Are observers reducible to structures?

Ovidiu Cristinel Stoica
Dept. of Theoretical Physics, NIPNE—HH, Bucharest, Romania.
Email: cristi.stoica@theory.nipne.ro, holotronix@gmail.com
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Physical systems are characterized by their structure and dynamics. But the physical laws only express relations, and their symmetries allow any possible relational structure to be also possible in a different parametrization or basis of the state space. I show that, if observers are reducible to their structure, observer-like structures from different parametrizations would identify differently the observables with physical properties. They would perceive the same system as being in a different state. This leads to the question: is there a unique correspondence between observables and physical properties, or this correspondence is relative to the parametrization in which the observer-like structure making the observation exists?

I show that, if observer-like structures from all parametrizations were observers, their memory of the external world would have no correspondence with the facts, it would be no better than random guess. Since our experience shows that this is not the case, there must be more to the observers than their structure. This implies that the correspondence between observables and physical properties is unique, and it's manifest in the observers. This result applies to both quantum and classical physics, so it is independent of the measurement problem. It has implications for structural realism, philosophy of mind, the foundations of quantum and classical physics, and quantum-first approaches.

Keywords: Foundations of quantum and classical physics; observers; structural realism; philosophy of mind;

I. INTRODUCTION

In this article I'll look into two apparently unrelated but strongly intertwined questions. The first question is

Question 1. *Is there an unambiguous correspondence between observables and physical properties?*

Here by "observable" I mean the operator or function that represents a physical property.

This question is relevant for various research programs aiming to recover the three-dimensional space and other physical properties and structures from the quantum structure alone [5, 12, 31] or from relations only [27].

But we will see that this problem occurs even in standard Physics. The physical laws alone don't give an unambiguous answer, because they only express relations. This makes them invariant with respect to a large group of reparametrizations of the state space. In Classical Physics, these are the canonical transformations. In Quantum Physics, they are unitary transformations.

The answer is given by the observers. Observers make experiments, and establish a correspondence between observables and physical properties. By "observers" I don't necessarily mean observers that "collapse" the wavefunction or play any role attributed to them to solve the measurement problem. In fact, the same problem appears in both Classical and Quantum Physics.

But the observers are physical systems, so they should also be subject to the physical laws. This is often understood as implying that the observers should be completely reducible to their constituting relations, to their structure. We will see that if this were true the observers would not be able to ascribe uniquely physical properties to the observables. This leads us to the second question,

Question 2. *Are observers reducible to their structure?*

The answer to Question 1 depends on the answer to Question 2. But how can we answer this question?

I will prove that, if the answer to Question 2 were affirmative, there would be no correlation between the observer's memory and the properties of external objects. In other words, observers would know nothing about the external world. The reason boils down to the fact that the symmetry of the state space dissolves any such correlation, by allowing the external world to appear as having different properties in different parametrizations obtained without changing the structure of the observer. This doesn't happen if the only observers are the observer-like structures from a unique parametrization.

Section §II contains a review of the quantum formalism that will be applied in the rest of the article. Section §III reviews the symmetry transformations of the state space and the physical laws. Section §IV shows how these symmetries imply that the same structures can occur in any parametrization of the state space, and that the same state can appear as different structures in different parametrizations. I prove the main result in Section §V. Some physical aspects of the proof are discussed in Section §VI. The article concludes with a discussion of some of the implications of the result in Section §VII.

II. PHYSICAL LAWS AND STRUCTURES

Any structure of any system in the world, including the structure of the observers, should be a physical structure. Therefore, here I review the physical structures and their dynamics, and the formalism used in the article.

I will use the quantum formalism, because the world is quantum, and all existing systems are ultimately quantum. But Classical Physics can be expressed in the same formalism [20], and we will get the same result.

The state of a quantum system is represented by a *state vector*, a complex vector $|\psi\rangle$ of unit length in an infinite-dimensional vector space $\mathcal{H}$ called here *state space*. The state space is a *Hilbert space*, a special kind of vector space endowed with a complex scalar product with the Hermitian property $\langle\psi|\psi'\rangle = (\langle\psi'|\psi\rangle)^*$, where $*$ is the complex conjugation. The length of a vector $|\psi\rangle$ from $\mathcal{H}$ is $||\psi\rangle| := \sqrt{\langle\psi|\psi\rangle}$.

But since all unit vectors are identical under the symmetries of the state space, the vector alone is not sufficient to describe the structure and properties of a system. The properties are represented by linear operators $\widehat{\mathbf{A}} : \mathcal{H} \to \mathcal{H}$, that are *Hermitian*, *i.e.* $\langle\psi|\widehat{\mathbf{A}}|\psi'\rangle = \left(\langle\psi'|\widehat{\mathbf{A}}|\psi\rangle\right)^*$ for all $|\psi\rangle, |\psi'\rangle \in \mathcal{H}$. We call them *observables*. The property represented by $\widehat{\mathbf{A}}$ has a definite a for a state vector $|\psi\rangle$ if and only if $|\psi\rangle$ is an *eigenvector* of $\widehat{\mathbf{A}}$ with the *eigenvalue* a, that is,

$$\widehat{\mathbf{A}}|\psi\rangle = a|\psi\rangle. \tag{1}$$

Since $\widehat{\mathbf{A}}$ is Hermitian, it has only real eigenvalues.

In Quantum Physics, for a given state, only some of the properties have definite values.

The state space admits a special basis, consisting of vectors uniquely identified by the combination of definite values of positions, components of spin, and internal degrees of freedom (d.o.f.s) of different particles. (I use the word "particles" because I work in the "particle representation" based on positions, but we will get wavefunctions, not point-particles.) What physical property each of these d.o.f.s represents is very important, but for simplicity I will denote these values uniformly by $q_1, q_2, \ldots$, and the basis vectors by $|q_1, q_2, \ldots\rangle$. The values $q_1, q_2, \ldots$ are eigenvalues of the operators $\widehat{q}_1, \widehat{q}_2, \ldots$, representing positions, components of spin, and internal d.o.f.s. Each basis vector satisfies the equation

$$\widehat{q}_j|q_1, q_2, \ldots\rangle = q_j|q_1, q_2, \ldots\rangle \tag{2}$$

for each of these operators $\widehat{q}_j$ and the corresponding eigenvalue q_j. The number and kind of each operator $\widehat{q}_j$ required to "fill the slots" of a vector $|q_1, q_2, \ldots\rangle$ depend on the number and type of particles whose state is represented by $|q_1, q_2, \ldots\rangle$.

All possible combinations of eigenvalues $(q_1, q_2, \ldots)$ of the operators $(\widehat{q}_1, \widehat{q}_2, \ldots)$ form a *parameter space* $\mathcal{C}$, usually called *position configuration space*. Since the number and kind of the operators $\widehat{q}_j$ depend on the number and type of particles, $\mathcal{C}$ is not a connected manifold, but a union of manifolds of various dimensions, each of them being the parameter space for systems of different numbers and types of particles with definite values for the spin and internal d.o.f.s.

The *wavefunction* for a state vector $|\psi\rangle \in \mathcal{H}$ is a complex function $\psi : \mathcal{C} \to \mathbb{C}$ defined by

$$\psi(q_1, q_2, \ldots) := \langle q_1, q_2, \ldots |\psi\rangle. \tag{3}$$

With the notation $\boldsymbol{q} = (q_1, q_2, \ldots)$, the wavefunction is

$$\psi(\boldsymbol{q}) = \langle\boldsymbol{q}|\psi\rangle. \tag{4}$$

But how exactly are the other physical properties represented by operators? For each d.o.f. q_j there is a *canonically conjugate operator*. If q_j is a position d.o.f., its canonically conjugate operator is the momentum operator $\widehat{p}_j := -i\hbar\frac{\partial}{\partial q_j}$, where $\hbar$ is the reduced Planck's constant. All other physical properties depend on the operators $\widehat{q}_j$ and $\widehat{p}_k$, so they are represented by operators of the form $\widehat{f}(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})$, where $\widehat{\boldsymbol{q}} = (\widehat{q}_1, \widehat{q}_2, \ldots)$ and $\widehat{\boldsymbol{p}} = (\widehat{p}_1, \widehat{p}_2, \ldots)$.

The *evolution equation* is, for any $|\psi(0)\rangle$ and any $t \in \mathbb{R}$,

$$|\psi(t)\rangle = \widehat{\mathbf{U}}_t|\psi(0)\rangle. \tag{5}$$

The *evolution operators* $\widehat{\mathbf{U}}_t$ are defined by $\widehat{\mathbf{U}}_t = e^{-\frac{i}{\hbar}\widehat{\mathbf{H}}t}$, where $\widehat{\mathbf{H}}$ is the *Hamiltonian operator*. The operators $\widehat{\mathbf{U}}_t$ are *unitary operators*, that is, they preserve the structure of the state space $\mathcal{H}$, including the scalar product. Equation (5) gives the solutions of the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \widehat{\mathbf{H}}|\psi(t)\rangle. \tag{6}$$

The propagation of the wavefunction on the parameter space $\mathcal{C}$ is therefore given by $\psi(\boldsymbol{q}, t) = \langle\boldsymbol{q}|\psi(t)\rangle$.

Remark 1 (Physical structures)**.** The wavefunction $\psi(\boldsymbol{q}, t)$ contains the complete information about the system. This is true even if we interpret $\psi(\boldsymbol{q}, t)$ probabilistically and include collapses in its evolution. Therefore, an omniscient being somewhat similar to the Laplace demon, let's call it the *metaobserver*, should be able to read everything about the system by examining the patterns of $\psi(\boldsymbol{q}, t)$. The structure of any system is manifest in these patterns. For example, the structure of a separable state of two particles is different from that of an entangled state, and this is visible in the pattern of the wavefunction of the separable state because the wavefunction is factorizable, *i.e.* the variables q_j of the first particle are separable from those of the second particle. □

Remark 2 (Physical space)**.** For a particle, we can identify as physical space the space parametrized by (q_1, q_2, q_3). If there is another particle with position space (q_4, q_5, q_6), and the interactions between them depend on the distance $\sqrt{|q_4 - q_1|^2 + |q_5 - q_2|^2 + |q_6 - q_3|^2}$, this determines an identification between the spaces (q_1, q_2, q_3) and (q_4, q_5, q_6). Similarly, the spaces of other other particles are identified with (q_1, q_2, q_3). In nonrelativistic Quantum Mechanics, this works because the potential depends on the distance [1]. In Quantum Field Theory fields interact by local coupling, which allows the identification between the spaces of the fields. Also see [31]. □

III. SYMMETRIES OF THE PHYSICAL LAWS

When we examine the wavefunction on the parameter space $\mathcal{C}$, we assume a preferred basis for $\mathcal{H}$, consisting of vectors of the form $|q_1, q_2, \ldots\rangle$. But it is possible to change this basis by applying a *unitary transformation* (a linear transformation that preserves the structure of the state space). A unitary transformation $\widehat{\mathbf{S}}$ transforms any operator $\widehat{\mathbf{A}}$ into another operator $\widehat{\mathbf{A}}' = \widehat{\mathbf{S}}\widehat{\mathbf{A}}\widehat{\mathbf{S}}^{-1}$.

In particular, we get $\widehat{q}'_j = \widehat{\mathbf{S}}\widehat{q}_j\widehat{\mathbf{S}}^{-1}$ and $\widehat{p}'_k = \widehat{\mathbf{S}}\widehat{p}_k\widehat{\mathbf{S}}^{-1}$. The eigenvalues of the operators $\widehat{q}'_j = \widehat{\mathbf{S}}\widehat{q}_j\widehat{\mathbf{S}}^{-1}$ correspond to a parameter space $\mathcal{C}'$, in general different from $\mathcal{C}$, which I will denote by $\widehat{\mathbf{S}}(\mathcal{C})$.

Any operator $\widehat{f}(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})$ is transformed into an operator

$$\widehat{f}'(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}}) = \widehat{\mathbf{S}}\widehat{f}(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})\widehat{\mathbf{S}}^{-1} = \widehat{f}(\widehat{\boldsymbol{q}}', \widehat{\boldsymbol{p}}'), \tag{7}$$

where $\widehat{f}(\widehat{\boldsymbol{q}}', \widehat{\boldsymbol{p}}')$ depends functionally on $\widehat{\boldsymbol{q}}'$ and $\widehat{\boldsymbol{p}}'$ in the same way $\widehat{f}(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})$ depends on $\widehat{\boldsymbol{q}}$ and $\widehat{\boldsymbol{p}}$.

The Schrödinger equation (6) is transformed into $i\hbar\frac{d}{dt}|\psi'(t)\rangle = \widehat{\mathbf{H}}'|\psi'(t)\rangle$, where $|\psi'(t)\rangle = \widehat{\mathbf{S}}|\psi(t)\rangle$ and $\widehat{\mathbf{H}}' = \widehat{\mathbf{S}}\widehat{\mathbf{H}}\widehat{\mathbf{S}}^{-1}$. Therefore, a quantum system remains a quantum system under symmetry transformations.

Remark 3. For a state vector $|\psi\rangle$, the wavefunction $\langle\boldsymbol{q}'|\psi\rangle$ on $\mathcal{C}'$ contains, in general, different structures than the wavefunction $\langle\boldsymbol{q}|\psi\rangle$ on $\mathcal{C}$. For example, plane waves in the position space appear as wavefunctions concentrated at points in the momentum space (the Fourier transform of the position space), and vice versa. Only Gaussian wavefunctions appear as Gaussians in both the position space and the momentum space, though not in all other parameter spaces.

Similarly, the wavefunction $\langle\boldsymbol{q}'|\psi'\rangle$ on $\mathcal{C}'$, corresponding to another state vector $|\psi'\rangle = \widehat{\mathbf{S}}|\psi\rangle$, looks exactly like the wavefunction ψ on $\mathcal{C}$. This is because the unitary operator $\widehat{\mathbf{S}}$ preserves the scalar product, so the scalar product between $|\boldsymbol{q}'\rangle = \widehat{\mathbf{S}}|\boldsymbol{q}\rangle$ and $|\psi'\rangle = \widehat{\mathbf{S}}|\psi\rangle$ is equal, for all $\boldsymbol{q} \in \mathcal{C}$, to the scalar product between $|\boldsymbol{q}\rangle$ and $|\psi\rangle$,

$$\langle\boldsymbol{q}'|\psi'\rangle = \langle\boldsymbol{q}|\psi\rangle. \tag{8}$$

Then, any structure possible on the parameter space $\mathcal{C}$ is also possible on any other parameter space $\widehat{\mathbf{S}}(\mathcal{C})$. □

There is a special type of symmetry transformations:

Definition 1. An *isonomy* is a symmetry transformation $\widehat{\mathbf{S}}$ that preserves the form of the physical laws.

If $\widehat{\mathbf{S}}$ is an isonomy, we say that $\widehat{\mathbf{S}}\widehat{\mathbf{A}}\widehat{\mathbf{S}}^{-1}$ is *isonomic* with $\widehat{\mathbf{A}}$, and also that $\widehat{\mathbf{S}}(\mathcal{C})$ is isonomic with $\mathcal{C}$.

A transformation $\widehat{\mathbf{S}}$ is an isonomy if and only if it commutes with the Hamiltonian operator $\widehat{\mathbf{H}}$, *i.e.* $\widehat{\mathbf{S}}\widehat{\mathbf{H}} = \widehat{\mathbf{H}}\widehat{\mathbf{S}}$. Then, $\widehat{\mathbf{S}}\widehat{\mathbf{H}}\widehat{\mathbf{S}}^{-1} = \widehat{\mathbf{H}}$, so indeed the dynamics follows the same law as in equations (5) and (6).

IV. META-RELATIVITY OF OBSERVERS

We attribute unique physical meanings to the operators $\widehat{q}_j$. But due to its symmetries, the quantum formalism alone doesn't distinguish them from other choices $\widehat{q}'_j$. It doesn't allow any parameter space $\mathcal{C}$ to be special among all possible parameter spaces $\widehat{\mathbf{S}}(\mathcal{C})$. What breaks the huge symmetry of the state space?

We attribute physical meaning to the operators because of the experiments. But how would this work?

Suppose that the wavefunction $\langle\boldsymbol{q}|\psi\rangle$ on $\mathcal{C}$ contains observer-like structures performing experiments. If they are conscious, they would consider that the parameters q_j correspond to positions in the physical space.

Let $\mathcal{C}' = \widehat{\mathbf{S}}(\mathcal{C})$ be another parameter space, parametrized by the eigenvalues of $\widehat{q}'_j = \widehat{\mathbf{S}}\widehat{q}_j\widehat{\mathbf{S}}^{-1}$. On $\mathcal{C}'$ there are wavefunctions $\langle\boldsymbol{q}'|\psi'\rangle$ that also contain observer-like structures. For example, if $|\psi'\rangle = \widehat{\mathbf{S}}|\psi\rangle$, the structures from $\langle\boldsymbol{q}'|\psi'\rangle$ are identical to those from $\langle\boldsymbol{q}|\psi\rangle$, and if $\widehat{\mathbf{S}}$ commutes with $\widehat{\mathbf{H}}$, they evolve identically.

Therefore, if observers are reducible to the structures, no experiment can tell them in what parameter space they live. It could be $\mathcal{C}$ or any other parameter space $\widehat{\mathbf{S}}(\mathcal{C})$. In particular,

Proposition 1. Observer-like structures from different parameter spaces identify different physical spaces.

Proof. Observer-like structures from different parameter spaces $\mathcal{C}$ and $\mathcal{C}'$ use different sets of operators $\widehat{\boldsymbol{q}}$ and $\widehat{\boldsymbol{q}}'$ to represent positions in space. Then, the identification of the physical space from Remark 2 leads to different results. The parameter spaces $\mathcal{C}$ and $\mathcal{C}'$ coincide only if the operators $\widehat{\boldsymbol{q}}'$ commute with all $\widehat{\boldsymbol{q}}$, and this happens when they are all functions of $\widehat{\boldsymbol{q}}$ independent of $\widehat{\boldsymbol{p}}$. Otherwise, from Remark 2, the resulting physical spaces will be different. □

Consequently, they perceive the same state as differently structured with respect to the physical space.

Remark 4. Note that even if there is an objectively unique physical space, even if its associated observables are ontologically special, whatever this means, Proposition 1 implies that observer-like structures from $\mathcal{C}$ and $\mathcal{C}'$ still identify different physical spaces. What is physical space to an observer-like structure on $\mathcal{C}$, to an observer-like structure on $\mathcal{C}'$ it appears as a space consisting of other three d.o.f.s, associated to different physical properties. And the same happens for all other observables. □

Let's extract these findings in the form of a principle.

Principle 1 (Meta-Relativity)**.** Observer-like structures on any two parameter spaces $\mathcal{C}$ and $\mathcal{C}'$ agree upon the laws of physics if and only if $\mathcal{C}$ and $\mathcal{C}'$ are isonomic. But in general they disagree about the physical properties associated to the observables and about the physical space.

Neither the relations that we can extract from experiments, nor the theory can determine the physical meaning of the operators. The physical meaning of the operators is relative to the parameter space, in the sense that observers from a parameter space $\mathcal{C}$ have a different physical interpretation of the operators compared to the observers from another parameter space $\mathcal{C}'$. But all observers from the same parameter space agree upon the physical meaning of the operators.

Principle 1 is very similar to the Principle of Relativity. For isometric coordinate transformations in space from (x, y, z) to (x', y', z'), different observers agree upon the physical laws. But there is no way to tell that the coordinates (x, y, z) are special compared to (x', y', z'). The Poincaré transformations that appear in Special Relativity are particular isonomies.

Principle 1 extends the Principle of Relativity, so I chose to name it "the Principle of Meta-Relativity". But it extends the Principle of Relativity only for structures, not for their physical meaning. Principle of Relativity remains true about the physical meaning of spacetime.

All we can access by *intersubjectively verifiable experiments* are the relations. Relations allow us to build mathematical models of the world. The nature of the relata is outside the realm of intersubjectively verifiable experiments. This truth was noticed in one form or another by various philosophers, notably Poincaré [24] and Russell [28], and it is called *epistemic structural realism* [21]. Epistemic structural realism seems to apply to science, because

1. No intersubjectively verifiable experiment can go beyond the relations.

2. No theoretical model can go beyond relations. Logically consistent theories admit faithful mathematical models in terms of mathematical structures [6, 16]. But mathematical structures themselves are nothing but sets and relations [14].

Remark 5. Even if $(\widehat{q}_1, \widehat{q}_2, \ldots)$ have a special ontic status compared to other choices $(\widehat{q}'_1, \widehat{q}'_2, \ldots)$, this can't be assessed by intersubjectively verifiable empirical means. These means can only establish that different observer-like structures from the same parameter space assign the same physical properties to the observables.

If there were objective means to determine $(\widehat{q}_1, \widehat{q}_2, \ldots)$ as the preferred properties, then we would be able to obtain them from the abstract state vector $|\psi\rangle$ and the abstract Hamiltonian $\widehat{\mathbf{H}}$, where "abstract" means that they are not expressed in a preferred basis or parametrization. But it's impossible to obtain them uniquely [31].

We can claim that the properties represented by the operators $(\widehat{q}_1, \widehat{q}_2, \ldots)$ associated to our parameter space are ontologically "more real" than other possible choices $(\widehat{q}'_1, \widehat{q}'_2, \ldots)$. But this doesn't solve the problem, since all properties $\widehat{f}(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})$ are as real as $\widehat{q}_j$ and $\widehat{p}_k$. In particular, any other choice $(\widehat{q}'_1, \widehat{q}'_2, \ldots)$ is as real as $(\widehat{q}_1, \widehat{q}_2, \ldots)$, because any $\widehat{q}'_j$ is also a functions $\widehat{q}'_j = \widehat{q}'_j(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})$. □

Remark 6. Even if there are some unknown hidden structures that break the symmetry so that $(\widehat{q}_1, \widehat{q}_2, \ldots)$ are special compared to $(\widehat{q}'_1, \widehat{q}'_2, \ldots)$ the problem remains. If we don't have empirical access to those hidden structures, they are useless. And if we will be able to access them, we will have to complete our theory. But the new theory will also be relational, so it will have its own large symmetry group, hence it will have the same problem.

For example, consider that we add point-particles with definite positions, as in the *pilot-wave theory* [3]. Then, the symmetry transformations are canonical transformations of the phase space of point-particles, done in tandem with unitary transformations of the pilot wave. The system of point-particles can be described using the quantum formalism (see Corollary 1), coupled with the quantum system of the pilot wave. Therefore, the quantum formalism and the discussions from this article apply and adding point-particles can't avoid it. □

V. ARE OBSERVERS REDUCIBLE TO STRUCTURES?

Given that there is no physical way to determine if an observer-like structure is special compared to an isomorphic structure from another parameter space, Principle of Meta-Relativity leads to the following question:

Question 3. *Are the observer-like structures from all parameter spaces conscious?*

A more specific variant of Question 3 is the following one. If the wavefunction on a parameter space $\mathcal{C}$ contains observer-like structures that are conscious, and if isomorphic structures can be found on another parameter space $\mathcal{C}'$, are the latter structures conscious as well?

If consciousness is reducible to structures, the Principle of Meta-Relativity implies that all observer-like structures in all parameter spaces are conscious. But if it turns out that not all observer-like structures in all parameter spaces are conscious, this would mean that there is more to consciousness than just the structure.

Theorem 1. *If observers were reducible to structures, they would know nothing about the external world.*

Proof. I will prove the theorem in three steps:

Step 1.1. Show that for any observer-like structure whose memory is correlated with its environment, there are parameter spaces on which the wavefunction of the same state contains identical observer-like structures, but having any possible environment.

Step 1.2. Show that the alternative parameter spaces with different environments are uniformly distributed.

Step 1.3. Show that there is no correlation between the observer's memory and these environments.

The observer is a subsystem of the universe. We can represent the state of the universe by

$$|\psi\rangle = |\psi_\omega\rangle|\psi_\varepsilon\rangle, \tag{9}$$

where $|\psi_\omega\rangle \in \mathcal{H}_\omega$ represents the observer, $|\psi_\varepsilon\rangle \in \mathcal{H}_\varepsilon$ represents the rest of the world, and $\mathcal{H} \cong \mathcal{H}_\omega \otimes \mathcal{H}_\varepsilon$. The following proof can be adapted easily for an observer entangled with the environment.

The parameter space decomposes as a Cartesian product $\mathcal{C} \cong \mathcal{C}_\omega \times \mathcal{C}_\varepsilon$. The wavefunction is therefore

$$\psi(\boldsymbol{q}_\omega, \boldsymbol{q}_\varepsilon) = \psi_\omega(\boldsymbol{q}_\omega)\psi_\varepsilon(\boldsymbol{q}_\varepsilon), \tag{10}$$

where $\boldsymbol{q}_\omega \in \mathcal{C}_\omega$ and $\boldsymbol{q}_\varepsilon \in \mathcal{C}_\varepsilon$.

Step 1.1. Since the observer can only access directly her own present state of mind, it is sufficient that the state vector $|\psi_\omega\rangle$ represents the brain of the observer. However, to humor anyone who would object to this, let us assume that $|\psi_\omega\rangle$ represents a more extended system that contains the observer. For example, suppose that $|\psi_\omega\rangle$ represents a room in which the observer is right now.

The observer's memory contains information about various properties of various external systems. For example, if she remembers that in the corner of her kitchen there is a table, her memory contains information about the approximate size of the kitchen and the table, and their relative position. So our observer expects that, if she goes to the kitchen, she will find these to be true.

But how does the wavefunction of the total state vector $|\psi_\omega\rangle|\psi_\varepsilon\rangle$ appear on other parameter spaces?

Consider a symmetry transformation that preserves the form of the observer-like structure $\psi_\omega(\boldsymbol{q}_\omega)$, but freely changes the structure of the rest of the world $\psi_\varepsilon(\boldsymbol{q}_\varepsilon)$. Such a transformation is represented by a unitary operator of the form

$$\widehat{\mathbf{S}} = \widehat{\mathbf{S}}_\omega \otimes \widehat{\mathbf{S}}_\varepsilon, \tag{11}$$

Here $\widehat{\mathbf{S}}_\omega : \mathcal{H}_\omega \to \mathcal{H}_\omega$,

$$\widehat{\mathbf{S}}_\omega|\psi_\omega\rangle = |\psi_\omega\rangle, \tag{12}$$

and therefore

$$\psi_\omega(\boldsymbol{q}'_\omega) = \psi_\omega(\boldsymbol{q}_\omega). \tag{13}$$

The operator $\widehat{\mathbf{S}}_\varepsilon$ is any unitary operator on $\mathcal{H}_\varepsilon$, so the external world can be transformed to look on $\mathcal{C}'_\varepsilon = \widehat{\mathbf{S}}_\varepsilon(\mathcal{C}_\varepsilon)$ like any other state $|\psi'_\varepsilon\rangle \in \mathcal{H}_\varepsilon$ looks on $\mathcal{C}$,

$$\widehat{\mathbf{S}}_\varepsilon|\psi_\varepsilon\rangle = |\psi'_\varepsilon\rangle. \tag{14}$$

That is, for any wavefunction $\psi'_\varepsilon(\boldsymbol{q}_\varepsilon)$ on $\mathcal{C}_\varepsilon$, we can choose $\psi_\varepsilon(\boldsymbol{q}'_\varepsilon)$ to look on $\mathcal{C}'_\varepsilon$ like $\psi'_\varepsilon(\boldsymbol{q}_\varepsilon)$ looks on $\mathcal{C}_\varepsilon$:

$$\begin{aligned}\psi_\varepsilon(\boldsymbol{q}'_\varepsilon) &= \langle \boldsymbol{q}'_\varepsilon|\psi_\varepsilon\rangle = \langle \boldsymbol{q}_\varepsilon|\widehat{\mathbf{S}}|\psi_\varepsilon\rangle = \langle \boldsymbol{q}_\varepsilon|\psi'_\varepsilon\rangle \\ &= \psi'_\varepsilon(\boldsymbol{q}_\varepsilon).\end{aligned} \tag{15}$$

Therefore, the total state vector appears on $\mathcal{C}' = \widehat{\mathbf{S}}(\mathcal{C})$ as the wavefunction

$$\psi_\omega(\boldsymbol{q}'_\omega)\psi_\varepsilon(\boldsymbol{q}'_\varepsilon) = \psi_\omega(\boldsymbol{q}_\omega)\psi'_\varepsilon(\boldsymbol{q}_\varepsilon). \tag{16}$$

This shows that indeed the observer-like structure remains unchanged under symmetry transformations $\widehat{\mathbf{S}}_\omega \otimes \widehat{\mathbf{S}}_\varepsilon$ under the condition (12), and the rest of the world can change freely to any possible state.

Step 1.2. We need to find out the probability distribution of possible alternative wavefunctions $\psi_\varepsilon(\boldsymbol{q}'_\varepsilon)$ of the environment. Since with any wavefunction ψ_ε all other wavefunctions ψ'_ε are equally present on different parameter spaces, this measure has to be the uniform measure on the set of unit vectors from $\mathcal{H}_\varepsilon$. This is the *Fubini-Study measure*, the unique measures invariant under the unitary symmetries, up to a numerical factor [2].

Any other measure would break the unitary symmetry of the state space, and would introduce preferred parameter spaces by a sleight of hand.

Step 1.3. Let us find out the probability as the ratio between the measure of "favorable cases" and the measure of "all possible cases". Denote by $\mu_\approx$, respectively $\mu_{\not\approx}$, the measure of the set of unit vectors $|\psi'_\varepsilon\rangle \in \mathcal{H}_\varepsilon$ consistent, respectively inconsistent with the observer's memory. The probability that the memory of the observer-like structure contains accurate data about the external world is

$$\mu_\approx/(\mu_\approx + \mu_{\not\approx}). \tag{17}$$

As seen in Step 1.2, the measure μ is uniform. This implies that the probability from equation (17) is the same as the probability that the observer-like structure guesses the properties of the external world by pure chance. In other words, *there is no correlation between the observer's state and the environment, so the observer knows nothing about the external world.*

Let us take a more concrete look. Consider a property of the external world, represented by $\widehat{\mathbf{A}}_\varepsilon$, about which the observer has in her memory the information that it has a definite value a. Denote the measure of the states with this property by $\mu_\approx(\widehat{\mathbf{A}}_\varepsilon|\psi'_\varepsilon\rangle = a|\psi'_\varepsilon\rangle)$. The ratio between $\mu_\approx(\widehat{\mathbf{A}}_\varepsilon|\psi'_\varepsilon\rangle = a|\psi'_\varepsilon\rangle)$ and the measure of the set of all unit vectors is zero. Because the same observer-like structure $\psi_\omega(\boldsymbol{q}_\omega)$ can have any possible environment $\psi_\varepsilon(\boldsymbol{q}'_\varepsilon) = \psi'_\varepsilon(\boldsymbol{q}_\varepsilon)$, it's memory contains zero information about the value of $\widehat{\mathbf{A}}_\varepsilon$.

However, perhaps we shouldn't consider as "all possible cases" all possible environments, but only those for which $\widehat{\mathbf{A}}_\varepsilon$ has a definite value. Suppose that $\widehat{\mathbf{A}}_\varepsilon$ has n eigenvalues, and the corresponding eigenspaces are unitarily equivalent. Even restricted like this, the probability still has to be uniform, because together with any eigenvector $|\psi_\varepsilon\rangle \in \mathcal{H}_\varepsilon$, any other eigenvector $|\psi'_\varepsilon\rangle \in \mathcal{H}_\varepsilon$ is equally obtainable by a symmetry transformation. We see that if all structures identical with the observer's structure were conscious regardless of their parameter space, the probability (17) would be $1/n$, so the observer would know about the property represented by $\widehat{\mathbf{A}}_\varepsilon$ exactly what is allowed by random guess.

So indeed if observers were reducible to structures, they would know nothing about the external world. □

The result is true at any moment of time. But one may think that somehow the changes endured by the observer in a non-zero time interval can avoid this. They can't.

Consider the system containing the observer to be practically isolated, coupled very weakly with the external systems. Then the Hamiltonian is approximately of the form $\widehat{\mathbf{H}}_\omega \otimes \widehat{\mathbf{H}}_\varepsilon$. As long as the coupling is weak, the observer's evolution is left unchanged by $\widehat{\mathbf{S}}$ if $[\widehat{\mathbf{S}}_\omega, \widehat{\mathbf{H}}_\omega] = 0$. But $\widehat{\mathbf{S}}_\varepsilon$ may very well commute with $\widehat{\mathbf{H}}_\varepsilon$, because this will change only the evolution of the environment. So indeed this doesn't avoid the result.

Remark 7. In this article, by "observer" I don't necessarily mean somebody making a quantum measurement. Although any observer inevitably makes quantum observations, the result doesn't rely on the measurement problem. The following Corollary should clear any doubt. □

Corollary 1. Theorem 1 is true in a classical world too.

Proof. Koompan and von Neumann showed how to formulate Classical Physics using the quantum formalism [20, 37]. But there are some differences. The momentum operators $\widehat{p}_j$ are not the form $-i\hbar\frac{\partial}{\partial q_j}$, they are independent of $\widehat{q}_j$. Since now all operators $\widehat{q}_j$ and $\widehat{p}_k$ commute, the constant $\hbar$ plays no role. The resulting parameter space $\mathcal{C}$ is the classical *phase space*, containing both q_j and p_k. The physical states are represented only by basis vectors $|\boldsymbol{q}, \boldsymbol{p}\rangle$. The wavefunctions are localized at points $(\boldsymbol{q}, \boldsymbol{p}) \in \mathcal{C}$. On the state vectors $|\boldsymbol{q}, \boldsymbol{p}\rangle$, all observables $\widehat{f}(\widehat{\boldsymbol{q}}, \widehat{\boldsymbol{p}})$ have definite values. Because the state always remains classical, the evolution operators $\widehat{\mathbf{U}}_t$ from equation (5) maps basis vectors to basis vectors. The symmetry transformations of the phase space, called *canonical transformations*, are represented only by unitary transformations that map basis vectors to basis vectors.

Since these restrictions don't affect the proof of Theorem 1, the result applies to classical worlds too. □

Corollary 2. The parameter space supporting observers is unique up to spacetime and gauge symmetries.

Proof. The proof of Theorem 1 shows that only in some parameter spaces the wavefunction ψ_ε of the external world corresponds to the memory of the observer represented by ψ_ω. Any unitary transformation $\widehat{\mathbf{S}}_\varepsilon$ as in the proof of Theorem 1 introduces an ambiguity in some physical properties. Then, as in the proof of the Theorem, the observer is unable to know these properties. The observer's memory contains information about a very limited number of properties, so it doesn't fix the wavefunction ψ_ε of the external world. Consequently, the parameter space $\mathcal{C}_\varepsilon$ is far from being completely determined, as seen in equation (14). However, the observer can make experiments to determine any physical property of a system. This means that all these properties should be accessible to the observer, given the right experiments. Therefore, the properties that can't be known to the observer even in principle have to be "nonphysical", *i.e.* dependent of the reference frame or the gauge. The properties that can be known identify a unique parameter space supporting observers, up to spacetime and gauge symmetries. □

Theorem 1 answers Question 2 negatively, and, based on this, Corollary 2 answers Question 1 affirmatively.

VI. DISCUSSION

We take for granted the existence of a unique correspondence between the operators representing properties, and the physical properties themselves. But we have seen that, if all observer-like structures were observers, experiments couldn't ensure uniquely this correspondence. This correspondence is absent from the theoretical description too. It only seems to be part of the theory because we give different names to the various operators, we label them with different symbols, and we all follow the convention, so we agree with one another about their meaning. The uniqueness of this correspondence is ensured by the fact that only observer-like structures from a particular parameter space can be observers, which is proven in Theorem 1. Without this, no observer would be able to know anything about the external world.

We also take for granted that our memory holds correct information about the external world automatically, just because we interacted with it in the past. This predisposition may make the proof of Theorem 1 more difficult to understand. But the evolution equations of physics are reversible, and if we remember our past interactions, we should equally remember our future interactions. Or rather there should be no relation between the content of our between memory and the external world at all, because all state vectors $|\psi_\omega\rangle|\psi_\varepsilon\rangle$ are equally "legal" under the laws of physics. The states containing brains with memories that don't correspond to facts about the external world are as "legal" as those with reliable memories, and even overwhelmingly outnumber them. Without special conditions that ensure the reliability of our memories, most observers would fluctuate ephemerally into existence by accident and then dissipate ([9], p. 65). The chance of not being a *Boltzmann brain* would be practically zero.

Fortunately, the initial state of the universe was extremely special. The Boltzmann entropy was extremely low, and it increases steadily in time, ensuring the validity of the Second Law of Thermodynamics [4, 10]. This is believed to also ensure the relative reliability of our memories. And even though, as shown in [33], reliable memories require more fine tuning of the initial state than just starting in a low-entropy state, we live in such a friendly universe. Penrose estimates it to be one in $10^{10^{123}}$ [23]. Our memories about the properties of external systems are reliable because we are part of a universe in such a select state.

But this is not true of most states in which the universe could be, including most states resulting from its state by unitary transformations $\widehat{\mathbf{S}} = \widehat{\mathbf{S}}_\omega \otimes \widehat{\mathbf{S}}_\varepsilon$ as in equation (11). No matter how friendly our universe appears on our parameter space, it will not appear friendly at all to the observer-like structures from most alternative parameter spaces. Therefore, if observers were reducible to structures, any observer should expect that in the very next moment the universe containing it will turn out to be crazy. There would be rare instances when the observer-like structure survives for a brief period of time, and even then, in most cases, it would experience a surrealist reality. Every time when this doesn't happen to us is a subtle reminder that we are more than the structure.

VII. IMPLICATIONS

It is sometimes believed that the state vector and the Hamiltonian are sufficient to recover everything else, the space structure, the tensor product structure (necessary for the existence of subsystems), and the correspondence between observables and physical properties. This seems to be needed by various quantum-first approaches to Quantum Gravity, see [5] and other references in [31]. In [31, 34] it was shown that this is possible only for properties that are constant across time and across states. Any quantum-first approach is extremely ambiguous, resulting in infinitely many solutions representing physically different worlds. But we could hope that this ambiguity is harmless and consistent with the empirical data. However, Theorem 1 shows that this isn't true. If it were true, the observer-like structures would not be able to know anything about the external world. This would contradict the most basic empirical facts and would make science impossible.

Theorem 1 also rejects the thesis that physical properties are purely relational, as proposed by Rovelli [27], since this also implies that observers would know nothing about the external world.

Proposals like the above may come from the implicit assumption that *ontic structural realism*, the thesis that only the structure exists, that there are only relations and no relata, is true [21]. Theorem 1 shows otherwise: not all isomorphic structures are created equal. Epistemic structural realism proposes that even if things have a nature of their own, this is inaccessible to us through science (see Section §IV). But Theorem 1 shows that, without knowing something in addition to the structures, we would know nothing. Something makes only some of the isomorphic observer-like structures be observers. So whatever breaks structural realism, this is manifest through the observers, or rather to the observers.

But what is an observer? In this article I had in mind the human observers as a directly verifiable example familiar to all of us. An observer-like structure is any structure isomorphic with the structure of an observer. But to restrict observers to humans would be anthropomorphism, so if there is a way to characterize observers in a non-anthropomorphic way, we should adopt it. But whatever the definition of an observer is, such an observer must have a structure. And regardless of the characteristic of the structure of the observers, Theorem 1 shows that not all structures isomorphic with it are observers.

This article makes no claim to define or elucidate what kind of structure a system must have to be a conscious observer. This problem belongs to other fields, from Neuroscience to philosophy of mind. But the results from this article inform these fields that observers, whatever they are, are not reducible to their structure. For example, the *computational theory of mind* proposes that mind is reducible to a computation [7, 25]. If "computation" means what is understood in Computer Science, this is already rejected [35], and if it also means the internal structure of the machine implementing it, this would contradict Turing universality [35] and Theorem 1. *Functionalism* [22] proposes that mind reduces to its function. If by "function" we understand structure and its dynamics, this is in conflict with Theorem 1. *Illusionism* proposes that *phenomenal consciousness*, experience itself, is an illusion of the computation or function of the structure [8, 11]. Even if we go down to the brain's finest structural details, as in the *identity theory* [29], we can't avoid Theorem 1. The structure of a Carbon atom or even of any particle can exist in any possible parameter space, hence the problem remains. On the other end of the spectrum we find the proposals that mind is not reducible to structure. *Panpsychism* proposes that even the elementary particles have such mental properties [13]. A naive rejection of panpsychism is that it adds new properties unknown in physics, and this should lead to different predictions than, for example, Particle Physics. But this article shows that such properties correspond in fact to the already known physical properties. Corollary 2 shows that this correspondence has to be unique (up to Poincaré and gauge symmetries), and it should go down to the complete set of basis observables. This implies a full identification between mental and physical properties, suggesting a form of monism. *Neutral monism* proposes that the intrinsic nature of things appears externally as physical properties, and internally as mental properties [28]. *Idealism* [15, 17, 18, 30] is a monistic position that identifies the physical properties and the physical laws as the structure and dynamics of a fundamental consciousness. Another position is *dualism* [26], stating that both matter and mind are fundamental and either interact or mirror one another. This would unnecessarily duplicate both the ontology and the structures, so it would be redundant. A monistic position wouldn't have this problem. Whatever the explanation is, it should take into account that observers are not reducible to their structure, and Corollary 2 shows that this should go all the way down. There is an essentially unique parameter space supporting observers, and observer-like structures from other parameter spaces are *philosophical zombies* [19]. This difference seems to elude both the theoretical description and the intersubjectively verifiable experiments. However, as shown by Theorem 1 and Section §VI, this is revealed empirically by the fact that at any instant we could turn out to be observers in a crazy surrealistic world completely unrelated to our memories, but every time we find ourselves in a friendly one. As if the universe is so friendly that it reassures us at every instant about this fact. Indubitably, structure remains important, and trying to characterize the structure of conscious systems is essential in advancing our understanding of observers.

The existence of an ontologically special basis beyond structure and relations was conjectured previously because it allows reasoning about the self-location of the observer in a way that leads to the Born rule [32] and endows the Many-Worlds Interpretation with genuine probabilities and a local ontology [36]. Other proposals that don't use a fixed ontic basis fail to get the Born rule ([36], Proposition 1, §6). Theorem 1 justifies conjecturing such ontic differences, by showing that this is unavoidable.

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