Underdetermination in Classic and Modern Tests of General Relativity

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Abstract

Canonically, 'classic' tests of general relativity (GR) include perihelion precession, the bending of light around stars, and gravitational redshift; 'modern' tests have to do with, *inter alia*, relativistic time delay, equivalence principle tests, gravitational lensing, strong field gravity, and gravitational waves. The orthodoxy is that both classic and modern tests of GR afford experimental confirmation of that theory *in particular*. In this article, we question this orthodoxy, by showing there are classes of both relativistic theories (with spatiotemporal geometrical properties different from those of GR) and non-relativistic theories (in which the lightcones of a relativistic spacetime are 'flattened') which would also pass such tests. Thus, (a) issues of underdetermination in the context of GR loom much larger than one might have thought, and (b) given this, one has to think more carefully about what exactly such tests in fact *are* testing.

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1. Introduction

Einstein's general theory of relativity (GR) is our best theory of space and time, and has captured both the popular and the academic imagination. How could it not? The theory tells a compelling story, informing us not only that our most commonly held intuitions concerning space and time are wrong, but also that space and time themselves are inextricably interwoven, both together and with the material content of the world. According to GR, gravity is not *really* a force, but rather a manifestation of spacetime curvature, which governs the motion of all matter and energy in the universe.[4]

Furthermore, the story goes, GR is one of the most well-confirmed theories in all of science as it has been subjected for over a century to rigorous tests and held up to the strictest scrutiny. Such testing and empirical success is understood to vindicate the theory in its entirety. But does this mean that all of GR's radical conclusions regarding the nature space, time, and gravity are thereby confirmed conclusively? In this article, we analyze closely the classic and modern tests of GR in order to explore the nature of these claims and examine what these tests in fact do manage to confirm conclusively—we find that the situation is more delicate than one might have thought.

To be specific, here are the two classes of tests of GR which we consider in this article:

Classic tests perihelion precession, bending of light, and gravitational redshift.

Modern tests strong field gravity, gravitational lensing, FLRW cosmological solutions, gravitational waves, and black holes.

While this story of empirical success is certainly remarkable, we argue that the limitations of what these tests have actually shown are not widely appreciated. Indeed, while the extent to which these tests either disconfirm or severely constrain theories of gravity that are known to be *empirically inequivalent* alternatives to GR is well-appreciated—this has been a major theme in modern gravitational research!

[4]We mean this in a loose sense—we wish to remain agnostic about which object in GR represents the gravitational field *sensu stricto*. For more on this latter issue, see (Lehmkuhl 2008).

(see e.g. (Will 2014))—what is not appreciated is that there are alternative theories that can be understood to be *empirically equivalent* to GR in important ways and for which these tests offer far less in terms of conclusive analysis. In particular, the classic tests of GR are significantly less constraining than is often appreciated—often, for example, they amount to tests of a particular geometric solution *within* GR; a solution that can readily be reproduced by a number of alternative theories. Although modern tests are in a certain sense more stringent, the data resulting from these tests which is taken to confirm GR can likewise be accounted for by a number of alternative theories. In what follows in this article, we analyze how and to what extent both classes of test can be passed by theories of gravity other than GR. Thereby, we demonstrate that there are still persistent underdetermination issues even within the context of what we consider to be one of the most well-tested and rigorously-confirmed theories of physics.

The structure of the article is as follows. In §2, we remind the reader of the issues in general philosophy of science relevant to this article—in particular, issues of underdetermination of theory by evidence and of theory equivalence. In §3, we present the classic tests of GR. In §4, we introduce the distinction between projective and conformal structures—these structures will be useful in later sections when it comes to classifying spacetime theories alternative to GR. In §5, we consider the 'geometric trinity' of relativistic alternatives to GR (reviewed by Jiménez et al. (2019)), and demonstrate that all theories in this 'trinity' would pass both the classic and modern tests of GR. In §6, we consider recently-developed non-relativistic spacetime theories (reviewed by Hansen et al. (2020)) which also pass the classic tests of GR, as well as (almost!) all the modern tests of GR. In §8, we consider how to respond to the issues of underdetermination presented by these cases. In §9, we wrap up.

2. Underdetermination and Theory Equivalence

In this section, we review issues of underdetermination of theory by evidence (§2.1) and of theory equivalence (§2.2).

2.1. Weak and strong underdetermination.

Underdetermination of theory by evidence comes in different stripes. 'Weak underdetermination' (also known as 'transient underdetermination') refers to underdetermination with respect to the currently available data (Ladyman 2001, p. 246). This occurs between distinct theories that give different predictions for at least some empirical phenomena. However, these predictions either have not yet been tested or cannot currently be tested, meaning that such underdetermination could conceivably be broken in the future. This is a familiar theme in current gravitational research, and in particular within cosmology. For example, the standard model of cosmology, dubbed the ΛCDM

('Λ cold dark matter') model, describes a universe completely consistent with the FLRW (Friedmann-Lemaitre-Robertson-Walker) solution of GR. Yet, puzzles concerning the nature of dark matter and dark energy have led cosmologists to explore a vast number of theories which explain Λ and/or CDM, in ways that modify the typical understanding of these entities within a GR framework. A few (amongst many) examples include Brans-Dicke gravity, $f(R)$ gravity, quintessence, and certain relativistic extensions of modified Newtonian dynamics (MOND).[5] Another cosmological example can be found in examining attempts to model the early universe, where there are a number of theoretical frameworks in play such as inflation, bouncing cosmologies, and string gas cosmology.[6] These theories can and do give rise to distinct predictions that differentiate them from each other, but current observational constraints are consistent with a number of possibilities. The hope is that future observations will break this underdetermination.

In contrast to weak underdetermination, 'strong underdetermination' refers to underdetermination with respect to all empirical data that will ever be available (Ladyman 2001, pp. 261-262). This is often discussed in terms of distinct but empirically equivalent models within the same theory, where this refers usually to empirically equivalent models related by a symmetry. A famous example invites us to consider two Newtonian models of the universe, one at absolute rest and the other boosted by a constant velocity (van Fraassen 1980, pp. 46-47). This example illustrates that there is a plurality of distinct Newtonian models of the universe which are compatible with the empirical data, yet an observer embedded in any of the worlds represented by those models could never distinguish empirically between them because the theory itself indicates that such absolute standards of motion are unobservable.[7] However, strong underdetermination need not be restricted to empirically equivalent models of a particular theory. Strong underdetermination can also exist between models of distinct yet empirically equivalent *theories*. This has been considered in the recent philosophy literature in the context of theories related by dualities—see e.g. Butterfield (2021), De Haro and Butterfield (2017), Matsubara (2013), and Read (2016b).

Strong underdetermination is understandably seen as a serious threat to scien-

[5]See Clifton et al. (2012) and Joyce et al. (2016) for some physics reviews on various modified gravity proposals and Duerr and Wolf (forthcoming) and Martens and Lehmkuhl (2020) for some relevant philosophical analysis.

[6]See Brandenberger and Peter (2017), Guth et al. (2014), and Ijjas and Steinhardt (2016) for physics discussions of these theories and Dawid and McCoy (n.d.), Wolf (n.d.), and Wolf and Thébault (forthcoming) for philosophical analyses of some of the extra-empirical philosophical issues at play in these debates.

[7]For some recent discussions as to whether absolute velocities really are empirically unobservable in Newtonian mechanics, see Jacobs (2022) and Murgueitio Ramírez and Middleton (2021). Note also that one might still be able to distinguish between (some) empirically indistinguishable worlds using the linguistic resources of indexicals—for some discussion of this point, see Cheng and Read (2021), but we won't go into this further in this article.

tific realism because the existence of empirically equivalent yet putatively distinct models calls into question the extent to which our scientific models can correspond fully to reality—surely, the realist thought goes, at most *one* of those models can correspond to reality, so what gives? If multiple, ontologically distinct models can correctly describe the same phenomena, what license do we have for adopting a realist attitude towards the non-observable structures in the models of our theories? There is thus a serious motivation to deny that instances of strong underdetermination truly exist.

In this article, we argue that there are instances of both strong and weak underdetermination within gravitational physics at the level of the most significant regimes and tests that most would consider to be conclusively settled in the favour GR. For example, there are theories that are known to be dynamically equivalent to GR, but which postulate very different kinds of geometrical structures in their description of gravity. These are the 'Teleparallel Equivalent to General Relativity' (TEGR), which is by now somewhat known in the philosophical literature and describes gravitational effects using torsion in a flat spacetime, and the 'Symmetric Teleparallel Equivalent to General Relativity' (STGR), which is substantially less well-known in the existing philosophical literature and describes gravity as a manifestation of non-metricity in a flat spacetime. As we shall see, no empirical test could ever discriminate between these theories,[8] which are collectively known as the 'geometrical trinity' of relativistic gravitational theories (Jiménez et al. 2019). Modulo an interpretive move to collapse this trinity into one theory, this is arguable a live case of strong underdetermination.

In addition to the geometric trinity, however, there is a case of *weak* underdetermination in the context of the tests of GR which has to this point gone largely unnoticed within the literature (although Hansen et al. (2019b) do provide some discussion of these issues). There is an even less familiar theory, known as 'Type-II Newton Cartan Theory' (NCTII),[9] which describes gravity in terms of a torsionful, fully non-relativistic spacetime (Hansen et al. 2020). While this theory differs from the theories in the geometric trinity in important ways that do end up distinguishing their empirical claims from each other (but—interestingly—only in the context of some very recent modern tests of GR, to do with gravitational wave physics, more on which below), this theory can also be understood to pass *all* the classic tests

[8] At least within the regime of classical physics: the theories do differ by boundary terms (see Wolf and Read (2023) for philosophical discussion) which might manifest as instantionic effects after path integral quantisation—this, however, deserves to be worked out in detail, and we won't discuss it further in this article, save for some brief remarks in §9. It's also worth making the technical point here that in making this claim, we're restricting to GR models set on parallelizable manifolds: doing so undermines none of the points which we'll go on to make in this article.

[9] Partly this theory is not well known because it first appeared in the literature only in 2014—see (Christensen et al. 2014); moreover, the action principle for the theory appeared in the literature only in 2019, with (Hansen et al. 2019a).

of GR. Furthermore, this theory does not merely pass these tests in the same way that, say, Brans-Dicke theory can be understood to pass solar system tests of GR by tuning its coupling parameters. Rather, as we shall see, NCTII gives identical predictions as GR for these important solar system tests. This leads to the bizarre conclusion that this instance of weak underdetermination between relativistic theories of gravity and non-relativistic theories of gravity has only been broken far more recently than most would imagine—indeed, with the advent of LIGO!

2.2. **Theory equivalence.** As should be clear from the preceding discussion of underdetermination, it is important to establish what is meant by 'equivalence' with regard to determining whether two theories are truly distinct from each other, or in fact are merely different formulations of the same theory. For example, the standard line goes that Heisenberg matrix mechanics and Schrödinger wave mechanics are not only empirically equivalent, but in fact are different formulations of the same theory of quantum mechanics.[10] How do we determine whether or not our gravitational theories introduced above are truly equivalent to one another? The literature concerning questions on theoretical equivalence is vast and offers many possible answers to this question (Weatherall 2019a,b). Of particular relevance to us are notions of 'empirical equivalence' and 'interpretational equivalence', to both of which we have already alluded.

Empirical equivalence is generally taken to be a necessary but not sufficient condition for full theoretical equivalence. Complete empirical equivalence would mean that two theories have the same range of applicability regarding the empirical scenarios which they describe and provide indistinguishable predictions for said empirical scenarios. To be slightly more specific, we can understand (to use the terminology of van Fraassen (1980)) that models M of a theory T have 'empirical substructures', which can represent observable phenomena. Suppose, for every M of T, there is an M' of T', where the empirical substructures of M and M' are isomorphic. Then, T and T' can be understood to be empirically equivalent.

This standard for empirical equivalence distinguishes between the theories within the geometric trinity on the one hand and NCTII on the other because (as we'll see in detail below) NCTII appears not to be able to support the same claims as the geometric trinity with regard to the phenomena of gravitational waves. Thus, we have here a case of *weak* underdetermination that modern (but not classic!) tests of GR conceivably can break.

Reaching a conclusive verdict regarding the equivalence or inequivalence of the gravitational theories *within* the geometric trinity, on the other hand, is a far more subtle business. These theories differ quite substantially in terms of the structures

[10] Actually, it's questionable whether this claim is completely correct: see (Muller 1997a,b). But the example is sufficient to illustrate our point.

from which they are built, yet they can all be understood to be dynamically equivalent to each other as their actions differ only by boundary terms. With regard to dynamical content, these theories are equivalent full stop. As the empirical tests of GR that have been performed to date have only been sensitive only to dynamical content, we here set aside questions concerning demonstration of equivalence at the level of empirical claims regarding boundary-related content and phenomena (these issues are discussed further by Wolf and Read (2023)).[11] For our purposes here, the geometric trinity clearly seems to constitute an example of strong underdetermination, where both classic and modern tests that have been performed to date offer no hope of discriminating amongst the theories.

However, could we still not just say that all of these theories within the trinity are actually somehow the same 'theory', but merely dressed up in different mathematical details or formalisms? One way of doing this would be to demonstrate that these theories can be understood as interpretationally equivalent to each other. Interpretational equivalence in this sense holds when theories are understood to make all of the same claims about the phenomena they describe, going beyond purely empirical considerations (Coffey 2014). This would include claims about what kinds of entities exist in the world and what properties they have, as well as what the fundamental laws of nature are. If this can be done, this would offer an avenue towards breaking the underdetermination. However, arguing that the geometric trinity theories are interpretationally equivalent to each other in their current forms is not the only way to proceed. Indeed one could also move to new interpretive frameworks to cash out their equivalence. We return to this issue in §8, but before doing so we need to say more regarding how all the theories introduced up to this point interact with the tests of GR, both classic and modern.

3. Classic Tests of General Relativity

The three classic tests of GR were all identified by Einstein early on in his development of the theory (see (Einstein 1916) for an early summary and discussion of these tests). Given their temporal proximity to the development of GR, explanatory power, and novel nature, they were hailed as spectacular confirmations and christened a new paradigm for space, time, and gravitation. The classic tests are:

1. Perihelion precession of Mercury's orbit: It had long been known that Mercury's perihelion had an anomalous precession of about 43 arcseconds per century (Le Verrier 1859; Newcomb 1882). Orbital precession is predicted by

[11] Even if it does seem that claims of empirical equivalence can be supported regarding boundary phenomena (for example, both theories reproduce the GR result for black hole entropy as shown by Heisenberg et al. (2022) and Oshita and Wu (2017)), it is important to note that this is a non-trivial matter that requires further investigation and is often ignored in the literature on theoretical equivalence.

Newtonian gravity in the presence of perturbing forces and the total precession was an order of magnitude larger than the unexplained value as most of it had been accounted for by determining the perturbing affects of other solar system bodies. However, the 43 arcseconds remained persistently unaccounted-for. Given that this empirical result was already known, explaining this was a 'simple' matter of solving the Kepler two-body problem in GR and noticing that additional terms in the effective gravitational potential produce this effect.

2. Gravitational deflection of light: This is another effect that has a Newtonian analogue as it was understood that even in a Newtonian framework gravitational effects should bend the paths of light rays (Soares 2005). However, Einstein correctly used GR to predict that the bending of light due to the Sun's gravity should be twice the value expected from Newtonian gravity at roughly 1.75 arcseconds. This was first confirmed by Eddington during his famous expedition to measure a solar eclipse in 1919 (Dyson et al. 1920).

3. Gravitational redshift of light: This test does not have a Newtonian analogue. The basic idea is that the gravitational redshift of light will result from the fact that there is a difference in proper time depending on where observers are in a gravitational field. Early attempts offered some measurements of this effect in the spectral lines of stars (Wheeler 1957); however, gravitational redshift was more conclusively demonstrated by the famous Pound-Rebka experiments (Pound and Rebka 1959; Pound and Rebka 1960).

As both Wheeler (1957) and Dicke (1957) noted at the famous 1957 Chapel Hill conference, at that point in time, the experimental evidence used to infer support for GR was not significantly better than it was only a few years after the advent of the theory. Essentially the only significant work that had been done in this area involved incrementally improved versions of these classic tests. Peebles (2017) attributes the lack of attention afforded by physicists in the first half of the 20$^{\text{th}}$ Century to the further testing GR to a number of factors, including nuclear and particle physics consuming most of the oxygen within the community and the seeming absence of any technologically feasible alternative experiments. However, he also attributes this neglect the community's unreserved, overwhelming acceptance of GR due to its compelling theoretical architecture. Considering that the classic tests were the only firm empirical basis for GR for the better part of half a century, they evidently occupy a special place in the history of gravitational physics.

The standard view within physics is that these tests established definitively that gravity is described by a metric encoding spacetime curvature.[12] For example,

[12] In the case of gravitational redshift, the consensus is questioned by Brown and Read (2016),

Will (2014), in one of his many comprehensive reviews of the experimental status of gravitational physics, expresses the conventional wisdom that gravity must essentially be a manifestation of spacetime curvature, with any deviations from pure GR principles and predictions being at best incredibly small.[13] In what follows, we will examine closely how GR and other theories of gravity pass these tests successfully; in the process, we will highlight what these empirical studies *actually* test in our gravitational theories.

4. Projective and Conformal Structure in Gravitational Theory

Standard foundational presentations of general relativity typically proceed by specifying that kinematical possibilities of the theory are given by tuples $\langle M, g, \Phi \rangle$, where M is a four-dimensional differentiable manifold, g is a Lorentzian metric field on M, and Φ a placeholder for material fields. At the level of dynamics, these fields are specified to satisfy the Einstein equation plus any dynamical equations for the material fields (e.g., Maxwell's equations); moreover, typically textbooks make various interpretative stipulations—e.g., that metric distances and times are read off by ideal rods and clocks, respectively: see e.g. (Malament 2012, p. 136).[14]

In this article, we proceed somewhat differently, by treating the basic geometric object(s) of the theory not as a Lorentzian metric field g, but rather as projective structure $\mathcal{P}$, defined as the equivalence class of affine connections Γ

$$\Gamma \overset{\mathcal{P}}{\equiv} \Gamma' \quad \Leftrightarrow \quad \exists \text{ a 1-form } \psi \text{ s.t. } {\Gamma'^{\mu}}_{\nu\rho} = {\Gamma^{\mu}}_{\nu\rho} + {\delta^{\mu}}_{\nu}\psi_{\rho} + {\delta^{\mu}}_{\rho}\psi_{\nu},$$

and conformal structure $\mathcal{C}$, defined as the equivalence class of Lorentzian metrics g

$$g \overset{\mathcal{C}}{\equiv} g' \quad \Leftrightarrow \quad \exists \text{ a function } f \text{ on } M \text{ s.t. } g' = e^{f} g$$

—such a proposal has also been made by *inter alia* Stachel (2011). Since Weyl (1921), it has been known that a Lorentzian metric is fixed uniquely by its associated projective and conformal structures; since the seminal paper of Ehlers et al. (2012), it has also been known that a given $\mathcal{P}$ and $\mathcal{C}$, subject to some additional (supposedly innocuous) conditions, fix uniquely a Lorentzian metric.

who argue that the results of Pound-Rebka-type experiments can be accounted for by considering accelerating frames in SR. We won't go into these arguments further in this article.

[13]For the classic tests, this is usually cashed out from within the framework of the parameterised post-Newtonian ('PPN') formalism (Will 2014). In essence, the PPN formalism represents an expansion of the GR metric in terms of its Newtonian approximation and higher order terms that capture the GR effects from the spatial and temporal parts of the metric. This also provides a convenient formalism within which to facilitate comparisons with empirically inequivalent theories of gravity through the behavior of certain higher-order terms in the expansion.

[14]'Causal-inertial' constructivism *à la* Ehlers et al. (2012) offers an alternative to this 'chronometric' approach to the empirical interpretation of GR: see (Adlam et al. 2022; Linnemann and Read 2021a) for discussion.

The merit of working with the sub-metrical constituents $\mathcal{P}$ and $\mathcal{C}$ of a Lorentzian metric g is that doing so affords one greater flexibility to explore the results of modifying such structures in turn (a point also made by Stachel (2011)). Before we get to this, however, we should recall the canonical views on the physical significance of $\mathcal{P}$ and $\mathcal{C}$. Projective structure $\mathcal{P}$ identifies certain trajectories as being geodesics; empirically, it is supposed to be picked out by the paths of unforced test particles. Conformal structure, on the other hand, refers to the distribution of light cones, and is supposed to be identified empirically by the paths of light rays. (For further discussion of these empirical considerations, see again (Ehlers et al. 2012).) As Stachel (2011) writes, "the relation between conformal and projective structures reflects—and is reflected by—the relation between classical massless wave theories, which in practice means electromagnetism, and classical particle theories and their ensembles represented by the stress-energy tensors of ordinary matter."

In defining projective structure, it is helpful to find an object which is invariant under projective transformations. This can be found by realizing that the trace of the affine connection $\Gamma^{\kappa}_{\mu\kappa}$ transforms as $\Gamma'_{\mu} \to \Gamma_{\mu} + \psi_{\mu}$. Thus, we can construct an object $P^{\kappa}_{\mu\nu}$:

$$P^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \frac{1}{5}\left(\delta^{\kappa}_{\mu}\Gamma_{\nu} + \delta^{\kappa}_{\nu}\Gamma_{\mu}\right), \tag{1}$$

where this object is invariant under projective transformations. We can then say that when $P(\Gamma) = P(\Gamma')$ (indices suppressed), both connections are projectively equivalent and belong to the equivalence class $[\Gamma]$, which is identical to $\mathcal{P}$.

In the context of Lorentzian geometries, conformal structure $\mathcal{C}$ refers to an equivalence class of metric tensors $[g]$. This equivalence class of metric tensors determines the light-cone structure and leaves this structure invariant. This essentially amounts to singling out classes of null-hypersurfaces. For example, g' and g belong to the same equivalence class $[g]$ if there is a transformation $g' \to \Omega^2 g$, where $\Omega(x)$ is a function of scale, that leaves the distribution of light-cones unchanged. Another way of thinking about this is that such scale transformations alter concepts like lengths and volumes, but leave angles intact. Of course, one might reasonably ask what conformal structure outside of the context of Lorentzian geometries could possibly amount to—we'll return to this question in §6, when we introduce non-relativistic theories of gravity.

5. Classic Tests and the Geometrical Trinity

5.1. **General Relativity.** Since a Lorentzian metric field g is fixed uniquely (up to a constant factor) by its associated projective and conformal structure, one can (completely uncontroversially) rewrite the kinematical possibilities of GR as tuples $\langle M, \mathcal{P}_{GR}, \mathcal{C}_{GR}, \Phi\rangle$. Here, the projective structure $\mathcal{P}_{GR}$ is associated with the equivalence class of affine connections defined by the Levi-Civita connection Γ and the conformal structure $\mathcal{C}_{GR}$ is associated with the equivalence class of metric tensors g differing by (spacetime-dependent) scale transformations. Φ is a placeholder for material fields.

The Levi-Civita connection, whose components in a coordinate basis are

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\rho} \left(\partial_{\lambda} g_{\rho\nu} + \partial_{\nu} g_{\rho\lambda} - \partial_{\rho} g_{\nu\lambda} \right), \tag{2}$$

is crucial to the conceptualization of GR as a theory because it is the only affine connection that realizes the unification of gravity and inertia. That is, it is this choice that leads to the conclusion that gravity is *not* a force, but rather is a manifestation of spacetime curvature. To see this, let us briefly consider how this principle manifests itself in the mathematical formalism of GR and its description of gravity. Our first point of contact—and indeed the starting point for conducting many of the actual tests of GR—is the geodesic equation,

$$\frac{d^2 x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\nu\lambda} \frac{dx^{\nu}}{d\tau} \frac{dx^{\lambda}}{d\tau} = 0, \tag{3}$$

where τ parameterizes the curve in terms of proper time, and x^{μ} are the coordinates in use. This defines the equations of motion that would apply to a massive body or photon that is experiencing gravity as described by GR. In Newtonian physics, a test body undergoing inertial motion will naturally move on straight lines in Euclidean space unless or until acted upon by a force. (3) generalizes this notion of inertia, or straight-line motion, to include curved spaces as the affine connection represents how basis vectors change given an arbitrary curved manifold. Thus, we can understand the geodesic equation as unifying both gravity and inertia: a body experiencing gravitation is no longer understood to move in a path that deviates from straight lines in Euclidean space due to a gravitational force, but rather is now understood to move in straight lines within a curved spacetime geometry created by gravity.[15]

What's important to us to note about the Levi-Civita connection is that it is the unique connection which is (i) torsion free and (ii) metric compatible. These conditions, respectively, are $\Gamma^{\lambda}_{[\mu\nu]} = 0$ and $\nabla_{\rho} g_{\mu\nu} = 0$ (Wald 1984, Ch. 3.1). The first condition ensures that vectors that are parallel transported along each other will form a closed parallelogram and the second condition ensures that the lengths of vectors do not change during parallel transport (Jiménez et al. 2018). This connection defines the projective structure $\mathcal{P}_{GR}$ of GR; to anticipate, other theories in the geometric trinity will modify this projective structure (so, in light of what we've said above, those theories will be interpreted as force theories of gravity).

[15]More technically, one could say this: since the difference between any two connections is a tensor, if gravity and inertia are unified in the Levi-Civita connection, then they will not be with respect to any other connection, for which the RHS of (3) will contain an additional tensorial piece, to be interpreted as a gravitational force. We will see more of this below.

The full dynamical content of GR is expressed by the Einstein field equations, which are those equations obtained by varying the following action—the Einstein-Hilbert action—with respect to the metric:

$$S_{EH} = \frac{1}{2}\int d^4x\sqrt{g}R; \qquad \delta S_{EH} = 0 \implies R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = kT_{\mu\nu}. \tag{4}$$

Here $R_{\mu\nu}$ is the Ricci curvature tensor, R is the Ricci scalar curvature, $g_{\mu\nu}$ is the metric, and $T_{\mu\nu}$ is the stress-energy tensor for matter fields. The Ricci curvature tensor $R_{\mu\nu}$ is built out of the coefficients of the Levi-Civita connection, which in turn are built out of the metric $g_{\mu\nu}$ and its derivatives.

Thus, we see that the LHS of the Einstein field equations is entirely composed of the metric, and first and second derivatives of the metric, and these are then organized into mathematical objects that quantify concepts like curvature. All together, these structures afford substance to the idea that GR geometrises gravity in terms of spacetime curvature. That is, gravity is described by a dynamical metric tensor g; this metric defines a curved spacetime that is determined by the distribution of mass-energy content, and motion under the influence of gravity conforms to inertial trajectories within this curved spacetime.

5.2. **Classic tests of GR.** In basic terms, the classic empirical tests of GR require (i) solving the Einstein equation to obtain the metric g, (ii) predicting how objects (massive bodies and photons) should behave in such a spacetime environment, and (iii) testing those predictions. The classic tests of GR are:

1. perihelion precession of Mercury's orbit,
2. gravitational deflection of light, and
3. gravitational redshift of light.

It turns out that all of these tests are essentially probes of the Schwarzschild metric of GR. The Schwarzschild metric is the solution to the Einstein field equations which describes the gravitational field outside of a spherical mass M with no electric charge or angular momentum. The solution is obtained by solving the field equations for a metric that is spherically symmetric, static, and in vacuum. Considering these properties, it is an excellent candidate for many astrophysical applications, including modelling the trajectories of planets in the gravitational field of the sun and modelling the trajectories of photons in the gravitational fields of the sun or earth. The form of the Schwarzschild metric is

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)c^2dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{5}$$

Once one has the form of the Schwarzschild metric, one can feed this into the geodesic equation in order to calculate how test particles like masses or photons would behave in this system. Upon using symmetries to simplify the set of coupled differential equations and imposing that motion happen in the equatorial plane, one obtains the following (Carroll 2019, Ch. 5):

$$\frac{1}{2}\left(\frac{dr}{d\lambda}\right)^2 + V(r) = \frac{1}{2}E^2, \tag{6}$$

which is a simple equation describing the classical energy E of a mass with a kinetic energy and potential energy $V(r)$ as a function of radius r. Here $V(r) = \frac{1}{2}\epsilon - \epsilon\frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GML^2}{r^3}$, where L refers to angular momentum and $\epsilon = g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$ is a constant of motion, which is $\epsilon = 1$ for massive particles and $\epsilon = 0$ for photons.

Following for example Wald (1984, Ch. 6) or Carroll (2019, Ch. 5), one can then solve these equations in order to carry out the classic tests of GR for massive bodies and photons, respectively. In particular, for massive bodies ($\epsilon = 1$) GR introduces an extra term (beyond the standard Newtonian terms) in the effective radial potential in the two body problem that is proportional to $\sim r^{-3}$, which induces the additional 43 arcseconds of precession for Mercury's perihelion that had yet to be accounted-for. Similarly, one can explore this problem for photon orbits ($\epsilon = 0$) and conclude that they should expect to observe roughly ~ 1.7 arcseconds of deflection around the sun. This is twice the expected Newtonian value, which comes from the fact that a Newtonian analysis of light deflection in effect only considers the temporal component of the metric tensor,[16] while leaving out contributions from the spatial components that are present in the above equation. Finally, the gravitational redshift of light can be seen from considering two stationary observers in a Schwarzschild geometry, one who observes the emission of the photon and another who observes the emitted photon far away from the first observer. There is a difference in proper time between the observers who are at different radii r; consequently, they will measure different frequencies for the emitted photons depending on where they are in the gravitational field.

5.3. **Geometric Trinity.** These tests were all taken to offer spectacular evidence for GR and to confirm that gravity is indeed a manifestation of relativistic spacetime curvature. After all, these tests probe a specific solution for the spacetime metric g, which within the framework of GR encodes spacetime curvature. How-

[16]The simplest way to understand the effective Newtonian limit of GR is to consider the Newtonian limit where gravity is weak and speeds are low compared to the speed of light. This procedure identifies the effective (Newtonian) gravitational potential as $\frac{2GM}{r}$, or the second term in the temporal component of the Schwarzschild metric. While this is a good approximation in many contexts, it clearly does not work when considering photon geodesics because the fact that the speed being considering is the speed of light means that the spatial components of the metric are just as significant as the temporal component that approximates the Newtonian potential.

ever, does this evidence truly single out GR as the correct theory of gravitation for our universe? In fact, we can consider other theories which do not rely upon notions of spacetime curvature. In particular, we can consider how the 'Teleparallel Equivalent to General Relativity' (TEGR) (Aldrovandi and Pereira 2013; Bahamonde et al. 2023; Golovnev 2018) and the 'Symmetric Teleparallel Equivalent to General Relativity' (STGR) (Jiménez et al. 2018; Nester and Yo 1998) account for the phenomena associated with the classic tests.

TEGR is a theory in which gravity is a manifestation not of spacetime curvature, but rather of spacetime *torsion*. Curvature can be quantified as the angle by which a vector rotates when it is parallel transported along a closed path, or in other words, how the tangent spaces of the geometry roll along the curve. Likewise, torsion can be understood as the way that tangent spaces twist along the curve and is mathematically described by the torsion tensor $T^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{[\nu\lambda]}$, which we already know vanishes in GR as one of the conditions that defines the unique Levi-Civita connection. TEGR replaces the zero torsion condition in the connection with a zero curvature condition. This results in what is known as the Weitzenböck connection, for which (i) $R^{\lambda}{}_{\mu\nu\sigma} = 0$ and (ii) $\nabla_{\rho} g_{\mu\nu} = 0$, but $T^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{[\nu\lambda]} \neq 0$ (Jiménez et al. 2018). Thus, we can define TEGR as a spacetime theory with models of the form $\langle M, \mathcal{P}_{TEGR}, \mathcal{C}_{GR} = \mathcal{C}_{TEGR}, \Phi \rangle$, where $\mathcal{P}_{TEGR}$ is the projective structure associated with an equivalence class of affine connections containing as an element Γ_{TEGR}, which is the Weitzenböck connection (which replaces the Levi-Civita connection of GR).[17] The theory is given by the action

$$S_{TEGR} = \frac{1}{2} \int d^4x \sqrt{g} T, \tag{7}$$

where T is the torsion scalar in analogy with the Ricci curvature scalar.

Similarly, STGR is a theory in which gravity is a manifestation of spacetime *non-metricity*, and curvature and torsion vanish. Non-metricity can be understood as the geometric effect whereby the act of parallel transporting a vector changes the length of this vector. That is, STGR obeys the conditions (i) $R^{\lambda}{}_{\mu\nu\sigma} = 0$ and (ii) $T^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{[\nu\lambda]} = 0$, but $Q_{\rho\mu\nu} := \nabla_{\rho} g_{\mu\nu} \neq 0$ (Jiménez et al. 2018). Thus, we can define STGR as a spacetime theory with models of the form $\langle M, \mathcal{P}_{STGR}, \mathcal{C}_{GR} = \mathcal{C}_{STGR}, \Phi \rangle$, where $\mathcal{P}_{STGR}$ is the projective structure associated with an equivalence class of affine connections containing as an element Γ_{STGR}, which is the non-metric connection of the theory (which replaces the Levi-Civita connection of GR). This theory is given by the action

$$S_{STGR} = \frac{1}{2} \int d^4x \sqrt{g} Q, \tag{8}$$

[17] TEGR is sometimes formulated in terms of vielbeins rather than a metric as this makes the gauge structure of the theory more apparent—see e.g. (Aldrovandi and Pereira 2013). We will mostly stick to the metric formulation in this article.

where Q is the non-metricity scalar, again in analogy with the Ricci curvature scalar and the torsion scalar.

At first blush, these theories appear to be very different to GR. After all, they are built out of entirely different geometrical structures; they reject firmly a fundamental tenant of GR that gravity is a manifestation of spacetime curvature because both of these theories mandate that spacetime is necessarily flat! Nevertheless, we can concisely understand all of these theories in terms of systematically altering the projective structure used in the initial construction of GR. To see this, consider that the most general affine connection has the following decomposition:

$$\Gamma^{\mu}{}_{\nu\lambda} = \overset{\circ}{\Gamma}{}^{\mu}{}_{\nu\lambda} + K^{\mu}{}_{\nu\lambda}(T^{\mu}{}_{\nu\lambda}) + L^{\mu}{}_{\nu\lambda}(Q_{\mu\nu\lambda}), \tag{9}$$

where $\overset{\circ}{\Gamma}{}^{\mu}{}_{\nu\lambda}$ is now defined as the Levi-Civita connection, and $K^{\mu}{}_{\nu\lambda}(T^{\mu}{}_{\nu\lambda})$ and $L^{\mu}{}_{\nu\lambda}(Q_{\mu\nu\lambda})$ are known as the contorsion and distorsion tensors respectively and are functions of their respective torsion and non-metricity tensors (Ortin 2004, p. 9). The conditions relating to curvature, torsion, and non-metricity that are applied in constructing all of these gravitational theories are effectively constraining the form of the affine connection used in the particular gravitational theory of interest. That is, $\Gamma_{TEGR} = \overset{\circ}{\Gamma}{}^{\mu}{}_{\nu\lambda} + K^{\mu}{}_{\nu\lambda}(T^{\mu}{}_{\nu\lambda})$ and $\Gamma_{STGR} = \overset{\circ}{\Gamma}{}^{\mu}{}_{\nu\lambda} + L^{\mu}{}_{\nu\lambda}(Q_{\mu\nu\lambda})$ and the affine connections of these respective theories can be understood as being composed of the initial Levi-Civita part along with a non-vanishing piece from torsion or non-metricity.

While the projective structure utilized in GR has clearly been altered, the conformal structure has not as the spacetime metric—which encodes said structure—has not changed. In the process of building these theories we have only manipulated properties of the affine connection, while retaining the same notions of timelike, spacelike, and null intervals inherited from the basic relativistic conception of causal structure. So, this family of theories all share the same conformal structure such that $\mathcal{C}_{GR} = \mathcal{C}_{TEGR} = \mathcal{C}_{STGR}$.

It is one thing to construct a gravitational theory that is fundamentally different from GR, but another thing entirely for that theory to account for the same empirical phenomena in an equally satisfactory manner. Can TEGR and STGR actually achieve this? The above decomposition of the affine connection gives us a common language that not only applies when relating the affine connections used within the respective theories, but which can also be used in order to translate between all of the structures used within these theories. If one is interested in investigating the dynamical content of these theories and comparing them to each other, then one can use these relationships to write the actions of each theory in terms of the geometrical structures of one of the others. For example, if we want to compare the trajectories of particles in GR to those in TEGR, we can use these mathematical relationships to write the TEGR action in the language of GR, which amounts to translating between torsion and curvature. This procedure yields the following translation between the actions of GR and TEGR: (Aldrovandi and Pereira 2013, Ch. 9.2)

$$S_{TEGR} = \frac{1}{2}\int d^4x\sqrt{g}R + \int d^4x\sqrt{g}\nabla_\mu T^{\alpha\mu}_{\alpha}. \tag{10}$$

In other words, the TEGR action is equivalent to the Einstein-Hilbert action up to a total divergence term. The same procedure for STGR yields:

$$S_{TEGR} = \frac{1}{2}\int d^4x\sqrt{g}R + \int d^4x\sqrt{g}\nabla_\mu (Q^{\alpha\mu}_{\alpha} - Q^{\mu\alpha}_{\alpha}). \tag{11}$$

Again, the STGR action is equivalent to the Einstein-Hilbert action up to a total divergence term (Jiménez et al. 2018).

The upshot of this is that, despite the fact that all three theories utilize entirely different geometric structures to describe gravity, *they are all dynamically equivalent to each other*. This follows from the fact that total divergence terms, also known as boundary terms, do not affect the equations of motion when variational procedures are used to construct the dynamics of these theories. This means that all three theories (GR, TEGR, and STGR) obey the exact same Einstein field equations (albeit written in terms of different geometric quantities). The three classic tests of GR test only the trajectories of massive bodies and photons according to the Schwarzschild solution of the Einstein field equations. Consequently, any theory whose dynamics satisfy the Einstein field equations will give identical predictions for any such tests. Indeed, TEGR and STGR reproduce this same Schwarzschild solution for the spherically symmetric static spacetimes that we consider for these tests (Adak et al. 2013; Aldrovandi and Pereira 2013). This challenges the notion that these tests offer confirmation for *specifically* GR, and the view of gravity as a manifestation of *specifically* spacetime curvature. TEGR and STGR give empirically equivalent descriptions for all dynamical tests, while describing gravitational degrees of freedom as following from torsion or non-metricity in a flat spacetime environment.

To say more on this: consider moving from the geodesic equation in GR to the geodesic equation in one of these alternative theories:

$$\frac{d^2x^\mu}{d\tau^2} + \overset{\circ}{\Gamma}{}^{\mu}{}_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = \frac{d^2x^\mu}{d\tau^2} + (\Gamma^{\mu}{}_{\nu\lambda} - K^{\mu}{}_{\nu\lambda}/L^{\mu}{}_{\nu\lambda})\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = 0, \tag{12}$$

depending on whether we are working in the framework of TEGR or STGR. That is, generalized inertial trajectories are given by the two terms involving the second time derivative of the coordinates and the affine connection (as in GR), leaving a third term proportional to the contorsion/distorsion tensor that is readily interpreted as a force that would direct test masses and photons away from geodesics. The actual empirical trajectories that test masses and photons follow are *equivalent* regardless of which theory we work in because the dynamical equations of motion are identical in each theory. However, in GR these trajectories are interpreted as inertial motion following geodesics in curved spacetime, whereas in TEGR and STGR these same trajectories are interpreted as non-inertial motion resulting from gravitational forces that direct motion away from geodesics in a flat spacetime.

6. Classic Tests and Non-relativistic Gravity

6.1. **Newton-Cartan Theory.** It is fascinating that there are alternatives to GR which can account for the classic tests using geometric properties different from curvature. As we have seen, this can be understood concisely as altering the projective structure encoded by the Levi-Civita connection, while leaving unmodified the conformal structure of the models of GR. Surely, maintaining this conformal structure would be essential to passing these tests as it was these very tests that distinguished Einstein's relativistic theory from Newton's non-relativistic theory? As we shall see, even this seemingly unassailable statement does not hold.

It is well-known that non-relativistic theories of gravity can be geometrized in analogy with GR. 'Newton-Cartan theory' (NCT) typically refers to a geometric theory equivalent to Newtonian gravity. As standardly presented, this theory is given by models of the form $\langle M, \tau, h, \nabla, \Phi \rangle$, where M is a differentiable manifold as usual, τ is a (degenerate) temporal metric, h is a (degenerate) spatial metric, and ∇ is a Newton-Cartan connection which is compatible with both τ and h (Malament 2012, p. 248). Some important features of this theory are as follows:

1. The temporal and spatial metrics obey the orthogonality condition $\tau_\mu h^{\mu\nu} = 0$.
2. The connection is assumed to be symmetric, which ensures that torsion is zero (from this it follows that e.g. $\nabla_{[\mu}\tau_{\lambda]} = 0$).
3. The equation of motion is the geometrised Poisson equation $R_{\mu\nu} = 4\pi G\rho\tau_\mu\tau_\nu$, where $R_{\mu\nu}$ is the Ricci curvature.

Taken together, test particles in NCT are understood to follow geodesics in a curved spacetime (where the curvature is located in the temporal metric) in a manner that is empirically equivalent to Newtonian gravity,[18] but in very close analogy to GR regarding its geometric structure. Essentially, NCT repackages Newtonian gravity in differential-geometric language similar to that of GR.

[18] In fact, for full equivalence with standard Newtonian gravity, further curvature conditions must be imposed in NCT—see e.g. (Malament 2012, p. 268). These further conditions won't be relevant to us in what follows.

6.2. **Type II Newton-Cartan Theory.** As should be obvious given its empirical equivalence with standard Newtonian gravity, NCT would never pass the classic tests of GR. However, *qua* geometric, non-relativistic theory of gravity, NCT as presented above is not the only game in town. One recent approach begins with a relativistic theory and uses a careful $1/c$ expansion in order to isolate a novel non-relativistic spacetime theory. A natural place to start would be to see what non-relativistic theory emerges from GR itself. This is precisely what is done by Hansen et al. (2020). These authors decompose the GR metric into temporal and spatial metrics, expand these respective objects in order $1/c^2$, rewrite the Einstein-Hilbert Lagrangian using these objects, and find the non-relativistic limit to give an action of the form

$$S = \int d^4x L(\tau_\mu, h^{\mu\nu}, m_\mu, \Phi_{\mu\nu}), \tag{13}$$

where of course τ_μ and $h^{\mu\nu}$ refer to the temporal and spatial metrics respectively, while m_μ is known as the mass gauge field (a peculiar feature of Newton-Cartan geometries that ensures that Newton-Cartan geometric structures are invariant under non-relativistic gauge transformations) and $\Phi_{\mu\nu}$ can be understood as a geometric/tensorial generalisation of a Newtonian potential. The action itself is somewhat cumbersome and its exact expression need not concern us here (see (Hansen et al. 2020) for the full expression); suffice it to say here that this defines a new fully non-relativistic spacetime theory dubbed *Type-II Newton-Cartan theory* (NCTII).

Consistent with our approach with the previous theories, we will discuss both the projective and conformal structure of NCTII in order to highlight how it differs from the theories in the geometrical trinity. As with the geometrical trinity, there is a relationship between the projective structures of normal GR and NCTII. That is, one begins with the Levi-Civita connection, decomposes the object into temporal and spatial parts, expands in terms of c, and takes the leading order term in the expansion (Hansen et al. 2020, sec 2.3):[19]

$$\Gamma^\rho_{\mu\nu} = -v^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma} \left(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}\right), \tag{16}$$

where τ_μ and $h^{\mu\nu}$ refer to temporal and spatial metrics respectively and v is an inverse of the temporal metric, in the sense that $\tau_\mu v^\mu = 1$ (note that this inverse is not unique; likewise, the inverse of $h^{\mu\nu}$ is not unique). Formally, (16) seems very

[19]The idea is to take the Levi-Civita connection

$$\Gamma^\rho{}_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} \left(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}\right), \tag{14}$$

and expand it in perturbations of $1/c$

$$\Gamma^\rho{}_{\mu\nu} = c^2 \Gamma^\rho_{(-2)\mu\nu} + \Gamma^\rho_{(0)\mu\nu} + \frac{1}{c^2} \Gamma^\rho_{(2)\mu\nu} + O\left(\frac{1}{c^4}\right), \tag{15}$$

obtaining the leading order term.

similar to the NCT connection when written in components. However, the NCT connection has zero torsion, i.e. $\partial_{[\mu}\tau_{\lambda]} = 0$. Without imposing this condition by fiat, the connection described in (16) naturally has torsion. Indeed different versions of Newton-Cartan geometries can be categorized by different torsion conditions; NCTII uses the 'twistless torsional' condition $\tau_{[\mu}\partial_{\nu}\tau_{\rho]} = 0$, which allows for torsion, but ensures that the spacetime admits of a foliation into equal time slices (Hansen et al. 2020).[20] The projective structure $\mathcal{P}_{NCTII}$ is then defined by this twistless, torsional connection (see (Hansen et al. 2020) for a detailed discussion of geodesics in NCTII).

The conformal structure of this theory can be understood by considering the limit taken in the construction of this theory. The slope of the light cone in Lorentzian geometries is $1/c$, and the process of taking this limit can be visualized as 'flattening' out the light cone as we are expanding around $c = \infty$ (Hansen et al. 2020, §2.1).[21] This is a notable departure from any of the theories we have considered so far. Once the light cones are 'flattened' in this way, there no longer remains a lightcone structure defined by a null interval on a spacetime metric. Given this, one might wonder what becomes of conformal structure in such spacetimes. We can, however, continue to speak of conformal transformations that preserve the direction of the degenerate temporal and spatial metrics, in close analogy to how conformal transformations with a relativistic metric preserve angles while altering lengths and volumes. Following Duval et al. (2017), we can associate the conformal structure of this particular variant of Newton-Cartan theory $\mathcal{C}_{NCTII}$ with an equivalence class of metrics $[t]$ and $[h]$, whose directions are independently preserved by scale transformations $\Omega(x)$.[22]

Bringing this all together, models of NCTII can be represented in the following way: $\langle M, \mathcal{P}_{NCTII}, \mathcal{C}_{NCTII}, \Phi\rangle$. Given that both the projective and conformal structures of this theory are so different from both those of GR and those of the other relativistic theories we have considered up to this point, and instead are associated closely with structures of other non-relativistic theories, it would be striking if this theory could pass most or all of the tests that were taken to confirm GR, and to refute conclusively Newton's non-relativistic theory. In fact, however, it turns out that the twistless torsional condition in NCTII opens up solutions that

[20]The classification of Newton-Cartan geometries by torsion conditions is the following:

Newton-Cartan geometry No torsion, i.e. $\partial_{[\mu}\tau_{\nu]} = 0$.

Twistless torsional Newton-Cartan geometry $\tau_{[\mu}\partial_{\nu}\tau_{\rho]} = 0$.

Torsional Newton-Cartan geometry No constraints on τ.

[21]More technically, $c = \hat{c}/\sqrt{\sigma}$, where σ is a small dimensionless parameter which is expanded around 0 (Hansen et al. 2020, §2.1).

[22]For some philosophical discussion of conformal transformations in non-relativistic spacetime settings, see (Dewar and Read 2020).

were otherwise excluded in the torsionless version NCT. Moreover, it is these solutions that encode the kinds of strong gravitational effects to which the classic tests are sensitive—phenomena for which the traditional Newtonian theory could not account (Hansen et al. 2019b).

The procedure for determining dynamical trajectories of test particles in NCTII is very similar to that outlined in GR. Solving the equations of motion for the metrics in a background that is static, spherically symmetric, and in a vacuum yields

$$\begin{aligned} \tau_\mu dx^\mu &= \sqrt{1-\frac{2GM}{r}}dt, \\ h_{\mu\nu}dx^\mu dx^\nu &= \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2. \end{aligned} \tag{17}$$

While the metrics are degenerate, the above two equations bear an unmistakable resemblance to the standard Schwarzschild solution in GR. Indeed, one can think of the $(1-\frac{2GM}{r})^{1/2}$ in the temporal component as a lapse function, which is equivalent to the lapse function in a standard ADM 3+1 split in GR. Upon utilizing the geodesic equation and imposing motion in the equatorial plane, Hansen et al. (2019b) show that this produces results equivalent to those of GR. That is, the resulting set of equations of motion is identical to (6) (which is now no longer a surprise given the form of the metric), and this theory produces both perihelion precession and light deflection in accordance with expectations from GR. Furthermore, from the form of the temporal component we can also see immediately that this theory will produce a gravitational redshift equivalent to GR as it reproduces the exact difference in proper time between observers at different radii r.

7. Modern Tests

Although it is indeed remarkable that a non-relativistic theory of gravity can yield equivalent descriptions to GR for the empirical tests which were most responsible for ushering in the paradigm change to the latter, one might nevertheless ask at this stage: how far does this underdetermination really stretch, given the preponderance of modern tests of GR? After all, the bleak empirical status of gravitational theory bemoaned by Wheeler and Dicke did not persist for long following the renewed emphasis on the subject (largely due to their efforts). Ever since, there has been a flurry of new empirical tests, driven both by developments in theoretical understanding and technological advancements. However, it turns out that almost none of these tests can discriminate between GR (along with the other geometric trinity theories) and NCTII!

Among these, there are a number of prominent tests that were designed to discriminate between GR and alternative theories of gravity that are known to be empirically distinct from GR. For example, Eötvös-type experiments are designed to measure the difference between inertial and gravitational mass and Dicke's group improved these results by orders of magnitude (Roll et al. 1964), which has continued to this day with satellite experiments (Touboul et al. 2022). Despite these impressive modern results, experiments of this type were available even in Newton's day because the equivalence of gravitational and intertial masses holds in any Newtonian gravitational theory as well as GR. Similarly, lunar laser ranging experiments have been used to demonstrate that the gravitational constant does not vary in time (Merkowitz 2010; Muller and Biskupek 2007), a key prediction of many alternative theories of gravity that violate the strong equivalence principle.[23] Again, this is something on which the theories with which we are concerned can agree, because none of them invoke features that would lead to the kinds of signals these experiments are meant to probe.

Many of the other prominent tests of GR amount to further tests that depend on the Schwarzschild solution. These include Shapiro time delay (Shapiro 1964), strong field tests of the gravitational redshift of light outside of a supermassive black hole (Abuter et al. 2018), strong field tests of the perihelion precession of a star orbiting a supermassive black hole (Abuter et al. 2020), and gravitational lensing (Bartelmann and Maturi 2016) (with light deflection being the first example of this kind of test). All of these tests use the Schwarzschild solution as a starting point for determining the predictions that the experiments probe and consequently, NCTII will agree with GR and with the geometric trinity more generally. On the other hand, the Schwarzschild solution is not the only GR solution that has undergone rigorous testing. The Friedmann-Lemaitre-Robertson-Walker (FLRW) solution of GR is a foundational cornerstone of modern cosmology and has likewise been studied empirically extensively (Aghanim et al. 2020). Yet, here again as Hansen et al. (2020) show, NCTII likewise has a solution that reproduces FLRW cosmology in much the same way that it reproduces the results of the Schwarzschild solution. Thus, NCTII and GR are also in agreement regarding all of these modern tests.

Where NCTII and the geometric trinity seem to diverge is in the prediction of gravitational waves. There has been indirect evidence of gravitational waves since observations indicated that the Hulse-Taylor binary was undergoing an orbital decay consistent with predictions from GR that such systems should radiate away gravitational energy (Hulse and Taylor 1975; Taylor and Weisberg 1982). However, the first direct detection of gravitational waves occurred in 2016 when the LIGO and Virgo collaborations observed the in-spiral of a binary black hole system (Abbott et al. 2016). This test essentially probes the Kerr metric (the solution for rotating black

[23]Constraints on the time variations of the gravitational constant also come from indirect and direct tests of gravitational waves. See (Belgacem et al. 2019; Manchester 2015; Wolf and Lagos 2020) for further details.

holes—believed to be a good description for actual astrophysical black holes)[24] and perturbations on it in the form of gravitational waves. The result was a waveform consistent with both the in-spiral and merger of a binary black hole system and the ringdown of the final Kerr black hole. This is one of the most important recent empirical successes in gravitational research and has constrained significantly the kinds of alternatives theories of gravitational that remain viable (Baker et al. 2017). Yet, the existence, production, and properties of gravitational waves are—unsurprising given their dynamical equivalence—significant empirical consequences upon which all theories in the geometric trinity agree (Abedi and Capozziello 2018; Bamba et al. 2013; Hohmann 2018; Jiménez et al. 2020; Soudi et al. 2019).

On the other hand, Hansen et al. (2020) have found that NCTII does not admit gravitational wave solutions due to the nature of the expansion taken in deriving the theory: *in this sense*, NCTII is neither dynamically nor empirically equivalent to GR. This is consistent with the conventional wisdom that Newtonian theories (both standard Newtonian gravity and traditional NCT) do not admit gravitational waves.[25] While it should not be surprising that empirical results have decided conclusively in favour of relativistic theories of gravity over non-relativistic theories, the extent to which the weak underdetermination between GR and NCTII can be pushed is striking. Indeed, this leads to the admittedly bizarre conclusion that the empirical results from experimental gravitational physics have only recently offered conclusive discrimination between relativistic and non-relativistic theories of gravity, given that NCTII passes so many of the other tests (both classic and modern) of GR.

8. The Geometric Trinity as a Case of Strong Underdetermination

Having argued that (a) the theories which constitute the geometric trinity are empirically equivalent, and that therefore (b) if one of them passes any of the myriad tests conceived of in experimental gravitational physics then all three will do so, it seems that there is a good case to be made that the geometric trinity does indeed present a case a strong underdetermination. Granting this in what follows, we here survey some of the possible responses to this underdetermination.

Recall from Section 2 that when presented with theories which are empirically equivalent, the first question to ask is: are those putatively distinct theories in fact

[24] We also note that important tests constraining possible deviations from the Kerr solution have also been recently carried out by the Event Horizon Telescope (Psaltis et al. 2020, 2021).

[25] Although it should be noted that the extent to which wave solutions exist in Newtonian theory is not settled conclusively. The main issue is that the standard Poisson equation, on which Newtonian gravity, NCT, and NCTII all agree, is an elliptic equation and these types of equations are not typically understood to have propagating solutions. See Linnemann and Read (2021b) and Dewar and Weatherall (2018) for alternative interpretations of the Poisson equation, on which it can be understood as admitting propagating solutions.

merely different ways of expressing the same theory? That is: are they theoretically equivalent? Since one component of theoretical equivalence is interpretative equivalence, here we must also ask: are they interpretationally equivalent, in the sense that that they postulate the same underlying ontology, make identical claims about the objects which they describe, etc.? Exploring the structural differences which exist between the theories which constitute the geometric trinity as we have done deflates any attempt to brush off their differences as merely cosmetic, however. All of these theories postulate different mathematical and geometric structures as underlying our description of gravity; moreover, all three theories disagree on the fundamental character of inertial motion, which is succinctly captured in the differences between their projective structures. In this sense, these theories are interpretationally distinct.

Given that this first move does not seem promising, we are left with (at least) four distinct approaches to dealing with cases of strong underdetermination between empirically equivalent yet distinct theories: roughly following the terminology of Le Bihan and Read (2018), we call these the 'common core', 'overarching theory', 'discrimination' and 'conventionalist' approaches; here, we discuss each in turn in the context of the geometric trinity.[26]

8.1. **Common core.** The common core approach advocates moving to a new interpretive framework that allows one to break the underdetermination. This would involve isolating the 'common core' that is shared between GR, TEGR, and STGR, and then interpreting this shared common core as a distinct, ontologically viable theory of its own.

To see how this might work, it is helpful to recall a previous example of underdetermination and the successful application of this approach. Recall the famous example of empirically equivalent Newtonian models of the universe which differ by a kinematic shift. Here, one can isolate the shared common core between the Newtonian models, while purging what is not shared between them (i.e., transtemporal identities of spacetime points *qua* spatial points), in order to arrive at the by-now standard Newtonian mechanics set in Galilean spacetime. Furthermore, the remaining structure can be readily interpreted as a Galilean spacetime rather than a traditional Newtonian spacetime. This provides a clear 'common core' interpretation of the two models (one at rest and one with a constant velocity) whereby they completely agree about the structure they attribute to the world (on this, see (Butterfield 2021)).

This strategy does not seem viable for GR, TEGR, and STGR. When we examine the structures of these theories, the part of this process whereby we isolate

[26] Le Bihan and Read (2018) call their fourth class of response 'pluralism'; we'll replace this with 'conventionalism' (in the sense of Duerr and Read (2023)), since the latter is a little more precisely-defined, which will suit our purposes.

a common core and purge the extraneous structure would leave remaining a very impoverished ontology. After all, given that all three theories disagree on projective/affine structure and that it is this very structure which encodes gravitational effects, its not clear what structure is left to actually describe gravity once this is expunged from the common core. In the example above, the only content that was purged was Newtonian notions of absolute rest that were already known to be superfluous from considerations to do with dynamical symmetries.

8.2. **Overarching theory.** The common core and overarching theory approaches are both similar in that they require moving to a new interpretive framework that allows one to break the underdetermination. Yet, rather than isolating a shared 'common core' amongst the theories, the overarching approach would involve a synthesis of the entirety of the theoretic structures contained within GR, TEGR, and STGR. That is, the overarching approach seeks to embed these theories into a new framework that would allow us interpret them as different facets of same underlying ontology.

In contrast with the common core approach, the overarching theory approach to the geometric trinity does not seem to be blocked automatically. A good example of a successful application of this approach can be witnessed in the equivalence of the Jordan and Einstein 'frames' of Brans-Dicke theory (Duerr 2021; Lobo 2016).[27] Brans-Dicke theory can be cast into two formulations related by a conformal transformation (Brans and Dicke 1961; Dicke 1962). One of these is known as the 'Jordan frame', in which a scalar field is non-minimally coupled to the Ricci scalar and test matter still follows geodesics of the metric. The other is known as the 'Einstein frame', in which the gravitational part of the action takes the usual Einsteinian form but the scalar field exhibits a universal coupling with matter, leading to test masses being forced away from the natural geodesics of this other metric (not coupled to the scalar field). Historically, there was debate regarding which frame was 'correct', or whether they were in fact equivalent (see e.g. Duerr (2021) and Weinstein (1996) for some philosophical discussion), but it was later realized that one could understand the equivalence of the Jordan and Einstein frames by moving away from Riemannian geometry and reformulating the theory within the richer framework of an integrable Weyl geometry (Lobo 2016; Romero et al. 2012). In so doing, it becomes apparent that both of these frames—seemingly representing different geometric objects within a Riemannian geometry—are merely different representations of the same invariant geometric objects within the new Weyl geometry.

While this is no doubt an exciting example of successful application of the

[27]'Frames' here is not to be understood in the sense of frames of reference. The use of the word 'frame' in the context of Brans-Dicke theory refers to different ways of mathematically representing the theory.

overarching theory approach which might *in principle* lend hope to a similar interpretive strategy in the present case, some tempering of this enthusiasm is probably in order, for acknowledging the possibility that there may exist some overarching theory into which GR, TEGR, and STGR can be embedded is of course very different from actually finding this overarching theory. Indeed, there is no guarantee that such a theory or framework exists actually exists, or *if* it exists that it can be found.[28] The proof here is in the pudding, and at present we are—to the best of our knowledge—lacking a framework which would be suitable for employing the overarching theory approach.

8.3. **Discrimination.** In the case of the geometric trinity, the discrimination strategy—of preferentially favouring the ontological claims associated with one theory—appears to be both (a) viable, and (b) deployed actively when physicists and philosophers discuss these theories. That said, its application in this particular instance is understandably fraught with controversy and differing opinions. Indeed, there are a number of ways that this approach can be applied, deploying as they do both philosophically- and physically-motivated criteria (and often a mix of both).

For example, Knox (2014) has argued that (traditional) NCT is the correct spacetime setting for Newtonian physics, because it has less 'surplus' structure (by the same kinds of dynamical symmetry considerations as mentioned above in the case of the move to Galilean spacetime) than standard flat-spacetime Newtonian gravity. This position—that we should prefer a theory or framework with less surplus structure—is clearly a substantive philosophical position (albeit currently a popular one: see Dasgupta (2016) for further discussion). As Knox (2011) subscribes to this position, she argues that we can discriminate between GR and TEGR in favor of GR. This is because TEGR possesses an additional 'internal' freedom to perform Lorentz transformations (Read 2016a), meaning that TEGR has additional surplus structure when compared to GR.

In addition to endorsing this reasoning, Knox (2011) furthermore—and separately—maintains that TEGR is parasitic upon GR because its inertial structure is in fact still that of GR, because gravitating but otherwise force-free bodies still follow geodesics of the Levi-Civita connection. Indeed, given that in STPG one also decomposes the Levi-Civita connection into a distinct connection plus a correction term (this time in terms of the distorsion tensor), Knox's points presented in the case of TEGR carries over straightforwardly to the case of STGR—thereby, she can maintain that gravitating but otherwise force-free bodies in STGR still follow geodesics of the Levi-Civita connection, and therefore STGR, like TEGR, is parasitic upon GR.

[28] Perhaps, for two arbitrary theories, one can prove that there invariably exists some overarching theory into which both can be embedded. But this is conjecture; we'll leave putting meat on the bones as a task for future pursuit.

So, up to this point, we have two motivations which might underlie the discrimination approach in the case of the geometric trinity:

1. Fewer surplus degrees of freedom in one theory versus another.
2. Inertial structure of one theory remaining 'physically significant' in the other.

But there are yet further motivations which might underlie the discrimination approach. For example, some physicists who work on TEGR believe that this theory offers non-trivial *benefits* due to its gauge structure. While in this paper we have chosen to present GR, TEGR, and STGR in their standard formulations (the tradition formulation of GR and the Palatini formulations of TEGR and STGR) due both to their familiarity and the ease of comparison that this facilitates, all of these theories can alternatively be formulated in terms of vielbeins (Aldrovandi and Pereira 2013; De Felice and Clarke 1992; Jiménez et al. 2019). We will not dwell on the details here, but within the vielbein formulation of TEGR it becomes apparent that TEGR is a gauge theory of the translation group (Aldrovandi and Pereira 2013, Ch. 3). Furthermore, preferring this structure can be understood by way of a unificationist perspective:[29]

> Three of the four known fundamental interactions of nature—namely, the electromagnetic, the weak and the strong interactions—are described [...] as gauge theory. Only [...] general relativity, does not fit in such a gauge scheme. Teleparallel gravity [...] fits perfectly in the gauge template. Its advent, therefore, means that now all four fundamental interactions of nature turn out to be described by one and the same kind of theory (Aldrovandi and Pereira 2015).

STGR also has its own adherents and they express similar unificatory motives. In particular, STGR can be expressed in what is known as the 'coincident gauge'. Here, the theory can be expressed in such a way that its action resembles the Einstein $\Gamma\Gamma$ action, but with the conceptual difference that the connection can now (again, the claim goes) be interpreted as encoding a gauge theory of translations (Jiménez et al. 2018). This motivates a similar perspective in terms of its closer conceptual unity with the rest of fundamental physics. Thus, we have the following further motivation which might be taken to underlie the discrimination approach:

3. One theory better accords with the architecture of the rest of physics than another, in terms of having the same mathematical structure.

[29] As Aldrovandi and Pereira (2013) acknowledge (and as Wallace (2015) has also registered), TEGR is not quite a standard Yang-Mills gauge theory due to the presence of soldering. We won't go into this further here.

We make no claim that (1)-(3) exhaust the kinds of philosphical/physical considerations which might weigh in favour of the discrimination approach.[30] In any case, though: very few (if any) physicists take an absolutist tone when discussing these theories and potentially discriminating between them. However, there are principled reasons that factor into their decisions to work within one framework or the other, particularly when it comes to working with formalisms like TEGR or STGR that are far less popular and understood than the dominant GR paradigm. In this sense, they are using some combination of physically-motivated principles and philosophical positions to discriminate gently in favor of their preferred leg of the trinity.

8.4. **Conventionalism.** Another strategy which might be deployed in the context of apparent case of strong underdetermination raised by the geometric trinity is *geometric conventionalism*. This strategy is explored (both in general and indeed with specific reference to the geometric trinity) by Duerr and Read (2023), so we will accordingly keep our remarks here somewhat brief. Suffice it to say that geometric conventionalism is a programme on which one simply abstains from assigning truth values to propositions to do with the geometrical degrees of freedom of the theories under consideration (here, the theories which constitute the geometric trinity): much like choosing a coordinate frame rather than another, one can choose one geometric convention rather than another, but no such choice should be afforded 'deep' ontological significance. (Note, though, that this is not the same as declaring all propositions to do with geometrical degrees of freedom to be false, which would be more in line with the common core approach.) As articulated by Duerr and Read (2023), geometric conventionalism appears to be an attractive option in the case of the geometric trinity, in the absence of a 'common core' or overarching theory; moreover, insofar as (as discussed above) physicists are generally *not* dogmatic about their preferred geometrical formulation, it is perhaps not unreasonable to place many into the conventionalist camp.[31]

9. Conclusions

There is no doubt that both classic and modern eras of experimental gravitational physics have proven to be enormously successful in terms of their fruitfulness for theory development, confirmation, and elimination. However, despite this success, there are still somewhat surprising gaps in the knowledge that this renaissance of

[30]In fact, we can think of several other salient considerations right off the bat—but there's little point in extending this list *ad nauseum*.

[31]Is being a conventionalist sufficient to present a 'metaphysically perspicuous characterisation' (Møller-Nielsen 2017; Read and Møller-Nielsen 2020) of one's ontological commitments? It's not obvious, but here we'll set aside that question.

experimental gravitation has left us, which is essentially due to the ability of alternative theories to reproduce exactly the dynamical solutions that serve as the starting point for conducting experimental tests in the first place. However, this is not necessarily a negative, as the most common reactions to instances of underdetermination would indicate.

Consider first the issue of strong underdetermination within the geometric trinity. We take the underdetermination exhibited by the geometric trinity as an invitation for further exploration. We advocate a broadly pluralistic attitude in the absence of strong reasons to pursue any of the articulated responses to strong underdetermination as it seems that these competing theoretical frameworks have something to offer to physicists and philosophers alike. These potentially fruitful avenues include but are not limited to the following:

1. Intrinsic conceptual and interpretive clarity with respect to key physical quantities: Both TEGR and STGR seem to offer conceptual advantages in terms of defining concepts like energy-momentum density. That is, both have the resources to define tensorial energy-momentum densities because, unlike GR, the structure of the theories allows one to separate inertial and gravitational effects (Aldrovandi and Pereira 2013; Jiménez et al. 2018, §18.2.3). To give another example, TEGR arguably offers a clear conceptual understanding of black hole entropy. Within the TEGR framework, black hole entropy can be expressed as a volume integral rather than in terms of area, which is more consistent with our typical thermodynamic understanding of the concept of entropy (Hammad et al. 2019).

2. Calculation facility: Both TEGR and STGR have actions which include only first derivatives of metric, as opposed to GR, the action for which includes both first and second derivatives of the metric. This means that TEGR and STGR are more natural for problems where boundary terms are important as these actions have well-defined variations for Dirichlet boundary conditions (this was explicitly shown for TEGR by Oshita and Wu (2017) but also applies to STGR (Jiménez et al. 2018)). On the contrary, to use GR in the same applications one must supplemented GR with the Gibbons-Hawking-York boundary term to remove the problematic second derivatives of the metric by hand (Gibbons and Hawking 1977). A particularly interesting application of this feature is that one can use these theories to calculate black hole entropy (which agrees with the GR prediction), but in an arguably more straightforward manner (Heisenberg et al. 2022; Oshita and Wu 2017).

3. Extra-theoretic considerations: e.g. quantisation. For example: (a) quantising GR versus TEGR via a path integral might lead to different (empirically significant) instantonic effects (due to the actions of the theories differing

by a boundary term); insofar as one *quantised* theory might thereby be preferred on empirical grounds, one might take this to carry over to the classical case (where the theories are empirically equivalent). And (b): if one theory (e.g. TEGR) can be cast into the form of a Yang-Mills theory (say), and quantising said theories is unproblematic, this might be taken as a reason to prefer the formalism of said theory (again, say TEGR) over another (say GR).

4. Larger theory space: While GR, TEGR, and STGR are (again) equivalent to each other, in pursuit of cosmological and modified gravity theories that might explain dark energy or offer plausible realizations of inflation it is common to build extensions out of higher order geometric scalars. These classes of theories go by the names of $F(R)$, $F(T)$, and $F(Q)$ gravity respectively, where for example $F(R)$ represents general functions of the curvature tensor and its contractions (*mutatis mutandis* for its torsion and non-metricity counterparts). However, when these theories are extended in this way they are *not* equivalent and this results in a new space of theories to explore in modified gravity and cosmological applications (Bahamonde et al. 2023, §5.3).

While the above has focused on all the reasons we should be interested in exploring the geometric trinity, what can be said for Newton-Cartan theory in its various formulations? Of course it is interesting that a version of this theory can pass all of the classic tests of gravity and many of the modern ones, but as a non-relativistic theory of gravity how much value does it have besides being of academic interest? It turns out that these non-relativistic geometries have found important applications in physics:

1. (Torsional) Newton-Cartan theories constitute essential tools in the study of condensed matter systems, for example by using NCT as a background for modelling the unitary Fermi gas (Bekaert et al. 2012; Son and Wingate 2006) or the quantum Hall effect (Geracie et al. 2016; Son 2013; Wolf et al. 2023). On the latter: remarkably, it was found that NCT is the correct background on which to model the quantum Hall effect. This is, roughly speaking, because condensed matter systems are non-relativistic and the structure of NCT provides a natural geometric setting that respects the underlying symmetries of the system.

2. There is a non-relativistic version of the AdS/CFT correspondence, within which (Torsional) Newton-Cartan geometry plays a crucial role. Indeed, in the non-relativistic AdS/CFT correspondence, the bulk spacetime is a so-called Lifshitz spacetime (instead of AdS) while the boundary theory is a CFT on a NCTII background. This correspondence can be used to model many condensed matter systems (Hartong et al. 2016).

3. (Torsional) Newton-Cartan geometry is the cornerstone of two different initiatives in the pursuit of a quantum gravity theory: first, it is fundamental for non-relativistic string theory (Andringa et al. 2012; Harmark et al. 2017), in which non-relativistic symmetries assume the same role as Poincaré symmetries for the relativistic string (that is, they are the global symmetries of the string worldsheet). Second, dynamic (T)NC geometries have been shown to be the geometrised version of Hořava-Lifshitz gravity (Hartong and Obers 2015), a gravitational theory first proposed by Hořava (2009).[32] Hořava-Lifshitz gravity is power-counting renormalisable, making it possible for it to be quantised canonically.

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[32] It has been suggested that Hořava-Lifshitz gravity can be useful for describing certain cosmological phenomena (Mukohyama 2010).

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