Evaluating the Symmetry to Reality Inference: Not All Symmetry Signals Redundancy

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The symmetry to reality inference (*StRI*) is concerned with how fundamental theories in physics describe/represent genuine features of the world. *StRI* asserts that symmetry related models of a theory describe/represent/correspond to the same physical reality. Accepting *StRI* implies that theories in mathematical physics routinely involve redundancy in their descriptive apparatuses since it is a commonplace that the symmetries of field equations/equations of motion are not shared by typical solutions. There is a good motivation for *StRI* in cases that physicists would classify of gauge symmetries; namely, if the gauge variables had worldly counterparts then determinism in guise of a good Cauchy problem for the field equations/equations of motion would fail. It is argued that outside of such cases *StRI* leads to a distorted picture of how theories of mathematical physics represent the physical world. However, the way I propose to draw the line between gauge and non-gauge symmetries can be blurred by clever reformulations of theories. In particular, in some instances it appears to be all too easy to make diffeomorphism invariance look like a gauge symmetry. It seems more productive to discuss such issues than to tilt with the special pleading for *StRI*.

1 Introduction

As construed here the symmetry to reality inference (*StRI*) is concerned with how fundamental theories in physics describe/represent genuine features of the world. *StRI* asserts that symmetry related models of a theory describe/represent/correspond to the same physical reality. In all of the examples considered here the laws of the theory are in the form of differential equations, and a "model" of the theory can be taken to mean a solution of these equations. The *StRI* is accompanied by a principal that can be viewed either as a corollary of $StRI$ or as a motivating posit; namely, P: Only the symmetry invariant quantities in the theory have worldly counterparts, i.e. only these invariant quantities can represent/correspond to real properties of the world. An apparently liberalized version of P appeals to the notion of supervenience.[1] P^*: Only those quantities in the theory that supervene on the symmetry invariant quantities have worldly counterparts. Full throated and unhedged endorsement of $StRI$ and P/P^* is hard to find in the philosophical literature.[2] My strategy here is to take on the full-on versions of $StRI$ and P/P^* and try to determine how much backtracking and hedging is called for.[3]

There is one class of symmetries—gauge symmetries—where physics supplies a clear motivation for $StRI$ and P/P^* and where it is pellucid that and why $StRI$ and P/P^* are correct. Some philosophers want to deploy $StRI$ and P/P^* for what practicing physicists would not count as gauge symmetries, such as Galilean invariance for Newtonian theories or Poincaré invariance for special relativistic theories, where neither the physical theories themselves nor the way they are applied supplies this motivation. Philosophers try to make up the deficit by special pleadings. But while these pleadings lead to interesting issues, they have little to do with Galilean invariance or Poincaré invariance symmetries per; and worse, the attempt to enforce $StRI$ and P/P^* for these non-gauge symmetries lead to a distorted picture of how theories of mathematical physics represent the physical world.

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[1] Roughly the supervenience notion is captured by the slogan that X supervenes on Y just in case no difference in X is possible without a difference in Y. Such supervenience comes in different strengths depending on what sense of possibility is used. Here the relevant sense of possibility would seem to be possible according to the laws of physics.

[2] Dasgupta's endorsement of P is cautious and hedged: "The idea is that if a putative feature is variant in laws that we have reason to think are true and complete, then this is some reason to think that the feature is not real" (Dasgupta 2016, 840). Saunders (2007, 453) identifies spacetime relationism with commitment to what he calls the invariance principle: "only quantities invariant under exact symmetries are real." See also Greaves and Wallace (2014) and Baker (2010) for other endorsements. Baker was once an enthusiast for $StRI$ and P; he now has a much more nuanced view (see Baker 2023).

[3] The focus here is on Newtonian and classical relativistic theories. Quantum theories require a separate treatment.

2 Some reasons for pause

There is something undeniably attractive about $StRI$ and P/P^*: they provide tools philosophers can use to interrogate the symmetry properties of physical theories that, on the assumption those theories are true and complete, would allow the identification of the real properties of the physical world. But before blindly applying these tools take a deep breath and let it out slowly while reflecting on some reasons to hesitate.

One reason for pause is the realization that some equations of motion admit as symmetries a group of transformations that act transitively; that is, for any two solutions there is an element of the group that takes one solution to the other (see Belot 2013). One is reminded here of Hermann Weyl's remark:

> If nature were all lawfulness then every phenomenon would share the full symmetry of the laws of nature as formulated by the theory of relativity. The mere fact that this is not so proves that *contingency* is an essential feature of nature. (1952, 26)

The advocate of $StRI$ may wish to bravely declare the courage of his[4] convictions (and disbelieve his lying eyes), exclaiming "If the theory in question were true and complete then there would be no contingency—all would be lawfulness in the strongest sense that, despite surface appearances, there would be only one physically distinct situation allowed by the laws since, by $StRI$, all of the solutions describe/represent/correspond to the same physical reality." Indeed a brave stance but also one that is potentially unstable since any indication that different solutions correspond to different physics can bring it tumbling down.

Less brazenly, the advocate of $StRI$ may note that examples of the sort under discussion tend to occur for simple "toy" systems and then opine that the examples are unlikely to be repeated for the laws that govern the more complex systems that make up our world. While this may prove to be the case, the usefulness of toy systems in providing a test drive for $StRI$ and P/P^* should not be downplayed.

The symmetries to be discussed below are of the continuous variety. But if $StRI$ works then it should work as well for discrete symmetries, such a time reversal invariance and mirror image reflection. Such an application,

[4] "His" not "her" since only toxic masculinity can produce such boastfulness.

however, has jarring consequences. It would imply, for example, that if the true fundamental physical laws all obey time reversal invariance then not only would it be the case that "running the film of the 20th century backwards" would produce a scenario that is compatible with the laws; but it would also be the case that while it would be true to say that WW II occurred temporally between WW I and the Korean War, it would not be true say that WW I, WW II, and the Korean War occurred in that time order rather than the reverse order. Since, by $StRI$, the time reversed models correspond to the same physical reality, that reality cannot on pain of inconsistency include an earlier-to-later ordering. Again the advocate of $StRI$ may simply declare the courage of his convictions. Alternatively, $StRI$ can be saved from embarrassment by giving an account of how events acquire their time order that does not make the order hostage to $StRI$; or one could opine that although events are perceived to be time ordered the events themselves are not so ordered. Take your choice, and welcome to it.

This is all by way of pot-shotting. Let us begin serious discussion with an example where $StRI$ and P/P^* are transparently correct. Identifying the features that make them correct will help us to see where and how P/P^* can go wrong, bringing $StRI$ into question.

3 Maxwell source-free electromagnetism (on Minkowski spacetime) as a gauge theory

The symmetries of interest here are symmetries of the laws of a theory, assumed to take the form of differential equations. A symmetry transformation is a transformation of the variables in the laws such that solutions of equations are carried to solutions. But not every willy-nilly one-one mapping of the set of solutions onto itself will count as a symmetry transformation in the intended sense. What conditions are necessary and sufficient to capture the intended sense is a difficult question to answer in full generality. Here a partial answer is provided by the decision to focus on laws that are derivable from an action principle so that the laws are the Euler-Lagrange (EL) equations resulting from extremizing the action. So the focus here will be on divergence symmetries—transformations that leave the action invariant up to a total divergence $\partial_\mu X^\mu$ where X^μ vanishes on the boundary of the domain of integration defining the action. Such a symmetry carries solutions of the EL to solutions.[5]

There are more gentle introductions to gauge theories using toy examples. But let's not be timid—let's jump right in with source-free Maxwell theory, which is arguably the simplest among the physically interesting examples of a gauge theory. *Notation*: Greek indices μ, ν,... run from 0 to 3 while Latin indices i, j, ... run from 1 to 3. Raising and lowering of indices is done with the Minkowski metric $\eta_{\mu\nu}$, which in inertial coordinates reads $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$.

Introduce the electromagnetic four-potentials $A_\mu(x) = A_\mu(x^0, x^1, x^2, x^3)$ (or in other commonly used notation $A_\mu(t, \mathbf{x})$) and define the Maxwell field tensor $F_{\mu\nu}$ by

$$F_{\mu\nu} := \partial_{[\mu} A_{\nu]} \tag{1}$$

From this definition we have the Bianchi identity

$$\partial_{[\gamma} F_{\mu\nu]} = 0 \tag{2}$$

The Lagrangian density for the source-free Maxwell field is

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{3}$$

and the action is

$$\mathcal{A}_M(\eta_{\mu\nu}, F_{\mu\nu}) = \int \mathcal{L}_M \sqrt{-\eta} d^4x \tag{4}$$

where $\eta := \det(\eta_{\mu\nu})$. The resulting Euler-Lagrange equations are

$$\partial_\mu F^{\mu\nu} = 0 \tag{5}$$

The equations (2) and (5) are the Maxwell equations governing the source-free electromagnetic field. A model of the theory has the form $(\mathcal{M}, \eta_{\mu\nu}, F_{\mu\nu})$, where $\mathcal{M} \simeq \mathbb{R}^4$ is the manifold on which the Minkowski metric $\eta_{\mu\nu}$ and the Maxwell field tensor $F_{\mu\nu}$ reside and where $F_{\mu\nu}$ satisfies the Maxwell equations.[6]

[5] The reverse inference from symmetry of the EL equations to a divergence symmetry of the action is not valid in general, as illustrated by scaling transformations.

[6] For details see Torre (2020, Ch. 5).

There is a problem with this formulation of Maxwell electromagnetism if the potentials A_μ represent/correspond to real properties of electromagnetic reality: determinism fails in the sense that there is not a good Cauchy problem in terms of the A_μ. The transformations

$$A_\mu \mapsto \overline{A}_\mu = A_\mu + \partial_\mu \Lambda, \tag{6}$$

where $\Lambda(x^0, x^1, x^2, x^4)$ is an arbitrary function of the spacetime coordinates, leave the Maxwell tensor invariant—$\overline{F}_{\mu\nu} = \partial_{[\mu}\overline{A}_{\nu]} = F_{\mu\nu}$. Further, these transformations are divergence symmetries of $\mathcal{A}_M$ and, therefore are symmetries of the Euler-Lagrange (EL) equations. Such transformations are referred to in the literature as "local" or "non-rigid": the former because the transformation can be different at different spacetime locations and non-rigid because the transformation can be chosen to be the identity for $x^0 \leq 0$ but non-identity for $x^0 > 0$. The non-rigidity plus the fact that the transformations (6) are symmetries of the EL equations means that there are solutions of (2) and (5) that have the same Cauchy data for the A_μ—the same values of A_μ and $\partial_0 A_\mu$—at $x^0 = 0$ but different values of A_μ for $x^0 > 0$.

An example of "non-local" (aka "global") or "rigid" transformations are the Poincaré transformations (= inhomogeneous Lorentz transformations) which compose the symmetry group of the background Minkowski spacetime. Because of the rigidity of this symmetry group, Minkowski spacetime is a priori friendly to a good Cauchy problem—when formulated in terms of the right variables.

There is no canonical account in the physics literature of what counts as a gauge theory and a gauge symmetry, but for theories that admit a Lagrangian/Hamiltonian formulation there are two generally accepted markers for gauge freedom. Most authors count a theory as a gauge theory if the Lagrangian admits as a symmetry an infinite dimensional Lie group involving arbitrary functions of the independent variables—in the present example, the spacetime coordinates. As we have seen, this feature makes for a problem with determinism. In addition Noether's second theorem applies and the conservation laws entailed by the infinite dimensional symmetry group are trivial. A conservation law has the form $\partial_\mu N^\mu = 0$ where N^μ is constructed locally from the field variables. The law is said to be "trivial" if I: $\nabla_\mu N^\mu \equiv 0$ or II: $N^\mu = 0$ on all solutions to the EL equations. The conserved Noether current in the source-free Maxwell case is $N^\mu = \partial_\mu F^{\mu\nu}$, and the conservation law is trivial in both senses. The Noether charge Q_Σ associated with any

Cauchy surface Σ of Minkowski spacetime (e.g. the hypersurface $x^0 = 0$ with (x^0, x^1, x^2, x^4) an inertial coordinate system), $Q_\Sigma := \int_\Sigma N^\sigma d\Sigma_\sigma$, is zero for any solution. For a finite dimensional Lie group Noether's first theorem applies and the associated symmetry is non-trivial—in the case of the Poincaré group the entailed conservation laws include conservation of energy and momentum.

Other physicists take as necessary and sufficient for non-trivial gauge freedom the condition that the symmetry group of the Lagrangian implies that the Lagrangian (or more particularly the Hessian of the Lagrangian) is singular (which will be the case if the Lagrangian admits as a symmetry an infinite dimensional Lie group involving arbitrary functions of the independent variables), generating constraints on the canonical momenta in the Hamiltonian version of the theory. The topic of constraints and gauge transformations is fraught with formidable technical and interpretational issues. Even what was once considered a cornerstone of the approach—Dirac's principle that primary first class constraints generate gauge transformations—is open to debate (see Barbour and Foster 2008). I will keep clear of these controversies here, and will not discuss Hamiltonian formulations of theories.[7]

Once gauge is acknowledged, the interpretative move of taking gauge variables as involving descriptive redundancy provides a route to restoring a good Cauchy problem. In the case of source-free Maxwell theory the idea would be that, yes, initial Cauchy data for the gauge variables—the values of A_μ and $\partial_0 A_\mu$ at $x^0 = 0$—fail to fix a unique evolution; but this failure is not indicative of a failure of determinism for source-free electromagnetic fields because solutions of the Maxwell equations (2) and (5) that are connected by a gauge transformation (6) correspond to/represent the same physical process.

This idea can be substantiated in the present case by a constructive degauging of the theory. Begin by noting that the Maxwell field strength tensor $F_{\mu\nu}$ is gauge invariant. Then instead of using the potentials A_μ as the field variables, use the electric and magnetic fields, where in any inertial frame the components of the electric and magnetic fields are given respectively by $E^i := F^{0i}$ and $B^i := \epsilon^{ijk} F_{jk}$ with ϵ^{ijk} the totally antisymmetric tensor with $\epsilon^{123} = +1$. Using these variables equation (5) can be pulled apart into a pair of equations more readily recognizable as two of the source-free Maxwell

[7] This dodge does not work when quantizing classical gauge theories via the route of canonical quantization.

equations

$$\text{div}\,\overrightarrow{E} = 0 \tag{5a}$$

$$\text{curl}\,\overrightarrow{B} - \partial_0\overrightarrow{E} = 0 \tag{5b}$$

Similarly, equation (2) can be separated into another pair of equations more readily recognizable as the other two of the source-free Maxwell equations:

$$\text{div}\,\overrightarrow{B} = 0 \tag{2a}$$

$$\text{curl}\,\overrightarrow{E} + \partial_0\overrightarrow{B} = 0 \tag{2b}$$

Equations (2a) and (5a) impose constraints on the Cauchy data, which now consists of the values of the gauge invariant quantities $\overrightarrow{E}$, $\overrightarrow{B}$ and their time derivatives $\partial_0\overrightarrow{E}$, $\partial_0\overrightarrow{B}$ at $x^0 = 0$. For any such Cauchy data satisfying the constraints at $x^0 = 0$ there is a unique solution to the evolution equations (2b) and (5b) for $x^0 > 0$ and, furthermore, this solution satisfies the constraint equations at each $x^0 > 0$. The Cauchy problem doesn't get any better than this.

In the present example the $StRI$ and P/P^* have a clear and compelling motivation: if P/P^* is rejected for the gauge symmetries of source-free Maxwell then determinism in the form of a good Cauchy problem fails; accepting P/P^* secures determinism and makes $StRI$ transparently valid. However, once we move beyond symmetries that physicists would classify as gauge symmetries $StRI$ and P/P^* become contentious. In effect, the proponents of $StRI$ and P/P^* want to treat other symmetries, such as Galilean invariance for Newtonian theories and Poincaré invariance for special relativistic theories, as gauge symmetries in the sense that these symmetries relate solutions having the same worldly counterparts. They are in for a heap of trouble in promoting such want-a-be gauge symmetries.

4 Want-a-be gauge

Once we move away from symmetries falling under the semi-official definition of gauge symmetries as involving an infinite dimensional symmetry group with arbitrary functions of the spacetime variables, the motivation of avoiding a failure of determinism by seeing redundancy of descriptive apparatus of the theory is lost. Furthermore, as already noted, finite dimensional symmetry groups fall under Noether's first theorem and lead to non-trivial conservation laws. This leaves the proponent of applying the $StRI$ and P/P^* to such symmetries the unpleasant task of explaining how non-trivial conservation laws can follow from descriptive redundancy in the theory.

The untenability of applying P/P^* to Galilean and Poincaré invariance is easily demonstrated. Here we concentrate on Poincaré invariant field theories in the special relativistic context, but similar morals can the drawn for Galilean invariant particle theories in the Newtonian context. Symbolizing a Poincaré transformation by $\mathcal{P} = (a, L)$ where a and L stand respectively for a spacetime translation and a proper Lorentz transformation (rotations and velocity boosts), the action of $\mathcal{P}$ on the Maxwell field tensor is given by

$$^{(a,L)}F_{\mu\nu}(x) = L^{\sigma}_{\mu} L^{\rho}_{\nu} F_{\sigma\rho}(L^{-1}(x - a)) \tag{7}$$

(see Combe and Sorba 1975). If $F_{\mu\nu}(x)$ is a solution to Maxwell's equations then so is $^{(a,L)}F_{\mu\nu}(x)$. But in general $^{(a,L)}F_{\mu\nu}(x) \neq F_{\mu\nu}(x)$. The equality required for invariance under spatiotemporal translation requires that the fields are constant in time and uniform in space, and the invariance under proper Lorentz transformations further narrows the admissible fields—the Lorentz invariants are all functions of the two invariants $\overrightarrow{B}^2 - \overrightarrow{E}^2$ and $\overrightarrow{B} \cdot \overrightarrow{E}$ (see Escobar and Urrutia 2014).

In short, the subset of Maxwell fields exhibiting Poincaré invariance is a very meager set, and presumably the fields one would expect to encounter in a source-free electromagnetic world would rarely belong to this subset.[8],[9] Again one is reminded of Weyl's remark quoted above. Here the point being

[8] What needs to be provided is: a precise characterization of a "generic" solution to the Maxwell equations; a demonstration that the Poincaré invariant solutions are non-generic; and an argument for the assumption that the type of solution one would typically encounter is generic. This is a large promissory note, but I am confident that an able researcher can cash it.

[9] The point applies to what one would expect to encounter on a local basis as well as in the large because Maxwell's equations have the locality property that if the $F_{\mu\nu}$ of the model $(\mathcal{M}, \eta_{\mu\nu}, F_{\mu\nu})$ satisfies Maxwell's equations and $\mathcal{O} \subset \mathcal{M}$ is an open subset then $F_{\mu\nu}|_{\mathcal{O}}$ also satisfies Maxwell's equations.

Using hypothetical source-free electromagnetic worlds admittedly weakens the point, but the same point could be made in terms of more realistic cases by adding electromagnetic sources, as for example by coupling the Maxwell field and a charged scalar field.

that contingency of nature as expressed in the variety of typical electromagnetic fields satisfying the Maxwell equations speaks against the notion that electromagnetic phenomena must share the full symmetry of the laws and, thus, speaks against the application of P which implies that the worldly counterparts of solutions of Maxwell's equations can differ only in values of the Poincaré invariants of $F_{\mu\nu}(x)$. To put the point in terms of P^*, the supervenience basis of Poincaré invariants is too slender to support what one expects to see in source-free electromagnetic worlds. Take the case of null electromagnetic fields where the Lorentz invariants $\overrightarrow{B}^2 - \overrightarrow{E}^2$ and $\overrightarrow{B} \cdot \overrightarrow{E}$ are both equal to 0 and, consequently, where $\overrightarrow{B}$ and $\overrightarrow{E}$ are perpendicular and equal in magnitude, as in the case of plane wave solutions of the source-free Maxwell equations. With $\overrightarrow{B}$ and $\overrightarrow{E}$ constant in time and uniform in space, as required by Poincaré invariance, the Poincaré-invariant supervenience basis can't support differences in the array of null fields solving Maxwell's equations. (Of course, this line of criticism needs to be supplemented by an account of how non-Poincaré invariant elements of the electromagnetic fields are measured. This matter will be taken up shortly.)

If electromagnetic worlds were governed by strengthened Maxwell laws that have as solutions only Poincaré invariant $F_{\mu\nu}$ then electromagnetic nature would be all lawfulness, and the proponents of P/P^* would be correct that super-Maxwell theory represents the world by means of Poincaré invariants. But our world is Maxwellian not super-Maxwellian, and Maxwell theory represents the world not primarily by means of Poincaré invariant quantities but by the gauge invariant but non-Poincaré invariant $F_{\mu\nu}$, which is constructed from the non-Poincaré invariant and, indeed, non-Lorentz invariant quantities $\overrightarrow{E}$ and $\overrightarrow{B}$. Although non-Lorentz invariant these quantities are Lorentz *co*variant, and their transformation properties under change of inertial coordinates guarantees that the form of Maxwell's equations (2a)-(2b) and (5a)-(5b) is preserved.

The principles P/P^* should not be confused with the legitimate empiricist demand that physical theories represent the world by means of quantities denoting observable/measurable features. But if P/P^* is rejected for Maxwell theory, how is the theory to be squared with the empiricist demand? The answer emerges from answering another question: How do observers make contact with features of Maxwell worlds? Not by possessing a magic meter that records values of the Poincaré invariants of the electromagnetic field. In minimalist terms an observer can be modeled by a timelike world line or the tangent vector field V^μ of the worldline.[10] The introduction of an inertial observer breaks Poincaré and Lorentz invariance by introducing a distinguished reference frame (the rest frame of the observer) and a distinguished location in space (the location of the observer in her rest frame) making it possible for the observer to measure non-Poincaré and non-Lorentz invariant quantities. The observer can measure $\overrightarrow{E}$ and $\overrightarrow{B}$ by using a test charge q that is sufficiently small that its backreaction on the field can be neglected and then using the Lorentz force law, which for a unit mass test charge with four-velocity U^ν has the form $qF^{\mu\nu}U_\nu$.[11] In three-vector notation this force is $q(\overrightarrow{E} + \overrightarrow{u} \texttt{ x } \overrightarrow{B})$ where $\overrightarrow{u}$ is the three-velocity of the test charge. If the charge's instantaneous rest frame ($\overrightarrow{u} = 0$) coincides with the rest frame of the said inertial observer then the Lorentz force divided by q serves as a measure of $\overrightarrow{E}$ for said observer. In the case of a purely magnetic field ($\overrightarrow{E} \equiv 0$) measuring the value of Lorentz force and the velocity $\overrightarrow{u}$ of a moving test charge suffices to determine the value of $\overrightarrow{B}$ for said observer. In more general cases determining the value of $\overrightarrow{B}$ is a more elaborate procedure, but this need not detain us here. The covariance properties of $\overrightarrow{E}$ and $\overrightarrow{B}$ determine what values other observers will measure.

In response one might say that what is being measured *are* invariants; namely, the values of $\overrightarrow{E}$ and $\overrightarrow{B}$ relative to an observer/reference frame. But this way of preserving idea that only invariant quantities are measurable changes the game. Accompanying the idea is the insinuation that because quantities lying on the variant side of the variant vs. invariant distinction are not measurable they do not have worldly counterparts, as P asserts. But now it turns out that to account for the fact that physics courses and laboratory reports speak of measuring non-Poincaré invariant quantities, such as $\overrightarrow{E}$ and $\overrightarrow{B}$ in electromagnetism and (three-) momenta of particles in particle mechanics, these quantities are described in a way that pushes them from the variant to the invariant side of the line. This seems to be a retreat from the original claims that only the Poincaré invariants of $F_{\mu\nu}$ have worldly counterparts and that, therefore, $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ and $(\mathbb{R}^4, \eta_{\mu\nu}, {}^{(a,L)}F_{\mu\nu})$

[10] In measuring vector valued quantities even more structure needs to be associated with the observer; in particular an orthonormal tetrad $\Upsilon^\mu_{(\alpha)}$, $\alpha = 0, 1, 2, 3$, of vectors parallel propagated along the observer's worldline with $\Upsilon^\mu_{(0)} = V^\mu$. Let this nicety be understood in what follows without adding it explicitly to an already complicated notation.

[11] Here we are drawing on an auxiliary theory. But in interpreting one theory it is typical to require the help of another.

represent/correspond to the same physical situation even when the equality $F_{\mu\nu}(x) = {}^{(a,L)}F_{\mu\nu}(x)$ does not hold for all x. The worry here is not so much that P is false but rather that that in defending P from falsity it is rendered murky.

A favorite example used to motivate the idea that only invariant quantities are measurable is absolute velocity. Granted, absolute velocity is not measurable in either the setting of Newtonian or special relativistic physics. But it is misleading to say that it is not measurable because it is variant under Galilean or Poincaré transformations. It is not measurable because there is nothing to measure. Absolute velocity means velocity relative to absolute space in the guise of a distinguished inertial frame, which is not to be found in either neo-Newtonian or Minkowski spacetime.[12] One could add additional structure to these spacetimes to mark out a distinguished inertial frame, reducing the spacetime symmetry groups so that velocity boosts are killed, making absolute velocity a symmetry invariant. Occam's razor militates against such a move since the additional structure is not needed to support the laws of Newtonian or special relativistic physics. In short, the story of absolute velocity can be made into a morality play about what goes wrong when there is a mismatch between the symmetries of the laws and the symmetries of the background spacetime. But when there is no mismatch—as in the cases discussed here—the morals of the story have little to say about which quantities are measurable and which have worldly counterparts.

5 The symmetry to reality inference assessed

P/P^* was found wanting. What of $StRI$ and the idea that symmetry related models represent/correspond to the same physical situation? For our Maxwell example there are two readings of $StRI$, one straightforward but nocuous, the other less direct and innocuous but also unhelpful. If the symmetry at issue is Poincaré invariance then the straightforward reading is that $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ and $(\mathbb{R}^4, {}^{(a,L)}\eta_{\mu\nu}, {}^{(a,L)}F_{\mu\nu})$ $(= (\mathbb{R}^4, \eta_{\mu\nu}, {}^{(a,L)}F_{\mu\nu})$ since ${}^{(a,L)}\eta_{\mu\nu} = \eta_{\mu\nu})$ represent/correspond to the same physical situation. All would be well if P/P^* were true and only Poincaré invariant Maxwell fields where ${}^{(a,L)}F_{\mu\nu}(x) = F_{\mu\nu}(x)$ for all $x \in \mathbb{R}^4$ have worldly counterparts. But we concluded above that even non-Poincaré invariant $F_{\mu\nu}(x)$ do have worldly

[12] See Sec. 7 below for a description of neo–Newtonian spacetime.

counterparts, and since the counterparts of ${}^{(a,L)}F_{\mu\nu}(x)$ and $F_{\mu\nu}(x)$ attribute incompatible properties to any spatiotemporal location $p \in \mathbb{R}^4$, $x(p) = (x^0(p), x^1(p), x^2(p), x^3(p), x^4(p))$, when ${}^{(a,L)}F_{\mu\nu}(x) \neq F_{\mu\nu}(x)$, $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ and $(\mathbb{R}^4, {}^{(a,L)}\eta_{\mu\nu}, {}^{(a,L)}F_{\mu\nu})$ cannot, on pain of inconsistency, represent the same physics.

The less direct and innocuous reading simply repeats in the present setting familiar truisms about model isomorphisms. For any model $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ of Maxwell theory and any Poincaré transformation $P = (a, L)$ (construed as a point mapping of $\mathbb{R}^4$ onto itself) we can create an isomorphic copy by "shifting" $F_{\mu\nu}(x(p))$ to $F_{\mu\nu}(x(P(p)))$ for all $p \in \mathbb{R}^4$. Anything true in the model $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ about electromagnetic happenings at p will be true in the shifted model about the electromagnetic happenings at its counterpart $x(P(p))$, and vice versa. This can be generalized to include observers. Include our inertial observer as represented by the velocity field V^μ in the models and consider the effect of a pure Lorentz velocity boost ℓ: any model $(\mathbb{R}^4, \eta_{\mu\nu}, V^\mu, F_{\mu\nu})$ is transformed to $(\mathbb{R}^4, \eta_{\mu\nu}, {}^{(0,\ell)}V^\mu, {}^{(0,\ell)}F_{\mu\nu})$, and the boosted inertial observer will experience the same electromagnetic physics in her inertial frame as her unboosted counterpart experienced in her frame. In *this* sense the Lorentz velocity boost symmetry related models represent/correspond to the same physics; but this sense does not get us $StRI$ or P/P^*.

What holds for Maxwell theory is just a special case of a much more general feature of any theory whose models have the form $(\mathcal{M}, X_{(1)}, X_{(2)}, ..., X_{(N)})$ where the $X_{(n)}$ are geometric object fields—scalar, vector, tensor fields—on $\mathcal{M}$. Let $d : \mathcal{M} \to \mathcal{M}$ be a diffeomorphism (a one-one onto C^∞ map whose inverse d^{-1} is C^∞). We can use d to create an isomorphic copy $(\mathcal{M}, {}^dX_{(1)}, {}^dX_{(2)}, ..., {}^dX_{(N)})$ by d-shifting the $X_{(n)}$ so that anything true in $(d(\mathcal{M}), X_{(1)}, X_{(2)}, ..., X_{(N)})$ $(= (\mathcal{M}, X_{(1)}, X_{(2)}, ..., X_{(N)})$ since $\mathcal{M}$ and $d(\mathcal{M})$ have identical manifold structure) about $p \in \mathcal{M}$ will be true in $(\mathcal{M}, {}^dX_{(1)}, {}^dX_{(2)}, ..., {}^dX_{(N)})$ about $d(p)$, and vice versa.[13]

By themselves these truisms about model isomorphisms cut no ice with respect to the issue of how to use facts about the symmetries of laws of a theory to draw conclusions about which quantities in the theory represent/correspond to real worldly properties. To reach such conclusions requires some special pleading. The pleading used by $StRI$ proponents some-

[13]For detailed construction of the d-shift operation on tensor fields see Appendix C.1 of Wald (1984).

times takes the form of an indistinguishability argument. It asserts that the worlds represented by the shifted models are physically indistinguishable from the original and from each other—i.e. indistinguishable by means of any physical measurement—and then in good positivist fashion it concludes that because they are observationally indistinguishable there is no distinction—they are all the same world. For good measure accuse those who say otherwise of committing a sin with a Latin name—believing in *haecceitism*.[14] Finally, note that when ${}^{(a,L)}F_{\mu\nu} \neq F_{\mu\nu}$ the isomorphic models $(\mathcal{M}, \eta_{\mu\nu}, F_{\mu\nu})$ and $(\mathcal{M}, {}^{(a,L)}\eta_{\mu\nu}, {}^{(a,L)}F_{\mu\nu})$ can represent the same world only if only the shift invariants of $F_{\mu\nu}$ have worldly counterparts, underwriting P.

I want to do a modus tollens where the proponent of *StRI* wants us to do a modus ponens. I see a reductio of the special pleading for Maxwell theory since we have agreed (I trust) that non-shift invariant $F_{\mu\nu}$ do have worldly counterparts. The proponent of *StRI* must give a non-question begging and convincing reason to break this agreement along with an account of what quantities do have worldly counterparts. The challenge (call it the *StRI challenge*) to those who want to treat Poincaré invariance as a gauge symmetry—as connecting the same real physical state under different descriptions—is to reformulate Maxwell theory in a form which (i) rejects the tensor-fields-on-manifolds formulation of classical field theory in favor of a different set of variables on which the Poincaré transformations act as the identity while (ii) showing that the modified theory functions to predict and explain electromagnetic phenomena at least as well as the standard theory; or else (i′) modifies the natural semantics for theories formulated in terms of tensor fields on manifolds—in particular, it would have to reject the Tarskian condition "$F_{\mu\nu}(x)$" is true iff the value of the Maxwell field tensor at x is $F_{\mu\nu}(x)$, while (ii′) showing that under the new semantics the theory is able to predict and explain electromagnetic phenomena at least as well as under the standard semantics. There is an implied research program here that *might* bear fruit not only for philosophy of physics but also for physics itself. It is up to the proponents of $STRI$ to carry out the program.

Alternatively the proponents of $STRI$ might try to finesse the challenge by stipulating that the (timeless) state of a source-free electromagnetic system corresponds to an equivalence class of Poincaré related $F_{\mu\nu}$, and then

[14] Sins with Latin names are presumed to be especially egregious. *Haecceitism* has truck with "bare particulars" and holds that worlds can differ non-qualitatively without differing qualitatively.

declaring job well done and declining to say anything further.[15] But quietude here is unacceptable. The natural question arising is: what are the truth conditions for the obtaining of the worldly counterpart corresponding to an equivalence class of Poincaré related $F_{\mu\nu}$? If the truth conditions are stated in terms of the values of a set of variables then those variables are the very ones that are to be used to answer to the challenge, and so the challenge has not been finessed.[16] If the truth conditions are specified in another manner, what is it and how does it illuminate how the theory represents real features of the world?

I have a suggestion for an alternative to trying to use $StRI$ and P/P^* to get insight into what theories of mathematical physics are trying to tell us about what is real. Proceed as follows. Work with the semi-official sense of gauge freedom sketched above to identify the gauge freedom in the theory; and having identified it, factor it out.[17] Then find the smallest subset of the

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[15] One is reminded here of some versions of the view labeled "sophistication about symmetries"; see Dewar (2019).

[16] Yet another alternative for the proponents of $StRI$ is to escape some of the problems raised above by playing up the anti-*haecceitism* angle and asserting that spatiotemporal locations, points $p \in \mathbb{R}^4$ or neighborhoods thereof, have no self-identity apart from the roles they play in the description of the electromagnetic field; conclude that since $x(p)$ in the original model and $x(P(p))$ in a Poincaré $P = (a, L)$-shifted model play qualitatively identical roles in their respective models they denote the same spatiotemporal location and, thus, the original model and all of its P-shifted counterparts represent the same world. Note that if the $F_{\mu\nu}$ in $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ is a Poincaré invariant Maxwell field then any $p, p' \in \mathbb{R}^4$ are qualitatively identical; but this hardly means that $x(p)$ and $x(p')$ denote the same spatiotemporal location. We can and do specify spatiotemporal locations independently of specifying the values of physical fields at these locations. A (not very precise) spatiotemporal location is picked out by Lucy's declaration "The time is now and the spatial location is where I stand," and when she adds "Here now in my rest frame the magnetic field is zero and the electric field is intense and rapidly oscillating" she has given a (not very precise) specification of the value of the electromagnetic field at a designated spatiotemporal location. And in doing so she has not committed any sin, Latin or otherwise. And Lucy knows perfectly well that she is making an assertion about the electromagnetic field values at, say, $x(p)$ (her here-now) rather than $x(P(p))$ (her there-later).

[17] There is an obvious motivation for taking this step in the context of explanation where we are looking for the difference makers for the occurrence or non-occurrence of a physical effect: a difference in the values of gauge variables cannot be such a difference maker since it involves only a difference in description and not a difference in physical state. This is entirely compatible with retaining the gauge variables for future developments of the theory. For example, the gauge variables discussed above in source-free Maxwell electromagnetism serve (in Carlo Rovelli's phrase) as handles for coupling the electromagnetic

remaining variables that have a good Cauchy problem. If there is a unique such smallest set assume as a defeasible working posit that these variables have worldly counterparts.[18] Use the theory and any needed auxiliary theories to explain how observers can measure values of these counterparts. If successful, conclude that the posit is correct. If unsuccessful rethink. This procedure lacks the elegance and simplicity of $StRI$ and P/P^*. But it has the virtue of being deaf to the siren song sung by $StRI$ and P/P^* of a quick and easy an easy path to a knowledge of what the theory is trying to tell us about what is real. While the procedure makes no overt appeal to symmetries and symmetry invariants, they play an indispensable role in determining the outcome since they are baked into the equations of motion.

6 The devil's advocate

Having criticized $StRI$ let me now play the devil's advocate. Where there is genuine gauge symmetry $StRI$ is golden. The devil whispers that gauge symmetry for the types of theories under discussion is easy to come by and, in particular, the diffeomorphism group $\mathfrak{G}_{\mathbb{R}^4}$ of $\mathbb{R}^4$ is a gauge group of source-free Maxwell theory. Any $d \in \mathfrak{G}_{\mathbb{R}^4}$ is a symmetry of the action—indeed, we have $\mathcal{A}_M(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu}) = \mathcal{A}_M(\mathbb{R}^4, {}^d\eta_{\mu\nu}, {}^dF_{\mu\nu})$—and, thus, it is a symmetry of Maxwell's equations. And since $\mathfrak{G}_{\mathbb{R}^4}$ involves arbitrary smooth functions of the spacetime coordinates $\mathfrak{G}_{\mathbb{R}^4}$ comprises gauge symmetries. And from the meaning of gauge, it follows that the d-shift related models $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ and $(\mathbb{R}^4, {}^d\eta_{\mu\nu}, {}^dF_{\mu\nu})$ represent/correspond to the same physical state of affairs, just as $StRI$ would have it. Those who follow the devil's line need to provide an account of what the diffeomorphic invariants are and how the worldly counterparts of these quantities serve as a basis for explaining the phenomena of Maxwellian worlds.

Resistance to the devil's line starts from the sentiment that it fails to take seriously the intended interpretation of special relativistic theories, wherein Minkowski spacetime serves as a fixed backdrop against which phenomena perform. On this picture, a d-shift should be applied to the Maxwell field tensor $F_{\mu\nu}$ but not to the Minkowski metric $\eta_{\mu\nu}$, yielding $(\mathbb{R}^4, \eta_{\mu\nu}, {}^dF_{\mu\nu})$ rather than $(\mathbb{R}^4, {}^d\eta_{\mu\nu}, {}^dF_{\mu\nu})$ as the d-shift counterpart of $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$. Of course, if d is a symmetry of $\eta_{\mu\nu}$ (i.e. ${}^d\eta_{\mu\nu} = \eta_{\mu\nu}$) then $\mathcal{A}_M(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu}) =$

field to other fields.

[18] If there is more than one smallest subset some choices have to be made.

$\mathcal{A}_M(\mathbb{R}^4, \eta_{\mu\nu}, {}^d F_{\mu\nu})$. But in general $\mathcal{A}_M(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu}) \neq \mathcal{A}_M(\mathbb{R}^4, \eta_{\mu\nu}, {}^d F_{\mu\nu})$, not even up to a total divergence; and in general with $\eta_{\mu\nu}$ held fixed it need not be the case that $(\mathbb{R}^4, \eta_{\mu\nu}, {}^d F_{\mu\nu})$ is a solution to Maxwell's equations whenever $(\mathbb{R}^4, \eta_{\mu\nu}, F_{\mu\nu})$ is.

Such resistance has an appeal. But the resistance fades as the strength of the fixed background structure fades, until no resistance is left when a fixed background structure becomes non-existent. This is a theme that will be explored in the following sections. The exploration, I claim, raises issues about how theories of mathematical physics represent the world that are more interesting than any amount of sparing with the advocates of $StRI$.

7 Finding more gauge freedom

The proponents of $StRI$ and P/P^* should be on the hunt for legitimate gauge freedom, as signaled by something other than a question-begging loyalty to $StRI$ and P/P^*. Where to look for it? How to create it if you don't initially find any? A return to basics is in order.

We saw that a principal reason to treat the electromagnetic potentials in Maxwell's theory as gauge variables is that if they did represent/correspond to genuine physical magnitudes then a good Cauchy problem is not possible. And correspondingly a reason not to treat Galilean invariance in Newtonian theories and Poincaré invariance in special relativistic theories as gauge symmetries is that a good Cauchy problem is at hand without the need to posit redundancy in the descriptive apparatus of the theory. The reason that a good Cauchy problem is thus attainable is that in these setting there is a fixed spacetime background—neo-Newtonian spacetime and Minkowski spacetime respectively—and that these spacetimes have sufficient structure so that the symmetry groups of the background spacetime—the inhomogeneous Galilean group and the Poincaré group respectively—have the property that if a group element is the identity on an initial value hypersurface $t = 0$ (say)—a plane of absolute simultaneity in neo-Newtonian spacetime or a plane of simultaneity for an inertial frame of Minkowski spacetime—and thus preserve the initial data, then it is the identity for $t > 0$.

If the background spacetime structure is weakened the possibility of a good Cauchy problem can be undercut, and the restoration of the possibility can be sought in the detection of gauge freedom. The point can be illustrated by a toy example that starts with neo-Newtonian spacetime and then weakens its structure. The structure of neo-Newtonian spacetime is much more complicated than Minkowski spacetime which is characterized by a single geometric object, the Minkowski metric. Neo-Newtonian spacetime has several pieces—a preferred foliation of time slices (absolute simultaneity), a Euclidean $\mathbb{E}^3$ spatial metric for the simultaneity slices, a metric that gives the time lapse between events on different simultaneity slices (absolute time/duration), and a family of inertial frames. When these pieces are carefully assembled and fitted together the resulting symmetry group of the spacetime is composed of the inhomogeneous Galilean transformations

$$\begin{aligned} \mathbf{x} \mapsto \mathbf{x}' &= \mathbf{R}\mathbf{x} + \mathbf{u}t + \mathbf{a} \\ t &\mapsto t' = t + b \end{aligned} \qquad (Gal)$$

where $t, \mathbf{x}$ are inertial coordinates with t now absolute time, $\mathbf{R}$ is a constant Euclidean rotation matrix, and $\mathbf{u}$, $\mathbf{a}$, b are constants.

An appropriate Lagrangian density for a Newtonian system of non-interacting unit mass point particles is

$$\mathcal{L}_{\mathbf{x}} = \frac{1}{2}\sum_{n=1}^{N} \dot{\mathbf{x}}_n^2, \;\; \dot{\mathbf{x}}_n := \frac{d\mathbf{x}_n}{dt} \qquad (8)$$

where $\mathbf{x}_n$ is the position of the nth particle. The Galilean transformations are divergence symmetries of $\mathcal{L}_{\mathbf{x}}$ and, therefore, symmetries of the EL equations, which in this instance are Newton's equations of motion for free particles. Since the Galilean transformations are "rigid"/"global" there is a good Cauchy problem with the values of the positions $\mathbf{x}_n$ and velocities $\dot{\mathbf{x}}_n$ of all the particles at $t = 0$ fixing a unique solution of the EL equations for $t > 0$.[19] Noether's first theorem applies, yielding the familiar conservation laws for energy and momentum.

The dreary story of the griefs of attempting to apply *StRI* and P/P^* to the Poincaré symmetry of Maxwell theory can be retold here for an attempt to apply *StRI* and P/P^* to the Galilean invariance of Newtonian particle theory. But this is left as an exercise to the reader who will undoubtedly conclude that the theory represents a Newtonian particle world world not exclusively by means of Galilean invariant quantities, such as magnitude of particle acceleration $\ddot{\mathbf{x}}_n$, but also by means of Galilean *co*variant quantities $\mathbf{x}_n$ and $\dot{\mathbf{x}}_n$.

[19] At least locally in time. Global existence and uniqueness of solutions is another matter.

Rather than trying to use $StRI$ and P/P^* promote $\mathbf{x}_n$ and $\dot{\mathbf{x}}_n$ to gauge quantities, let's do a legitimate promotion by changing the background spacetime structure to accommodate the notion that all motion is the relative motion of bodies, a notion that found many followers among natural philosophers in the 17th-19th centuries. We can do this by removing the inertial structure from neo-Newtonian spacetime while leaving the other structures intact. In the resulting spacetime there is no "absolute" (invariant) notion of acceleration for individual particles. To further simplify the discussion suppress two spatial dimensions. The symmetry group of the resulting two-dimensional spacetime is composed of transformations of the form

$$\begin{aligned} x &\rightarrow x' = x + f(t) \\ t &\mapsto t' = t + b \end{aligned} \tag{Rel}$$

where $f(t)$ is an arbitrary function of t. A modified Lagrangian density that admits (Rel) as a symmetry group is

$$\widetilde{\mathcal{L}}_x = \frac{1}{2}\sum_{n=1}^{N-1} (\dot{x}_{n+1} - \dot{x}_n)^2. \tag{9}$$

(Here I am adopting the example given in Rovelli 2014. Rovelli does not discuss the spacetime setting for his example, but I am assuming that he would find congenial my situation of his example.) Since the transformations (Rel) are "local"/"non-rigid" we are in the land of gauge because the EL equations for the x_n do not have a good Cauchy problem.

We can restore determinism by recognizing the x_n as gauge variables and by reformulating the theory using the gauge invariant quantities $a_n := x_{n+1} - x_n$. The Lagrangian density $\widetilde{\mathcal{L}}_{\mathbf{x}}$ of (8) can be rewritten as

$$\widetilde{\mathcal{L}}_a = \frac{1}{2}\sum_{n=1}^{N-1} \dot{a}_n^2. \tag{10}$$

There are $N-1$ EL equations, $\ddot{a}_n = 0$ for the $N-1$ gauge invariant a_n, giving a good (if physically uninteresting) Cauchy problem.

8 General relativity theory

All of the examples considered thus far concern the behavior of fields or particles in a fixed spacetime background. In Einstein's general theory of relativity (GTR) and related theories there is no fixed background spacetime, and the spacetime metric $g_{\mu\upsilon}$ becomes a dynamical variable that evolves according to the theory's field equations. The original source-free Einstein field equations read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0 \tag{11}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor (defined in terms of $g_{\mu\nu}$ and its first and second derivatives) and $R := R^{\mu}_{\mu}$ is the Ricci curvature scalar. Einstein later considered the addition of a cosmological constant term that changes (11) to

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu} = 0 \tag{12}$$

where λ is a constant. This complication will be ignored for present purposes.

When Einstein presented his gravitational field equations in 1915-16 the concepts of gauge freedom and gauge invariance were not part of the physics vocabulary.[20] Nevertheless, his desire to explain gravitational phenomena in terms of a dynamical spacetime metric led him to what, by our lights, counts as a gauge theory.[21]

Consider a general relativistic spacetime $\mathcal{M}, g_{\mu\nu}$ where $\mathcal{M}$ is a four-dimensional C^∞ manifold and $g_{\mu\nu}$ is a Lorentzian metric defined on all of $\mathcal{M}$. (Minkowski spacetime is the special case where $\mathcal{M} \simeq \mathbb{R}^4$ and $g_{\mu\nu} = \eta_{\mu\nu}$.) One sees directly that if $d : \mathcal{M} \to \mathcal{M}$ is a diffeomorphism and $g_{\mu\nu}$ satisfies the Einstein equations for all $x \in \mathcal{M}$ then so does the d-shifted ${}^{d}g_{\mu\nu}$ although in general $g_{\mu\nu}(x) \neq {}^{d}g_{\mu\nu}(x)$. Furthermore, the Einstein equations (11) are the EL equations of the Einstein-Hilbert action with Lagrangian density

$$\mathcal{L}_{EH} = R\sqrt{-g}, \quad g := \det(g_{\mu\nu}) \tag{13}$$

and $g_{\mu\nu} \mapsto {}^{d}g_{\mu\nu}$ is a divergence symmetry of $\mathcal{L}_{EH}$ and, thus, a symmetry of the EL equations (11).[22] So for given $\mathcal{M}$ the group of diffeomorphisms $\mathfrak{G}_{\mathcal{M}}$

[20]See O'Raifeartaigh (1997) for the history of gauge theories.

[21]For an account that situates GTR within the class of gauge theories, see Lee and Wald (1990). The Hamiltonian formulation of GTR supports the classification of GTR as a gauge theory. But which of the Hamiltonian constraints generate gauge is under dispute.

[22]Actually, because of the second-order nature of the Einstein equations the Einstein-Hilbert action has to be slightly modified to get rid of an unwanted boundary term that appears in the extremization of the action under variations of $g_{\mu\nu}$ with $\delta g_{\mu\nu}$ vanishing on the boundary. See Wald (1984, p. 458).

of $\mathcal{M}$ may be considered the gauge group of the theory.

Since the gauge group involves arbitrary smooth functions of spacetime position there is not a good Cauchy problem for the gauge variable $g_{\mu\nu}$. Posing the Cauchy problem for GTR is much more delicate and involved than for theories with a fixed spacetime background. Here I will give only a brief outline, and readers wanting more details are referred to Wald (1984, Section 10.2). The Cauchy data for Einstein's equations is specified by a three-dimensional C^∞ manifold S, a Riemannian metric $h_{\mu\nu}$ on S (which gives the spatial geometry of S), and a symmetric tensor field $K_{\mu\nu}$ on S, called the second fundamental form or extrinsic curvature of S (which specifies the "time derivative" of $h_{\mu\nu}$). In analogy with Maxwell's equations, Einstein's equations imply constraints on the initial data. So take a Cauchy data set to be a triple $(S,\, h_{\mu\nu}, K_{\mu\nu})$ with $h_{\mu\nu}, K_{\mu\nu}$ satisfying the constraints. A Cauchy development of a Cauchy data set $(S,\, h_{\mu\nu}, K_{\mu\nu})$ consists of a spacetime $\mathcal{M}, g_{\mu\nu}$ with $g_{\mu\nu}$ satisfying the Einstein equations (11) for all $x \in \mathcal{M}$ together with an embedding of S into $\mathcal{M}$ as a spacelike hypersurface such that $g_{\mu\nu}$ induces on (the imbedded image of) S the metric $h_{\mu\nu}$ and the extrinsic curvature $K_{\mu\nu}$, and such that every inextendible causal curve in $\mathcal{M}, g_{\mu\nu}$ intersects S (making S a "Cauchy surface" of $\mathcal{M}, g_{\mu\nu}$). Choose a $d \in \mathfrak{G}_{\mathcal{M}}$ that is the identity on a neighborhood $N(S) \subset \mathcal{M}$ of S but non-identity otherwise. Then $\mathcal{M},^{d} g_{\mu\nu}$ is a Cauchy development of the same Cauchy data set that agrees with $\mathcal{M}, g_{\mu\nu}$ on $N(S)$ but disagrees outside $N(S)$. However, as a saving grace there is gauge-good Cauchy problem: it is proved, first, that for any given Cauchy data set $(S,\, h_{\mu\nu}, K_{\mu\nu})$ there is Cauchy development and, secondly, that that there is a unique up to diffeomorphism (i.e. up to a gauge transformation) maximal Cauchy development (again see Wald 1984 for details).

The no-go result for a good Cauchy problem is not peculiar to Einstein's GTR but applies to any spacetime theory which does not use a fixed spacetime background and which has the diffeomorphism group as its gauge group. Whether not such a theory has a gauge-good Cauchy problems depends, of course, on the details of the theory.

9 Still more (much more) gauge?

In the above examples the distinction between gauge symmetries (in the semi-official sense) and non-gauge symmetries as well as the distinction between theories formulated against a fixed background spacetime and theories in which the spacetime structure is dynamical were presented as if are they are clean cut. But this was before reckoning without clever reformulations of theories, as is illustrated by the Sorkinizing our running example of source-free Maxwell theory which.[23]

In reformulating the theory, instead of raising and lowering indices using a fixed Minkowski metric $\eta_{\mu\nu}$ use a general Lorentz signature metric $g_{\mu\nu}$ which is treated as a dynamical variable whose value is not specified a priori but is to be determined by the field equations, and replace derivatives ∂_μ with respect to inertial coordinates by covariant derivatives ∇_μ determined by the metric $g_{\mu\nu}$. Introduce a tensor field $\lambda^{\mu\nu\gamma\delta}$ having the same symmetries as the Riemann tensor $R_{\mu\nu\gamma\delta}$ of $g_{\mu\nu}$ and add the term $\lambda^{\mu\nu\gamma\delta}R_{\mu\nu\gamma\delta}$ to the the Lagrangian density (13) to give a total Lagrangian

$$-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}\sqrt{-g}d^4x \ + \int \lambda^{\mu\nu\gamma\rho}R_{\mu\nu\gamma\rho}\sqrt{-g}d^4x \qquad (14)$$

where the components of the tensor fields are expressed in an arbitrary coordinate system.[24] This action admits diffeomorphism invariance as a divergence symmetry, which makes it a gauge symmetry of the equations of motion. Variation of the action with respect to $\lambda^{\mu\nu\gamma\delta}$ gives

$$R_{\mu\nu\gamma\rho} = 0 \qquad (15)$$

implying that $g_{\mu\nu}$ is a flat Minkowskian metric. Equation (2) (the Bianchi identity) is still in force as a consequence of (1) but now with the derivatives ∂_μ replaced by covariant derivatives ∇_μ. Variation of the action with respect to the potentials A_μ gives a version of (5) again with covariant derivatives replacing ordinary derivatives and indices raised by $g^{\mu\nu}$ and lowered by $g_{\mu\nu}$. But in both of these cases the equations reduce to their Minkowski form after taking into account $R_{\mu\nu\gamma\rho} = 0$.

The only new equation of motion results from varying the modified action (14) with respect to the spacetime metric $g_{\mu\nu}$. In parallel with Sorkin's example this variation yields the equation

[23] Here I apply the techniques described by Sorkin (2002) for the scalar Klein-Gordon field. As the title of Sorkin's paper indicates his example was directed towards the issue of the status of the requirement of general covariance, whether the requirement is a substantive constraint or mere formal/notational constraint on theories. Here the example is repurposed to probe the distinction between gauge and non-gauge symmetries.

[24] Aficionados of action principles will recognize that $\lambda^{\mu\nu\gamma\rho}$ is serving as a Lagrangian multiplier.

$$T^{\mu\nu} = \nabla_\gamma \nabla_\rho \lambda^{\mu\nu\gamma\delta} \tag{16}$$

where $T^{\mu\nu}$ is the the stress-energy tensor of the electromagnetic field given by $F^{\mu\sigma}F_\sigma^{\ \nu} - \frac{1}{4}g^{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}$.[25] In view of symmetries of $\lambda^{\mu\nu\gamma\delta}$ (16) yields the conservation law

$$\nabla_\mu T^{\mu\nu} = 0 \tag{17}$$

And, again taking into account $R_{\mu\nu\gamma\delta} = 0$, this law reduces to its Minkowskian form.

If the source-free Maxwell electromagnetic theory sketched in Section 3 were physically equivalent to the Sorkinized theory outlined in the present section then the status of diffeomorphism invariance as a gauge symmetry along with the distinction between a gauge and non-gauge symmetry would be shown to be slippery and murky matters indeed.[26] But arguably the two theories are physically distinct in interesting ways. Most notably the Sorkinization of Maxwell theory was achieved by the introduction of what is, in effect, a new physical field $\lambda^{\mu\nu\gamma\delta}$ which is sourced by the electromagnetic field and whose response to this source is governed by the field equation (16), much like the Einstein gravitational field's response to the presence of matter-energy fields is described by putting the stress energy-energy tensor of these fields on the rhs of equation (11).

However, the need to introduce the field $\lambda^{\mu\nu\gamma\delta}$ was driven by the demand that the field equations be derivable from an action principle, a demand that physicists have generally accepted since early in the 20th century and has been accepted as the basis of our discussion. While an action formulation is certainly desirable if one has an eye towards quantization, it would require more than a little argumentation to establish that an action formulation is a

[25] This expression for the stress-energy tensor of the electromagnetic field comes from taking the functional derivative $\frac{\delta}{\delta g_{\mu\nu}}$ with respect to $g_{\mu\nu}$ of the first term (the "matter term") of the action integral in (14). This definition of the stress-energy tensor $T^{\mu\nu}$ guarantees that it is symmetric with respect to μ and ν. The stress-energy tensor arising from Noether's theorem need not be symmetric, but it can be made symmetric by adding a "trivial" conservation term.

[26] Sorkin (2002) remained noncommittal about the physical equivalence of the theories in his original example—a scalar Klein-Gordon field in Minkowski space and its Sorkinized counterpart—but then he goes on to make the point I am about to make for Maxwell example.

priori essential to a theory that aspires to the status of a fundamental physical theory or that it is essential to understanding the physical status of variables in the theory. If the demand is rejected we could do a skinny Sorkinization that obviates the need to introduce the field $\lambda^{\mu\nu\gamma\delta}$ by taking the theory to consist of equations (2) and (5), with a general Lorentz signature metric $g_{\mu\nu}$ in place of the Minkowski metric and covariant derivatives with respect to $g_{\mu\nu}$ in place of ordinary derivatives, and equation (15). Admittedly this skinny version has the air of a mathematical trick that does nothing but produce a not very well disguised version of the standard Minkowski spacetime version of Maxwell theory. But something has gone seriously amiss if the seemingly substantive issues that have concerned us turn on judgments of what is or isn't a mathematical trick.

In any case, if either the fat or the skinny Sorkinization of Maxwell theory is accepted as a true and complete theory of source-free electromagnetism and diffeomorphism invariance is a gauge invariance of these theories then the $StRI$ applies. But this victory entails the obligation to respond to an analog the $STRI$ *challenge* outlined above in Section 5; and the obligation falls not just on the proponents of $StRI$ but to all who accept these theories as true and complete.

10 Conclusion

Theories of mathematical physics do not interpret themselves. Philosophers of physics are eager to offer a helping hand; indeed some of them see it as their main function to lay out the interpretive options and to adjudicate their merits and demerits. In this enterprise philosophers are under no obligation to follow the opinions of physicists on these matters, but it would seem prudent to take heed of these opinions and pause when the interpretative principles they propose to follow do not comport with practices in physics. In their rush to endorse $StRI$ and P/P^* philosophers ignore this caution in two related ways. First, they propose to treat a wide array of symmetries as gauge symmetries in the broad sense of connectioning different descriptions of the same physical state of affairs, whereas physicists are apt to see gauge freedom in a much narrower range of cases. Second, a familiar fact about theories in mathematical physics is that the laws of theory—in the form of field equations/equations of motion—exhibit symmetries that are not exhibited by typical solutions. Physicists generally do not find this disconcerting;

Hermann Weyl, for example, took this commonplace to be an expression of contingency in nature. But the proponents of $StRI$ and P/P^* think that it is a symptom of the redundancy in the mathematical apparatus these theories use to represent the world.

I agree, of course, with the ethos of P/P^* and the $StRI$ for gauge symmetries in the narrow sense used by physicists, where the gauge variables are identified by the fact that taking different values of these variables to imply a difference in the physical state would entail that the field equations/equations of motion do not have a good Cauchy problem. But for other symmetries I can find no good motivation for P/P^* and the $StRI$. However, I recognize that the way I propose to draw the line between gauge and non-gauge symmetries can be blurred by clever reformulations of theories. In particular, in some cases it appears to be all too easy to make diffeomorphism invariance a gauge symmetry by Sorkinizing the theory. My plea is to devote energy to discussing such issues and leave those enamored of P/P^* and the $StRI$ alone to seek therapy.

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