A Note on Landauer’s Principle

R. E. Kastner (rkastner@umd.edu)

and Andreas Schlatter

July 4, 2023

ABSTRACT. A form of Landauer’s Principle is shown to hold for thermal systems by reference to the joint entropy associated with conjugate observables. It is shown that the source of the compensating entropy for irreversible physical processes is due to the irreducible uncertainty attending values of such mutually incompatible observables. The relevant irreversibility is argued to be that of quantum measurement rather than erasure of classical memory devices, as commonly assumed.

Landauer’s Principle, in the form addressed here, can be defined as the requirement for N bits of entropy increase in an operation physically implementing N bits of logical irreversibility corresponding to entropy decrease (i.e., reduction in accessible phase space volume).[[1]](#footnote-1) Landauer provided some heuristic arguments in support of his principle, but ultimately appealed to the Second Law of Thermodynamics (Landauer 1961, 187). Earman and Norton (1999) have correctly pointed out that attempts to use Landauer’s Principle (LP) to save the Second Law from violation by such devices as “Maxwell’s Demon” are circular whenever they invoke the Second Law in support of LP. EN (1999) also argue that attempts to save the Second Law and/or LP without resort to the Second Law are subject to counterexamples and are not universal or conclusive. In addition, Norton (2013) argues that the information-theoretic approach to thwarting the Demon is similarly circular and fails to identify a fundamental and universal principle.[[2]](#footnote-2)

Meanwhile, Bennett (1982) claimed that the relevant irreversible process is that of erasure of memory devices rather than measurement, which he portrayed as copying. Bennett’s formulation has become the Received View in the literature on this topic, and a consensus has developed that Bennett refuted Brillouin’s identification of measurement as the source of the entropy cost attending the entropy-reducing sorting of the Demon (Brillouin, 1951). However, the copying model of measurement applies only to the classical limit and is thus inadequate at a fundamental level. If a system is prepared in a state *y(x)* with spread *Dx,* measuring its position is an irreversible process that, by the uncertainty principle, expands the uncertainty associated with its momentum (and vice versa). It appears that the Received View has neglected to take this point into account, and it is the focus of the present Note.

Before dealing with further specifics, we first consider the important concern of Norton (2013) that one cannot simply assume that ``information entropy'' *I* can be automatically identified with thermodynamic entropy *S*, a practice that has contributed to confusion and arguably to overstated claims about the findings of experiments (see, e.g., discussion of an experiment by Bérut *et al* in Norton (2013, 3.7). However, the present work supports the relation of *I* to *S* under appropriate physical conditions. In teasing out the relation of *I* to *S*, we must distinguish between epistemic uncertainty (in which a system is in a determinate but unknown outcome-state) and ontological uncertainty (in which a system cannot be said to possess a property corresponding to an outcome-state). Let us denote the former, epistemic form of *I* by *IE*and the latter, ontological form of *I* by *IO*. *IE*, as Norton notes, needs to be distinguished from *S*: the fact that we do or don't happen to know which state a classical memory device is in does not affect its accessible phase space volume.[[3]](#footnote-3) In contrast, *IO* can be interpreted as a generalized form of thermodynamic entropy due to the complementary relationship between incompatible observables in quantum systems and the fact that such systems do not possess any particular determinate value independently of measurement.[[4]](#footnote-4) In this spirit, we re-label *IO* as *IQ* .

Thus, for the quantum situation, the traditional Boltzmann and other forms of classical thermodynamic entropy must be generalized to accommodate the fundamental quantum level at which Nature operates, since those traditional forms assume values of incompatible observables to be simultaneously determinate, as in the classical phase space description (and tacitly assume all uncertainty as epistemic). Instead, we must treat the position and momentum state spaces separately, while taking into account their mutual constraint due to their Fourier-transform relationship (which implies the uncertainty principle). Thus, if one is dealing with a quantum system in some prepared state, the entropy associated with that state is properly understood as a measure of uncertainty that is irreducibly a *joint entropy* corresponding to incompatible sets of observables.

The concept of joint entropy applicable to the quantum case was studied by Hirschman (1957) and Leipnik (1959), who built on the pioneering work of Weyl (1928). These studies yielded a quantitative expression for the joint entropy for Fourier transform pairs such as position and momentum. Specifically, we have the relation:[[5]](#footnote-5)

(1)

where the left-hand side is the joint entropy *L*. The associated relation for dimensionless entropy, i.e., the joint quantum information *IQ* is defined as

(2)

Note that this quantity reflects the accessible position and momentum state spaces corresponding to an objective uncertainty or spread in values, since for general there is no fact of the matter about the system's possession of any particular value of either observable. For a Gaussian, (1) and (2) become equalities. In this case, it is clear that any reduction in the entropy associated with either observable must be compensated by an increase by the same amount in the complementary observable. Since thermal states correspond to Gaussian distributions, the relation (2) implies Landauer’s Principle (interpreted ontologically as above) for any quantum system in thermal equilibrium. We thus conclude that the appropriate quantum-level definition of thermodynamic entropy is . Moreover, arguably it is the quantum form of Landauer's Principle that underlies the Second Law.

This point--that it is quantum uncertainty that yields Landauer's Principle--has been obscured in the literature because the states of quantum systems in thermal equilibrium are usually given only in terms of the energy (momentum) basis, and the position basis is neglected; thus, the constraint represented by (1) and (2) is generally not evident and not taken into account. Moreover, the typical state description is that of an ostensibly proper mixed state such as

, (3)

which misrepresents the general situation in which a system is in a pure state of indeterminate energy (as in the situation pertaining to (1)), or in an improper mixed state (as in the case of entanglement with other quantum systems). In either case, it is not legitimate to view a subsystem as being in a determinate (pre-existing) but merely unknown energy state, so that measuring its energy is not a reversible copying procedure. Instead, measurement is, in general, irreversible.[[6]](#footnote-6)

As a simple example of the relevance of (1), consider a single gas molecule in a large box of length *l*. This is a quantum system described to a good approximation by a pure state Gaussian , such that its momentum space wave function is Using the molecule to do work via a piston would require detecting which side of the box it occupies, an irreversible position measurement that reduces the x-uncertainty such that the resulting state is . By the uncertainty principle, there is an accompanying increase in the spread of the momentum space wave function such that , and the joint entropy remains (at least) unchanged. This situation, rather than any considerations about memory erasure (which, as noted earlier, does not actually affect physical entropy) or dissipation effects (to which there are arguably always exceptions, cf. Earman and Norton (1999)), is what enforces Landauer’s Principle.

The increase in momentum-entropy corresponds to heating up the molecule, and indeed, in order to detect its position we must put provide energy in some form. This is the familiar observation that there is no detection without the involvement of photons, and that the more precisely we wish to measure the position, the higher the energy of the photon(s) required. In practice, the resulting increase in momentum spread thwarts the appropriate insertion of the piston/partition.

The case of momentum measurement corresponds to that of Maxwell’s Demon, who is conceived as sorting faster and slower molecules into each half of the box in order to violate the Second Law. It is traditionally assumed that the molecules have a well-defined but unknown momentum, but this is not the case at the quantum level; there is never a real "Maxwell's Demon" as Maxwell originally envisioned. Thus, treatments that presuppose this classical picture must inevitably fall short of the relevant physics.

Any real "Demon" must deal with the quantum nature of the gas molecules. This means that his measurement projects their indeterminate momentum state of into some more-determinate state , where . For thermal states described by Gaussians, the joint entropy expression (1) tells us that any reduction in the momentum-entropy effected by the Demon’s measurement of momentum is physically compensated by a corresponding increase in the position-entropy. In practice this means that any success the Demon has in measuring the momentum of a molecule must leave its position so indeterminate as to prevent him from getting it through the trap door. In other words, he now knows where to put it, but it has been so delocalized as to prevent actual physical sorting.

It should be noted, however, that the Second Law is defended from these sorts of challenges not by any argument about the specific pragmatics of thwarting the various mechanical manipulations involved in any particular scenario, but by the fact that Landauer’s Principle is upheld by the joint entropy constraint (1), which is an inviolable property of any Fourier transform pair. This prevents, in principle, violation of the Second Law, and constitutes an "exorcism of the Demon" that does not depend on illicit circular reference to the Second Law.

Finally, let us summarize the fundamental nature of Landauer's Principle, appropriately generalized to the quantum case. It can be formulated as comprising two basic principles in terms of (objective) quantum information *IO,X*  pertaining to a particular observable *X* and entropy *S*:

a) , where *X* is a particular observable of a conjugate pair *(X,Y)*

b) A reduction of is necessarily compensated by minimally an equal increase of

so that the total entropy *S,* defined by , does not decrease.

In conclusion, it has been shown that a form of Landauer’s Principle corresponding to ontological (as opposed to epistemic) information is implied for a thermal system by the joint entropy corresponding to the quantum uncertainty principle, independently of the Second Law. This remedies the current situation, in which Landauer’s Principle has often been invoked as a surrogate for the Second Law when it is the Second Law itself that is being challenged. It also arguably provides the true physical basis for the Second Law.

References

Bennett, C. (1982) "The thermodynamics of computation—A review," *Int. J. Theor. Phys.* *21*, 905–940.

Bérut, A.; Arakelyan, A.; Petrosyan, A.; Ciliberto, S.; Dillenschneider, R.; Eric, L.E. (2012). Experimental verification of Landauer’s principle linking information and thermodynamics. *Nature* **483**, 187–189.

Brillouin, L. (1951) "Maxwell’s Demon Cannot Operate: Information and Entropy. I," *Journal of Applied Physics 22*, 334–337. Reprinted in Leff and Rex (1990), pp. 134–137.

Earman, J.; Norton, J.D. (1999). Exorcist XIV: The wrath of Maxwell’s demon. Part II: From Szilard to Landauer and beyond. *Stud. Hist. Philos. M. P.* *30*, 1–40.

Hirschman, I. I. (1957). “A Note on Entropy,” *American Journal of Mathematics* 79:1, 152-156.

Kastner, R. E. (2017). "On Quantum Collapse as a Basis for the Second Law of Thermodynamics," *Entropy 19*(3): 106. https://doi.org/10.3390/e19030106

Landauer, R. (1961). "Irreversibility and Heat Generation in the Computing Process," *IBM* *J. Res. Dev. 5,* 183.

Leipnik, Roy (1959). “Entropy and the Uncertainty Principle,” *Information and Control 2*, 64-79.

Norton, J. (2013). "All Shook Up: Fluctuations, Maxwell’s Demon and the Thermodynamics of Computation," Entropy **2013**, 15(10), 4432-4483; [**https://doi.org/10.3390/e15104432**](https://doi.org/10.3390/e15104432)

Norton, J.D. (2005). "Eaters of the lotus: Landauer’s principle and the return of Maxwell’s demon." *Stud. Hist. Philos. M. P.* *36*, 375–411.

Schlatter, A. and Kastner, R. E. (2023). "Gravity from Transactions: Fulfilling the Entropic Gravity Program," J. Phys. Commun. **7** 065009. https://iopscience.iop.org/article/10.1088/2399-6528/acd6d7.

Weyl, H. (1928) *Theory of Groups and Quantum Mechanics*. New York: Dutton. (77, 393-4)

1. One might consider this a form of Szilard's Principle if measurement is a physically irreversible process; the present work argues that it is. [↑](#footnote-ref-1)
2. The information-theoretic tradition invokes a stronger version of Landauer's Principle than has been stated here, in which the logical structure of the computation is taken as dictating the entropy cost, rather than its actual physical implementation. [↑](#footnote-ref-2)
3. This is why erasure of a classically-modeled memory device does not correspond to a true phase space compression, as Norton notes (2013, 3.6): "Erasure reduces logical space but not physical phase space.''

   See also Norton (2005). [↑](#footnote-ref-3)
4. Even under an assumption of hidden variables {*l*}, *IO* applies to aquantum system in state , since a putative property-possession *l* does not define a state . [↑](#footnote-ref-4)
5. The differential form can in principle yield negative entropy values, but may be viewed as an idealization, since the notion of a position and momentum continuum can also be viewed as idealizations. In additional, there is no true position observable at the fully relativistic level, so that localization is likely limited to a Planck volume (cf. Schlatter and Kastner, 2023). Thus, (1) may be viewed as an excellent approximation for the macroscopic level. [↑](#footnote-ref-5)
6. Quantum measurement and its relation to entropy increase is discussed in detail in Kastner (2017). Of course, accounts insisting that quantum theory is always unitary are subject to the same irreversibility objections and lack of true state space compression as the classical theory, as well as the measurement problem, which may be why the results offered herein have thus far been overlooked. [↑](#footnote-ref-6)