The Lorentz transformation in a fishbowl: a comment on Cheng and Read’s “Why not a sound postulate?”

Daniel Shanahan
PO Box 3423, Newstead, Queensland 4006, Australia
June 24, 2023

Abstract

In support of their contention that it is the absence of a subsisting medium that imbues the speed of light with fundamentality, Bryan Cheng and James Read discuss certain “fishbowl universes” in which physical influences evolve, not at the speed of light, but that of sound. The Lorentz transformation simulated in these sonic universes, which the authors cite from the literature of analogue gravity, is not that of Einstein, for whom an aether was “superfluous”, but that of the earlier relativity of Lorentz and Poincaré, which did suppose such a medium. The authors’ intention is not to argue analogically, but simply to contrast the situation of light with that of sound. However, I argue that these universes are too successful as analogues to support the authors’ case. By reducing Lorentzian relativity to its bare essentials, they provide a compelling demonstration of the viability and explanatory strengths of the earlier theory. They show how a thoroughly wave-theoretic treatment of the elementary particles would explain why all aspects of matter transform in like manner, thereby avoiding a difficulty that was a significant reason for the demise of Lorentzian relativity after 1905. Importantly, these sonic universes suggest a unifying explanation, not only of the Lorentz transformation and de Broglie wave, but of the principle of relativity, which was merely postulated, rather than explained, by Einstein in 1905.

Keywords
analogue gravity · Occam’s razor · Einstein’s “constructive” theory · Lorentzian relativity · de Broglie wave · principle of relativity

1 Introduction

In their intriguingly entitled paper, “Why Not a Sound Postulate?” [1], Bryan Cheng and James Read argue that it is the self-subsistence of electromagnetic waves that imbues the speed of light with fundamentality. They explain:
By the ‘fundamentality’ of a theory of waves, we mean this: the theory describes waves which are understood to be self-subsistent, rather than to be oscillations in some medium. In this sense, sound waves are not fundamental, as they can be understood as higher-level descriptions of oscillations in some antecedently-given ontology - viz., the air. By contrast, electromagnetic waves are fundamental, for they are not to be understood as higher-level descriptions of oscillations in some antecedently-given ontology .... One understands that the electromagnetic field just is the wave: unlike air in the case of sound, the electromagnetic field here is not ontologically prior to the wave.

But I suggest that the speed of light has a fundamental significance that is far more interesting than any lack of a supporting medium: the Lorentz transformation implies that the velocity \(c\) is not merely the limiting velocity for the transport of energy and information, but the underlying velocity of evolution of all physical influences, by which I mean any effect involved in the structure or interactions of an elementary particle\(^1\).

Nor should it be necessary to stray too far from orthodoxy to question the authors’ assertion that there is no supporting medium. Einstein vacillated on this issue, see generally Kostro [2], and it was when he was declaring the aether “superfluous” that he was also finding it necessary to postulate rather than explain the principle of relativity [3].

The authors discuss model universes (see their footnote 16) in which the materials from which clocks and other measuring devices are constructed are “governed by sonic Poincaré invariant laws”. They refer in particular to the models of Barceló and Jannes [4] and Todd and Menicucci [5], “situated in the broader context of ‘analogue gravity’ - the investigation of relativistic physics by way of surrogate physical systems”\(^2\).

My primary objective in this paper will be to follow Cheng and Read into these curiously unorthodox “fishbowl universes”\(^3\) to demonstrate how this wave-based modelling of the elementary particles might explain, whereas the current orthodoxy does not, the physical origin of the Lorentz transformation.

In these model universes, there is by construction only one constituent velocity. The waves from which particles are formed and the influences passing through and between those particles are sound waves. In these rudimentary particle structures, it will be possible to discern, not only the physical origin

---

\(^1\)This is not to suggest that the authors are denying that \(c\) might also have the significance I am arguing for here. Indeed they discuss, particularly in their Sect. 5.3, various senses in which this elusive concept of fundamentality might be considered.

\(^2\)For comprehensive introductions to analogue gravity, the authors refer to Barceló et al [6] and Volovik [7], and for philosophical discussions, Crowther et al [8] and Dardashti et al [9].

\(^3\)For the expression “fishbowl universe”, see Barceló and Jannes [4].
of the Lorentz transformation, but also that of the mysterious de Broglie wave, and of the principle of relativity itself.

But as the originators of these models inform us\textsuperscript{4}, the Lorentz transformation being simulated in these analogues is not that of the special relativity of Einstein, for whom the luminiferous aether was “superfluous”, but that of the earlier theory, associated primarily with Lorentz and Poincaré, that is generally thought to have favoured the existence of a light-supporting medium and privileged frame of reference.

It is clearly not the intention of Cheng and Read to argue analogically for the revival of that Lorentzian relativity or of its luminiferous aether. They have invoked these sonic models in a cause rather different from the usual concern of analogue gravity, which is to assess the plausibility of some theoretically predicted, but empirically inaccessible, phenomenon by investigating an analogue in which the corresponding feature \textit{is} accessible. For instance, a notable interest of analogue gravity is Hawking radiation, which was predicted theoretically and would have significant consequences for the thermodynamics of black holes, but is thought to be too weak for detection by methods currently available.

In discussing these fishbowl universes, the authors’ intention is merely to contrast the situation of electromagnetic waves with that of sound waves. As the authors explain, both waveforms satisfy a wave equation that is itself Poincaré invariant. But in the universe that we inhabit, it is only the electromagnetic wave that actually realizes the requirements of Poincaré invariance, that is to say, the invariance of the velocity in question under spatial rotations and Lorentz transformations. In the case of sound, and as the authors also explain, the symmetries of the Poincaré group are broken: the velocity of sound relative to an observer is not invariant, but varies with the velocity of that observer with respect to the medium.

In these sonic analogues, there is necessarily a sound-carrying medium and, for sound waves, a privileged frame of reference. Cheng and Read dismiss at the outset the possibility that electromagnetic waves might also require a supporting medium. Citing Occam’s razor, they argue that “there was for Einstein, and indeed still is, neither theoretical reason nor experimental evidence for implementing an ether in electromagnetism”.

But when William of Occam asserted that “it is vain to do with more what can be done with less”\textsuperscript{5}, he was assuming a “less” and a “more” of otherwise equal plausibility. From a consideration of these simple fishbowl universes, in which the workings of covariance have been laid bare in a manner that leaves

\textsuperscript{4}They cautiously deny that they are actually advocating an aether model, see Todd and Menicucci [5], at p. 1271, and Barceló and Jannes [4], Sect. 4, who state that whereas everything arises from the medium in their model, “the luminiferous ether supposedly only affected electromagnetic phenomena”. Yet Poincaré [10] [11], at least, had seen that all fundamental forces must evolve at the velocity $c$, referring specifically to gravity, which was the only other fundamental force then known.

\textsuperscript{5}As quoted by J. McFadden in his recently published book on Occam’s razor [12], at p.51
little room for disanalogy, I will argue that it is in fact the theory of Lorentz and Poincaré that is the ontologically less extravagant, and that the advantages in explanatory unification suggested by the earlier theory are in any case of far greater import than any consideration of simplicity or parsimony.

I will conclude by comparing Lorentzian relativity with three later conceptions of the Lorentz transformation, these being the special relativity of Einstein, the Minkowskian spacetime approach, and Brown’s “dynamic relativity” [13].

2 The fishbowl universes

Consider a closed laboratory (a fishbowl) in which the velocity $c_s$ of sound is measured by devices formed from material in which all physical influences evolve at the same velocity as sound. Barceló and Jannes [4] describe a Michelson-Morley interferometer in which the arms comprise arrays of equally spaced quasiparticles, which in a collective oscillation produce disturbances that evolve at the velocity $c_s$ in all directions. They show that when this quasi-interferometer moves at a velocity $v < c_s$ with respect to the laboratory, it must experience a Fitzgerald-Lorentz contraction with a Lorentz factor,

$$ \gamma = \left(1 - \frac{v^2}{c_s^2}\right)^{-\frac{1}{2}}, \quad (1) $$

based on the velocity $c_s$ rather than that of light.

Todd and Menicucci [5] show in their model how all the curious changes predicted by the Lorentz transformation - not only the contraction, but also the dilation of time and the failure of simultaneity - might be replicated by a chain of sound clocks, these being akin to light clocks except that the return journey between opposed mirrors is made by sound waves rather than electromagnetic waves.

Solid matter is thus simulated in both models by the superposition of counterpropagating sound waves. As will be discussed in Sect. 5, such a superposition forms a standing wave when stationary, but if it is to move through the subsisting medium must adopt the form of a travelling wave, specifically a carrier wave, giving rise as it does so to the de Broglie wave as a modulation (a dephasing or beating) of the moving wave structure (see, for example Feynman et al [14], Vol. I, Chap. 48). However, my concern in the present section will be confined to a consideration of how this wave-based simulation of a moving object would be perceived by an observer moving in parallel with the moving wave structure.

That will depend on whether this moving observer is within the fishbowl or outside it. As the wave structure moves through the enclosed medium at some velocity $v < c_s$, sonic signals from behind will reach the wave structure (the moving object) at the relative velocity $c_s - v$, and those from ahead at the relative velocity $c_s + v$. If an internal observer - an observer within the fishbowl - were capable of distinguishing these velocities, Poincaré invariance would be lost.
But since everything within the fishbowl is similarly constituted from underlying sound waves of velocity $c_s$, this internal observer and any measuring device accompanying the observer will suffer in like manner the changes described by the sonic Lorentz transformation, and in consequence neither observer nor measuring device will be capable of detecting the difference in relative velocities.

On the other hand, an observer who is moving in parallel with the moving wave structure, but is outside the fishbowl, will be able to perceive the difference in relative velocities and will recognize that the sound-carrying medium constitutes a privileged frame of reference for what is occurring within the fishbowl. That this medium is effectively non-existent for the internal observer is thus a form of illusion induced by the Lorentz transformation.

I suggest, with respect, that these models provide no support for the authors’ contention that $c$ owes its fundamentality to its independence of a subsisting medium. The issue here is not whether light would be unique if it required no medium (for clearly it would be), but whether light does in fact require no medium. Rather than supporting the authors’ position on that question, these fishbowl universes tend against it by demonstrating that if such a subsisting medium does exist, the covariance of the Lorentz transformation will have rendered it undetectable.

That Lorentz covariance has this effect is well-recognized, and it was largely with this result in mind that the Lorentz transformation was conceived, see, for instance Poincaré [10] and [11]. However, the manner in which the medium is rendered undetectable in Lorentzian relativity is demonstrated in a particularly transparent manner when we are encouraged to imagine the workings of covariance from the perspective of an external observer.

3 Einstein’s despair

But these sonic analogues take the argument for Lorentzian relativity a good deal further than simply providing an explanation for the apparent absence of a subsisting medium. They suggest the solution to a problem that confounded, not only the proponents of Lorentzian relativity, but Einstein himself.

It had been realized by the end of the nineteenth century, notably by Lorentz [17], that if matter changes in accordance with the Lorentz transformation, this would explain the observed invariance of the speed of light. It had also been
understood by then that these same changes would ensure that the laws of physics are the same in all inertial frames in accordance with the principle of relativity, which had been enunciated by Poincaré [18] as an empirical but unexplained fact. But it was not apparent why matter should change in exactly that fortuitous manner, or indeed that it actually did so.

Lorentz had sought an explanation for these changes in classical electromagnetic theory, but what could not be explained is why all aspects of matter - electrons, atoms and the forces between them - should all transform in precisely the same manner and degree. What seemed to be required was some all-encompassing explanation of the constitution of matter that would explain the universal effect of the Lorentz transformation, irrespective of how matter might ultimately be found to be constituted.

From Einstein's later writings, it is apparent that prior to 1905 he too had been seeking an explanation of the Lorentz transformation based on the constitution of matter, a theory that he termed a "constructive theory" in contrast to the "principle theory" that he was actually to present in 1905. As he recalled many years later, this constructive theory eluded him:

I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts. The longer and more despairingly I tried, the more I came to the conviction that only the discovery of a universal formal principle could lead us to assured results (Einstein [19])

From illustrations provided by Einstein - thermodynamics as a principle theory, and the kinetic theory of gases as the constructive theory - it can be seen that what he believed to be lacking in his theory of 1905 was an explanation that would account for the transformation of matter from the behaviour of underlying microprocesses, which, ironically perhaps, is also the kind of explanation that Lorentz had sought in his theory of electrons.

It has been suggested that Einstein "settled for a theory of principle because he was confident that the two postulates on which he built his theory would survive the quantum revolution he saw coming" (Balashov and Janssen [20]). I will argue, when considering the principle of relativity in Sect. 6 of this paper that the vagaries of quantum mechanics are not in fact a problem for a constructive theory.

It will suffice to notice here that Einstein did avoid any mention of the quantum in his paper of 1905. Showing an impressive confidence in the elegance of physical law, he simply adopted as postulates, the principle of relativity and what he referred to as the principle of the constancy of the velocity of light, namely, that "light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body" [3]. He showed how these assumptions lead via the Lorentz transformation to a self-consistent treatment of the electrodynamics of moving bodies.
In effect, Einstein recognized in the principle of relativity, a fundamental symmetry of Nature, see for instance Martin [21], and in so doing, was able to side-step the difficulties that had frustrated attempts to explain why all material objects should transform in like manner. But in 1905, Einstein neither explained, nor purported to explain, why light has the same velocity for all inertial observers or why the laws of physics are the same in all inertial frames. And having failed to explain the physical basis of those postulates, nor therefore had Einstein provided a satisfactory explanation of the physical origin of the Lorentz transformation.

Pursuing some earlier insights of Poincaré [10] [11], Minkowski then showed in 1908 how the Lorentz transformation could be treated as a rotation in a four-dimensional manifold [22]. It is Minkowski who must be thanked for worldlines and light cones and the convenient and intuitive picture provided by his spacetime diagrams.

It has been argued that Minkowski’s geometric treatment also provided the constructive explanation sought by Einstein (Balashov and Janssen [20]). But while Einstein did eventually adopt Minkowski’s geometric approach in the formulation of general relativity, his concern at the absence of a constructive theory seems to have continued unabated, see, for instance, Einstein [19] and [23]. As Harvey Brown noted in his Physical Relativity,

[T]here is a theme running through Einstein’s writings .... that what he called “elementary” foundations were unavailable to account for the stable structure and cohesion of matter, and that this was the reason he constructed [special relativity] in the way he did [13], at p. 113.

4 A “constructive” principle

What these fishbowl universes are now suggesting is that a theory of the kind that Einstein had despaired of finding in 1905 might be be based on what might be termed a principle of construction, namely that everything must necessarily transform in like manner if everything is formed in like manner from underlying influences of the same fundamental velocity.

These fishbowl universes may themselves seem hopelessly unlikely, but in the universe that we actually inhabit, it must likewise be assumed, both in special relativity and in Lorentzian relativity, that there is only the one fundamental velocity, that of light\(^7\). If other such velocities were involved in the constitution of matter, each would have its own Lorentz transformation and corresponding Lorentz factor \(\gamma\), and the laws of physics could not be the same from one inertial frame to the next. Nor then could matter retain the stability of its characteristic form under a change of inertial frame.

\(^7\)In an interesting discussion Cheng and Read do in fact consider the possible consequences for causality of a second fundamental velocity [1], Sect 5.2; and see also Geroch [24].
There are of course velocities that differ from $c$, those for example of massive objects, sound waves, and refracted light. But in each case, the velocity in question must be considered the net effect of underlying influences that do evolve at velocity $c$. Unlike $c$, such a velocity does not remain unchanged on a change of inertial frame, but as Einstein explained in 1905 [3], transforms in accordance with the relativistic formula for the composition of velocities. In the parlance of fundamentality, see Tahko [25], these other velocities are not fundamental, but existentially and compositionally dependent on the velocity $c$.

On the evidence of the Planck-Einstein relation,

$$ E = h\omega_0, \quad (2) $$

which associates a characteristic frequency $\omega_0$ with an elementary particle of energy $E$, it would then seem reasonable to assume that, as in these sonic models, these underlying influences of velocity $c$ must be wave-like and have the characteristic frequency of the particle in question.

Let us suppose then, as it has been in these sonic analogues, that the elementary particles and the forces between them comprise underlying wave-like influences of velocity $c$, having a characteristic frequency $\omega_0$, which would suggest in turn that a massive particle must comprise in its rest frame some form of stationary or standing wave comprising counter-propagating waves of velocity $c$.

This of course is far from being an explanation of the elementary particles, and even further from being an explanation of the complexities of the standard model, but it is suggested nonetheless that the all-encompassing ambit of the Lorentz transformation provides a compelling case for this understanding of the underlying nature of matter. And insofar as this conception of matter concerns the constitution of the elementary particles, it can also be characterized, I believe, as a constructive theory. But it is not a theory that Einstein could have proposed in 1905, for to do so he would have had to anticipate by some eighteen years, Louis de Broglie’s insight of 1923 that matter like light is wave-like.

Yet there was already a strong hint in Einstein’s paper of 1905 that any effect involved in the constitution or interactions of the elementary particles must evolve at velocity $c$. In that paper, as in his thought experiments involving moving trains and railway stations and the like, Einstein deduced the Lorentz transformation from a comparison of times taken by light signals making return trips, transversely and longitudinally, with respect to the direction of movement of some solid object. In effect, Einstein assimilated the relativistic behaviour of matter to that of a superposition of counter-propagating light waves. What Einstein could not then deduce, and could not have done so prior to de Broglie, is that matter will necessarily behave like counter-propagating waves of velocity $c$ if it comprises counter-propagating wave-like influences of that velocity.

In the next section, I will show how this conception of matter replicates the Lorentz transformation in such a way that the de Broglie wave emerges as an
integral element of the moving superposition and need not be thought of as something ontologically distinct from the wave-structured particle itself. But as in the fishbowl universes, this will be a Lorentzian relativity that suggests the existence of a privileged frame of reference.

The principle theory that Einstein presented in 1905 had no need of such a frame or of the subsisting medium that would define that frame. As Einstein famously declared,

> The introduction of a ‘luminiferous ether’ will prove to be superfluous inasmuch as the view here to be developed will not require an ‘absolutely stationary space’ provided with special properties.

On the authority of Einstein, the authors have applied Occam’s razor to summarily dismiss the possibility of that subsisting medium. But while special relativity and Lorentzian relativity may be empirically indistinguishable, they do differ significantly in what they suggest regarding the structure of space and time (a point recently stressed by Bradley [26], and see also the interesting paper of Knox [27]). As I will argue in the balance of this paper, they also differ in their ability to explain the Lorentz transformation, the de Broglie wave, and the principle of relativity.

The wave-theoretic approach proposed here has the further merit of effecting a unification of matter and radiation, for which indeed there is a good deal of evidence, for instance the decay of positronium into counter-propagating photons, and the observance of the Planck-Einstein relation by massless as well as massive particles.

## 5 The de Broglie wave

In the absence of a generally accepted understanding of what the de Broglie wave is and of how it comes into existence, it may not seem surprising that this wave has no analogue in the fishbowl universes discussed above or, it would seem, in any other model discussed in the literature of analogue gravity.

But this is a serious deficiency when one of the concerns of analogue gravity is the possibility of a theory of quantum gravity. Were it not for the wave characteristics defined for a massive particle by its de Broglie wave, there could not have been a quantum mechanics in which the dynamic properties of a massive particle are treated in terms of evolving wave characteristics.

In his thesis of 1923 [28], de Broglie showed that if the Planck-Einstein relation,

\[
E = h\omega,
\]

\[\text{(3)}\]

For instance, the otherwise comprehensive “living review” by Barceló et al [6] makes no reference at all to the de Broglie wave.
for the photon were extended to solid matter and equated with Einstein’s statement,

\[ E = mc^2, \]  

of the equivalence of mass and energy, a massive particle could be associated in its rest frame with a characteristic frequency,

\[ \omega_0 = \frac{mc^2}{\hbar}, \]

where \( m \) and \( \hbar \) are respectively the rest mass of the particle and the reduced Planck constant (de Broglie [28], Chap. 1, Sect I).

As the particle moves at velocity \( v \), the Planck-Einstein relation (3) becomes,

\[ E = h\omega_E = \gamma \hbar \omega_0, \]  

(where \( \omega_E \) is the Einstein frequency) and, as de Broglie also predicted, the moving particle acquires in accordance with what is now known as the de Broglie relation,

\[ p = h\kappa_{dB} = \hbar \gamma \omega_0 \frac{v}{c^2}, \]  

an associated wave number \( \kappa_{dB} \), thus defining for the moving particle, its associated de Broglie wave,

\[ \psi_{dB} = e^{i(\omega_E t - \kappa_{dB} x)}. \]  

It was the association established by Eqns. (5) and (6), between the dynamic properties of the particle and its corresponding wave characteristics, that made it possible to consider the evolution and interactions of a massive particle as if it were a wave. But for a particle moving at velocity \( v \), the wave (7) has the velocity,

\[ v_{dB} = \frac{\omega_E}{\kappa_{dB}} = \frac{E}{p} = \frac{c^2}{v}, \]

which evidently exceeds the limiting velocity \( c \) of light.

De Broglie was able to recover the classical velocity of the particle by locating it within a suitably contrived superposition of waves of differing frequencies (de Broglie [28], Chap. I, Sect. II). But such a wave packet spreads rapidly with time and very soon the particle could be almost anywhere at all. It is also a matter for suspicion that the de Broglie wave is not itself a covariant relativistic object. The wave is a creature of relativity - specifically, as de Broglie himself stressed, a consequence of the relativity of simultaneity - but it does not display the full effects of a Lorentz transformation.

The de Broglie wave has been largely supplanted in significance by the wave functions that emerge as solutions to the Schrödinger and other wave equations of quantum mechanics. It is now perhaps more likely to be discussed in the
context of de Broglie-Bohm interpretations of quantum mechanics than with reference to interactions such as scattering and interference. But the Schrödinger equation was itself conceived as an equation for the de Broglie wave (see Bloch [29]). For a massive particle that, as in these fishbowl universes, is moving freely in the absence of a potential, the solution of the relativistic Schrödinger equation (the Klein-Gordon equation),

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi - \nabla^2 \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0,$$

just is the de Broglie wave (7).

It will thus be instructive to consider where and how the de Broglie wave has gone missing from these fishbowl universes. I will first show that a wave factor with the characteristics $\omega_E$ and $\kappa_{dB}$ of the de Broglie wave is necessarily induced whenever a moving particle (or its fields) are represented, as they have been in these sonic universes, by counter-propagating waves that are of equal velocity with respect to a subsisting medium.

It should be said that this is not an original way of deriving the de Broglie wave. The wave emerges from an underlying standing wave in two of the three demonstrations of the Broglie wave in de Broglie’s thesis of 1923 [28], one of these being a treatment in Minkowski spacetime [28], Chap. I, Sect. III, and the other an intuitively more accessible toy model comprising an array of oscillating springs [28], Chap. I, Sect. II. This interpretation was subsequently discussed by Wolff [30], and provided the basis for an argument in favour of Lorentzian relativity that appeared in this journal in 2014 [31].

Consider, as in the model of Barceló and Jannes [4], a quasi-particle comprising the superposition of incoming and outgoing rays, which will be assumed in this case to have the velocity $c$ (of light), frequency $\omega_o$, and wave number $\kappa_o$, that is,

$$\psi(\mathbf{r}, t) = \sin \kappa_o r \cos \omega_0 t$$ (9)

which has the idealized$^9$ form of a spherical standing wave centred at $\mathbf{r} = 0$.

Following a Lorentz boost in the $x$-direction, and switching now from polar to Cartesian coordinates, spherical wave (9) becomes (see Shanahan [32]) the travelling wave,

$$\Psi(x, y, z, t) = \sin \kappa_o \sqrt{\gamma^2 (x - vt)^2 + y^2 + z^2} \cos[\omega_0 \gamma (t - vx/c^2)],$$ (10)

of which the first factor,

$$\sin \kappa_o \sqrt{\gamma^2 (x - vt)^2 + y^2 + z^2},$$ (11)

$^9$While this simple structure is obviously unphysical, it can be shown that all standing waves, of whatever form, give rise to a de Broglie wave under the Lorentz transformation, see [32].
is a carrier wave, which is evidently moving in the \(x\)-direction at the velocity \(v\) and can be seen from the inclusion of the Lorentz factor \(\gamma\) to have the ellipsoidal form described by the Lorentz-Fitzgerald contraction.

The second factor

\[ \cos(\omega_0 \gamma (t - vx/c^2)) \]

becomes, with the assistance of Eqn. (5),

\[ \cos(\omega_E t - k_d x), \]

which, when expressed in exponential form, is the de Broglie wave of Eqn. (7), not here an independent wave, but a sinusoidal modulation (a dephasing or beating) advancing through the carrier wave (11) at the superluminal velocity \(c^2/v\).

In Eqn. (10), the de Broglie wave is no longer a mysterious superluminal wave of unknown ontology, but has emerged in a manner well-known in wave theory as the modulation or beating of a travelling wave (Feynman et al [14], Vol. I, Chap. 48). And unlike the de Broglie wave considered alone, the full modulated wave (10) is a manifestly covariant relativistic object, capable in principle of taking its place in the tensor equations of relativistic physics. The Lorentz-Fitzgerald contraction appears in the carrier wave (11), while the dilation of time and failure of simultaneity are described by the modulation (12).

In this wave-theoretic interpretation of matter, it now becomes apparent why simultaneity must fail and how it does so. The effect of the modulation is that the parts of the moving wave are no longer cresting in unison, but in sequence, those ahead lagging in phase those behind. The modulation thus describes a progressive loss of phase in the direction of travel corresponding exactly in effect to the failure of simultaneity in that direction predicted by the Lorentz transformation.

Why then has the de Broglie wave gone missing from these fishbowl universes? The necessary wave structure was there. In both models, the predictions of the Lorentz transformation (though in the interferometer of Barceló and Jannes [4], only the Fitzgerald-Lorentz contraction) were deduced from changes in what were in effect superpositions of counter-propagating waves of velocity \(c_s\) (as in Eqns. (9) and (10)). In the interferometer of Barceló and Jannes, these were wave-like disturbances propagating from each member of an array of quasiparticles. In the chain of sound clocks described by Todd and Menicucci [5], the counter-propagating waves comprised sequences of sound pulses making return trips between opposed reflectors. These structures were thus capable in principle of replicating the de Broglie wave as a modulation, as might have become apparent had the derivations proceeded in terms of the changing frequencies and wave numbers of those counter-propagating waves.

But the derivations presented in these papers are effectively pre-quantum. In each case, the authors of these models have changed horses in midstream, so to
speak, by assimilating the behaviour of the superimposed waves to that of rigid rods and ideal clocks, and then proceeding very much as Einstein did in 1905. Todd and Menicucci [5] tell us that their sound clocks are separated by “spacing arms”, while Barceló and Jannes [4], at 194, employ “emergent vector fields and sources to produce a rigid bar”. In each case, the comparison undertaken is not between the differing wave characteristics of signals propagating longitudinally and transversely with respect to the direction of motion, which would have revealed the de Broglie wave, but between the differing times taken for return trips in those directions, which was the way in which Einstein proceeded in 1905.

When the existence of the de Broglie wave was predicted by de Broglie in 1923, it was interpreted as being something ontologically distinct from the associated particle, which seems to have been thought of at the time as some form of small solid or point-like object. Efforts, notably by Schrödinger (see Dorling [34]) to unify particle and wave in a thoroughly wave-theoretic interpretation of matter were at that time unsuccessful.

What then might this de Broglie wave be telling us regarding the fundamentality of the velocity $c$? I suggest that the existence of this wave provides compelling corroboration for the conclusion reached in Sect. 4, on the evidence of the Lorentz transformation and the Planck-Einstein relation (Eqn. (5)), that matter comprises underlying wave-like effects of velocity $c$.

If this is so, it becomes possible to look within the ideal rods and clocks of Einstein’s paper of 1905 to locate these curious changes of length, time and simultaneity in the wave-like constitution of matter.

6 The principle of relativity

Those changes of length, time and simultaneity are precisely of the form and degree necessary to ensure that the laws of physics are the same in all inertial frames of reference. This is so in both special relativity and Lorentzian relativity, but in the context of special relativity, where Einstein found it necessary to assume the principle of relativity, the Lorentz transformation would seem a fortuitous but unexplained property of the given universe.

But if it is not spacetime that is transformed, but what is occupying space and time, the question that must instead be asked is why matter should change in this manner, and this is a question that does suggest an answer.

Let us assume that, as in the fishbowl universes, a massive particle is in some sense a standing wave and that there is an inertial frame in which it is not only observed to be a standing wave, but in fact is that standing wave. It will be assumed, in other words, that there is a privileged frame, in which the velocity

---

10 De Broglie himself referred only to the electron rather than to particles generally. Until Rutherford’s discovery of the proton in 1919 and its naming in 1920, the electron had been the only massive particle known, see Romer [33].
of light is not only observed to have the velocity $c$, but (as with the velocity $c_s$ in the fishbowl) does in fact have this velocity. 

(In any other inertial frame, this wave structure will have acquired the modulated form described by the Lorentz transformation (as in Eqn. (10)), but the covariance of the transformation will ensure that to an observer within that frame, the modulation goes unnoticed and light presents nonetheless with the velocity $c$, as is the case for an internal observer in a fishbowl universe.)

If a massive particle is a standing wave in only the one privileged frame, we need to consider why it should adopt the distorted and modulated form described by the Lorentz transformation in any other frame. The short answer is, I suggest, that this is the only form it can adopt if it is to move with respect to the privileged frame.

That it adopts this form, rather than some other form, is in part at least a consequence of trigonometry. If a massive particle comprises waves having the velocity $c$ with respect to a privileged frame of reference, and these waves converge on, or diverge from, a point that is stationary in that privileged frame, the orientation of the waves must experience the effects of aberration if they are to continue to be convergent on, or divergent from, that same point as it moves with respect to the frame. These waves must also experience the changes in frequency and wave number described by the Doppler effect. The result is that the standing wave changes so as to form the modulated travelling wave described by Eqn. (10).

Considered in this way, Lorentz covariance is itself a consequence of trigonometry. To an observer moving with the moving particle, the point on which those constituent waves converge, and from which they diverge, will again be stationary, so that to that observer, the particle will appear to have the form that it took in its rest frame. Indeed, it will have exactly that form, thus explaining the perfection of the illusion induced by Lorentz covariance. Conversely, the observer moving with the particle, will observe a second particle that has remained in the stationary frame to be changed in the same manner, but in the opposite sense, to the change that has in fact occurred in the moving particle.

But a further question needs to be asked. Why, when subjected to an impressed force, does not the standing wave simply fall apart? Of course, when a particle is subjected to sufficient force it does fall apart. It disintegrates. While the standard model with its many free parameters does not entirely explain why the elementary particles exist in a limited number of characteristic forms, it can be understood that a particle owes its persistence of form to a binding energy that may involve a complex interplay of internal effects - forces, constraints, symmetries, topologies - some perhaps as yet unknown - and even when known, imperfectly understood.

Except in the privileged frame, the wave structure of the particle will experience the distortions described by the Lorentz transformation. But Lorentz covariance and the conformal nature of the transformation will ensure that the
equations governing the evolution and interactions of the particle remain form-invariant. Those interactions will thus depend, not on the particle’s velocity and disposition with respect to the privileged frame, but on its velocities and dispositions relative to everything that surrounds the particle.

According to this view, the laws of physics remain the same from one frame to the next, not because of some fortuitously structured spacetime, but because the survival of a particle in its characteristic form, and the survival in turn of its binding energy, demand that the structure of the particle change from one inertial frame to the next in the manner described by the Lorentz transformation.

In this scheme, the form-invariance of the laws of physics and the covariance ensured by the Lorentz transformation are no longer brute, unexplained, properties of the given universe, or ontologically prior to the elementary particles, but a consequence of the manner in which those particles must adapt to a change of inertial frame.

Could the same argument be put in the context of special relativity? The difficulty is that the arrow of explanation becomes reversed. In Lorentzian relativity, the argument is that the stability of matter requires that a particle rearrange itself between inertial frames in a manner that maintains the interrelationships that govern its existence. While it is true that the spacetime structure assumed by special relativity would also ensure those interrelationships, it is not apparent in the context of that theory why a spacetime structure should have precisely those properties necessary to ensure form-invariance, or indeed, what those properties might be.

The comparison can also be expressed in terms of fundamentality: in the context of special relativity, the principle of relativity is a fundamental symmetry of Nature, but this is essentially because it is a symmetry of unexplained origin; in the wave-theoretic treatment of Lorentzian relativity described above, the principle of relativity is not strictly speaking fundamental at all, but emergent along with the Lorentz transformation and the de Broglie wave from the underlying wave structure of matter and radiation.

One further issue is relevant here. As mentioned in Sect. 3, an inconsistency has been perceived between the precision suggested by the Lorentz transformation and the vagaries of quantum mechanics. However, concerns of this kind arise from the imposition of a geometrical idealization - a rotation of spacetime - on a real world that clearly does not accord with that ideal. It is inevitable that on a change of inertial frame, the likely fates of individual particles will change, as would the dispositions and trajectories of the molecules of the air in a spacecraft. It is also likely that an accelerated object will experience acceleration-induced stresses. But none of this should be of any more relevance to the operation of the Lorentz transformation than the possibility that an astronaut might spill their coffee under the stresses of acceleration. Nor should the uncertainties of quantum theory be of relevance. Probabilities may change,
but what the principle of relativity requires is not that the accelerated object
should survive unchanged, but that the laws of physics should do so.

This leaves Einstein’s other postulate, the light postulate, to be explained.
However, in the context of Lorentzian relativity, this is not the puzzle that it is
in a theory in which the relative velocity of light is the same for all observers.
In Lorentzian relativity, it is only necessary to suppose the invariance of $c$ with
respect to a privileged frame of reference, its observed invariance in other frames
being a consequence of the covariance described by the Lorentz transformation.

7 Minkowski and Brown

For the moment at least, Lorentzian relativity has been left behind - dismissed
as the foolishness of an earlier physics. Thus in the preface to his *Physical
Relativity* [13], Brown saw the need to emphasize, “from dire experience”, that
his dynamical approach “does not involve postulating the existence of a hidden
preferred inertial frame” and “is not a version of what is sometimes called in
the literature the *neo-Lorentzian interpretation* of special relativity”.

What is now topical, at least in the philosophical literature, is a contest be-
tween Brown’s dynamical approach and the geometry-first spacetime approach
initiated by Minkowski - neither of which subscribe to a subsisting medium.
Pooley [35] has described the issue as follows:

One of the guiding intuitions behind the dynamical approach
concerns explanatory priority. Consider, for example, the relativis-
tic phenomenon of length contraction. Do rods behave as they do
in virtue of the spatiotemporal environment in which they are im-
mersed, or are facts about the geometrical structure of spacetime
reducible (inter alia) to the behaviour of rods?

The argument has also been phrased in terms of symmetries\textsuperscript{11}. Are the
symmetries in the transformation of matter a consequence of the Minkowskian
structure of the spacetime in which matter resides or is it the other way around?
According to the geometrical view, as described by Balashov and Janssen [20],
“the geometrical structure of spacetime is the *explanans* and the invariance of
the forces, the *explanandum*”, whereas Brown describes the Minkowskian metric
as no more than a “codification of the behaviour of rods and clocks [13], p. 9.

But neither side has explained the origin of these symmetries. Brown favours
a constructive theory, such as that sought by Einstein, but concedes that as yet
“the Lorentz covariance of all the laws of physics is an unexplained brute fact”
[13], p. 143. Brown also argues, under the heading, “mystery of mysteries”,
that there is a similar lack of explanation in the spacetime approach [13], p.
143:

\textsuperscript{11}Indeed, a contest on several fronts - between substantivalism and relationalism, between
principle and constructive theories, as also between the dynamical and geometrical approaches
considered in this section. For a concise review of the various issues, see Read [36].
Talk of Lorentz covariance ‘reflecting the structure of space-time posited by the theory’ and of ‘tracing the invariance to a common origin’ needs to be fleshed out if we are to be given a genuine explanation here ....

I will return to Brown below, but will first say something regarding this need for the “fleshing out” of spacetime theories. What seems to be assumed in these theories is not that spacetime is transformed and carries matter with it\textsuperscript{12}, but that spacetime has a Minkowskian structure that ensures Lorentzian invariance, see for instance, Balashov and Janssen [20], and Maudlin [37], at p. 69. But while these theories ascribe to this Minkowskian structure the function of preserving Lorentz invariance, they provide no hint as to how this is achieved.

Even if nothing more were known of a phenomenon, it could be useful to at least know its function. Such knowledge could even have explanatory merit (see Knox [27] and Read and Menon [38]). But the difficulty for the geometry-first approach is that Lorentzian relativity not only claims to fulfil the same function, but as the fishbowl universes demonstrate, is able to explain how it does so.

An example of what is not addressed by these spacetime theories is the distinction that exists under the Lorentz transformation between what actually changes and what is merely observed to change. It is true that when one of two observers changes inertial frame, each observes\textsuperscript{13} in the other the effects of the transformation, but it is only the observer who has changed inertial frame who can have actually changed physically. The change that this accelerated observer perceives in the other observer, and indeed in the surrounding universe, is evidently a form of illusion. This feature of the Lorentz transformation is effectively obscured by Minkowski’s description of the Lorentz transformation as a “projection” from a (presumably unchanging) “absolute world” [22].

That a change of inertial frame is accompanied by physically real consequences in the object that has changed inertial frame is evidenced by the lack of mutuality in twin effect - the slower aging of the twin who undertakes a return trip as compared with that of her brother who stays at home. It is the travelling twin who experiences the “twin effect”.

That it is the change of inertial frame that induces such effects, and that they are induced solely in the object that undergoes that change of frame, should

\footnote{\textsuperscript{12}A notion that would require an unlikely spacetime that is able to contract in one way for one object and in a different way for another that is moving through the same space but in a different direction.}

\footnote{\textsuperscript{13}Where “observed” is used here, in the sense conventional in special relativity, to denote not what is “seen” at a particular instant, but what the observer considers to have occurred at that instant. What is seen includes light from events that occurred progressively earlier the further they were away, an effect here irrelevant. What an observer “sees” is also affected by the Penrose-Terrell rotation, an apparent rotation of objects as they are passed. A curious effect of the rotation is that an ellipsoidally contracted spherical object appears to have retained its spherical shape [39]. And, of course, I am also ignoring here the less consequential changes arising from the Earth’s own variations of inertial frame of reference.}
suffice in itself to raise the suspicion that the transformation could be the result, not of some unexplained geometry, but of changes occurring in matter, and that of course is the Lorentzian interpretation. Which brings me to John Bell’s comment, made in the course of advancing the pedagogic merits of Lorentzian relativity (or as he referred to it, the relativity of Fitzgerald, Larmor, Lorentz and Poincaré), that all the effects of special relativity can be understood from a single frame of reference Bell [40], (and see also Brown [13], at p. 124 et seq).

To appreciate the significance of Bell’s comment, consider two observers, Sally whose spacecraft is moving away from the Earth at velocity \( v \), and Harry who has remained at home. Harry observes light to be overtaking Sally from behind at the relative velocity \( c - v \), and to be meeting her from ahead at the relative velocity \( c + v \). Harry thus understands that since Sally herself is observing light to have the velocity \( c \) from whatever direction it arrives, she must have undergone the changes described by the Lorentz transformation, which as he also knows, are precisely of the form and degree necessary to induce in Sally the perception that it is not she who has changed, but all that surrounds her.

It is thus as Bell said: Harry is able to understand from a single inertial frame, not only the changes in Sally, but why it seems to Sally that it is not she who has changed, but the universe about her.

From the standpoint of the Lorentzian relativity that was of interest to Bell, it is a sufficient explanation of the changes observed by both Harry and Sally that Sally (and not Harry) has become transformed in the manner described by the Lorentz transformation. The wave-based version of Lorentzian relativity illustrated by the fishbowl universes shows, not only how this change in Sally is able to occur, but why it is such as to induce in her the illusion that it is not she but Harry who has experienced these changes.

From the standpoint of special relativity, one might instead contemplate a purely relational approach in which it is always the other party, who not only seems to have changed, but in some sense has actually done so. There is thus a symmetry in this relational approach that does not exist in Lorentzian relativity. But to invoke an unexplained Minkowskian spacetime structure to explain how this symmetry is achieved would be to explain one mystery by another. I will say something more regarding these problems with the relational approach in the next section.

Perhaps a more fruitful way of considering the geometry-first approach would be to ask, not how it might be explained, but why it has achieved the acceptance it has, and this is a question that will also be considered in the next section.

I return now to Brown for whom the Minkowskian metric is a “codification of the behaviour of rods and clocks” [13], at p. 9. Citing Einstein’s dissatisfaction with special relativity, Brown argues, and in my view, does so convincingly, that the elementary particles and the fields and forces by which those particles bind to, and interact with, other particles, including the quantum aspects of those particles, fields and forces, transform between inertial frames in such a manner
that it \textit{seems} that space and time have changed in the manner contemplated by Minkowski in 1908.

I also agree with Brown’s arguments for a constructive theory. But what he seems to be advocating is a Lorentzian relativity without the aether. Like Einstein, Brown dispenses with that undetectable medium and the privileged frame that it would define, but also like Einstein, he is unable to explain the Lorentz transformation, which as discussed above, he regards as “an unexplained brute fact” \cite{13}, at p. 143.

Having rejected the privileged frame, Brown seems to be contemplating some form of relational scheme (for discussions of Brown’s stance see Pooley \cite{35}, Knox \cite{27} and Read \cite{36}). But it seems to me that a relational approach would present a conundrum for Brown’s dynamic relativity. In special relativity, the velocity of light is postulated to be $c$ in all inertial frames, which makes it possible to argue that, as the relational view requires, all inertial frames have equal status. But in a theory in which the Lorentz transformation is explained by the symmetries of matter fields, which is to say, by changes in matter and the fields constituting matter, it becomes an illusion induced by covariance that the velocity is $c$ in every frame. In such a theory, the only way in which all frames could have equal status would be for the invariance of $c$ to be an illusion in all frames, which makes no apparent sense, and thus the conundrum.

In a wave-theoretic treatment of matter, such as that modelled by the fishbowl universes, the existence, at least locally, of a privileged frame would seem inevitable. How to imagine a particle comprising counter-propagating waves without supposing that there is some inertial frame of reference in which those waves are of equal velocity and frequency?

There is also at least one reason to suppose the existence of the subsisting medium that would define that privileged frame. When Einstein declared the aether to be superfluous, he explained that “the view here to be developed will not require an ‘absolutely stationary space’ provided with special properties”. But space does have one "special property" that suggests the existence of such a medium. Unlike massive particles, one photon never overtakes another - not at least in a vacuum. Photons pass through space at a common rate of progress, and their passage is thus similar in this respect to the propagation of a wave through an elastic medium, where the velocity of the disturbance is determined by the nature of the wave and the properties of the medium.

Perhaps there is some phenomenon, other than a subsisting medium, that is constraining light to the velocity $c$, but no one has come forward to suggest what this might be.

\section{Explanantia}

In exploring the fishbowl universes, I have considered four conceptions of the Lorentz transformation:
• Lorentzian relativity, the original proposal associated primarily with Lorentz and Poincaré, in which the transformation is the result of changes in the structure of matter, and which assumes a privileged inertial frame of reference and subsisting medium.

• Special relativity, the theory of Einstein, in which the transformation is derived from light and relativity postulates, and in which the privileged frame and subsisting medium are “superfluous”

• Minkowski’s geometric interpretation, which accepts Einstein’s postulates and the absence of a subsisting medium and privileged frame, but explains the transformation as a rotation in a four-dimensional spacetime.

• Brown’s dynamical approach, in which Minkowskian spacetime is a “codification” of the laws governing matter and, as in special relativity, there is no privileged frame or subsisting medium.

Lorentzian relativity is very much the outlier here, and although Brown’s Physical Relativity [13], and his collaboration with Pooley [41] and [42], have been the subject of significant scholarly interest, it is the Minkowski spacetime interpretation of Einstein’s special relativity that is standard and entrenched.

However, in comparing the four, several precepts of scientific explanation seem relevant. One is the requirement that the explanation proceed from well-established premises and that, as Hempel insisted [43], it comprise a sound deductive argument in which the phenomenon to be explained, the explanandum, follows as a logical and lawful consequence from the explanation, the explanans.

I suggest that none of the last three contenders meets these requirements.

Einstein’s paper of 1905 essentially asserted as brute fact what Lorentz and others had sought unsuccessfully to prove, namely the universal application of the Lorentz transformation. Einstein provided no explanation of the light and relativity postulates from which he derived the transformation. While his bold assertion of those postulates constituted a significant breakthrough, it is surely of relevance to how we now assess the explanatory force of his theory that he himself regarded his principle approach as inferior to a constructive theory.

While Brown’s arguments for a constructive theory seem cogent and convincing, he has not himself succeeded in formulating such a theory, and on his own admission, has left the Lorentz transformation, brute and unexplained.

However, I agree with Brown that the spacetime approach is also explanatorily inadequate, which raises the question as to why it has gained the acceptance it has. I suggest that it has succeeded because it is in one sense wholly admirable - an illustration par excellence of van Fraassen’s pragmatic account of explanation [44], according to which it is necessary to consider the context in which the explanation is sought. If one were to ask for an explanation of the
Lorentz transformation in conceptually simple terms, Minkowski’s transformation in a four-dimensional spacetime would suit very well.

As demonstrated by the fishbowl universes discussed by Cheng and Read, Lorentzian relativity does provide a viable explanation of the Lorentz transformation. And also of relevance here are those accounts of scientific explanation that stress the role of unification, see Friedman [45], and particularly Kitcher, who emphasizes [46], p. 432 that:

Science advances our understanding of nature by showing us how to derive descriptions of many phenomena, using the same patterns of derivation again and again, and, in demonstrating this, it shows us how to reduce the number of facts we have to accept as ultimate (or brute).

As an indication of the insistent nature of the enquiry, I also mention the words, written in 1896, of the eminent zoologist and autodidact, T. H. Huxley [47], at p.165:

In the ultimate analysis everything is incomprehensible, and the whole object of science is simply to reduce the fundamental incomprehensibilities to the smallest possible number.

I believe that these fishbowl universes of Cheng and Read have shown how the number of these “incomprehensibilities”, or as Brown calls them, “miracles”, would be significantly reduced by a thoroughly wave-theoretic treatment of matter in the context of Lorentzian relativity. Assuming a wave-based origin for both matter and radiation, these curious universes suggest a unifying explanation for the de Broglie wave, the Lorentz transformation, the principle of relativity, and the stability and cohesion of the elementary particles.

9 Concluding remarks

In the complexities of the actual universe, it may not be as readily apparent as it is in these simple fishbowl worlds that there is only the one underlying velocity and that all other velocities are existentially dependent on that velocity. But the all-encompassing ambit of the Lorentz transformation implies that here in the actual world there is likewise only one such velocity, the velocity $c$ of light. I have argued that it is this, rather than the apparent absence of a subsisting medium, that imbues $c$ with fundamentality.

As these sonic universes have illustrated, the detection of such a medium would be difficult to reconcile with the covariance maintained by the Lorentz transformation, as also with the principle of relativity, which is the source of that covariance. It would thus seem prudent to regard the existence or otherwise of this medium as an open question.
And yet, had the authors been arguing analogically for the existence of a subsisting medium, a sonic universe might well have provided a plausible analogue. As argued above, the Lorentz transformation could be seen as a consequence of the way in which the wave structure of a massive particle must rearrange itself to move through the medium, the dephasing described by the de Broglie wave would emerge as evidence of that wave structure, and the workings of Lorentz covariance could be imagined from an external perspective free of the illusions induced by that covariance.

Statements and declarations

The author wishes to thank an anonymous referee for their careful and very generous assistance in bringing this project to fruition.

The author declares that no funds, grants, or other support were received during the preparation of this manuscript.

The author has no relevant financial or non-financial interests to disclose.

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

References


