The Structure of Space and Time, and Physical Indeterminacy

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ABSTRACT. I introduce a sequence which I call *indefinite*: a sequence every element of which has a successor but whose number of elements is bounded; this is no contradiction. I then consider the possibility of space and time being indefinitely divisible. This is theoretically possible and agrees with experience. If this is space and time's structure, then even if the laws of nature are deterministic, the behaviour of physical systems will be probabilistic. This approach may also shed light on directionality in time, a source of the Uncertainty Principle, and the reality of chaos.

Introduction

The discontent with Quantum Mechanics (QM) is interpretative, not empirical: the empirical results are in excellent agreement with the theory. We do not understand, however, the physical reality which the theory describes. In this paper I shall provide a new understanding of the structure of space and time and derive some additional aspects of physical reality from it. This will offer a different interpretation of some features of QM, as well as of chaotic phenomena, hopefully removing some concerns.

Moreover, I shall try to make the physical reality as described by QM comprehensible without adding anything to the theory or modifying it. For instance, unlike Bohm (1952), I shall not claim that there are various relevant physical quantities not expressed by the theory, the so-called 'hidden variables'; and unlike GRW (1985), I shall not claim that some sort of 'spontaneous collapse' hypothesis should be added to the theory. Some such additions might be compatible with my interpretation, but it does not need them in order to make sense of physical reality. There is no empirical motivation for the claim that QM is incomplete, as no observations are incompatible with what the theory predicts or show any regularity beyond what it predicts.

Indefinite Sequences

I start by describing a logical possibility, *an indefinite sequence*.

This is a sequence of elements, each of which has a successor, but whose number of elements is bounded, less than some natural number.

$\mathcal{S}$ = $a_1, a_2, a_3 \ldots$; for each $a_i \in \mathcal{S}$, there is $a_{i+1} \in \mathcal{S}$; and for some N, for each $a_i \in \mathcal{S}$, $i < N$.

So, an indefinite sequence has no last element, but it is bounded.

Such a sequence is demonstrated by *sorites* sequences. If we start with a heap of sand and remove one grain after another, we end with grains which constitute no heap, so we have a bounded sequence of heaps. Yet the sequence of heaps has no last member: when a grain of sand is removed from a heap, the remaining collection is still a heap. The sequence of heaps is therefore an indefinite sequence. (More on this, including responses to some apparent difficulties, in Ben-Yami 2010).

This idea might initially appear inconsistent, but any attempt I have come across to find an inconsistency fails, as it presupposes at some stage that indefinite sequences are impossible. And if sorites sequences are indeed indefinite sequences, then the idea is proved consistent by having instances.

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– *But if each element has another one following it, why can't we add elements on and on until we have more than N elements?* – This 'can' is a physical one, not a logical one; and why a physical process should fail is not a question for logic. (In the case of sorites sequences it might fail because of the vague boundaries between, say, a heap and a collection which is not a single heap.)

The structure of space and time

We have no sufficient reason to hold that either space or time is *infinitely* divisible. This is not supported by any empirical fact or conceptual considerations. If the only alternative to infinite divisibility were *finite* divisibility, then this would be conceptually problematic, if not contradictory, for familiar reasons. But once the possibility of *indefinite* divisibility is acknowledged, an alternative without these conceptual difficulties is available. Whether space and time are infinitely or indefinitely divisible, should be decided by the consequences of each hypothesis and their agreement with observation.

What might it mean for the *distance* between two physical points, say, to be indefinite? Let us consider an elementary example (I have written on this in more detail, in Hebrew, in my 2012: §3.3).

Suppose the length of a rigid rod is L. We can use it as a unit of measurement – our ruler – and measure the distance D between points A and B as being, say, between $7L$ and $8L$. We next divide the rod into ten *roughly* equal units. In practice, we mark roughly equidistant notches on it. 'Roughly', since at this stage all we can say is that the new unit enters between the end of the rod more than 9 but less than 11 times – we have yet no more refined unit to improve on that determination. We again measure the distance between points A and B and get, for instance, the result that $7.3L < D < 7.5L$. Further divisions, by further notches or by other means, will help us improve the accuracy of the measurement. Perhaps we can determine that $7.472 < D < 7.481$ units. But the bounded accuracy in determining what counts as the end points of the rod and the edges of A and B will make it impossible for us to proceed much beyond this accuracy.

These or related limitations will face us whatever we use as unit and whichever distances we measure. Whatever is our standard s and whichever is the distance d we measure, the distance won't be determined much beyond some Δ, such that $d = (x \pm \Delta)s$.

This Δ is *not* an expression of limitations of knowledge, but of bounded accuracy of distances in nature. It's not epistemic, but ontic.

Δ is also determined only to an indefinite degree of accuracy; that is why I used 'much beyond' in the formulation above.

I shall assume in this paper that the structure of space and of time is indefinite, and explore the implications of this assumption. I am interested primarily in the conceptual possibility, and therefore I won't try to add to my assumption specifications derived from contingent, factual physics laws. But I shall compare the conceptual results to contemporary physics, to show that the indefinite-structure assumption can explain a variety of physical phenomena. Some details of the application of the conceptual possibility to contemporary physics will depend on specifications coming from contemporary theories and should give more exact results, but this will not be attempted here.

The representation of location

I focus for the moment on location in space. What I write is applicable also to location in time, and below I shall consider time, or space and time together where it is relevant.

In Newtonian physics and in Special Relativity, we represent the relative locations of bodies in space at a given moment by means of $\mathbf{R}^3$. In General Relativity the spatiotemporal manifold has additional complexities, which will not be addressed here, but the considerations below can be adapted to that more complex manifold. The considerations here are applicable to configuration-space as well, and probably with some adaptations, given Liouville's theorem, to phase space.

If distances between bodies are indefinite, then the precision that $\mathbf{R}^3$ offers is beyond that found in nature. Given any unit of length, the distance between any two bodies relative to it is determined up to a vague, bounded degree of accuracy – our Δ above – while if we ascribe to each body a location in $\mathbf{R}^3$, this distance is determined with infinite accuracy.

This is not the only case in which $\mathbf{R}^3$ offers more structure than there is in physical reality. Translation, rotation and any linear transformation on $\mathbf{R}^3$ are also meaningless: if you add the same number to all locations, say, you do not represent a different physical reality.

Accordingly, if we wish to represent the location of bodies and distances between them by means of $\mathbf{R}^3$, we should ascribe to each body a *vague* region as its location.

In practice, we can *approximate* this by having a probability function, perhaps some Gaussian function, ascribe location to a body. The standard deviation or RMS of the Gaussian will approximate the degree of accuracy Δ itself in the determination of the body's location.

The fact that we can represent location only *approximately* in $\mathbf{R}^3$ is not a flaw in this representation, since location *is* determined only up to a bounded accuracy. But this does not mean that there aren't better and worse approximations. Which method of mathematical representation will prove best is probably partly an empirical question.

The idea that the representation of location and of other physical quantities by real numbers is, in some sense, physically meaningless, is not new. It goes back at least to Max Born, who related it to a methodological principle dominant in modern physics and illustrated, for instance, by Einstein's Special and General Relativity:

> Statements like 'a quantity *x* has a completely definite value' (expressed by a real number and represented by a point in the mathematical continuum) seem to me to have no physical meaning. Modern physics has achieved its greatest successes by applying a principle of methodology, that concepts whose application requires distinctions that cannot in principle be observed, are meaningless and must be eliminated. (1955: 81)

The question is, what should replace the representation of quantities by **R**, and specifically of location by $\mathbf{R}^3$? One option, suggested by Nicolas Gisin, is to develop a new mathematics and a new concept of number – Gisin himself suggested replacing standard- by intuitionistic mathematics (2019: fn11; 2020). I find intuitionistic mathematics, at least in the version Gisin adopts, which introduces temporal concepts into arithmetic, insufficiently clear. But more importantly, the current mathematical practice works sufficiently well, in the sense that it provides accurate predictions and does not omit any regularity of which we are aware; there is therefore no need to modify it, and in fact these are good reasons *not* to do so. What is needed is a different interpretation of the mathematical representation, and this is achieved by reading Δ, which we formerly took as a reflection of lack of knowledge alone, as a representation of the indefiniteness in location as well.

Determination of the indefiniteness

We shouldn't think of the indefiniteness in location as provided by space and time independently of the bodies' distribution in them. Space and time are not inert receptacles. Rather, the degree of indefiniteness is determined by the interactions between the bodies. For instance, one possibility is that the greater the number of interactions and the closer the bodies, the lesser the indefiniteness. Also, the kind of interaction, for instance how energetic it is, might also affect the degree of indefiniteness in location.

The exact form of the dependence of indefiniteness on the interaction and distribution of bodies is to be empirically determined.

The indefinite structure of space-time and the distributions of the bodies in it are determined together. This resembles the way it is in General Relativity, in which the curvature of space-time and the distribution of bodies in it are determined together.

A principle of complementary inaccuracy or "uncertainty"

Since distance and duration have a bounded degree of accuracy, so does velocity, as velocity is the ratio between the distance a body has covered and the time it took to cover that distance.

Suppose a moving body's place is determined to a degree of accuracy of about ΔL, which for simplicity we suppose does not change during the interval of its motion that we are considering. Accordingly, since both beginning and end locations are determined to a degree of accuracy of about ΔL, the distance it travels is determined to a degree of accuracy of about 2ΔL. Let us designate it, L ± ΔL. Similarly, if the time a body is at a given location is determined to a degree of accuracy of about ΔT, the time it takes it to travel between beginning- and end locations is determined to a degree of accuracy of about 2ΔT. We designate it, T ± ΔT.

The velocity of the body between the two points is then,

$$v = (L \pm \Delta L) / (T \pm \Delta T)$$

Velocity is also determined to a bounded degree of accuracy. We shall say that velocity is *inaccurate*. This inaccuracy in velocity is *not* a matter of ignorance on our part, but an inaccuracy in nature.

We also see that *there is an inverse relation between the degree of accuracy of place and that of velocity*.

Suppose a body travels with a roughly uniform velocity, in the sense that its average speed over large distances is constant. The smaller the intervals L ± ΔL we consider, the lesser the time T it takes it to travel the distance, and the greater the relative inaccuracy 2ΔT in that time T. The inaccuracy in velocity, Δ*v*, is therefore larger. And over very small distances, when 2ΔT approaches T, the inaccuracy approaches infinity. By contrast, for larger intervals L ± ΔL, T is larger and 2ΔT negligible relative to T. The less the place of the body is determined, the more determinate is its average velocity over that interval.

I shall call this principle, *The Inaccuracy Principle*.

The Inaccuracy Principle is reminiscent of Heisenberg's Uncertainty Principle (Heisenberg originally used *Ungenauigkeit* – *inaccuracy* or *imprecision* – and not *Unsicherheit*). There are, however, some important differences between the two. First, the corresponding complementary variables of the Uncertainty Principle are not *velocity* and place, but *momentum* and place. The Uncertainty Principle states, for instance, that $\Delta p_x \Delta x \geq h/4\pi$, while the Inaccuracy Principle conjugates Δv_x and Δx. Secondly, the Uncertainty Principle specifies a quantity, h/4π, which serves as the minimum of the multiplication of the uncertainties or inaccuracies, while the Inaccuracy Principle is more qualitative in this respect, providing no quantitative minimum. Lastly, the Uncertainty Principle is a result of an empirical theory, based on many non-obvious observations. It is contingent in nature. The Inaccuracy Principle, by contrast, is a result of very general reflections on the nature of space and time.

Still, the Inaccuracy Principle, resulting from the indefinite structure of space-time, might be at the root of the Uncertainty Principle, although the latter adds some empirical, contingent determinations to the Inaccuracy Principle. The occurrence of momentum and not speed as the complementary variable of the location might indicate that the distribution of the masses in space-time determines its indefiniteness.

Laws, Indeterminacy and Directionality

If the structure of space and time is indefinite, then the location and velocity of a body at each moment – and a moment is not a point in time but a vague interval – these location and velocity are of bounded accuracy. Consequently, even if the laws of nature that describe the motion of bodies as a function of time are deterministic, there will be indeterminacy in the course of nature. (The laws can be deterministic in the sense that with the initial conditions of the system represented by *real* numbers they determine uniquely the future and past states of the system, again represented by real numbers, for any time.)

Suppose at a time t_0 – we ignore for the moment the inaccuracy of time – a body is in region $x \pm \Delta x$ with a velocity $v \pm \Delta v$. Then, given this inaccuracy in both quantities, the laws will provide a range of values for both x and v at any future time t. And as a rule, the larger is t, the greater the range which the laws will specify for both x and v. However, the location and speed of the body may continue to be inaccurate only up to the same Δx and Δv. Namely, there will be lower inaccuracy in place and speed than the laws specify. This additional determination of location and speed will be *random*, since it is not determined by any law.

If we represent the inaccurate location in space and time by a Gaussian function on $\mathbf{R}^3$, whose standard deviation approximates the degree of accuracy Δ in the determination of the body's location, then the Gaussian assigns different probabilities to different locations, and this will show in the probabilities of future specific developments. The probability that if a body started at region *a* it will arrive after a time *t* at region *b* might be greater than that of its arriving at region *c*. So this representation could give us a probabilistic distribution of the chances of the body being found at different regions.

If at a time t_0 bodies 1, … *n* are in regions $x_i \pm \Delta x$ with velocities $v_i \pm \Delta v$, then the ranges of places and velocities that the laws determine at time *t* include as a rule more possibilities with higher entropy than with lower one (I think this is the situation with current laws of nature). Since the specific regions and velocities of these ranges that the bodies will have at *t* are random (with the possibility of a probability given by something like the option mentioned in the previous paragraph), the entropy of the system will increase as a function of time. The arrow of time comes with an arrow of entropy.

These approach and result resemble those of coarse graining (e.g. Robertson 2018). However, the standard coarse graining approach does not see the coarse grain in location and velocity as a fact of nature, while the approach developed here, which assumes that the structure of space and time is indefinite, introduces *its* version of coarse grain as a means of the representation in $\mathbf{R}^3$ or any other manifold of approximating the indefiniteness of the quantities in nature.

Here and elsewhere, the indefinite structure of space and time turns what in other systems is lack of knowledge into lack of fact.

Chaos

Chaos theory examines the behaviour over time of systems that obey deterministic laws. In a vast variety of systems, the behaviour of their elements displays sensitive dependence on initial conditions. Namely, in most cases, a difference in the initial conditions grows exponentially with time. If a body started its motion with a slightly different speed, say, then this slight difference would grow exponentially until very soon the body's place and speed would be far removed from what they actually were. And in case the range of magnitudes is bounded – for instance, if the body is confined to some volume, or its speed is bounded – then there will be no general functional dependence between the two results after some time. The place in which the body ends up after a time *t* has nothing to do with the place it would have occupied after that time had it started its motion with a slightly different speed, say. Such behaviour is called *chaotic*.

Our ability to predict the development over time of chaotic systems is limited. We always know the initial conditions of a system up to certain accuracy. If the system is chaotic, this accuracy will diminish exponentially with time. And if the range of the measured magnitudes is bounded, then after some time any possible result would be compatible with what we know of the initial conditions. Although the laws that describe the development of the system over time, as well as that development itself, are deterministic, we can in principle predict it with exponentially decreasing accuracy.

Chaos acquires an additional meaning if the structure of space and time is indefinite. Although the laws that describe a chaotic system's development over time are deterministic, the initial conditions of the system are fixed only up to a certain bounded accuracy. Since the place of a particle at a certain moment, say, is inaccurate, the particle's future behaviour can be any of the behaviours it would have had, had its place been accurate and compatible with its actual inaccurate place. This yields a range of possible behaviours, the difference between any two of which grows exponentially with time. Accordingly, the indefiniteness of space and time yields indeterminacy in the future state of the system, indeterminacy which grows exponentially with time. Moreover, in case the relevant magnitudes are bounded, after some time any possible state is compatible with any initial conditions.

We can therefore ascribe an additional meaning to what we understand by *chaos*. Suppose we are given a system with deterministic laws that is chaotic in the above defined sense. Suppose further that some of the magnitudes relative to which the system displays chaotic behaviour are inaccurate. Then the system is chaotic also in the sense that the indeterminacy in its future state, given specific initial conditions, grows exponentially with time. And if the relevant magnitudes are bounded, there is no general functional dependence between the state the system is in at a given time and the state it ends up in after some time. Chaotic indeterminacy is not only an essential feature of our knowledge, but is also found in nature.

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