never discusses [the measurement problem]" [Zinkernagel, 2016, p. 13] and by Camilleri and Schlosshauer, who write that Bohr "said very little" about the quantum-to-classical transition [Camilleri and Schlosshauer, 2015, p. 9].

Bohr is of the opinion that quantum mechanics is really the fundamental description of the world. He e.g. writes that quantum mechanics "has, in fact, yielded a quite new basis for the understanding of the intrinsic stability of atomic structures, *which, in the last resort, conditions the regularities of all ordinary experience*" [Bohr, 1939b, p.24] [my italics] and refers to the classical realm as an idealisation. He often writes, that this idealisation is applicable when a system is "big enough" relative to the Planck's constant [Bohr, 1946, p. 165]. This is a rather confusing point, since Bohr, as shown in the account of his ideas given above, also talks of the classical description being applicable in certain well-defined contexts, and the classical idealisation being a result of imposing as-if separability in these contexts. Planck's constant was only discovered in experiments with atomic systems. The limitation of the simultaneous definition of complementary variables is marked by this small constant, and the idea is, that if one looks at macroscopic objects, one can effectively pretend that $\hbar = 0$, and thus that the complementary observables commute, and there is no entanglement.[10] I find it hard to see where this way of thinking leads in relation to Bohr's other ideas, and will leave the issue here as a bit of a conundrum.

Bohr also argues, that quantum mechanics only applies to *closed phenomena*, which are phenomena, which includes a irreversible process of some kind (such as markings on a photographic plate) [Bohr, 1955c, p. 89-90]. This could be used in regards to

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[10] Raggios theorem (see section 2.3) is again useful in connecting these different features, as they are proven to be equiavalent.

the measurement problem, as the presence of the the irreversible phenomena could be used to explain at what point some sort of effective collapse takes place. Howard e.g. does this in his account of how to address the measurement problem in a Bohrian world view, or, as he terms it, avoid one of the paradoxes of quantum mechanics: “the problem of the reduction of the wavepacket” [Howard, 1979, p. 202]. He argues, that an irreversible amplification process must necessarily take place in an observational apparatus, and that this process closes the phenomenon. This makes a boundary for where the quantum formalism should be applied [Howard, 1979, p. 203]. However, I do not consider the meaning of Bohr’s statement to be very clear. What marks out markings of photographic plates as irreversible compared to other interactions? And what is the meaning of calling the phenomena in which these occur closed? As far as I can see, this is just another way of saying that one gets physical content (statistical predictions) when one has classical reference. Or in other words, when specifying a context one has a phenomena, and here quantum mechanics applies in that it gives statistical predictions for this phenomena.

Zinkernagel also mentions that irriversibility necessary in recording, and that quantum mechanics only applies to closed phenomenon. But to Zinkernagel, it appears that the closedness is just a consequence of the as-if classicality; as one must treat something classically, one gets the irreversible process and the closed phenomenon. A marking on a photographic plate is therefore irreversible, as it cannot be analysed in further detail due to the fact that it is a result of the nonseperability of the instrument and measured system [Zinkernagel, 2016, p. 11-13]. The “irreversible process” changes the context, thus resulting in a kind of effective collapse, which is *not* a physical process, but a “a formal (as opposed to physical) notion in which a superposition is replaced by one of its components.” [Zinkernagel, 2016, p. 11]

The nonseperability of the quantum states results in the uncontrollable interaction between e.g. a measured system and a measurement instrument. This is different from the Schrödinger evolution (which is causal and controllable in this sense), which describes the evolution of the wavefunction that gives the statistical predcitions at any time. But it is not a physical interaction between two systems, and therefore there is no physical collapse included in the theory. That the effective collapse is not a physical process causing the superposition to go into an eigenstate is a point which is often misunderstood regarding Bohr. The idea of the formal collapse, which has been expressed here, corresponds to Bohr not endorsing the e-e link, as a measurement outcome corresponding to some eigenvalue does not necessitate a system which is definitely in the corresponding eigenstate. This is a clear difference between Bohr and the von Neumann account of measurements.

After this more general comment on Bohr's attitude towards the measurement problem, I will move on to how the specific formulations presented in section 2 can be addressed. The minimalist measurement problem, asks any interpretation of quantum mechanics how it makes sense of states of composite systems, such at equation 2.10, which could be state of the total object-instrument system in measurement situations.

$$\left(\sum_i \alpha_i|\alpha_i\rangle\right)|\beta_0\rangle \rightarrow \sum_i \alpha_i|\alpha_i\rangle|\beta_i\rangle \tag{2.10}$$

The central point in this type of state would in a Bohrian view be the fact that it is entangled, which shows how the state of the instrument is not separable from the state of the object. Importantly, this does not mean that the instrument in itself is in a superposition of different outcomes. With respect to the problem of interpretation, the meaning of such a state is therefore that it is a symbolic representation, which illustrate the non-separability between interacting systems by being an entangled state. The physical content is in the (statistical) predictions it gives for what the state of the system will be relative to any desired context, i.e. basis. To get these one would, in accordance with the doctrine of classical concepts, need to impose as-if separability in order to have the state of the instrument as a classical reference relative to which the a state can meaningfully be assigned to the measured system. An idea for a formal representation of these ideas will be given in section 6.

The problem of interference, concerns the question of why there is no interference at e.g. the end of measurements. In a Bohrian picture, the interference disappears due to doctrine of classical concepts and the nonseprability of quantum states. Classical terms are well-defined in certain contexts and we will always see things relative to a context. A measurement is a context in which some property becomes well-defined, and as such there can be no quantum interference. This is due to the nonseparability of the observed system and the measurement instrument. In his maximal Bohrian realism, Howard formalises this by arguing that quantum systems are described in terms of different mixtures relative to different contexts. In a measurement of spin in the z-direction, the system will be given as a mixture in the spin-z basis. The off-diagonal terms will be zero, and thus there will be no interference [Howard, 1979, p. 343, 345-346].

The problem of outcomes concerns the conflict between the time evolution as described in the quantum formalism and having definite outcomes of measurements (or just states with definite classical properties in general). In Bohr's quantum mechanics one does have well-defined definite properties relative to contexts in which the conditions for their definition are met with. Bohr does not really deny that there is an incompatibility, but says that the incompatibility comes from not considering the context-dependence of the definite outcomes. The outcomes occur relative to certain contexts and, most importantly for this problem, as a result of a fundamentally stochastic process. To Bohr, the question of how one definite eigenstate is obtained, is unanswerable due to the fundamentally statistical nature of quantum mechanics. The problem of outcomes is seen not as a bug, but as a feature of quantum mechanics, as it is, in a sense, a reformulation of these fundamental statistics. In Bohr's view quantum mechanics is not deterministic, and no underlying process leading to a specific outcome exists. The causal evolution leads to statistical predictions in different contexts/bases and no further.

The choice of basis in which the probabilities are found, are in Bohr's view given by the conditions of the experimental context, i.e. in the type of interaction the system has with the instrument of measurement. This constitutes Bohr's address of the problem of preferred basis. If an experiment is set to measure spin in the z-direction, the spin-z states will be well-defined, while the spin x-state cannot be defined at all as the spin-z and the spin-x operators do not commute, $[S_x, S_z] = -i\hbar S_y$ [Sakurai and Napolitano, 2017, p. 28], and therefore are complementary (i.e. the experimental conditions enabling the use of each of them are mutually exclusive). Though a state of a system can be written in many bases, the basis to which the Born rule applies in order to give statistical predictions is determined by the context. Bohr writes, that the wavefunction provides statistical content relative to different experimental contexts. This must mean that in the context in which an observable can be well-defined, the Born rule is applied to the wavefunction in the basis of that observable. As accounted for in section 2.2, a measurement involves an entanglement of a system, α, and the measurement apparatus, β,

$$\left(\sum_i \alpha_i|\alpha_i\rangle\right)|\beta_0\rangle \rightarrow \sum_i \alpha_i|\alpha_i\rangle|\beta_i\rangle \tag{2.10}$$

The decomposition version of the problem of preferred basis concerns the issue of which decomposition of this state rightly corresponds to the measured eigenstates. I will argue, that Bohr's answer to this question again lies in the need for a classical context. If we look at different compositions of an entangled state such as equation 2.10, we are looking at the measurement apparatus as an "object" to which quantum mechanics apply. In other words, we are treating the measurement apparatus structurally, as a physical system, and not as a classical reference for our measurement. In doing so the instrument cannot, according to Bohr, work as an instrument. To measure e.g. the spin of a particle, we must put the Stern-Gerlach apparatus in a definite orientation. The states of the instrument, with which the measured system becomes entangled must be classical, and relative to these a specific set eigenstates of the measured system becomes well-defined. In this sense a specific decomposition becomes the representation of the measurement context and the measured eigenstates.

The control problem is given as the mutual inconsistency of the unitary evolution of the wavefunction, determinate inputs, successful preparation and competent measurements. I imagine, that Bohr would assume definite inputs to be true, as these, like measurement outcomes, would have to be treated classically; one must be able to tell people what one did to prepare the state. As unitarity is also maintained, one arrives at the version of the control-problem corresponding to the no-cloning theorem: the limitation of the ability to prepare and measure states. I do not think, that Bohr would have a problem with implementing the no-cloning theorem as a feature of quantum mechanics rather than a problem. It follows from the quantum formalism, and the fact that it is not a part of classical mechanics, would not trouble Bohr.

In the problem of effect, an explanation of repeated measurements is sought. For Bohr, a repeated measurements is a new experimental setup, and thus a new phenomenon. This means that one cannot seek a causal explanation of how one measurement affects the future measurement, as this would mean seeking a causal explanation between different phenomena, which according to Bohr does not make sense. Thus, there can be a phenomena in which a single measurement is performed or one in which two measurements are performed. But still, one needs to account for the fact that the two experiments give the same outcome, and here the argument of van Fraasen [van Fraassen, 1997] is of use. As accounted for in relation to the solution of the problem of effect in decoherence (in section 4.2), he argue that one does not need a proper collapse to account for repeated measurements, as the two measurement instruments (B and C) enter into the entanglement with the measured system (A), in the following manner,

$$\Big(a\,|0\rangle_A|\phi\rangle_B + b\,|1\rangle_A|\phi_\perp\rangle_B\Big)\,|\phi\rangle_C \rightarrow a\,|0\rangle_A|\phi\rangle_B|\phi\rangle_C + b\,|1\rangle_A|\phi_\perp\rangle_B|\phi_\perp\rangle_C\,. \tag{4.10}$$

This means that in the phenomena where the system is measured two times, one can either measure one outcome twice or the other outcome twice and thus repeated measurements are accounted for.

To consider whether the problem of collapse can be meaningfully applied to Bohr's interpretation, one must consider whether it can be categorised as a collapse interpretation or not. I will argue that in some sense it can, and in some sense it cannot. The interpretation does not contain a collapse as in a physical type of non-unitary evolution that a system undergoes at certain types of interactions. On the other hand, it can be said to contain a kind of "effective collapse" as Bohr clearly is of the opinion that a quantum system do not always posses definite properties, but that it sometimes, or rather relative to some contexts, does. The key here is to consider in what contexts these different kind of states occur.

This is similar the point made by Zinkernagel, who uses the notion of wavecollapse as a solution to the measurement problem, but not in the sense of a real physical process taking place. A causal space-time account of the collapse is, according to Zinkernagel, not possible [Zinkernagel, 2016, p. 11]. As mentioned in the beginning of this section, a wavecollapse can be understood as a formal process in a Bohrian picture, which is due to a change in the context; a change in phenomenon.

The problem of statistics concerns the issue that the same initial conditions can lead to different outcomes (with probabilities given by the Born rule). In Bohr's view, this problem is definitly in the "not a bug, a feature"-category, as he describes quantum mechanics as fundamentally statistical in nature, and thus departs from a deterministic world view. He e.g. writes,

> The fact that in one and the same well-defined experimental arrangement we generally obtain recordings of different individual processes thus makes indispensable the recourse to a statistical account of quantum phenomena. [Bohr, 1962a, p. 25]

He thus argues, that the possibility of obtaining different outcomes from the same initial conditions is a physical fact, which has consequences for how one can understand the rules which govern quantum phenomena. This further underlines the limitations of the deterministic world view [Bohr, 1949c, p. 120]. I think this is on par with his attitude in general: he takes the physical results from investigations as they are and is willing to depart from the ideals of classical physics, rather than adding stuff to the theory in order to preserve them. In a poetic mood, Bohr mentions, that one can call it a "choice of nature", not referring, however, to nature as being something which is capable of make decisions, but to the impossibility of applying deterministic laws for individual quantum phenomena, i.e. the fundamentally statistical nature of the quantum world [Bohr, 1955d, p. 73].

Bohr also compares the fact that the same experiment with the same initial conditions can give different results, with the fact of different experiments on the same system can yield results that can seem to be contradictory (such as the wave-particle duality observed in a single particle double-slit experiment). He argues that these cases of apparently contradictory behaviour are solved by the nonseparability of the observed properties and the context of their observation, which leads to the impossibility of uniting different measurement results in a single picture like in classical physics. In other words, different measurements are considered to be different phenomenon, and are thus separate things [Bohr, 1960b, p. 30-31]. So how can this be relevant for understanding the different outcomes from the same initial conditions? Some of the uneasiness (to follow Rovelli's choice of words) is taken away by considering the idea of the separate phenomena. It is no longer the same evolution giving rise to different outcomes, but different outcomes occurring in different situations following statistical laws. This is like different coin tosses giving different results, but without the underlying deterministic account of the coin toss case, where the initial conditions are in fact *not* the same. Thus the problem of statistics is reduced to a problem of coming to terms with a fundamentally statistical theory.

In the formulation of the problem of statistics, Maudlin [Maudlin, 1995] states that completeness, determinism and different outcomes from the same initial conditions are in conflict. Maudlin must talk of determinism in the Northrop-definitions of determinism and causality from section 5.2.4, as he argues for the incompatibility with the different classical outcomes. It is therefore possible to maintain the *causal* (again in the Northrop-sense) evolution of the wavefunction even though the concept of *determinism* is abandoned under the assumption of the completeness of the wavefunction and the different outcomes from the same initial conditions. This, I argue, is what Bohr does.

Many features of quantum mechanics, such as the no-cloning theorem, is viewed in the "not a bug, but a feature"-way in modern quantum informational research. I argue that the same goes for the measurement problem in general in Bohr's view. The problems are all answered, though by (characteristically) giving up on ideas of how they should be answered from a classical point of view.

6 A synthesis of Decoherence and Bohr's ideas

In this section, I will investigate the positive contributions that decoherence and Bohr's ideas can make to one another. First, I will motivate the idea that each can contribute to the other, by drawing attention to several similarities between the two. Later, I will argue that each can point to an advantageous way of thinking about the other. Comparing Bohr with decoherence and vice versa is not a novel idea. It is done in e.g. [Camilleri and Schlosshauer, 2015] and [Howard, 2021], both of which contain observations which will be of use in this synthesis.

Camilleri and Schlosshauer argue that there has been a tendency to see decoherence as a challenge to the validity and/or relevance of Bohr's understanding of quantum mechanics. Decoherence does not need to make use of Bohr's concepts to have a starting point, and could therefore be seen as removing the need for Bohr's arguments by replacing them with dynamical explanations of the the same things, thus making Bohr's concepts either superfluous or wrong [Camilleri and Schlosshauer, 2015, p. 1-2]. However, Camilleri and Schlosshauer emphasise that decoherence can also be seen as a justification of Bohr's views; as an answer to some of the questions that can be raised regarding Bohr's ideas. Decoherence would here be considered to be a dynamical reformulation of Bohr's views [Camilleri and Schlosshauer, 2015, p. 2-3]. I would argue that though decoherence might contribute to a Bohrian view of quantum mechanics in such a manner, it cannot answer any questions if not combined with an interpretation, i.e. an understanding of what the density matrix and the reduced density matrix represents. This was seen in section 4.2, where decoherence as an answer to the measurement problem was discussed. Thus, if decoherence can be of use as a sort of formalisation of the Bohrian interpretation, this must be a version of decoherence which itself relies on the Bohrian interpretation in order to resolve its own ambiguities. Therefore the relationship between the two, if it exists, is necessarily one of mutual dependence. This idea of mutual beneficence will serve as the starting point of the following discussion.

6.1 (Dis)similarities

In the article by Camilleri and Schlosshauer [Camilleri and Schlosshauer, 2015] Bohr is presented as being primarily concerned with epistemological considerations, or rather epistemological considerations of experiments. They argue that in the doctrine of classical concepts, the choice of what is treated as the classical reference is due to functional, not dynamical considerations [Camilleri and Schlosshauer, 2015, p. 3-6]. Therefore Camerlleri and Sclosshauer come to the conclusion that Bohr's view is something separate from dynamical descriptions of the quantum-to-classical transition, and that it is compatible with the views on which decoherence is based; e.g. that the classical world is an approximation of quantum mechanics [Camilleri and Schlosshauer, 2015, p. 11-13]. They therefore conclude that Bohr (in their reading) might have a "a peaceful coexistence" with decoherence and this is what enables them to suggest that decoherence might serve as a justification of and contribution to Bohr's view [Camilleri and Schlosshauer, 2015, p. 13].

Howard [Howard, 2021] takes this one step further, and argues that decoherence is actually exactly what Bohr meant, when he was talking about complementarity and the doctrine of classical concepts. This is due to the fact that decoherence results in states (the reduced density matrices) which are observationally indistinguishable from the context-dependent mixtures, which Howard argues corresponds to a formal reformulation of Bohr's ideas. It is actually a question of mathematical equivalence of the statistical predictions given by each for the observables which are measurable in a particular context [Howard, 2021, p. 26]. According to Howard, the central idea in decoherence, i.e. that the appearance of classicality comes from a coupling of the degrees of freedom of the system and the degrees of freedom of the instrument and environment, was discussed long before the formulation of decoherence, in connection with Bohr's ideas [Howard, 2021, p. 4]. Thus Bohr is associated with the very essence of decoherence. Howard further argues, that, when understood properly, decoherence and complementarity are actually the same thing [Howard, 2021, p. 27]. Howard talks specifically of environment-induced decoherence. However, I consider his observations to also work for the general decoherence, as they concern the appearance of mixtures upon interaction of a system with other systems; something which is not restricted to environment-induced decoherence but is also characteristic of the general concept.

Similarly to both Howard and Camilleri and Schlosshauer, I will argue that several things points to a fundamental similarity between the kind of thinking which lies behind decoherence and Bohr's understanding of quantum mechanics. How far these similarities can go will be explored in the subsequent chapters, but first some points of similarity, and dissimilarity, will be pointed out.

The type of collapse in decoherence resembles that of Bohr's writings. In both cases there is no physical process that corresponds to a collapse, but one can still talk of a collapse in the sense of having a pure state in some situations and a mixture in others. That is to say, in both cases there is no physical collapse, but an "effective collapse", which occurs relative to some interaction of the system. I consider this characterisation likewise applicable to the "collapse" that takes place in decoherence - at least regarding the formal, rather than physical nature.

Both Zinkernagel [Zinkernagel, 2016] and Howard [Howard, 2021] considers this "collapse" in decoherence and in Bohr's interpretation to be associated with the concept of *practical irreversibility.* As mentioned, I find Bohr's use of this term somewhat ill-defined. What exactly characterises the irreversibility, and how does it tie in with the rest of his ideas? In docherence the term can refer to the fact that coherence cannot be regained except with a global interaction counteracting the entagnelment, which is practically impossible if the system is entangled with a macroscopical item/the environment. This might be a point, which is unclear in Bohr, but could be formalised by combining Bohr with decoherence.

The Born rule gives the probabilities for the different values of different observables, corresponding to different bases. Janssen writes, that the Born rule is "just a shorthand for a more complete description of the measurement process" [Janssen, 2021, p. 76]. The whole measurement process includes an interaction which correlates a set of eigenstates with the states of the measurement instrument. Thus the Born rule ends up applying to only one observable; i.e. "post measurement" (that is, after the interaction with the measurement instrument). Due to the nature of the correlations, the statistics will be the same as those given by the Born rule applied to the state of a system. This leads Janssen to quote Dieter Zeh: "the interaction with an appropriate measuring device defines an observable" [Janssen, 2021, p. 77]. Janssen is discussing decoherence, but this is also the exact point made by Bohr: that only relative to a specified setup can the different observables be defined and be attributed to the system.

This indicates a suggestion of a change in the role of the observer as compared with classical physics, which is present in both decoherence and in Bohr's interpretation. In Bohr's views it is reflected in the contextuality or reference frame dependence of all phenomena. In decoherence, it is more open what exactly the role of the observer is, but it is clear that inherent in the act of observation is the entanglement which can result in decoherence. That decoherence comes from tracing out parts of an entangled system leads Janssen to say that decoherence relies on "non-observation", a notion which clearly implies an essential part to be played by the observer in the procurance of the mixed state. I here talk of the general decoherence. In environment-induced decoherence, decoherence only occurs when the measurement is not ideal, i.e. not perfectly isolated or (at least in some accounts) when macroscopic measurement devices with a large number of degrees of freedom is used. Janssen's non-observation is actually the non-observation of the environment. I will however argue that her arguments apply equally well to the general decoherence, as she refers to the tracing out of some degrees of freedom, which is also characteristic of general decoherence. I will further argue, that what is to be understood by this "non-observation", depends on one's understanding of what the reduced density matrix represents. The context- and thus observer-dependence of Bohr's interpretation indicates that a Bohrian understanding might not be far-fetched.

In connection with the role of the observer, there is a similar attitude towards classicality in the two cases. The classical world is in both decoherence and Bohr's writings seen as an atypical kind of idealisation, i.e. one that successfully describes phenomena under certain circumstances, but is not made redundant by more information. Rather it requires the lack of information, or a context, to exist. Janssen writes, that in decoherence, classicality is a consequence of the quantum mechanical laws under certain conditions [Janssen, 2021, p. 66], a description which fits Bohr's notion of classicality as well. There is however, a difference in these conditions. Though the lack of information, the tracing out of parts of the total system, in decoherence might be compared with the classical treatment of measurement contexts (i.e. the disregarding of the quantum behaviour) in Bohr's writings, it would be difficult to consider them as the same thing. In decoherence, one ignores (in what sense will be discussed presently) an entire part of the world, by tracing out a whole subsystem from the density matrix. This is hardly what Bohr is alluding to when he talks of treating *aspects of* a measurement context as if quantum mechanics does not apply.

There also seems to be a reliance on a "cut" being made in both decoherence and Bohr's account of quantum mechanics. In both decoherence and in Bohr's interpretation, a cut between, or a separation of, the measured object and a measurement instrument, enables a description of the measured system in terms of a classical state. With Bohr one needs to impose as-if separability between any measured object and the measurement instrument, while in decoherence the reduced density matrix is obtained by tracing out some other system(s), thus making a "cut" between what is included and what is not. As mentioned, it is doubtful to what degree the tracing out of subsystems can be said to correspond to Bohr's specification of the experimental context. But suffice it for now to say that there is a similarity which warrants further investigation.

In both decoherence and Bohr's interpretation (the latter, at least in the opinion of Howard [Howard, 2021]) entanglement is not viewed as a micro-phenomenon, but as something which can exist with macroscopic objects as well. In decoherence, one has entanglement of a system with instrument- and environment degrees of freedom and Bohr talks of the non-separability between measured objects and their measurement instruments. This compatibility of entanglement and the systems which are to be described as classical (macroscopical measurement instruments etc.) is therefore a further point of similarity between Bohr and decoherence.

When discussing his definition of a phenomenon, Bohr writes that one must take "the *whole experimental arrangement*" [Bohr, 1949a, p. 50] [original italics] into account, when considering the conditions for the application of some concept. To define this more formally, the experimental arrangement which cannot be disregarded can be said to correspond to those systems with which the observed system is entangled. In decoherence, all entanglements also have an effect, and can be said to be necessary to take into account as they change the phenomena. By taking into account, I do not mean include in density matrix, but rather that there is an effect of tracing these out of the density matrix; the traced out parts create the observed phenomena. This serves the same role as the contexts, which according to Bohr must be taken into account, as they have an effect on the system, which cannot be compensated for, and therefore must be referenced in order to give an objective description. This is therefore a point in favour of viewing the tracing out in decoherence along the lines of Bohr's classical context.

The solutions to different formulations of the measurement problem given by decoherence and Bohr also show striking similarities. Regarding the minimalist measurement problem, it is in both cases emphasised that the key feature in e.g. the cat-and-atom state that it is not separable, that it therefore does not describe a cat in a superpostion. Both provide some explaination of how and when interference disappears, a basis is selected and an effective collapse occurs in accordance with the Born rule. The solution to the problem of effect in both cases makes use of van Fraaseens [van Fraassen, 1997] idea of repeated measurements. In the control problem a similar view is also expressed by the two. Both assume determinate inputs under certain conditions and ends up with the no-cloning theorem. The degree to which the conditions of one can clarify the meaning of conditions of the other will be discussed presently. But as mentioned in section 4.3, there is a problem with the no-cloning theorem in decoherence. Crull [Crull, 2013] argues, that measurements of superposed states, i.e. measurement outcomes in superposition states, are technically possible (though unlikely) after decoherence has occurred, as the off-diagonal terms are in general only heavily suppressed, and not exactly zero. Thus the no-cloning theorem is only approximate, and it seems like a less perfect measurement makes measurements of superpositions more likely. For Bohr, on the other hand, the no-cloning theorem is a fundamental feature of quantum mechanics. In a Bohrian view, it also would not make sense to say, that one can measure a superposition. In the doctrine of classical concepts it is argued that to communicate and describe things objectively one must use classical terms, which would not be possible for a measurement of a superposition. Quantum mechanics gives real physical content in the form of statistical predictions of definite outcomes, and these are the only measurement outcomes that would be meaningful. What I argue here is that in a Bohrian view it does not make sense to talk of “measuring superpositions”; a measurement instrument in a superposition of several outcomes could not constitute a measurement.

There is generally an issue with the fact that decoherence is approximate, or emergent, while Bohr seems to speak of very fundamental things. But it could be argued that decoherence, in the more general sense of the word, is not necessarily approximate. That is, the approximate nature could be said to be more of a practical issue, since it comes from e.g. the imperfectness of measurement situations.

Dissimilarities as well as similarities appear when holding decoherence and Bohr’s ideas up against each other. Janssen e.g. argues that the pointer basis is picked by the interaction Hamiltonian between the instrument and the environment, such that the basis which is least disturbed by the environment is the einselected basis [Janssen, 2021, p. 49-53]. Bohr, on the other hand, is of the opinion that the interaction between a measured object and the instrument/experimental setup determines which variables can be well-defined, and thus in which basis the wavefunction “collapses”. But as I read it, Janssen and Bohr are here talking of different things. As mentioned in section 4.2, Janssen, like Bohr, considers the observables to be defined by the interaction with the measurement device. She does not, however, consider the problem of preferred basis to be solved by this alone, as she still considers there to be an ambiguity. In accordance with Bohr’s ideas, she argues that diagonalisation of the density matrix occurs relative to the basis in which there are correlations with another system, but she seems to go further, and speaks of the environment as choosing what bases *can* be observables, these bases being where the correlations are stable with respect to the environment. I do not think that Bohr would ever consider this. Bohr is mainly concerned with analysing, understanding and explaining concrete measurement situations, and here the observables are the starting point; one sets out to measure some observable. He does concern himself with how it is that one can set up such and such experiments, but rather with what properties can be defined when one does. One can of course try to reconstruct what Bohr would have said if told of decoherence as an explanation of why the observables are what they are. He might have considered this as a good addition to the theory and thus his ideas could be reconciled with the idea of decoherence choosing the basis. On the other hand, he might have though that such explanations are beyond what can be expected of a scientific theory. In any case, such issues are connected with the idea of emergence of classicality, i.e. with the how quantum mechanics can account for why classical physics is the way it is. This subject is not the primary focus of this thesis, however, for which purpose it will suffice to mention that there does not seem to be a direct contradiction between the views held in Bohr and in decoherence on this point.

Janssen further points out that decoherence does not require an assumption of definiteness a priori, but rather arrives at this through a quantum mechanical description of what physically occurs during a measurement. [Janssen, 2021, p. 49, 65] This is at the least a different approach from Bohr, who takes the need for a classical description as an epistemological starting point in many of his arguments and as condition for a measurement to be a measurement. However, it does not necessarily follow that there is a contradiction, and that Bohr's a priori requirement cannot be associated with the physical results of decoherence. After all, Bohr talks of the physical theory of quantum mechanics as having well-defined classical definiteness relative to certain contexts, and thus of classical concepts as not only an epistemological requirement.

Lastly, most discussions of decoherence concern environment-induced decoherence, in which suppression of the off-diagonal terms takes place through interaction with environmental degrees of freedom, the states of which are not in general orthogonal. Bohr never talks of suppression of terms, but rather of a full "collapse", where different variables does have well-defined values relative to a context. But as mentioned before this question concerning the approximate nature of decoherence, might be explained by the fact that Bohr talks of ideal cases, while decoherence theorists generally does not. Decoherence, applied to a case of perfect entanglement between the orthogonal states of a pair of systems, gives off-diagonal terms of the reduced density matrix which are exactly zero.

All in all the many similarities between decoherence and Bohr's understanding of quantum mechanics are at the very least enough for motivating the project of gaining insight into each by considerations of the other.

6.2 Decoherence as a formalisation of Bohr's concepts

Bohr mostly expressed himself in sentences and ordinary language rather than through equations, and therefore his ideas do not appear very formal. In this aspect decoherence might be considered as a possible formalisation of (some of) Bohr's concepts, such as suggested by Camilleri and Schlosshauer [Camilleri and Schlosshauer, 2015].

Bohr e.g. writes, that one cannot account for the interaction between a measurement instrument and a measured object, as it is due to the nonseparability of the two and not a deterministic process. Decoherence could be seen as a formalisation of this, as it also concerns the relationship between e.g. an instrument and an object, which is one of entanglement. This is what leads to the "collapse", i.e. the state of the measured system becoming a mixture. The interaction between the two systems has therefore resulted in going from a pure state to a mixture, but not through some traceable evolution - such as Bohr argues.

Bohr often talks of a kind of classical limit when systems are $\gg \hbar$. Camilleri and Schlosshauer writes that this concerns the fact that macroscopic things, such as measurement instruments, cannot possibly be isolated from environment, and thus decohere [Camilleri and Schlosshauer, 2015, p. 11-12]. This might be a way of understanding this limit which is in line with the treatment of classicality in general.

The doctrine of classical concepts, which Howard puts equal to the necessity of treating e.g. a object and an instrument as if they were separable, could perhaps find a formal counterpart in decoherence, which requires the tracing out of subsystem with which the observed subsystem is entangled. Only considering some part of the total system on its own might be said to correspond to treating it as if it were separable from the other system. In this picture, the systems which are traced out correspond to those which are the references, and the interactions which selects the basis are the same in both. This seems supported by Howard's context-dependent mixtures being observationally indistinguishable from the results of decoherence. This exact point is mentioned by Howard as the grounds for concluding decoherence to be what Bohr was getting at. But, as mentioned, there is the issue of the difference between them; in decoherence one "ignores" a whole system, whereas in Bohr one has some aspect being treat as if it was classical. Being able to have a well-defined variable, requires that another, complementary variable cannot be defined. This complementary variable however, can not be considered to the the same as the traced out subsystem in decoherence, though it might be said to be "ignored", as it will be in another basis from that of the measured operator. The traced out system in decoherence, on the other hand, is in the basis of the measured operator.

As hinted in the previous section, the concept of practical irreversibility appears to have a clearer meaning in decoherence than in Bohr's interpretation. The irreversibility being associated with going from a pure state to a mixture fits well with the role it plays in Bohr's mentions of it, as marking out the phenomena for which the quantum state gives statistical predictions. Furthermore, the markings of photographic screens etc., which Bohr calls irreversible, would be cases where entanglement with a macroscopical objects would lead to decoherence, which would indeed be practically irreversible, as the interaction causing the entanglement would be practically impossible to counteract.

In decoherence the uncontrollable interaction between a measured object and the measurement instrument, as Bohr calls it, takes place upon the entanglement with and tracing out of the measurement instrument. Bohr writes, that the incontrollability is due to the need for a reference frame, or in other words the definition of phenomena; if the interaction was to be investigated, a new setup and thus a new phenomena would be needed. To look at an instrument as an object, a further instrument is needed, and then the first instrument cannot serve as object. This might be formalised in decoherence: if an instrument is to be included in reduced density matrix, then there is no "collapse". One therefore can treat both as objects of investigation, but then, the interaction does not constitute a measurement as it does not lead to a mixture. Using a second instrument to measure the interaction with the first instrument would be represented by an entanglement with yet another system. Tracing this out would give a different mixed state of the object-instrument system, i.e. a different "phenomena". This suggests the possibility of associating the reduced density matrix with a phenomena; a notion which could provide decoherence with the interpretational content which it is lacking.

6.3 Bohr's ideas as the interpretational basis of decoherence

If decoherence can in some sense serve the purpose of a formalisation of Bohr's concepts, it seems probable that Bohr's ideas can give a clue to the understanding of decoherence. In other words, Bohr's ideas can serve as the interpretational background which is lacking in decoherence. Though this is not done by Tanona [Tanona, 2013], who has a different agenda[11], he concludes that

> [...] to treat a system as decohered in some basis requires an implicit or explicit cut, and we should have justification for the pretermission cut that enables that description. [Tanona, 2013, p. 3627]

He argues, that through this cut, decoherence imposes as-if separability. This is much like the features of Bohr's account of the quantum mechanical formalism in general, which (in Bohr's view) marks out the contextuality of quantum mechanics. Thus it could be seen as motivating giving a contextual interpretation of decoherence as well. Tanona further writes, that one needs to identify the conditions for such a cut, and that one candidate for these can be Bohr's considerations leading to the "Copenhagen cut", i.e. imposing seperability on systems in non-separable states. In the version of Bohr,

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[11] This is that of proving that decoherence cannot account for classicality without relying on classicality, as mentioned previously.

presented in this thesis, what can be understood here is that the cut is determined by what kind of experimental setup is used in a measurement, or, more generally, what the context is and what kind of interactions take place; what kind of correlations between the eigenstates of the system and the states of the context are created by the interaction between them.

In a “Bohrian” reading of decoherence, the conditions for the classical idealisation (in decoherence) can be identified as concerning the type of context, such as Bohr describes. The effective classicality, which is in decoherence obtained by the tracing out of parts of a total system in an entangled state, can in a Bohrian interpretation be said to occur in specific contexts, which are represented by the reduced density matrix as suggested previously. By thus associating the reduced density matrices with Bohr’s context-dependent separable states, the classical idealisation is *not* just epistemological (determined by what we choose to “look at”, or something similar) but is the physical reality relative to the context, in which it can be defined.

The very similar type of collapse might also be said to occur under similar circumstances, thus giving an account of decoherence, which does not rely on subjective “non-observation”, but, like quantum mechanics in general in Bohr’s opinion, is saved from subjectivity by the reference frame dependence. Bohr argues, that the reference to the cut, or context, is what secures objectivity in spite of the observer-dependent nature of quantum mechanics. This is the motivation for the doctrine of classical concepts accounted for in section 5.2.2. If this was to be applied to decoherence, the mixtures that are the result of decoherence can be considered objective with reference to the traced out subsystems of the entangled state.

Even though there are differences between Bohr’s separable states and decohered states, my considerations suggest that it would be worthwhile trying to view Bohr’s phenomena as represented by a specific reduced density matrix, which occurs relative to what is traced out (i.e. the set up), and should therefore include a reference to that. The reduced density matrix could thus be said to have the same context-dependence as appears in Bohr’s writings. This approach is further motivated by Howard’s point of the indistinguishability of decohered density matrices and his context-dependent mixtures. [Howard, 2021]

In this picture, Bohr is a way out of the purely epistemological interpretation of the reduced density matrix, which seems to be implied in decoherence. Bohr writes that "any definable subdivision [of a phenomenon] requires *a change of the experimental arrangement* giving rise to new *individual* effects" [Bohr, 1949d, p. 99] [my italics]. In a one-slit experiment the choice between a measurement of either position or momentum is reflected in e.g. either bolting a one-slit apparatus to the lab or placing it on a spring. The physical setup is changed, and not (only) a state of mind of the physicist conducting the experiment. As was discussed in section 5.2.5, these examples clearly indicates that a phenomenon is defined by the physical context and not a cognitive process; a new setup means that a new, individual phenomenon is observed. Thus, if a reduced density matrix can be identified as a phenomenon in the Bohrian sense, it too can be interpreted in this non-subjective way, which does not depend on consciousness or "non-observation", but on the context. That being said, an observed phenomenon will be dependent on the observer qua a physical system; the reference frame dependence will be relevant in all observations.

Though it is clear that Bohr does not see the context (or the observer) as an epistemological notion (such as in e.g. QBism), it is not exactly clear how it can be understood. It does not seem in Bohr's writing as if he considers quantum mechanics as a relational theory between physical things (such as Rovelli [Rovelli, 1996] does). Rather it appears as if he considers some sort of subject (albeit a physical subject, describable in physical terms) to be present, when he talks of e.g. how to avoid subjectivity with a reference to the context. The doctrine of classical concepts also emerges through practical considerations of subjects that can learn and communicate. As argued in section 5.2.4, the understanding of the contextuality can perhaps benefit from a comparison with the reference frames of relativity, which, though they are more intuitive, are also hard to characterise as either ontological or epistemological.[12] It is clear however that Bohr's

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[12] How to understand Bohr's context-dependence is a difficult questions, and a final answer is beyond the scope of this thesis. As I have argued here, Bohr definitely considers people and measurements to be physical systems and processes, and that the contexts are not objects of the mind. But neither does it seem that he considers them as purely physical systems, as there is some epistemological or functional side to them as well. This is a question which deserves further investigation.

contexts are tied to concrete physical systems, and therefore Bohr can be useful in the above mentioned way, by giving a meaning to the (reduced) density matrices in decoherence, which does not involve human states of minds, but is rather concerned with descriptions in physical contexts serving as frames of reference.

In response to Tanona, who argues that decoherence cannot do without a classical assumption in the form of e.g. a Copenhagen cut, one can answer that maybe this is not such a bad thing (to which Tanona might quite possibly agree). In Bohr's view, relying on classical assumptions is an epistemological condition for doing objective science, which is reflected in the contextuality of quantum mechanics, as it is with reference to a (classical) context, in which the classical concepts can be defined. In a Bohrian light, the necessity of the cut holds a similar place in the theory to that of the necessity of a specified reference frame in relativity; *the reference frame dependence is fundamental, though the reference frames are not.*

In section 4.3, Wallace's and Tanona's opinions of the fundamentality of decoherence was discussed. Tanona argued that as decoherence relies on classical assumptions, it cannot have a fundamental role in the account of classicality. But as seen here, the need for classical assumptions can be considered as a fundamental feature of quantum mechanics in the Bohrian view, where a classical reference is necessary for the use of classical concepts. Wallace made a different point in arguing that decoherence cannot be fundamental due to it being approximate. This problem remains, but one might, as previously mentioned, consider the decoherence, more generally, as not just an approximate process, but an expression of the contextuality. In the idealised cases decoherence is not approximate, so the approximate nature might be viewed as pragmatic rather than fundamental.

In decoherence, the problem of outcomes is reduced to the qeustion of why one can apply the ignorance interpretation to the decohered mixture, but decoherence fail to give any further answer. However, a Bohrian, contextual way of thinking might help reconcile on with, if not solve, the problem of outcomes. In decoherence, one obtains mixtures, and not a definite outcome, and this can be considered as fitting with Bohr's idea of the fundamentally statistical nature of quantum mechanics. There is no determinism in decoherence: as mentioned in section 4.2, it was tried to make the discrete choice of measurement outcome depend on initial state of environment - but this was shown to not be possible. This suggests the possibility of regarding decoherence's lack of further explaination as a representation of the stochastic nature of quantum mechanics, rather than as a problem, and thus, Bohr can be of use in coming to terms with the problem of outcomes. In Bohr's view, the best theory one can get is one that gives statistical predictions; not because it is incomplete, but because of the contextuality. He accepts that quantum mechanics is not deterministic and gives no explanation of how one ends up in a specific eigenstate, as in his opinion there is none. The problem of outcomes is in this way addressed by the "not a bug, a feature"-approach through calling it fundamental statistics.

In section 4.2, Janssen's discussion of the problems in applying the ignorance interpretation to the reduced density matrix was discussed. She argues, that a) the eigenstates which one wish to apply the probabilities to are not the true eigenstates of the total system and b) the kind of ignorance in the ignorance interpretation is different from that which in decoherence leads to the pure state becoming a mixture. But these would not be a problem in the Bohrian view, where quantum mechanics is considered to be fundamentally statistical. In Bohr's view, the density matrix provides statistical predictions *relative to certain contexts*. In the Bohrian reading of decoherence, the contextual states are given by the reduced density matrix, whose off-diagonal elements are zero, thus allowing for the ignorance interpretation. With Bohr's reference frame dependence, therefore, it becomes no problem that the ignorance interpretation does not apply to the whole world, but only to the reduced density matrix. As reflected in Bohr's definition of a phenomenon, the dependence on a specific situation is a condition for applying any sort of concepts and only relative to these do the statistical predictions have meaning and definite properties are obtained. Regarding the second matter, it is clear that the fundamental statistical nature, means that the ignorance in the ignorance interpretation is not result of ignoring something, as would be the case in the classical statistical predictions, where the probabilities are the expression of lack of knowledge about the real, deterministic processes. Thus Bohr would agree with Janssen, that the two types of ignorance are different, but each is just an expression of different features (not bugs) of quantum mechanics; the tracing out of subsystems, which results in a state to which the ignorance interpretation *can* apply, represents the contextuality and the ignorance interpretation itself reflects the fundamentally stochastic nature of the theory.

In the discussion of decoherence as a solution to the problem of preferred basis (see section 4.2), the conclusion was that decoherence needs something additional to connect the preferred basis to the interpretational basis, i.e. the basis where the statistics have empirical meaning. This comes natural with a Bohrian understanding, as Bohr argues that quantum mechanics gives physical content relative to contexts, in which some operator (represented by some basis of the density matrix) can be well-defined.

The fact the decoherence is (usually) only approximate becomes problematic in basis selection. As Bohr is in general concerned with idealised cases, it is difficult to see if a Bohrian view might be helpful in this aspect. It could be argued that epistemological side to Bohr's arguments becomes useful in leading one to say that one must treat the state as if it was not approximate in a similar manner to how one must e.g. treat a bolted down one-slit apparatus as if momentum does not apply (even though nothing can be bolted down perfectly). However, the uncertainty concerning the exact meaning of Bohr's contexts makes these sort of considerations uncertain as well.

Decoherence also has a problem regarding choice of basis in the case of maximally entangled states, which becomes diagonalised in all bases and therefore provides no information (from the decoherence point of view) of which basis corresponds to an observation. A Bohrian understanding might be of use here, as it has a different starting point. Rather than deriving the basis from the interaction, one chooses (by one's experimental setup) a quantity, and then reduces the state of the system to that quantity, making the systems separable in the corresponding basis. If the density matrix becomes separable in other bases as well, one might argue, that this does not change the fact of what quantity becomes definite in the chosen context.

In section 4 it was concluded that decoherence cannot alone provide answers to questions of how to understand the quantum formalism, as it is itself a feature of the formalism. If decoherence is to have any explanatory power, it must first be coupled with an understanding of what the density matrix, and in particular the reduced density matrix, represents. Due to the many similarities between decoherence and Bohr's ideas, I have argued that a natural candidate for such an understanding is Bohr's. In a Bohrian picture, the density matrix can generally be considered to represent the inter-contextual state of a system. This is in Bohr's view a symbolic, abstract notion, which provides physical content in the form for statistical prediction for specific observables (represented by different bases) relative to specific contexts. This physical content can in "Bohrian decoherence" be associated with the decohered reduced density matrix, which is obtained as a result of measurement-like entanglement with another system and a type of cut, which determines what is the object of measurement. Thus, the reduced density matrix can be said to correspond to a phenomenon in the Bohrian sense, i.e. a description of some feature of the system relative to some context in which that feature can be regarded as well-defined and separable. In this picture, decoherence becomes the process of "becoming a phenomenon", i.e. going from the symbolic quantum state to the contextual mixture, in a basis given by the context, that is, by the correlations of the system with the classical reference.

Rovelli has the following to say about how Bohr's understanding of quantum mechanics would be translated into his relational account of quantum mechanics,

> Unitary evolution requires the system to be isolated, which is exactly what ceases to be true during the measurement, because of the interaction with the observer. If we include the observer into the system, then the evolution is still unitary, but we are now dealing with the description of a different observer. [Rovelli, 1996, p. 33]

He further adds that the apparent break with the unitary evolution of the Schrödinger equation is due to the interaction of the system with something *not taken into account*. Even without accepting Rovelli's version of quantum mechanics, I will argue that this observation is a good description of Bohr's views on the reference frame dependence or contextuality of quantum mechanics, as well as of decoherence seen in the Bohrian light. The concept of phenomena is exactly the consequence of Bohr's observation of the fact that when viewed from something which takes part in an interaction (as a measurement must do), one cannot get the "full picture", and due to the interaction leading to entanglement and non-separability, this will have an incontrollable effect on the measurement outcome, thus making what is observed depend on the situation in which it is observed. In Bohrian decoherence this comes to be represented by the tracing out of the measurement instrument which results in the state becoming a mixture. That is, the non-unitary evolution comes from the tracing out of subsystems to obtain the reduced density matrix, which embodies the necessity of observing things from a "reference frame". Therefore the unitary evolution is preserved in the total system. Rovelli talks of this total system as being described by a second observer. In a strictly Bohrian decoherence-sense the total state could be seen as a symbolic object, rather than being a description from a different observer, but to conduct measurements on this state, changes in the setup must be made to give entanglement with another measurement instrument. Therefore, Bohr argues that accounting for the interaction between a measured object and the measurement instrument is a new phenomena from that which is observed with the measurement instrument, and that a system cannot be both object and instrument of measurement at the same time. The quote above, however, also return to the difference in what is traced out in decoherence and what is disregarded in Bohr's interpretation. With Bohr, the "cut" corresponds to the reference being classical which can only be the case with respect to one observable at the time, while in decoherence, a subsystem as a whole is traced out.

7 Quantum Erasers: An analysis in terms of Bohr and decoherence

As the field of quantum mechanics develops, new phenomena are discovered, which were not known and therefore not considered in the early days of quantum mechanics when discussions of interpretational matters were at their highest. Such new developments in physics can therefore shed new light on interpretational questions, and might prompt considerations from different angles to those from which different interpretations were developed. Quantum erasers is such a new phenomenon. The main idea is, that which-way information, that destroys interference patterns, can be stored in a qubit and later erased, thus enabling the recovery of the interference. Apart from being an impressive piece of physics, this raises interesting questions on how to understand the relationship between different quantum phenomena. In this section, the theory behind quantum erasers as well as some experimental realisations will be presented, after which the phenomenon will be analysed in terms of the Bohrian reading of decoherence, and, primarily for the purpose of comparison, through Bohmian mechanics, the Everett interpretation and GRW Theory.

7.1 The physics of quantum erasers

The idea of quantum erasers is to recover quantum interference by erasing information, stored in the state of a second system (a qubit), which otherwise destroys the interference. As an example, one can look at single photons in a Mach-Zehner interferometer, as the one depicted in figure 3 [Gerry and Knight, 2005, p. 143]. The state of a photon send through such an interferometer can, between the two beamsplitters, be found with the use of input-output relations of annihilation operators in 50/50 beamsplitters.

Figure 3: A Mach-Zehnder interferometer. Light is send through a 50/50 beamsplitter (BS1), along an upper and a lower path of equal length, to the latter of which is added a variable phase ($\delta\phi$). The paths coinside at a second beamsplitter (BS2) after which coincidence counting is possible along either path (detector 1 or 2).

These are given by,

$$\hat{a}_2 = \frac{1}{\sqrt{2}}\left(\hat{a}_0 + i\hat{a}_1\right) \tag{7.1}$$

$$\hat{a}_3 = \frac{1}{\sqrt{2}}\left(i\hat{a}_0 + i\hat{a}_1\right) \tag{7.2}$$

$$\hat{a}_0 = \frac{1}{\sqrt{2}}\left(\hat{a}_2 - i\hat{a}_3\right) \tag{7.3}$$

$$\hat{a}_1 = \frac{1}{\sqrt{2}}\left(-i\hat{a}_2 + \hat{a}_3\right) \tag{7.4}$$

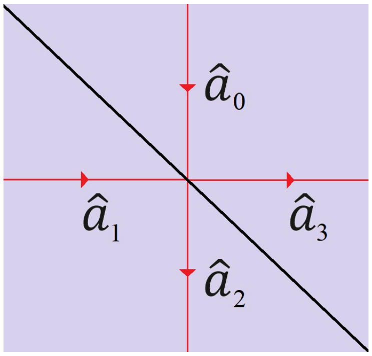

Figure 4: A beamsplitter with the annihilation operators of different modes marked.

where the numbering of the operators follows the notation in figure 4 [Gerry and Knight, 2005, p. 139]. An input state corresponds to $|0\rangle_0|1\rangle_1 = \hat{a}_1^\dagger|0\rangle_0|0\rangle_1$ when expressed as a number state. To find the state after the first beamsplitter, equation 7.4 is used. Furthermore the additional phase $\delta\phi$ is added to the upper path. This gives the state of the photon as

$$\hat{a}_1^\dagger|0\rangle_0|0\rangle_1 \rightarrow \frac{1}{\sqrt{2}}\left(i|1\rangle_2|0\rangle_3 + \mathrm{e}^{i\,\delta\phi}|0\rangle_2|1\rangle_3\right) \tag{7.5}$$

$$= \frac{1}{\sqrt{2}}\left(i|\mathrm{lower}\rangle + \mathrm{e}^{i\,\delta\phi}|\mathrm{upper}\rangle\right) \;, \tag{7.6}$$

where a change of notation, $|1\rangle_2|0\rangle_3 = |\mathrm{lower}\rangle$ and $|0\rangle_2|1\rangle_3 = |\mathrm{upper}\rangle$, has been made in the second line. This is a pure state with non-zero interference terms, as can be seen by looking at the density matrix,

$$\rho = \frac{1}{2}\left(i|\mathrm{lower}\rangle + \mathrm{e}^{i\,\delta\phi}|\mathrm{upper}\rangle\right)\left(-i\langle\mathrm{lower}| + \mathrm{e}^{-i\,\delta\phi}\langle\mathrm{upper}|\right) \tag{7.7}$$

$$= \frac{1}{2}\Big(|\mathrm{upper}\rangle\langle\mathrm{upper}| + |\mathrm{lower}\rangle\langle\mathrm{lower}| + \mathrm{e}^{-i\,(\delta\phi-\frac{\pi}{2})}|\mathrm{lower}\rangle\langle\mathrm{upper}| + \mathrm{e}^{i\,(\delta\phi-\frac{\pi}{2})}|\mathrm{upper}\rangle\langle\mathrm{lower}|\Big)\,. \tag{7.8}$$

Here the off-diagonal terms ($|\mathrm{lower}\rangle\langle\mathrm{upper}|$ and $|\mathrm{upper}\rangle\langle\mathrm{lower}|$) are dependent on $\delta\phi$. This interference is measured by either detector when varying the phase, $\delta\phi$, given to the upper path, as the probability of the photon being detected in either detector follows

$$P\begin{pmatrix} D1 \\ D2 \end{pmatrix} = \frac{1}{2}(1 \mp \cos(\delta\phi))\,. \tag{7.9}$$

Sending several single-photons through the interferometer while varying the phase added to the upper path should thus show this interference between $|\mathrm{upper}\rangle$ and $|\mathrm{lower}\rangle$ as a coincidence count in either detector which varies cosinusoidally with respect to the applied phase. [Gerry and Knight, 2005, p. 143-144]

If, however the state becomes entangled with e.g. a qubit, through an interaction that causes the spin-z of the qubits to change, $|\mathrm{lower}\rangle|\uparrow\rangle \rightarrow |\mathrm{lower}\rangle|\uparrow\rangle$ and $|\mathrm{upper}\rangle|\uparrow\rangle \rightarrow |\mathrm{upper}\rangle|\downarrow\rangle$, one get the following state,

$$\frac{1}{\sqrt{2}}\left(i|\mathrm{lower}\rangle|\uparrow\rangle + \mathrm{e}^{i\,\delta\phi}|\mathrm{upper}\rangle|\downarrow\rangle\right) \;. \tag{7.10}$$

The reduced density matrix of this state will be diagonal, with no interference terms:

$$\rho_{\text{photon}} = \frac{1}{2}(|\text{lower}\rangle\langle\text{lower}| + |\text{upper}\rangle\langle\text{upper}|)\,. \tag{7.11}$$

The state of the qubit could however, be expressed in another basis, such as the spin in the x-direction. The basis is changed using $|\rightleftarrows\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$, thus giving another expression of the state in equation 7.10:

$$\frac{1}{2}\left(\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle + i|\text{lower}\rangle\right)|\rightarrow\rangle + \left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle - i|\text{lower}\rangle\right)|\leftarrow\rangle\right) \tag{7.12}$$

Calculating the reduced density matrix of the photon gives,

$$\begin{aligned}
\rho_{\text{photon}} = \frac{1}{4}\,\mathrm{Tr}_{\text{qubit}}\Big(&\Big(\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle + i|\text{lower}\rangle\right)|\rightarrow\rangle + \left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle - i|\text{lower}\rangle\right)|\leftarrow\rangle\Big) \\
&\cdot\Big(\left(\mathrm{e}^{-i\,\delta\phi}\langle\text{upper}| - i\langle\text{lower}|\right)\langle\rightarrow| + \left(\mathrm{e}^{-i\,\delta\phi}\langle\text{upper}| + i\langle\text{lower}|\right)\langle\leftarrow|\Big)\Big) \qquad (7.13) \\
= \frac{1}{2}\Big(&|\text{upper}\rangle\langle\text{upper}| + |\text{lower}\rangle\langle\text{lower}| + (i\mathrm{e}^{i\,\delta\phi} - i\mathrm{e}^{i\,\delta\phi})|\text{lower}\rangle\langle\text{upper}| \\
&+ (i\mathrm{e}^{-i\,\delta\phi} - i\mathrm{e}^{-i\,\delta\phi})|\text{upper}\rangle\langle\text{lower}|\Big) \qquad (7.14) \\
= \frac{1}{2}\Big(&|\text{upper}\rangle\langle\text{upper}| + |\text{lower}\rangle\langle\text{lower}|\Big)\,. \qquad (7.15)
\end{aligned}$$

The interference terms are zero in this state as well. This makes sense, as nothing has been done except a basis change, and therefore measurement outcomes, such as seeing or not seeing interference in the detector as $\delta\phi$ is varied, should not change. If, however, one organises the data according to the spin of the qubit in the x-direction and only looks at one of the two outcomes, e.g. if one discards all the data where the spin of the qubit was found to be ← and only keeps those datapoints where the spin was measured as →, interference will be seen, as the (not normalised) reduced density matrix will in this case become,

$$\begin{aligned}
\rho_{\text{photon},\rightarrow} = \Big(&|\text{upper}\rangle\langle\text{upper}| + |\text{lower}\rangle\langle\text{lower}| + \mathrm{e}^{i(\delta\phi - \frac{\pi}{2})}|\text{upper}\rangle\langle\text{lower}| \\
&+ \mathrm{e}^{-i(\delta\phi - \frac{\pi}{2})}|\text{lower}\rangle\langle\text{upper}|\Big)\,. \qquad (7.16)
\end{aligned}$$

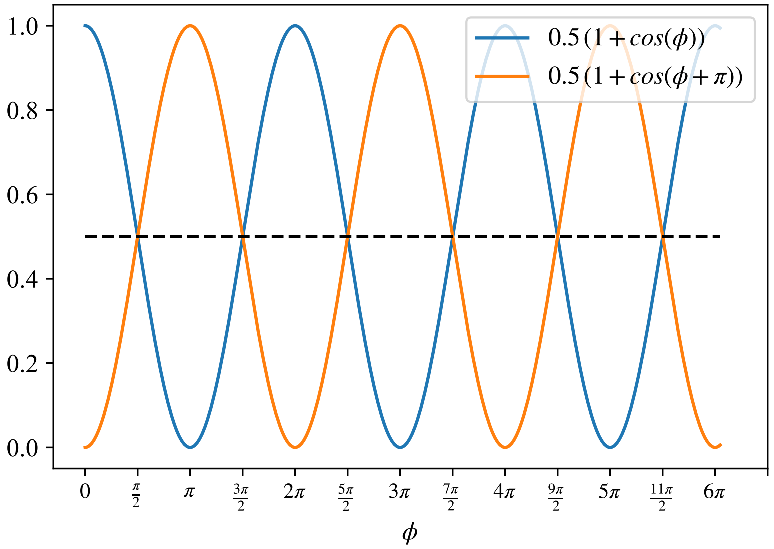

Figure 5: Cosine functions with a relative phase equal to π.

Similarly, for the other spin,

$$\rho_{\text{photon},\leftarrow} = \Big(|\text{upper}\rangle\langle\text{upper}| + |\text{lower}\rangle\langle\text{lower}| + \mathrm{e}^{i(\delta\phi+\frac{\pi}{2})}|\text{upper}\rangle\langle\text{lower}| + \mathrm{e}^{-i(\delta\phi+\frac{\pi}{2})}|\text{lower}\rangle\langle\text{upper}| \Big) . \tag{7.17}$$

Simply removing part one part of a state contradicts the lineary evolution in quantum mechanics, as Meehan points out when talking of state preparation [Meehan, 2019, p. 9]. But here we are not really talking of preparing a state, only about organising data.

The interference of these reduced density matrices is not visible, when looking at both spins together, since the interference pattern of one is shifted by π with respect to the other. This can be seen by a comparison of the off-diagonal terms in the two density matrices; one has $\delta\phi + \frac{\pi}{2}$ and the other $\delta\phi - \frac{\pi}{2}$ in the exponents. Thus, the cosine functions exactly annihilates one another, and the probability of detection of a photon in either detector is $\frac{1}{2}$ regardless of the phase added to the upper path. This is illustrated in figure 5, where two functions such as the one giving the probability of detection in one of the detectors, with a relative phase of π are shown.

This shows that, by performing a measurement on the qubit in a basis orthogonal to that of the basis states of the entanglement, one can recover interference, that was otherwise not present. This is called quantum erasure, as the qubit, when measured in the right basis, would provide information about which path of the interferometer, the photon took; information which is lost upon the measurement in the orthogonal basis. Thus, one can get the interference by giving up on the which-way information or gain which-way information without seeing the interference. It is however not possible to have both which-way information and interference. This exactly corresponds to conclusions drawn from ordinary quantum interferometers or the double slit experiment. There is however a major difference, as the choice between interference or which-way information can be made long after the photon actually went through the interferometer. This is due to the fact that it is made upon choice of basis for the measurement on the qubit, rather than being dictated by the experimental setup, i.e. whether a detector is placed or not in one of the arms. This aspect is what is known as *delayed choice*, and the type of quantum eraser which includes this, is therefore known as a *delayed choice quantum eraser* [Kim et al., 2000].

Different experimental realisations have been made of quantum erasers and delayed choice. An experiment [Kim et al., 2000] was performed in 2000, which constituted a delayed choice quantum eraser, where the choice was made randomly. The experiment involved two entangled photons emitted either from one atom or another. The photons travel through different paths towards different detectors in such a way, as to enable either detection of interference in photon 1, in case no information about which atom the photons were emitted from is gain by a measurement on photon 2, or vice versa. As mentioned, the choice between which-way information and interference is made randomly according to which detector the second photon is detected by. Due to the placing of the detectors, this can be made after the registration of the first photon, thus incorporating the element of delayed choice [Kim et al., 2000].

Two other experiments has demonstrated the delayed choice phenomenon; in [Jacques et al., 2007] with photons and in [Manning et al., 2015] with atoms. Both these uses Mach-Zehnder interferometers of different configurations to get interference or which-way information respectively. In [Jacques et al., 2007] single photons are sent into a Mach-Zehnder interferometer. The interferometer can be open (fiugre 3 with the second beamsplitter removed), where detection provides information about which path was taken by the photon but no interference pattern is seen over many counts as each path becomes correlated with a detector,

$$\frac{1}{\sqrt{2}}(i|\text{lower}\rangle|\text{D2 clicks}\rangle + \mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle|\text{D1 clicks}\rangle)\,. \tag{7.18}$$

It can also be closed (figure 3 with the second beamsplitter in place), where interference is seen over many counts but no which-way information is obtainable due to the presence of the second beamsplitter. The choice of configuration is made by a random number generator and is spatially separated from the point where the photon enters the interferometer, i.e. passes through the first beamsplitter. This is therefore a realisation of the delayed choice gedankenexperiment. This is not really a quantum eraser as nothing is erased, but it illustrates the same point; that the "choice" between phenomena can be made after the particle enters the interferometer and thus would choose one path or not.

The experiment in [Manning et al., 2015] is an atomic analouge of [Jacques et al., 2007]. There are several differences when using atoms rather than photons. The time in which to make the delayed choice is longer as the atoms travel slower than photons, and they couple more strongly to the environment and to gravitational fields. The latter of these circumstances, results in that "an atom can be thought of as a more classical particle than a photon" [Manning et al., 2015]. The experimental setup is made to be equivalent to the open and closed configurations of the Mach-Zehnder interferometer used in [Jacques et al., 2007]. Rather than different arms of the interferometer, the atom can be in two different momentum modes. The state of the atom, in the basis of these modes, are manipulated using π- and $\frac{\pi}{2}$-pulses corresponding to mirrors and beamsplitters respectively. The choice of configuration is also here made by a random number generator, which decides whether or not the second $\frac{\pi}{2}$-pulse is to be applided, and thus whether a phase-dependence is detected in the probability of the atom being in a specific momentum mode or not. This choice is not spatially separated (as was the case in [Jacques et al., 2007]), but is delayed in time relative to the atom "entering the interferometer".

These experiments show that if having no interference is a result of passing through one arm of the interferomter only, while interference is a result of passing through both, the choice of passing through one arm or through both arms is made after the atom begins its travel down the arm(s).

7.2 In Bohmian mechanics, the Everett interpretation and GRW Theory

In Bohmian mechancias the particle position is always definite, and the particle therefore goes through one arm of the interferometer only. But as the motion is guided by the wavefunction, which has phase-dependence, the probability of the particle reaching a specific detector shows interference with respect to the phase, $\delta\phi$. At the first beamsplitter, where the wavefunction becomes a superposition of being reflected and transmitted, the particle trajectory is determined by the exact location of the particle relative to the orientation of the beamsplitter. This is analogous to the Bohmian account given of the Stern-Gerlach apparatus in section 3.1.1. If the particle position is not measured, the wavefunction will interfere as the applied phase is varied, meaning that at the second beamsplitter, the the reflected (and transmitted) part of the wavefunction oscillates with the phase. If e.g. $\delta\phi = \pi$ the reflected part of the wavefunction is zero, and the particle will be transmitted, as its motion is guided by the wavefunction. In an interferomter, where the change in the definite particle trajectory takes place at the beamsplitters, the wavefunction comes to determine whether the particle is reflected or transmitted.

If the which-way information, i.e. the particle position, is measured the interference disappears. This can only be due to a change in the wavefunction made by the measurement, and is an expression of the Heisenberg-like understanding of the uncertainty relations in Bohmian mechanics: if the position is measured, the wavefunction is disturbed [Albert, 1992b, p. 164-165]. The particle becomes entangled with the qubit according to the state in equation 7.10. In Bohmian mechanics, a two particle system will have a position in a six-dimensional space, giving the coordinates of both particles (three coordinates each). If the qubit is at a position, correlated with e.g. $|\text{upper}\rangle$, this means that the two-particle system is located at a point in this six-dimensional space, where the $|\text{lower}\rangle$-part of the wavefunction is zero, and thus has no influence on the change in the particle's position [Albert, 1992b, p. 156-157]. But as I se it, this only occur if the spin of the qubit is correlated with its position by e.g. sending it through a Stern-Gerlach apparatus (i.e. a "spin measurement"). Antoher possibility is that as the particle trajectory is definite, the wavefunction of the qubit will also be definite. It will either have measured one thing or the other, and its wavefunction will therefore not be a superposition, but in an eigenstate. Therefore one of terms in the entangled wavefunction in 7.10 becomes zero, even though they are not correlated with different positions of the qubit. I consider the measurement performed by the qubit to be a useful thing to consider from a Bohmian point of view, as it is a maeaurement the outcome of which is not represented by a positional configuration - at least not until later (when its "spin" is "measured" in either basis). But it is a point better addressed by people with a greater knowledge of Bohmian mechanics.

Erasing the information stored in the qubit by "measuring the spin" in an orthogonal basis, will change the wavefunction of the qubit, and thus of the total two-particle system. This would, in turn, change the trajectory of the particle going through the interferometer (or rather, the second beamsplitter). But as this takes place after the particles has gone through the interferometer and reached the detectors, does this mean that the change in behaviour is transmitted backwards in time? Bohmian mechanics avoids the retrocausality caused by associating interference with a "choice" of going through both arms made upon entering the interferometer, as the particle trajectory is definite regardless of the configuration of the interferometer. But whether there is interference or not is determined by the wavefunction as the particle reaches the second beamsplitter, and it could seem like this must be changed by the future event to account for the interference after the erasure. On the other hand, this interference is found in the existing data, and can thus be said to be uncovered rather than created upon the erasure. As such, it might be due to correlations in the data with the qubit wavefunction. I will leave this as a question for people with more expert knowledge of Bohmian mechanics than myself to answer. As will be apparent also from the account made by the other interpretations, I consider quantum erasers to be a subject on which a discussion with the input of different interpretations would be highly beneficial.

As in Bohmian mechanics, there is no question in the Everett interpretation, of the particle passing through either one or both arms. In the Everett interpretation the universe contains both it passing through one and the other arm as described by the two terms in the wavefunction of the particle in the interferometer.

In the Everett interpretation, the argument for retrocausality implied by the delayed choice does not work, as the particle does not "choose" a specific kind of behaviour upon entering the interferometer. It passes through both arms regardless of the configuration. In the measurment of which-way information with the open configuration, the particle becomes entangled with the detector in a way which correlates each arm of the interferometer with a detector as shown in equation 7.18, giving rise to two branches: one in which detector one click, and one in which detector two click. Interference is observed if the second beamsplitter, which ensures that this correlation is not created, is in place. In the case of the quantum eraser, where the which-way measurement is done by entanglement with a qubit, the same explanation applies. The state of the system is given by one wavefunction describing the particle and whatever it is entangled with (i.e. the qubit), which always evolves according to the Schrödinger equation. The entanglement with the qubit, induces the branching of the wavefunction, with branches for the upper and the lower path, which do not interfere.

How this is formalised in terms of decoherence will be discussed more closely in the following section. Here I will just mention, that if decoherence is the basis for branching in the Everett interpretation, then the branching can be reversed. When a state has decohered into the environment, this is extremely unlikely, but as the decoherence in a quantum eraser is just due to the entanglement with a single particle, the qubit, it is feasible to make a measurement of this particle, which destroys the entanglement, and thus restores coherence and therefore the interference patterns. In other words, there is no practical irreversibility when only one particle, rather than a macroscopic measurement instrument, is involved. By performing a measurement on the qubit in a basis different from that of the entanglement (and branching), one can see different parts of the wavefunction; e.g. a part with no branching resultning from the interaction with the qubit.

This emphasises that the branches in the Everett interpretation cannot be viewed as different worlds in any sense of the word. What occurs in the case of the quantum erasers has nothing to do with reversing a splitting of the world, or even of bringing two branches back together. Rather, the measurement on the qubit allows one access a different part of the entangled state, where the interference remains. We can thus pick different bases of branches and get results accordingly. Since decoherence occurs upon measurement, one can, by measuring the qubit in another basis than that of the which-way-correlation, get another set of branches, relative to which there is interference. In my opinion, quantum erasers bring forward the uncertainty about what exact physical meaning the term "branch" is intended to convey. The Everett interpretation, in my opinion, comes to resemble Bohr's understanding a lot, as the "branches" seems to be an expression of how different phenomena (e.g. interference or which-way information) comes to be well-defined relative to different bases, or observables.

In GRW theory a collapse accounts for the disappearance of quantum interference, and therefore one might suspect that it is less resistant against the conclusion of retrocausality than e.g. the Everett interpretation. If one looks at the delayed choice experiments, where the configuration of the interferometer is chosen after the particle has entered it, however, I do not think that there is grounds for resorting to retrocausality. In GRW theory a collapse (most likely) occur upon entanglement with a system consisting of a great number of particles, such as the detectors in figure 3, and until then the state of the particle is described completely by its wavefunction, which goes through both arms of the interferometer. If the configuration is open, $|\text{upper}\rangle$ and $|\text{lower}\rangle$ becomes correlated with the two detectors as described in equation 7.18, it will upon interaction with the macroscopic detectors collapse into a configuration corresponding to a click in one of the detectors. The particle trajectory can be said to collapse into either $|\text{upper}\rangle$ or $|\text{lower}\rangle$. There is no interference, as relative to one detector clicking, there is only an eigenstate. In the closed configuration the entanglement with the detectors is different, so that the wavefunction corresponding to a click in one of the detectors, and thus the probability of a localisation in this configuration, oscillates with the phase. One would in GRW theory not expect a collapse to happen before the point of entanglement with the macroscopic detectors, and therefore the fact that the configuration is decided after the particle enters the interferometer does not become problematic.

In the quantum eraser case, however, the "change of configuration" is made *after* the particle has reached the detectors and been localised at a definite position, and this situation appears to be more problematic for GRW theory.

But firstly, the fact that the which-way information is here given in an entanglement with a qubit is a problem in itself. In GRW Theory, a collapse would not be very likely to happen upon entanglement with the a single particle, and therefore there would not be any reason for a collapse unless the which-way information is measured with a macroscopical instrument. So, if the qubit is just stored without entangleling the information with more particles, one would expect interference at the end of the interferometer, as if the qubit had not been there. I can not tell how one would account for the disappearance of the interference in such a case.

Setting this problem aside, the question of retrocausality arises. If the lack of interference is associated with the particle collapsing into a definite trajectory before reaching the second beamsplitter, while interference is associated with no such collapse, then it seems like GRW must have retrocausality to account for the later measurement on the qubit, allowing one to obtain interference patterns in the data. However, perhaps the interference found in this manner might be explained by correlations in the position of the collapses, if the location of the collapse also depends on the wavefunction in other bases of the entangled total wavefunction. I will leave this as a question for people with more expert knowledge of GRW theory.

7.3 In Bohrian decoherence

In their article, Manning et. al., write that their experiment “confirms Bohr’s view that it does not make sense to ascribe the wave or particle behaviour to a massive particle before the measurement takes place” [Manning et al., 2015], thus alluding to the fact that their experiments become problematic if one assumes that a particle must either pass through one or both arms of an interferometer and follow interference patterns or not accordingly. Whether this can really be said to be Bohr’s view is a topic of discussion in this section. Or rather; the implications of quantum erasers for what can be understood by “measurement” (or, in the terminology of this thesis, the context) in Bohr’s view will be investigated. As accounted for in section 5, Bohr argues that concepts of physical description can only be assigned to objects relative to classically described contexts. If these contexts are associated with measurement situations, one arrives at something like the above quote. As will be seen, quantum erasers problematise some ways of viewing this context. Firstly, the view that the particle either goes through one or both arms of the interferometer upon entering it depending on its configuration is contradicted by quantum erasers if one does not want to invoke retrocausality. This is what Manning et. al. alludes to, when writing that the particle can not either have a well-defined trajectory or wave behaviour when passing through the interferometer.

Jaques et. al. also refer to the kind of view expressed by Manning et. al., when writing that delayed choice experiments serve to rule out attempts at understanding complementarity as explained by a particle receiving information, upon entering an interferometer, about the setup which it is about to encounter, and behaving either as a particle, going through one arm, or a wave going through both, accordingly. [Jacques et al., 2007] This is the type of view which from delayed choice quantum erasers must conclude that there is retrocausality in quantum mechanics, as the setup is only determined after the particle has entered the interferometer, and the information, on which the particle’s behaviour was determined, must therefore have passed backwards in time. Jaqeus et. al. calls such as view a “too naive interpretation of quantum mechanical complementarity” [Jacques et al., 2007], thus agreeing with Manning et. al. that it is not Bohr’s view. Based on the discussion on Bohr in section 5, I would agree. This understanding of complementarity seems to suggest some sort of deterministic process in the interaction between the instrument and object of measurement, which is definitely not what Bohr is talking about. I think, Bohr's answer to the conclusion of retrocausality would be something like his answer to the EPR-paradox:

> Of course there is in a case like that just considered [the EPR-paradox] no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system. [Bohr, 1935, p. 700]

It is clear that Bohr does not consider complementary to be explained by one part of the system communicating via some sort of signals to the other, and thus determining its change in behaviour. Rather, the change in behaviour is due to a change in the circumstances, which constitutes a new phenomena where other variables can be well-defined. Though this kind of view may be hard to understand, it saves Bohr's interpretation from retrocausality in the form of signals going back in time and changing the behaviour of the particle as it passes through the interferometer.

As mentioned above, Manning et. al. associates Bohr with an idea of particle-like or wave-like behaviour not being ascribable prior to a measurement, thus implying that it becomes applicable upon measurement. I think, it would capture Bohr's ideas better to say that the particle-like or wave-like behaviour becomes applicable relative to measurement contexts in which the corresponding classical concepts (in this case position and momentum) can be well defined. Bohr's notion of phenomena therefore includes a reference to such a context, as mentioned in section 5.2.1. But this is a point where quantum erasers becomes relevant for the understanding of what these contexts are. It is e.g. important element in Bohr's interpretation that complementary variables cannot be well-defined relative to the same context:

> [...] their [refering to the wave-like or of the particle-like behaviour of light] closer analysis in mechanical terms demand mutually exclusive experimental

> arrangements. [Bohr, 1933a, p. 5]

That the experimental setup needed to study respectively wave-like and particle-like behaviour (and thus the contexts in which they are well-defined) exclude one another, could seem to be challenged by quantum erasers, where it appears as though both particle-like and wave-like behaviour can be observed using the same measurement arrangement, or, in other words, the same data. This therefore seems to be in conflict with Bohr's complementarity, where the use of different concepts requires a reference to different, mutually exclusive experimental setups. Bohr, however, emphasises the need for including all relevant parts of the experimental setup. As the state of the particle going through the interferometer becomes entangled with the qubit, this qubit, and whatever measurements are performed on it, must be taken into account when describing the phenomena. Thus, measuring the state of the stored particle in another basis to that in which the correlations with the interferometer-particle are (i.e. erasing the which-way information), constitutes a new phenomenon from that in which no measurements are performed on the qubit, or where the measurement is performed in the correlation basis (i.e. keeping the which-way information). It still holds that within one phenomenon only one concept can be well-defined; in one setup (which-way information is measured) the position is well-defined, while in the other (which-way information is erased) it is not. One can either have which-way information or interference, never both.

But the change in phenomena was made after the particles in the interferometer have reached a detector. The different behaviour is observed *in the same data.* This appears to suggest that another, complementary phenomena is somehow hidden in the first; the interference, which can be observed after the erasure of information, is present in the the particle-like observations. It appears like the complementary type of behaviour is always there, but is inaccessible. This, I think, puts another light on Bohr and might make his ideas appear less like contextual realism. In quantum erasers, the behaviour of the particle, the physics, has not been changed by the change in phenomena, only what patterns we can see in the data. This indicates that the contexts are to be viewed in a more epistemological way. Bohr's phenomena becomes something else than what physical behaviour appears relative to different physical systems. This is what e.g. Rovelli suggests is the right way in which to view quantum mechanics [Rovelli, 1996]. Quantum erasers at least seems problematic in a Rovelli-like interpretation of Bohr, though I will not exclude the possibility that Rovelli might give a satisfactory explanation of quantum erasers with a terminology different from Bohr's. These considerations of the context in the light of quantum erasers, does not mean that anything "un-physical" is going on or that our choices or states of mind in anyway effects the physical reality. Importantly, the state of the particles is not changed by the change of phenomena, as the data remains the same. What quantum erasers can be taken to show is that Bohr's "reference frames" cannot simply be set equal to a physical system.

The delayed choice experiments, though under some assumptions illustrating the same thing as quantum erasers (that if one holds the view that a particle either "chooses" to go through one arm or both, one must have retrocausality), does not have the same power in challenging and understanding of Bohr's phenomena as being purely physical. Different phenomena corresponding to the different configurations of the interferometer occur, and the fact that the configuration is only decided once the particle is in the interferometer does not seem to me to make much difference. If it did, it would be to one seeking a visualisation of the particle or, in other words, a deterministic account of how the phenomena is obtained. One is back at the idea of signals being passed to the particle about the setup it is about to encounter, and thus changing its behaviour, which is, as argued, not a correct description of Bohr's opinion. Delayed choice experiments are thus addressed by the same arguments for there not being retrocausality in quantum erasers, and as there is no question of recovering hidden interference in data already collected, they do not give rise to the above mentioned consequences for the Bohrian context.

Returning to quantum erasers, it is again emphasised that reference frames are tied to a physical context, and definitly not a state of mind, by the fact that the interference is destroyed already upon entanglement with the qubit, and not upon a measurement of the qubit, which would allow a person to access the knowledge of the path taken through the interferometer. The choice of observable, or stated differently the object-instrument cut, is thus being made by physical interactions of the system; with the qubit measuring the which-way information, the position of the particle becomes well-defined, and in this setup there can be no interference between the eigenstates corresponding to the different paths through the interferometer. The entanglement is a necessary condition for such a measurement, but as Bohr ccaptures in the doctrine of classical concepts (section 5.2.2), a further condition for a measurement is treating the measurement instrument (the qubit) as a classical, i.e. separable, reference in order for the position of the particle to be well-defined.

This is where, according to section 6, decoherence can come in as a formalisation of Bohr's phenomena. With the entanglement with the qubit, the reduced density matrix of the particle in the interferometer, corresponding to the contextual state of the particle in the Bohrian interpretation, is given as:

$$\begin{aligned}\rho_{\text{photon}} = \mathrm{Tr}_{\text{qubit}}\Bigg(&\frac{1}{\sqrt{2}}\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}]\rangle_{\text{photon}}|{\downarrow}\rangle_{\text{qubit}} + i|\text{lower}\rangle_{\text{photon}}|{\uparrow}\rangle_{\text{qubit}}\right)\\ &\cdot\frac{1}{\sqrt{2}}\left(\mathrm{e}^{-i\,\delta\phi}\langle\text{upper}|_{\text{photon}}\langle{\downarrow}|_{\text{qubit}} - i\langle\text{lower}|_{\text{photon}}\langle{\uparrow}|_{\text{qubit}}\right)\Bigg) \qquad (7.19)\\ = \frac{1}{2}\Big(&|\text{upper}\rangle\langle\text{upper}| + |\text{lower}\rangle\langle\text{lower}|\Big)\,. \qquad (7.20)\end{aligned}$$

The reduced density matrix of the particle in the interferometer thus goes from being in a pure state to, after the entanglement with the qubit, being in a mixture, as the reduced density matrix has no interference terms. This reduced density matrix can meaningfully be viewed as representing the phenomena in which the particle position is well-defined relative to the context of a which-way measurement. This is also an illustration of the point of the relevance of decoherence, which is not induced by the environment, as the interference is in this case destroyed upon entanglement with only one particle, and not upon entanglement with the environment.

In decoherence, as opposed to descriptions of measurement-like interactions involving a collapse, the possibility of regaining coherence remains. This cannot be done through any local interaction, i.e. by anything one can do with the decohered subsystem, but can happen globally, i.e. if something occurs to the total system, which counteract the entanglement. In general, this is highly unlikely, as the entanglement is usually with macroscopical systems or the environment, which have many degrees of freedom and cannot be isolated from further entanglement with other systems, which then also must be taken into account. But, as mentioned with regards to the Everett interpretation in the previous section, the entanglement in quantum erasers is with one particle only, and it becomes feasible to de-decohere the state. In quantum erasers, there is no question of bringing the state of the particle in the interferometer back to it's pre-measurement coherence by a global interaction with the total system. For one thing, the particles have already been further entangled with the macroscopical detectors, so such a procedure becomes practically impossible. Tanona writes that decoherence is not "undone" by the erasure, as the "Erasure does not really bring back the original interference pattern from the original coherent state but instead allows 'sorting' that uncovers hidden interference patterns" [Tanona, 2013, p. 3633]. One does not get the same coherent state (and thus the same interference) as one would have done if there was no qubit. However, one needs measurements of both subsystems to get the interference fringes, and one looses the which-way information which was gained by the entanglement with the qubit. This, only allows one to arrange the data differently, but still, it enables the recovery of *some* coherence. So though quantum erasers is not a case of "de-decohering" a state, one might say, that one counteracts the decoherence in an epistemological sense.

More precisely, when the qubit is measured in the x-basis, it becmoes entangled with the measurement instrument following a CNOT-interaction, $|{\rightarrow}\rangle_{\text{qubit}}|\alpha\rangle_{\text{ins}} \rightarrow |{\rightarrow}\rangle_{\text{qubit}}|\alpha\rangle_{\text{ins}}$ and $|{\leftarrow}\rangle_{\text{qubit}}|\alpha\rangle_{\text{ins}} \rightarrow |{\leftarrow}\rangle_{\text{qubit}}|\alpha_{\perp}\rangle_{\text{ins}}$, where $|\alpha\rangle_{\text{ins}}$ and $|\alpha_{\perp}\rangle_{\text{ins}}$ are orthogonal states of the measurement instrument. Thus the total state becomes

$$\frac{1}{\sqrt{2}}\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}]\rangle_{\text{photon}}|{\downarrow}\rangle_{\text{qubit}} + i|\text{lower}\rangle_{\text{photon}}|{\uparrow}\rangle_{\text{qubit}}\right)|\alpha\rangle_{\text{ins}} \tag{7.21}$$

$$\begin{aligned}\rightarrow \frac{1}{2}\Bigg(&\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle_{\text{photon}} + i|\text{lower}\rangle_{\text{photon}}\right)|{\rightarrow}\rangle_{\text{qubit}}|\alpha\rangle_{\text{ins}} \\ &+ \left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle_{\text{photon}} - i|\text{lower}\rangle_{\text{photon}}\right)|{\leftarrow}\rangle_{\text{qubit}}|\alpha_{\perp}\rangle_{\text{ins}}\Bigg),\end{aligned} \tag{7.22}$$

and the reduced density matrix of the qubit-photon system becomes,

$$\begin{aligned}\rho_{\text{photon,qubit}} = \frac{1}{4}\Big(&\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle_{\text{photon}} + i|\text{lower}\rangle_{\text{photon}}\right)|{\rightarrow}\rangle_{\text{qubit}} \\ &\quad\left(\mathrm{e}^{-i\,\delta\phi}\langle\text{upper}|_{\text{photon}} - i\langle\text{lower}|_{\text{photon}}\right)\langle{\rightarrow}|_{\text{qubit}} \\ &+\left(\mathrm{e}^{i\,\delta\phi}|\text{upper}\rangle_{\text{photon}} + i|\text{lower}\rangle_{\text{photon}}\right)|{\rightarrow}\rangle_{\text{qubit}} \\ &\quad\left(\mathrm{e}^{-i\,\delta\phi}\langle\text{upper}|_{\text{photon}} - i\langle\text{lower}|_{\text{photon}}\right)\langle{\rightarrow}|_{\text{qubit}}\Big)\,. \end{aligned} \tag{7.23}$$

This state could be called a mixture (in the qubit x-spin basis) of pure, non-entangled states, which each should show interference. Thus one can see the interference when looking only at only one part of the entangled state separately. This is the kind of "state preparation", which Meehan [Meehan, 2019] says is non-unitary; we cannot just remove parts of parts of the state. But as previously mentioned, there is no question of state preparation, only of organising data differently. If the qubit is traced out of the total entangled state, one gets equation 7.20, meaning that the data is the same as if no measurement on the qubit had taken place. This makes sense, as we would not expect a change in data upon a later measurement of the qubit.

Here it becomes apparent that the interference is not created, but uncovered by the measurement on the qubit, and again this pushes the Bohrian notion of complementarity towards being more about what can be accessed than what is. If the interference, or in general phenomena other than that which is measured, exist within the observered, complementary phenomena, it becomes more difficult to say, that it is not physically real in the context where its complementary counterpart is well-defined.

But it is not the same interference one recovers as the one which would be obtained if there was no qubit, and therefore it is not a case of both complementary behaviours being observed at once. The erasure of the which-way information becomes the context of a third phenomena, described in 7.23; a phenomena which does not manifest itself in a change of behaviour of a system, but in a pattern in the data of another phenomena. This is the feature of quantum erasers which makes it difficult to view Bohr's contexts as purely physical systems. Thus the concept of quantum erasers sharpens the suggestion made by the comparison with the reference frames of relativity theory in section

5.2.4; that it is hard to place these contexts in either of the categories *ontological* or *epistemological*.

8 Summary and Outlook

The frequency of questions that have been left open has been increasing throughout this thesis, and all in all there are several loose threads, which would warrant further investigation. Before giving an overview of these, however, I will summarise the contributions of this thesis to the vast and diverse subject of the interpretation of quantum mechanics.

The synthesis of Bohr's ideas and decoherence developed in this thesis is, to the best of my knowledge, more in-depth and detailed than what has been made elsewhere. The synthesis was motivated by a study of the similarities between decoherence and Bohr's interpretation. These include the treatment of classicality as a type of idealisation, the importance of entanglement which extends to macroscopic objects, the concept of an "effective collapse", the application of the Born rule relative to specific interactions (e.g. measurement interactions) and a suggestion of a change in the relationship between measurement instrument and measured system compared with classical physics. As Bohr's ideas and decoherence were analysed in terms of their respective address to the measurement problem, it was also possible to find similarities in their answers to several of the formulations of the measurement problem, such as e.g. the problem of effect and the problem of preferred basis.

Though some differences between decoherence and Bohr's interpretation also exist, I proceeded to use each to solve some of the remaining issues in the other. Decoherence can in many aspects serve as a formalisation of Bohr's concepts, such as the contextual states and the use of the term irreversibility. The concept of mixed contra pure states which are obtained through decoherence decoherence can be thought of in terms of Bohr's notion of context-dependent, well-defined classical states. Bohr's ideas, on the other hand, provides decoherence with a much needed interpretational basis. By associating the reduced density matrix with a Bohrian contextual state, decoherence can be understood in an objective way, which does not depend on choices of what to "look at", while Bohr's fundamental statistics can help reconcile decoherence with the problems of outcomes and statistics. Furthermore, the idea of decoherence having a fundamental place in quantum mechanics, can make sense in terms of Bohr's understanding of contextuality as a form of reference frame dependence.

The second novel contribution of this thesis is the account of quantum erasers made by the different interpretations. Both Bohmian mechanics and GRW theory seem to be challenged by the insight into quantum mechanics which the concept of quantum erasers provide. This implies that more investigation into quantum erasers might force these interpretations into something like retrocausality, as the appearance of interference in both cases would seem to be determined by the physical state of the particle upon reaching the second beamsplitter in the interferometer and upon detection respectively.

This thesis mainly concerns Bohr's interpretation and decoherence, or rather the synthesised Bohrian decoherence, and therefore the main open question might be said to concern the understanding of Bohr's notion of context in the light of quantum erasers. Though the concept of the phenomenon, which includes a reference to the experimental context, goes a long way towards making sense of quantum erasers and avoiding retrocausality, quantum erasers comes to sharpen the problem of how to view the Bohrian context, as it becomes problematic for some ways of talking about it. It is clear the contexts are defined by physical systems and their interaction with one another from e.g. the fact that interference is destroyed upon entanglement with a qubit which is not measured by any human. On the other hand, the fact that interference can be found in data showing no phase-dependence, by a subsequent measurement on this qubit, clearly shows that the context can not be viewed as the effect of a physical system upon the behaviour of another.

I think these issues demonstrate the fruitfulness of including quantum erasers in discussions on interpretational matters. Although I do not have answers to all these questions, I can therefore conclude that quantum erasers would be a good foundation for further investigations into the interpretation of quantum mechanics.

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