Branching with a Humean face

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Abstract

This paper investigates the prospects of developing a branching modal framework while keeping with the spirit of Humean Supervenience. It is argued that such an approach is bound to face hard problems regarding haecceitism and the notion of recombination. Possible directions for future philosophical developments of branching frameworks are suggested.

Keywords: Humean Supervenience, branching space-times, modal realism

Is a branching modal framework available for someone who adheres to Humean Supervenience? While this idea may seem controversial, in light of David Lewis' powerful rhetoric against branching frameworks, it is at least *prima facie* suggested—if not promoted—by the most up-to-date account of the most sophisticated modal branching theory, that is, in the recent monograph by Belnap et al. (2022) concerning *Branching Space-Times* (henceforth "BST"). There, the authors clearly state that the BST theory "belongs to metaphysics" (p. 14), and that in a version tailored to handle relativistic space-times its "concept of history is a case of a Humean mosaic" (p. 293; in the BST lingo it is the "histories", and not "worlds", that branch—more on that below).

It will be argued here that at least two serious problems may trouble Humean branching approaches to metaphysics that would take the current version of BST as their foundation. First, the resulting branching framework seems to allow differences of identity without any accompanying differences in quality. Second, there are situations in which—at least without being extended in some way—it is not able to offer a principled answer to questions about the *number* of possible worlds involved, and more fundamentally, which events are co-possible and which are not; this is in contrast with clear verdicts on such matters offered by Humean Supervenience coupled with D. Lewis' framework of divergent possible worlds. Hopefully, a serious examination of these issues will lead to some developments in the current branching frameworks.

The paper is structured as follows. In the next Section we sketch the motivation for investigating the aforementioned combination of ideas. Section 2 introduces the branching framework we will be dealing with, which combined with Humean Supervenience yields a position we'd like to call "Branching Humeism". In the final Section we discuss the two main problems we see for such an approach.

1 Motivation for Branching Humeism

Humean Supervenience (HS), which we will discuss in more detail at the beginning of Section 3, claims that every truth about the world depends exclusively on local, particular matters of fact. Various natural properties are instantiated at spacetime points; everything else supervenes on the facts about this arrangement. This position was originally promoted by D. Lewis. We will not defend it here; it's still considered to be a viable metaphysical stance, even though some certainly believe its problems to be insurmountable; see Weatherson (2015) and references therein. Instead, we would like to focus on the fact that there is a type of models of the Branching Space-Times theory (BST, originally proposed in Belnap (1992), recently given a monograph treatment in Belnap et al. (2022)), in which possible worlds[1] are copies of the 4-dimensional Minkowski space-time, every space-time point of which is assigned a set of qualities instantiated therein. These models may, then, initially seem to be a natural tool for someone attempting to develop a HS-inspired ontological framework; however, they involve possible worlds which *branch*, rather than *diverge*, as proposed by Lewis.

For us this is already enough to make exploring the combination of the two views worthwile. Still, let us consider additional sources of motivation. Why, then, would anyone want to move away from Lewis' framework of diverging worlds? Why should a Humean wish to adopt branching (ending up with a position we'd like to call 'Branching Humeism', BH for short)? One natural starting point is the so called 'open future intuition': you might think that the future is open, and that the openness in question is not merely epistemic—due to your lack of knowledge about the hidden workings of the world—but that there are genuinely two (or more) possibilities ahead of you. If you believe that you are literally a part of such two incompatible futures, and so perhaps subscribe to some form of realism with overlap (Torre, 2011; McDaniel, 2004), you might welcome a branching Humean framework.

This is a specific instance of a general reason a Humean might have for moving towards a branching approach: aversion to the idea of counterparts. A famous aspect of Lewis's framework is its rejection of trans-world identity of individuals; rather, nonidentical individuals from different worlds can be "coun-

[1] As already mentioned, BST proponents typically insist these objects be called "histories"; they would like the world to be "modally thick"; see e.g. (Placek et al., 2014, p. 408). Those with a general Humean outlook may believe this to be irrelevant. More on this issue in Section 2.1.

terparts", and when someone is saying that they could be watching a movie right now instead of reading this sentence, that modal statement, if true, is true because—roughly—in a different world a different individual, who bears the counterpart relation to them, is at that particular moment[2] watching a movie. There are a number of grounds on which a Humean might wish to eschew this idea, apart from subscribing to the open future intuition.

One might, for example, simply already be persuaded by the "Humphrey" examples by Kripke (1980) that one should assume trans-world identity of individuals. In fact, when one moves towards metaphysics from a logical background, one may be surprised that this position is controversial at all, since "[t]he notion of transworld individuals is presupposed by possible worlds semantics" (Akinci, 2004, p. 617). Typical textbooks, including the ones frequently read by philosophers, like (Garson, 2013), will introduce systems for quantified modal logic assuming that each possible world is assigned a subset of an overarching domain of individuals, without imposing any condition to the effect that these subsets be disjoint. Rigid designators will typically be introduced as ones which denote the same object in all possible worlds in which they denote anything at all; this suggests that it makes perfect sense to speak about one and the same individual existing in more than one possible world.[3] It seems natural, then, that a logician convinced by followers of HS about the merits of this metaphysical position would seek an ontological framework not requiring the thorough revision of quantified modal logic entailed by the usage of the notion of a counterpart. A branching framework, in which one and the same individual can belong to more than one possible world, may seem promising, then.

If this is till not enough, consider the apparent inconsistency in how Lewis treats his own arguments concerning temporary intrinsics and accidental intrinsics. Recall that Lewis's solution to the problem of persistence is that of perdurantism: an object exists at different times by having different temporal parts. It is not wholly present at any time it exists. The argument Lewis proposes for perdurantism refers to *temporary intrinsics* (Lewis, 1986b, ch. 2).

In a nutshell, the argument is as follows. Assume you have a workable distinction between intrinsic and extrinsic properties. Suppose that an individual possesses an intrinsic property at one time and does not possess it at another time. An endurantist faces a contradiction: which moment of existence of an individual we fix is irrelevant to whether the individual possesses an intrinsic property or not; therefore, since an individual is wholly present at all times of its existence, it both possesses that property and does not possess it. On the other hand, perdurantists can say that the individual has one temporal part which possesses the property, and another part which does not. Therefore an individual can satisfy temporally relative predicates "is R at time t_1" and "is not R at time t_2" in virtue of its t_1-part having the intrinsic property R and its

[2]Strictly speaking: at the moment which bears the counterpart relation to the moment at which the person is speaking in the actual world.

[3]You need to reach p. 290 in the Garson book to encounter the notion of counterpart; this is some time after rigid designators are introduced.

t_2-part not having it.[4]

An argument similar from the outset (but crucially different at the very end) refers to *accidental intrinsics*. The position to be refuted is that an individual can be wholly present in more than one world. An intrinsic property is, again, a property of an individual itself, and not, say, of ⟨ individual, world ⟩ pairs. Suppose that an individual possesses an intrinsic property at one world and does not possess it at another world. If it is wholly present at both of these worlds, then, since the property in question is intrinsic, then it both possesses the property and does not possess it. Lewis concludes that the right way to think about how it is possible for an individual to possess an intrinsic property it does not actually possess is that the property is possessed by a "counterpart" of that individual in a different world.

Both arguments can be rephrased to follow at first this abstract procedure: Assume R is an intrinsic property and that an individual I exists at n and m. Suppose that I is R at n and that I is not R at m. Observe that if I is wholly present at n, then, since R is an intrinsic property, from the fact that I is R at n we infer that I is R. Analogously, from the fact that I is not R at m we infer that I is not R. Therefore I is R and I is not R. Contradiction.

However, the arguments differ markedly in the conclusions drawn by Lewis. It would of course be trivial to say that, if correct, then these arguments show, granting that all the other assumptions hold, that an individual cannot be wholly present at n and m. This holds no matter whether those two are times or worlds. If n and m are times, Lewis would have us think that there is an overarching individual existing at both of those; it has thus at least two temporal parts, an n-part and an m-part. We can say that it is R at n because its n-part is R; we can say that it is not R at m because its m-part is not R. Still, it is *literally the same individual* at n and m; that individual is composed of temporal parts.

If, however, n and m are worlds, then instead of considering an overarching individual composed of modal parts, Lewis insists that two individuals need to be involved: one at n, and another, its counterpart, at m. The individual I existing at world n is R, and we can say that it is possible for it not to be R due to the fact that at world m its counterpart I' is not R.

One may justifiably be puzzled at this apparent philosophical inconsistency; why, based on the two arguments sketched above, should we accept the existence of individuals at various times in virtue of them having temporal parts, but at he same time oppose the existence of individuals in various worlds in virtue of them having modal parts? It has been recently argued that, in general, arguments from the realm of temporal discourse have modal parallels, and *vice versa* (Rini and Cresswell, 2012). To be consistent, then, we should either:

1. commit to modal counterparts, as Lewis does, but also to temporal counterparts, which Lewis does not;

   or

[4]Of course, there are also answers to this issue that are available to someone who does not want to subscribe to perdurantism; see e.g. Belnap et al. (2001).

2. commit to individuals existing at different times in virtue of having temporal parts, as Lewis does, but also to individuals existing at different worlds in virtue of having modal parts, which Lewis does not.

If we find the two arguments to be persuasive right to the moment in which we are to draw the conclusion, we should draw similar conclusions in both cases. And thus, those who are persuaded of perdurantism should adopt trans-world identity of individuals.[5]

Lewis devotes some time to argue that the temporal and modal cases are not analogous (starting on p. 218 of his 1986b). We believe, though, that his arguments beg the question against branching theorists. For example, his first argument refers to the impossibility of trans-world causation; this impossibility is perhaps arguable when worlds are separated spacetimes, but remains mysterious if worlds overlap. Pursuing these matters further would require a separate paper.

2 Branching

We hope that at this point we have managed to convince those subscribing to Humean Supervenience that a formally clear, counterpart-free approach to modal metaphysics, compatible with the needs of HS, deserves scrutiny. Consider, now, the following passage: "BST/BT are attempts to do metaphysics in a mathematically rigorous way, with the desideratum: be compatible with current physical theories" (Placek and Belnap, 2012, p. 468), where "BST" and "BT" denote the theories of Branching Space-Times and Branching Time, respectively. Spacetime features as an important concept in Lewisian metaphysics, therefore BST will be the theory on which we will focus; we have already mentioned that a class of models of this theory seems to offer a natural environment for a rigorously developed metaphysical framework adopting HS.

Of course, we have to note that the framework by Belnap and his followers is by no means the only branching approach on the market. One significant recent account has been proposed in Wilson (2020). In chapter 1.2 Wilson says initially that his account of supervenience "is, in effect, identical to Lewis's" (p. 31). Eventually, though, he leaves the validity of Humean Supervenience as an open question (p. 33). His approach requires a particular way of thinking about quantum theory, that is, the "diverging Everettian quantum mechanics" (*op. cit.*, chapter 2), and thus we will not discuss it here.

The current goal, then, to investigate certain philosophical prospects of BST, and not to prove theorems about it; therefore we will only present enough of its formal details as is sufficient for demonstrating some of its advantages and

[5] That said, some find at least the argument from temporary intrinsics not to be among the pearls of Lewis's output (e.g. Hawley, 2015, finds it to be "uncharacteristically weak"). Going in a different direction, Teller (2001) notes the similarity of the two arguments but ends up defending endurantism and overlap; McDaniel (2004) also advertises a modally realist option which eschews counterparts while committing to endurantism.

disadvantages when compared with the Lewisian approach.[6]

BST was introduced in (Belnap, 1992), with an extended "postprint" version appearing in 2003, and the theory arriving at its current form in the monograph (Belnap et al., 2022). It is Quinean in that the most basic elements of its structures are possible point events. It is Lewisian in that a world containing a point event encompasses all events which are "in suitable external relations" to it. It is not Lewisian in that the basic external relation is a branching causal order. It is not Quinean in that, as I will argue, it is not the case that every world has "its own space and time" (Hintikka, 1970, p. 127)[7].

The original motivation behind Belnap's 1992 field-opening paper was to "combine relativity and indeterminism in a rigorous theory" (p. 385).[8] Formally, the theory has two primitives, the nonempty set OW ("Our World") of possible point events, and the relation $\leqslant$, currently interpreted as a "pre-causal ordering" (Belnap et al., 2022, p. 25). No theory of causation is presupposed here; the ordering in some types of BST structures may end up being some sort of " 'time-like or light-like order', with the addition of a sense of 'direction' " (Belnap, 1992, p. 386); this is, indeed, what happens in the "Minkowskian Branching Structures" discussed below. A BST *history* is supposed to collect co-possible events; it generalizes the corresponding BT notion: instead of a chain w.r.t. the ordering $\leqslant$, in BST it is an upward-directed set. That is, any two events sharing a history have an element in that history which is above them both w.r.t. $\leqslant$. (The intuition is that if two point events belong to the same history, then if you think of them as light emissions, the light rays emitted by those two events will intersect at some future event in the same history.[9]) A simple and relatively current set of BST axioms runs as follows:

[6] Another reason for doing so is that it seems to me that some of the finer formal aspects are not essential to the philosophical merits of the theory, and rigorous alternatives with a similar general outlook have been considered. Some of those have found way to Belnap et al. (2021), which offers "new foundations" for the theory, but with which the philosophical community seems yet to have engaged. They also feature in the 2022 monograph by the same authors. Therefore it would not be wise to concentrate on its formal layer as a source of the theory's merits or flaws in this paper.

[7] It is probable that few proponents of branching frameworks would agree that their approaches are Quinean in any sense; however, the link to Lewis is explicit at least in (Belnap, 1992).

[8] It is important that "indeterminism" be understood modally in this quote. More concretely, the idea was that "a true description of our world requires fusing Einstein-Minkowski *space-time* with Prior/Thomason *branching time*" (p. 386). The reader will have noticed that "world" in the quote is in the singular and, in fact, the intention was to capture the vision of our world as a modally thick object, rife with internal possibilities, and not to describe a multitude of modally thin possible worlds. (Those searching for some sort of modal reductionism should probably look elsewhere, then.) That is, the "official" interpretation of a BST structure would have it contain a multitude of possible courses of events, "histories", as subsets of one world, the collection of possible point events, all of which are modally on a par, which the BST theory calls *Our World* (OW). We return to these matters later in the article.

[9] Two caveats: 1) the intuition is not, in fact, upheld in some BST frameworks in which, informally, "the world can end" before some two rays of light intersect; 2) therefore, a safer form of the intuition would be the contrapositive: if two light rays intersect, then their emissions belong to the same history. However, *that* intuition is of course too weak to support the *definition* of a history as an upward-directed set.

> "OW is a nonempty set, $\leqslant$ is a dense partial order on OW, each lower bounded chain has an infimum, and if upper bounded, has a supremum in each history containing it, OW has no maximal elements, and finally, for histories h_1, h_2, if E is a nonempty lower bounded chain in $h_1 \setminus h_2$, then some lower bound of E is maximal in $h_1 \cap h_2$. This last axiom (called the prior choice principle) entails that every two histories overlap and, moreover, that there is at least one element maximal in their overlap. Such an element we call a choice event of these histories." (Placek and Belnap, 2012, p. 447)[10]

Some BST structures can be equipped with the notion of a space-time location, in the sense which would allow us to speak about events taking place in "the same space-time point" in different histories.[11] In particular, there is a class of BST structures in which histories are isomorphic to Minkowski space-times.[12]

These, the "Minkowskian Branching Structures" (MBSs), look roughly as follows. Assume some set of physical properties P is given; properties which can be exemplified at spacetime points, like strengths of the electromagnetic field. Consider then Minkowskian spacetimes filled with content, that is, copies of $\langle \mathbb{R}^4, \leqslant_M \rangle$ ("scenarios"), where $\leqslant_M$ is the Minkowskian ordering[13], such that each spatiotemporal location in each scenario is associated with some set of elements of P by a function F called "property attribution". If that function is "proper", in a rigorous sense, points in regions of overlap of scenarios (that is, maximal downward-closed subsets of $\mathbb{R}^4$ over which the scenarios under consideration display no qualitative difference) can be identified by means of an equivalence relation, so that a tree-like structure of content-filled spacetimes emerges which can be shown to satisfy the BST axioms. (We skip the formal details since this

[10] The technical paper (Belnap et al., 2021) gives the BST theory its "new foundations", which are used extensively in the 2022 monograph. In particular, the new approach does away with choice events, switching to "choice sets", thanks to which the theory has better topological features and becomes compatible with the manifold topology of GR. This has no bearing for the arguments of the current paper.

[11] Note, though, that Müller (2005) showed that there are BST structures (i.e. pairs $\langle \mathrm{OW}, \leqslant \rangle$ satisfying the above axioms) which do not admit those: there is no sensible way of spatiotemporally "localizing" their point events.

[12] The Reader might wonder whether it is a good idea to use such an outdated notion as the Minkowski space-time. Shouldn't the true metaphysical theory about our world, and in particular about its spatiotemporal evolution, be more physically advanced? In particular, shouldn't it be based on general relativity (GR), at least? The answer is threefold. First, there is in fact a BST framework tailored to GR (Placek, 2014). Second, if we take modern physics seriously enough, it might be that we have to abandon Lewisian metaphysical approaches whatsoever (Wüthrich, 2020). Third: in fact, this outdated notion is continually referred to by modern physical projects, such as e.g. the causal set approach to gravity (see Henson (2009) for a concrete application and Dowker (2013) for an introduction to causal sets, which are to be discrete foundations of reality)—so, if it is alright for physicists, then philosophers should also be allowed to use it. Time will tell what physics brings us; it seems to us quite reasonable to investigate what a metaphysical approach compatible with a venerable *fragment* of physics could bring to the Humean table.

[13] First, define $<_M$ in the usual way: for x, $y \in \mathbb{R}^4$, $x <_M y$ iff 1) $x^0 < y^0$; and 2) $-(x^0 - y^0)^2 + \sum_{i=1}^{3}(x^i - y^i)^2 \leqslant 0$. Then set $x \leqslant_M y$ iff $x = y$ or $x <_M y$. See Section 9.1 of Belnap et al. (2022).

can be done rigorously in a number of ways.) The branches of the structures are called "histories" inside the BST framework, while we propose to consider, for the purpose of metaphysical talk about MBSs, calling them "possible worlds", a matter we return to in Section 2.1 below. We suggest, in other words, that thinking of the pluriverse as an MBS is an option open for modal realists with overlap: while they can both be modal realists, Lewis likes to keep his worlds apart, and a branching theorist considers them to have common "trunks".[14]

The main vehicles of MBSs are proper property attribution functions, which associate with each spatiotemporal location in a scenario a set of qualities exemplified at that point. A Humean should welcome such a perspective: the fundamental level of an MBS is an arrangement of local qualities, of local matters of particular fact. This seems to make it a perfect environment for developing a framework inspired by Humean Supervenience; in fact, as already mentioned, the authors of the 2022 monograph on BST do suggest that each history in an MBS is a Humean mosaic (p. 293).

Now, individuals. There seem to be only two papers written in the BST tradition which analyze this notion and not just assume that some domain of unanalyzed individuals is given, that is, (Belnap, 2011) and (Placek, 2012).[15] Both agree that individuals are sets of point events and, somewhat surprisingly for approaches proposed in the context of BST and not BT, that their intersection with any history is a (in some cases empty) chain (i.e. that they are not spatially extended); they differ, however, on additional constraints put on them. The former requires that individuals "come to life" and "die"; the latter allows e.g. for eternal elementary particles to be individuals, too. It is clear from the two papers that the requirement of lack of spatiotemporal extension is there just for reasons of simplicity and is to be done away with in some future developments of the approach; from now on we assume that the branching theorist has found some way of doing this. However, if we pursue this direction, it turns out that what we are left with are just two necessary conditions for something to be a BST individual (Placek, 2012, p. 30): that it is a subset of the universe of the structure and that its existence has no "gaps".[16] Still, even if some other requirements are proposed as necessary conditions, and possibly as forming, together with the previous ones, a sufficient condition, the fact will remain that individuals in BST structures will be subsets of the universe of these structures; we will see in Section 3 that this will already be enough to generate

[14] Note that we are *not* suggesting that any author who has been quoted here as a contributor to BST is a modal realist. To the contrary; from private communication we infer that they are rather opposed to the idea. In section 1.4 of the 2022 monograph it is suggested that the approach developed there—which, on the modal side, takes "real possibilities" as primitives—"sounds like an actualist position"; however, the jury is still out on the veracity of this verdict, at least until the theory says more about "states or properties" (p. 21).

[15] Interestingly, the topic is almost completely absent from the 2022 monograph. The issue of representing individuals is labelled on p. 7 as "controversial".

[16] Cf. Quine (1976, p. 859) on physical objects: they are "material content of any part of spacetime", "however scattered and discontinuous". Placek's notion does away with the second part and is close to transforming the whole into "discontinuous part of space time", linearity issues aside.

some problems for a Humean interpretation of the approach.

2.1 Worlds or histories?

This section is a terminological digression aimed mostly at branching enthusiasts. The main thread of the paper continues in Section 3.

We are using the term "world" in a way which departs from the intentions of most branching theorists, who follow Belnap in insisting that the structure they are talking about is a *single* world, which is to be thought of as "modally thick". We realise that what we are doing here can be seen as following Earman (2008) in identifying what branching theorists are careful to always label as "histories" with "possible worlds", and that Earman's identification was at least once described as "casual" (Placek and Belnap, 2012, p. 445, fn. 6). However, we insist that the identification we subscribe to in this paper is not casual; it is done after a careful scrutiny of most, if not all, papers written in the Belnapian branching tradition. We realise that one can decide to talk about a modally thick world with diverging histories (and to maintain that they are *not* worlds) for example because one believes that the Lewisian notion of world should be used (and his arguments against overlap should be resisted), while maintaining that the relation which holds a possible world together allows branching. A modally thick world is also what we seem to arrive at if we attempt to give a framework for "real possibilities" in at least one of the meanings the term currently has (see Rumberg, 2016b). This is now explicitly given as the philosophical bedrock for the BST theory in the Introduction to the 2022 monograph. However, we should notice that keeping histories distinct from the modally thick Our World is a philosophical choice—not a conceptual necessity. Interpreting BST structures as consisting of branching possible worlds considered in the vein of modal realists with overlap is simply a different option.

In our opinion, when one has a branching framework—with multiple histories forming a branching tree—in the back of one's mind, one will probably think that the role of being the object of "possible world talk" (Divers, 2002) is played by the histories, and not by the whole tree. For starters, if possible worlds correspond to those elements of a formal framework which represent possible courses of events, or "ways a world might be" (Lewis, 1986b, p. 2), or "ways things might have been" (Stalnaker, 2003, p. 27), then they cannot correspond to whole trees, since those represent *multiple* courses of events which are not corealizable—in contrast to individual histories. To continue the Stalnaker quote, possible worlds are "things that truth is defined relative to, things that our modal idioms may be understood as quantifiers over". Consider, then, that Our World (in the BST framework) or the whole tree in BT is not a parameter relative to which a proposition could be possibly true or possibly false. Nolan (2002) thinks that it is a "conceptual truth" that of anything that is to count as a possible world the following schema must hold, where P is any proposition:

> "Possibly P iff P has the "true according to" relation to some possible world, and

> Necessarily P iff P has the "true according to" relation to all possible worlds." (Nolan, 2002, p. 7)

Nolan writes this in an informal context; to assess which objects of a formal framework play the role of possible worlds we need to establish how exactly formulae are evaluated. When we do this for branching time we see that this is done in moment-history pairs (see e.g. Thomason, 1970). Taken literally, Nolan's scheme entails then that in BT possible worlds are moment-history pairs; strictly speaking, then, a history cannot be a part of a possible world, and *vice versa*. Remember, however, that we would like to be doing analytical metaphysics in a roughly Lewisian spirit, and so would like our worlds to be temporally extended. We propose then to interpret Nolan's scheme not completely literally, but to return to the idea from the above quote by Stalnaker: for some proposition P, how would we go about evaluating the proposition Possibly P in a BT framework in which histories are to represent possible courses of events? In particular, what should we quantify over? We certainly need moments of a BT structure, but what else? This of course depends on the language involved; for example, *stit*-logics might require a set of agents (Belnap et al., 2001). If we stick to a bare-boned temporal and modal language, though, the traditional answer is that what we quantify over histories: given a moment-history pair m/h, the proposition Possibly P is true if there is a history h' passing through m such that P is true at m/h', and Necessarily P is true if for all histories h' passing through m P is true at m/h'. The situation in BST, at least in those papers which properly introduce a language and valuation functions, is similar (see e.g. Müller, 2002). In branching frameworks, then, what we quantify over in our modal idioms are histories, not whole trees, which we take to be a strong suggestion argument for talking about them as possible worlds.

Some papers on branching theories suggest other, "smaller" or more "local" candidates for playing the semantic role outlined above: e.g. "continuations" (Placek, 2014) or "transitions" (Rumberg, 2016a). We cannot, however, take these as possible worlds, since they violate the prevalent intuition according to which a world is akin to a limit of a "series of increasingly more inclusive situations" (Menzel, 2023).

We concur, then, with the statement that "we should consider whole histories as the fundamental possibilities within branching time models, each such possibility specifying one way the complete current state of the world could be at each moment throughout its history" (Brown, 2014, p. 106). Brown does investigate considering many "Belnapian worlds", many whole trees of histories, as possible worlds; however, we would like to stay away from that option. Employing it would not only arguably reintroduce the problem of trans-world identity of individuals, avoiding which may be one of the key motivations for adopting a branching framework, but also would raise questions about the notion of possibility thus modelled and its difference from the one modelled by various histories inside a single tree (see e.g. remarks by Xu, 1997, p. 144).

3 Some problems of Branching Humeism

In this Section we shall finally investigate the prospects of Branching Humeism: is there a well-developed branching framework compatible with Humean Supervenience? In particular, how does BST fare in that regard?[17] Before we turn to the details, we should recall the basic tenets of HS. Let us first quote its creator himself (Lewis, 1986a, p. ix-x):

> "Humean supervenience is (...) the doctrine that all there is to the world is a vast mosaic of local matters of particular fact (...) We have geometry: a system of external relations of spatiotemporal distance between points. (...) And at those points we have local qualities: perfectly natural intrinsic properties, which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. There is no difference without difference in qualities. All else supervenes on that."

For a short, lucid formulation of the view we can also turn to Robinson (1989, p. 396): "We ask whether there exists a minimal sufficient set of intrinsic basic properties of points. Humean supervenience is, essentially, the thesis that there is such a set." Weatherson (2015) adds also the explicit requirement of anti-haecceitism: that there can be no difference consisting solely in a numerical difference between substrata, the "hooks", or "bare particulars" to which properties are attached. In the words of Kaplan (1975), a position cannot be anti-haecceitist—and so, we should infer, it cannot subscribe to Humean supervenience—if it allows that "distinct thisness may underlie great semblance" (p. 722-723).

Let us assume that the BST theorist adopts MBS' as the intended models for his or hers approach. Now, suppose he or she claims that what the proper property attribution does is precisely that it associates with spatiotemporal points local qualities instantiated at these points. Could this serve as a bedrock for developing of a Humean supervenientist metaphysical framework? In this Section we will study some of the issues such a framework will face and see how it may fare when compared to the Lewisian one.

3.1 Individuals and the risk of haecceitism

One issue BH needs to tackle relates to individuals and, in particular, a possible accusation of haecceitism. It indeed seems compatible with BH that there be two numerically distinct individuals even though there is no qualitative difference between them.

[17] Again, we do not suggest that branching theorists are modal realists. They usually seem to hold that there is no difference in the fundamental modal status of any two point events. However, for someone investigating the prospects of HS in branching frameworks, a modally realist outlook might be quite natural, especially if the hold the open future intuition, as mentioned above.

Recall that for BH an individual is a set of point events.[18] In MBSs, then, an individual is a set of spatiotemporal locations with which the property attribution function associates some family of qualitative properties, with the caveat that some of these locations may have different qualitative content in various worlds. The approach does not exclude MBSs in which the property attribution function associates exactly the same properties with disjoint, yet isomorphic branching-spatiotemporal regions (see the two figures). If no additional postulates regarding the notion of an individual are considered, it is therefore not ruled out that there be two numerically distinct individuals for which the point events which comprise their (branching) "life" instantiate exactly the same qualitative properties. BH seems to allow, then, distinct thisness underlying the perfect form of great semblance, that is, identity: and so, the accusation would be, it is haecceitistic, therefore non-Humean.

At first it may seem, however, that there is a response available to a Branching Humean. Consider this crucial portion of the HS definition given by Lewis: "There is no difference without difference in qualities". Suppose x and y are spatiotemporal locations. Notice that it does *not* follow from the definition just quoted that "There is no difference between the occupants of x and y without a difference between qualities instantiated at x and qualities instantiated at y". It fits the letter, if not the spirit of HS that two qualitatively identical individuals may be numerically distinct if there is a qualitative difference in their past. However, it is an easy mental exercise to reformulate the issues just discussed in the scope of a single possible world, with recurring patterns of instantiated qualities.

In fact, Lewis discusses similar cases himself in 4.4 of Lewis (1986b) (starting on p. 227, the "infinite sequence of epochs" example). However, his way of fighting the charge of haecceitistism is eventually to claim that the alleged "haecceitistic differences" are not differences between possible worlds, but between possibilities. This by itself would not be enough to sustain Humean supervenience. However, Lewis can make use of the fact that his approach contains the primitive, fundamental ontological kind of individuals (Bricker, 2006), and ultimately—roughly—claim that in such cases the individuals in question are related by the counterpart relation (and so we can say that the possibilities under discussion are "possibilities for the same individual"). Since abandoning that relation is a significant part of the motivation behind Branching Humeism, this move is not available to its proponents. However, we have to note here that when dealing with this problem, and in particular when making use of the counterpart relation, Lewis avails himself with tools from outside the HS toolbox—so perhaps a branching theorist should be allowed a similar move.

Still, it is hard for us to see a way around the problem open for a Branching Humean which would not involve introducing an individual as a primitive concept. This has been done fruitfully e.g. in branching-time approaches to agency

[18] At least until some better approach comes along. Remember that Belnap and Placek proposed only necessary conditions for such a set to be an individual; it is of course possible that researchers will suggest additional conditions, but until a fundamentally different concept of an individual is proposed for BST, individuals are, at their core, sets of point events.

(Belnap et al., 2001); however, we have not seen a similar move made by BST proponents.

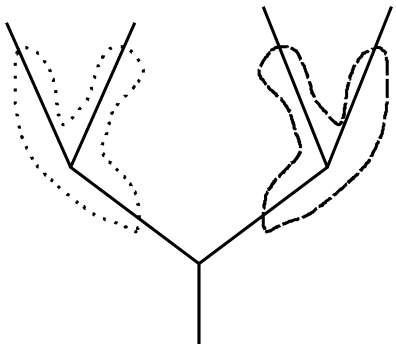

Figure 1: Four histories with two choice points each. Each maximal chain represents a Minkowski space-time. Two qualitatively identical individuals are located inside the regions delineated by the dotted and dashed lines—see next figure.

3.2 Modal saturation (absence of "funny business")

Consider two events which take place in space-like related regions of space-time—that is, none is in the forward light cone of the other—each of which has two possible outcomes. If you are following the BST literature, you can think here of spin measurements in different wings of an Einstein-Podolsky-Rosen-type (EPR) experiment, but we do not need to start with it: considering two coin tosses, Left and Right, such that neither is in the forward light cone of the other, will suffice for now.[19] Suppose that it is possible for the Left coin to come up heads and it is possible for it to come up tails; likewise for the Right coin. Abstract from anything that is happening anywhere else. How many possible worlds are there?

Lewis offers a clear answer here: there are exactly *four*.[20] This follows on any charitable reading of his "principle of recombination" (see chapter 1.8 of Lewis (1986b)); at a first approximation, Lewis offers it as saying that "patching together parts of different possible worlds yields another possible world. Roughly speaking, the principle is that anything can coexist with anything else, at least provided they occupy distinct spatiotemporal positions. Likewise, anything can fail to coexist with anything else".[21] The outcome 'Left-Heads' and the outcome

[19] Non-chancy events like choices of some agents would also be fine, but let us stick with coin tosses for the sake of the example.

[20] If other features of reality are taken into account, the Lewisian approach will still hold that all four types of possible worlds corresponding to the combinations of outcomes of the two coin tosses are nonempty.

[21] The principle is accompanied by a few enigmatic provisos. Metaphysicians have been

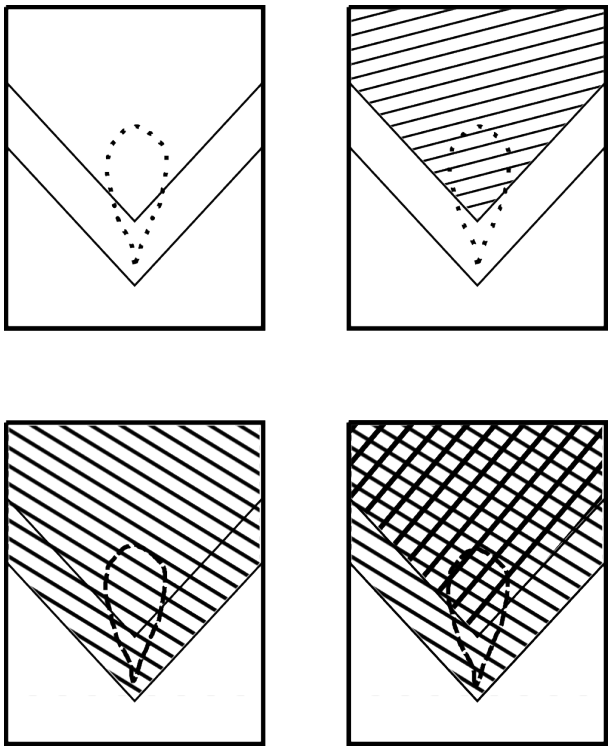

Figure 2: Four histories with two choice points each. Patterns inside regions represent numerical identity of point events, that is, the upper right history splits from the upper left one only at the later choice point etc. Two individuals are depicted, with dotted and dashed lines, each intersecting with two histories: while numerically distinct, they, as sets of point events, may instantiate exactly the same pattern of local qualities.

'Right-Tails' can coexist; they can also fail to coexist. The word "can" here is to be read as referring to metaphysical possibility. That is, each of the four combinations of outcomes occurs in some possible world.[22]

In contrast, the MBS framework offers no unique answer to this question:

struggling for more than three decades to unpack what those provisos should mean and what form the combinatorialism "axiom" should eventually take (for a modern discussion, situating Lewis's ideas in the context of Hume's "no necessary connections" doctrine, see Wilson, 2015). Saying that this struggle has been difficult would be to significantly understate the severity of the problem. It seems that only close to the end of the first decade of the current century philosophers arrived at proposals which stated the Lewisian combinatorialist idea in a logically clear way, so that what and where was quantified over was made evident. For an example of this see (Efird and Stoneham, 2008). However, the formulation those authors adopt has not been universally accepted.

[22]For the sake of brevity we do not cover the worlds in which at least one of the coins is not tossed.

there may be four worlds, there may be three, and there may be just two. In other words, the MBS approach does not exclude "modal correlations": for example, it may just be the case that the only two possible worlds are such that in one of them the Left coin comes up heads and the Right one comes up tails, while in the other the Left coin comes up tails and the Right one comes up heads.[23] This phenomenon—the pluriverse of possible worlds not containing some that would seem to be preliminarily suggested by combinatorial considerations—was initially considered not to be a disadvantage, but rather a welcome feature of BST. It was used in modelling of the Greenberger–Horne–Zeilinger theorem (Belnap and Szabó, 1996) and labelled "funny business" (Belnap, 2002). All versions of BST we have seen are able to provide clear and rigorous definitions of what this "funny business" is. However, for a broader philosophical project it is a burden, not a boon. We propose we change the label to some more illuminating expression, not least because we find nothing even remotely amusing about the pluriverse missing some combinatorially available possibilities. We suggest we say that there is no guarantee for MBSs to be *modally saturated*: it may be the case that some combinations of outcomes of space-like related events are not exemplified by any possible worlds.[24]

Philosophical applications of BST have largely depended on the assumption of modal saturation. This is the case e.g. for the results in Müller (2010), exploring a novel notion of indeterminism. The mathematical arguments therein explicitly require modal saturation, and this is not because there is some independent motivation for this assumption—rather, obtaining the results without it turned out to be very tricky[25]. Another example is the theory of causation developed in Belnap (2005). Some relevant structures arising in the BST models described therein correspond to the well-known INUS conditions discussed by Mackie; however, the theorem establishing the relationship crucially depends on modal saturation, and to our knowledge this assumption cannot be weakened without significantly modifying the result[26].

May an MBS theorist have any hope for harnessing the modal saturation phenomenon by means of some non-ad-hoc principles? Would an appeal to laws of nature be promising, for example? If we switch from coin tosses to spin measurements, and enter the field of EPR experiments, we may think we begin to see the light: nonprobabilistic variants of the relevant results (such as the Greenberger, Horne, Shimony and Zeilinger theorem discussed in Belnap and Szabó (1996)) may lead us to representing the EPR experiments as BST structures exhibiting failures of modal saturation (see e.g. Belnap (2002)). However, it is doubtful whether the route of referring to laws of nature in order to establish important aspects of the branching structure is indeed available here. The

[23] This does not, of course, change the fact that, for each coin, it is possible for it to land heads and it is possible for it to land tails. It also has no bearing on the non-extreme probabilities of outcomes of each toss.

[24] For a precise definition of "funny business" in the context of MBS', see e.g. Def. 9 in Placek (2021).

[25] Müller, personal communication.

[26] Chapter 6.4 of Belnap et al. (2022) includes a few relevant theorems, e.g. Theorems 6.3 and 6.4.

BST theory, of which MBS' are special kind of models, has only recently been given a modal-philosophical underpinning which is to shed light on the source of the branching involved. Namely, the 2022 monograph explicitly states that the project is rooted in the theory of "real possibilities": branching occurs when there are "multiple alternative possibilities open in concrete situations" (p. 6). It will not be fruitful to introduce this idea in more detail here, save for one crucial point concerning the place of laws of nature in the general picture: "real possibilities do not fall in the mold of law-given modalities" (p. 17). Moreover, Section 1.4 seems to suggest that what is really possible is fundamentally prior to what any laws dictate. Even if we do not go as far, the previous point should be enough to establish that in the most developed version of BST to date, one cannot answer questions about co-possibility, and so about the number of possible histories, by referring to laws.

The situation in the Lewisian picture is markedly different. Faced with the phenomena about which the GHZ theorem speaks, Lewisians will in principle inquire about the relevant physical laws from the Best System for the possible world in question (for an overview of the Best System approach to lawhood, see Eddon and Meacham (2015)). If these laws indeed imply the GHZ theorem or some variant to a similar effect, this will lead to definite conclusions about the kind of physically possible worlds there are. Some worlds which are at least *prima facie* metaphysically possible worlds according to the principle of recombination may turn out to be nomologically impossible. This direction of reasoning seems to be closed for BST theorists at this point. However, perhaps there is an option to consider: fiddling with the definition of "proper property attribution" in an MBS. Maybe this could be an "entry point" for laws into BST, so that they would indeed play a role in generating the branching structure. This is surely a matter for further research.

Haecceitism aside, then, your verdict on the relative merits of BH when compared to the Lewisian approach will depend on your stance regarding modal saturation. If you have some reasons to believe that the pluriverse is modally saturated, then in the case of the two coin tosses you will welcome the clear answer of Lewis's combinatorialism: there are four possible worlds, since there are four combinations of toss outcomes. Vague and controversial as it is, Lewis's combinatorialist axiom gives a clear answer in this case, while the MBS's sword of formal rigour turns out to be double-edged. One the one hand, it gives the formal researcher leeway which might be useful when modelling e.g. some takes on the EPR correlations, or previously coordinated choices by space-wise separated agents, when one might wish to exclude some prima facie possible worlds from the model. On the other hand, it precludes the theory from giving, or even suggesting an answer to the metaphysical question regarding *what possible worlds there are*. For example, in the case of the two coin tosses, the researcher simply chooses the number of scenarios to be considered right at the start: only later is the proper property attribution function defined, and the histories in the model are generated. There will be no mystery in that for the researcher; there will be exactly as many histories as there were scenarios. One can probably

maintain that each history behaves Humeanly, that is, everything happening in it supervenes on the values of the property attribution function. But before some additional postulates are added, we seem to be able to do no better than to decide the number of possible worlds arbitrarily, which is not an improvement on the Lewisian approach.

Formally, the assumption of modal saturation is just something which can make or break a mathematical theorem regarding BST structures. For someone seeking a metaphysical framework with a worked-out formal layer, though, it becomes a significant philosophical issue. BST theorists have so far been content with the fact that their theory is able to model both situations involving modal saturation and lacking it; they have been reluctant to take a definite stance on this; cf. e.g. (Belnap et al., 2022, p. 104): "we do not take it to be a settled matter that such modal correlations[27] exist in our world". Assuming modal saturation in general would bring the theory in concert with Lewis' principle of recombination. Not doing so invites the question: on what exactly does it depend whether two spatiotemporally separated pairs of two local outcomes involve two, three, or four possible worlds? It seems to us that if the BST theory is indeed to belong to the realm of metaphysics (as is explicitly claimed e.g. in Chapter 1.4 of the 2022 monograph), this issue should be among the first to be discussed.

4 Conclusions

Can you be a branching Humean? For someone who subscribes to Humean Supervenience, there are reasons for pursuing such an option. Perhaps for a modal realist who possesses the open future intuition the route is the shortest. But others may be interested in such an approach if they find the employment of the notion of counterpart and, maybe, the lack of formal rigour to be drawbacks of Lewis's account. In this paper we argued that a branching Humean faces at least two nontrivial charges: first, that of haecceitism, and second, that of not having a good answer to the question regarding what possible worlds there are.[28] In the second case, the seeming improvements in rigour offered by the modern branching theories seem to be offset by the lack of philosophical exploration of the branching variant of the Lewisian recombination postulate: the assumption of modal saturation.

We submit, then, that some branching Humean account will become feasible when branching theorists decide to use a fundamentally different concept of individual than the one currently employed (i.e., an individual as a set of point events), and when the metaphysical issues regarding modal saturation are paid more careful attention. We suggest that the current place the laws of nature hold in the BST theory might need to be reconsidered.

[27] In our parlance: lack of modal saturation.

[28] We have not discussed the issue of modal reductionism, because we do not consider it to be an advantage of Lewis' approach, and it is not a goal to which branching theories aspire.

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