Intervening is Imaging is Conditioning

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Abstract

I show that, in acyclic causal models, post-intervention probabilities are imaging probabilities which in turn are conditional probabilities.

1 Acyclic causal models

Let us consider an acyclic causal model $\mathcal{M}$ of the sort that is central to causal modeling (Spirtes et al. 1993/2000, Pearl 2000/2009, Halpern 2016, Hitchcock 2018). Readers familiar with them can skip this section.

$\mathcal{M} = \langle \mathcal{S}, \mathcal{F} \rangle$ is a *causal model* if, and only if, $\mathcal{S}$ is a signature and $\mathcal{F} = \{F_1, \ldots, F_n\}$ represents a set of n structural equations, for a finite natural number n. $\mathcal{S} = \langle \mathcal{U}, \mathcal{V}, R \rangle$ is a *signature* if, and only if, $\mathcal{U}$ is a finite set of exogenous variables, $\mathcal{V} = \{V_1, \ldots, V_n\}$ is a set of n endogenous variables that is disjoint from $\mathcal{U}$, and $R : \mathcal{U} \cup \mathcal{V} \to \mathcal{R}$ assigns to each exogenous or endogenous variable X in $\mathcal{U} \cup \mathcal{V}$ its *range* (not co-domain) $R(X) \subseteq \mathcal{R}$. $\mathcal{F} = \{F_1, \ldots, F_n\}$ *represents a set of n structural equations* if, and only if, for each natural number i, $1 \leq i \leq n$: F_i is a function from the Cartesian product $\mathcal{W}_i = \times_{X \in \mathcal{U} \cup \mathcal{V} \setminus \{V_i\}} R(X)$ of the ranges of all exogenous and endogenous variables other than V_i into the range $R(V_i)$ of the endogenous variable V_i. The *set of possible worlds* of the causal model $\mathcal{M}$ is defined as the Cartesian product $\mathcal{W} = \times_{X \in \mathcal{U} \cup \mathcal{V}} R(X)$ of the ranges of all exogenous and endogenous variables.

A causal model $\mathcal{M}$ is *acyclic* if, and only if, it is not the case that there are m endogenous variables $V_{i1}, \ldots, V_{im}$ in $\mathcal{V}$, for some natural number m, $2 \leq m \leq n$, such that the value of $F_{i(j+1)}$ depends on $R\left(V_{ij}\right)$ for $j = 1, \ldots, m-1$, and the value of F_{i1} depends on $R(V_{im})$. Importantly, dependence is just ordinary functional dependence: F_i depends on $R\left(V_j\right)$ if, and only if, there are arguments $\vec{w}_i$ and $\vec{w}_i{}'$ in the domain $\mathcal{W}_i = \times_{X \in \mathcal{U} \cup \mathcal{V} \setminus \{V_i\}} R(X)$ of F_i that differ only in the value from $R\left(V_j\right)$ such that their values under F_i differ, $F_i(\vec{w}_i) \neq F_i\left(\vec{w}_i{}'\right)$.

Let $Pa(V_i)$ be the set of variables X in $\mathcal{U} \cup \mathcal{V}$ such that F_i depends on $R(X)$. The elements of $Pa(V_i)$ are the *parents* of the endogenous variable V_i, that is, the set of variables that are *directly causally relevant* to V_i. Let $An(V_i)$ be the ancestral, or transitive closure, of $Pa(V_i)$, which is defined recursively as follows: $Pa(V_i) \subseteq An(V_i)$; if $V \in An(V_i)$, then $Pa(V) \subseteq An(V_i)$; and, nothing else is in $An(V_i)$. The elements of $An(V_i)$ are the *ancestors* of the endogenous variable V_i. A variable Y is a *non-descendant* of a variable X if, and only if, X and Y are different and X is not an ancestor of Y.

A *context* is a specification of the values of all exogenous variables. It can be represented by a vector $\vec{u}$ in the Cartesian product $R(\mathcal{U}) = \times_{U \in \mathcal{U}} R(U)$ of the ranges of all exogenous variables. A basic fact about causal models is that every acyclic causal model has a unique solution $w_{\vec{u}}$ for any context $\vec{u}$. Let $\mathcal{W}_0$ be the set of these "legal" possible worlds (Glymour et al. 2010). An acyclic causal model determines a unique directed acyclic graph whose nodes are the exogenous and endogenous variables in $\mathcal{U} \cup \mathcal{V}$ and whose arrows point into each endogenous variable V_i from all of the latter's parents in $Pa(V_i)$.

Acyclic causal models provide a semantics for some counterfactuals. The language includes atomic sentences of the form $V = v$ which say that endogenous variable V takes on a specific value v from its range $R(V)$, as well as the Boolean combinations that can be formed from these atomic sentences by finitely many applications of negation $\neg$, conjunction $\wedge$, and disjunction $\vee$. The variables must be endogenous. Sentences of the form $V \in S$, for a subset S of $R(V)$ with more (or less) than one element are not allowed. The antecedent of a counterfactual must by a finite conjunction $X_1 = x_1 \wedge \ldots \wedge X_k = x_k$ of one or more atomic sentences with distinct endogenous variables. The consequent must a Boolean combination ϕ of atomic sentences. Among others, this means that we cannot consider counterfactuals with a counterfactual in the antecedent or consequent.

An atomic sentence $V = v$ is true in $\mathcal{M}$ in $\vec{u}$ if, and only if, all solutions to the structural equations represented by $\mathcal{F}$ assign value v to the endogenous variable V if the exogenous variables in $\vec{\mathcal{U}}$ are set to $\vec{u}$. Since we are restricting the discussion to extended acyclic causal models which have a unique solution in any given context, this means that $V = v$ is true in $\mathcal{M}$ in $\vec{u}$ if, and only if, v is the value of V in the unique solution $w_{\vec{u}}$ to all equations in $\mathcal{M}$ in $\vec{u}$. The truth conditions for negations, conjunctions, and disjunctions are given in the usual way. The counterfactual $X_1 = x_1 \wedge \ldots \wedge X_k = x_k \mathrel{\Box\!\!\rightarrow} \phi$, or simply $\vec{X} = \vec{x} \mathrel{\Box\!\!\rightarrow} \phi$, is true in $\mathcal{M} = \langle \mathcal{S}, \mathcal{F} \rangle$ in $\vec{u}$, $\mathcal{M}, \vec{u} \models \vec{X} = \vec{x} \mathrel{\Box\!\!\rightarrow} \phi$ if, and only if, ϕ is true in $\mathcal{M}_{\vec{X}=\vec{x}} = \langle \mathcal{S}_{\vec{X}}, \mathcal{F}^{\vec{X}=\vec{x}} \rangle$ in $\vec{u}$.

The latter causal model results from $\mathcal{M}$ by removing the structural equation for X_i and by freezing the value of X_i at x_i, for each $i = 1, \ldots, k$. Formally, this means that $\mathcal{S}$ is reduced to $\mathcal{S}_{\vec{X}} = \langle \mathcal{U}, \mathcal{V} \setminus \{X_1, \ldots, X_k\}, \mathcal{R} \restriction_{\mathcal{U} \cup \mathcal{V} \setminus \{X_1, \ldots, X_k\}} \rangle$, where $\mathcal{R} \restriction_{\mathcal{U} \cup \mathcal{V} \setminus \{X_1, \ldots, X_k\}}$ is $\mathcal{R}$ with its domain restricted from $\mathcal{U} \cup \mathcal{V}$ to $\mathcal{U} \cup \mathcal{V} \setminus \{X_1, \ldots, X_k\}$; as well as that $\mathcal{F}$ is reduced to $\mathcal{F}^{\vec{X}=\vec{x}}$ which results from $\mathcal{F}$ by deleting, for each $i = 1, \ldots, k$, the function F_{X_i} representing the structural equation for X_i and by changing the remaining functions F_Y in $\mathcal{F} \setminus \{F_{X_1}, \ldots, F_{X_k}\}$ as follows: restrict the domain of each F_Y from $\times_{X \in \mathcal{U} \cup \mathcal{V} \setminus \{Y\}} R(X)$ to $\times_{X \in \mathcal{U} \cup \mathcal{V} \setminus \{Y, X_1, \ldots, X_k\}} R(X)$; and, replace F_Y by $F_Y^{\vec{X}=\vec{x}}$ which results from F_Y by setting $X_1, \ldots, X_k$ to $x_1, \ldots, x_k$, respectively.

2 Probability

Next let us consider a regular probability measure Pr on the power-set of $\mathcal{W}$. This means that every non-empty proposition over $\mathcal{W}$ receives a positive probability, including the singletons containing a possible world which I will identify with each other. The conditional probability $\Pr(\cdot \mid \mathcal{W}_0)$ is the probability measure conditional on the assumption that $\mathcal{M}$ is true and no intervention takes place. Note that $\Pr(w_{\vec{u}} \mid \mathcal{W}_0) = \Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}}\right)$, where $[\![\vec{U} = \vec{u}]\!]_{\mathcal{W}}$ is the proposition over $\mathcal{W}$ that is expressed by the sentence $\vec{U} = \vec{u}$. This means that $\Pr(\cdot \mid \mathcal{W}_0)$ allocates the entire probability mass of context $\vec{u}$ onto the single possible world $w_{\vec{u}}$; every other possible world that agrees with $w_{\vec{u}}$ on the values of the exogenous variables $\mathcal{U}$ receives probability zero.

If the set of exogenous variables $\mathcal{U}$ is probabilistically independent in the sense of $\Pr(\cdot \mid \mathcal{W}_0)$, Pearl (2000/2009: 30)'s theorem 1.4.1 applies. In this case $\Pr(\cdot \mid \mathcal{W}_0)$ satisfies the causal Markov condition for the directed acyclic graph that is determined by $\mathcal{M}$: each variable in $\mathcal{U} \cup \mathcal{V}$ is probabilistically independent of its non-descendants given its parents. In this case the pair $\langle \mathcal{M}, \Pr(\cdot \mid \mathcal{W}_0) \rangle$ is Markovian; it is semi-Markovian, if the set of exogenous variables $\mathcal{U}$ is not probabilistically independent in the sense of $\Pr(\cdot \mid \mathcal{W}_0)$. The significance of this theorem lies in connecting acyclic causal models to probability.

It is here that I am departing slightly from the approach usually taken. Usually (e.g., Pearl 2000/2009: ch. 3), one starts with a regular probability measure $\Pr_{\mathcal{U}}$ over the power-set of $R(\mathcal{U})$ and then extends $\Pr_{\mathcal{U}}$ to a unique regular probability measure $\Pr_{\mathcal{M}}$ over the power-set of $\mathcal{W}_0$. While

$$\Pr_{\mathcal{U}}\left([\![\vec{U} = \vec{u}]\!]_{R(\mathcal{U})}\right) = \Pr_{\mathcal{M}}\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}_0}\right) = \Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}} \mid \mathcal{W}_0\right) = \Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}}\right)$$

for every context $\vec{u}$, as well as, for every possible world $w_{\vec{u}}$ that is legal in $\mathcal{M}$, $\Pr_{\mathcal{M}}(w_{\vec{u}}) = \Pr(w_{\vec{u}} \mid \mathcal{W}_0)$, the sentence $\vec{U} = \vec{u}$ picks out different propositions over $R(\mathcal{U})$, $\mathcal{W}_0$, and $\mathcal{W}$. In addition, the probability measures $\Pr_{\mathcal{U}}$ and $\Pr_{\mathcal{M}}$ do not assign any probability to propositions comprised by possible worlds that are illegal in $\mathcal{M}$, while these propositions receive probability zero from $\Pr(\cdot \mid \mathcal{W}_0)$ and positive probability from Pr. It is this slight departure that enables me to prove my claims.

The post-intervention probability $\Pr(\cdot \mid \mathcal{W}_0)_{do(\vec{X}=\vec{x})}$ after intervening on the endogenous variables $\vec{X}$ and setting their values to $\vec{x}$ is calculated from the pre-intervention probability $\Pr(\cdot \mid \mathcal{W}_0)$ as follows: $\Pr(w \mid \mathcal{W}_0)_{do(\vec{X}=\vec{x})}$ equals

$$\Pr{}^*\left([\![\vec{X} = \vec{X}(w)]\!]_{\mathcal{W}}\right) \times \prod_{Y \in \mathcal{U} \cup \mathcal{V} \setminus \{X_1, \ldots, X_k\}} \Pr\left([\![Y = Y(w)]\!]_{\mathcal{W}} \mid ([\![Pa(Y) = Pa(Y)(w))]\!]_{\mathcal{W}} \cap \mathcal{W}_0\right),$$

where $\vec{X}(w)$ are the values of $\vec{X}$ in possible world w and the intervention-function $\Pr^*$ takes on value 1 for $\vec{X}(w) = \vec{x}$ and 0 otherwise. Of particular significance is the fact that the post-intervention probability $\Pr(\cdot \mid \mathcal{W}_0)_{do(\vec{X}=\vec{x})}$ satisfies the causal Markov condition for the directed acyclic graph that is determined by the acyclic causal model $\mathcal{M}_{\vec{X}=\vec{x}}$ if the pre-intervention probability $\Pr(\cdot \mid \mathcal{W}_0)$ satisfies the causal Markov condition for the directed acyclic graph that is determined by $\mathcal{M}$.

To establish my claims, let me amend a concept of Lewis (1973: 133)': *the interventionist theory of $\vec{X} = \vec{x}$ in context $\vec{u}$*, $IT\left(\vec{X} = \vec{x}, \vec{u}\right)$, is the set of sentences that would be true in $\mathcal{M}$ in $\vec{u}$ if $\vec{X} = \vec{x}$ were made true in $\mathcal{M}$ in $\vec{u}$ by an intervention that sets the value of $\vec{X}$ to $\vec{x}$:

$$\left\{\phi : \mathcal{M}, \vec{u} \models \vec{X} = \vec{x} \;\Box\!\!\rightarrow \phi\right\}.$$

$IT\left(\vec{X} = \vec{x}, \vec{u}\right)$ is true in precisely one possible world, viz. the unique solution $w_{\vec{u}}^{\vec{X}=\vec{x}}$ to all equations in $\mathcal{M}_{\vec{X}=\vec{x}}$ in $\vec{u}$. In the framework of Stalnaker (1968) whose central ingredient is a selection function f, the corresponding set of sentences picks out the unique possible world $f\left(\vec{X} = \vec{x}, w_{\vec{u}}\right)$ that is selected by f in the possible world $w_{\vec{u}}$ for the antecedent $\vec{X} = \vec{x}$ as the unique possible world that is closest or most similar to $w_{\vec{u}}$ and in which $\vec{X} = \vec{x}$ is true. In the slightly less demanding framework of Lewis (1973) the corresponding set of sentences may be empty if one does not make Lewis (1973: 19)'s "limit assumption" (Herzberger 1979). I can only speculate, but perhaps this is why, as far as I am informed, Lewis never considered the conditional probabilities introduced momentarily.

When one brings about $\vec{X} = \vec{x}$ by an intervention that sets the value of $\vec{X}$ to $\vec{x}$ and one assumes that $\mathcal{M}$ is true, the information one receives is the proposition expressed by the disjunction or intersection of all sets $IT\left(\vec{X} = \vec{x}, \vec{u}\right)$, for every context $\vec{u}$, i.e.,

$$IT\left(\vec{X} = \vec{x}\right) = \bigcap_{\vec{u}} \left\{\phi : \mathcal{M}, \vec{u} \models \vec{X} = \vec{x} \mathrel{\Box\!\!\rightarrow} \phi\right\}.$$

$IT\left(\vec{X} = \vec{x}\right)$ is true in all and only the possible worlds in $\mathcal{W}_0^{\vec{X}=\vec{x}}$, which is the set of legal possible worlds of the acyclic causal model $\mathcal{M}_{\vec{X}=\vec{x}}$. $IT\left(\vec{X} = \vec{x}\right)$ says that, assuming that $\mathcal{M}$ is true, what would be the case if $\vec{X} = \vec{x}$ were made true by an intervention that sets the values of $\vec{X}$ to $\vec{x}$ *is* the case. $IT\left(\vec{X} = \vec{x}\right)$ implies $\vec{X} = \vec{x}$ (but, in general, is not implied by it). This is so also in the frameworks of Stalnaker (1968) and Lewis (1973), as well as any other framework that validates $\vec{X} = \vec{x} \mathrel{\Box\!\!\rightarrow} \vec{X} = \vec{x}$ (such as Huber 2021's).

Note that, for every context $\vec{u}$,

$$\Pr\left(w_{\vec{u}}^{\vec{X}=\vec{x}} \mid \mathcal{W}_0\right)_{do\left(\vec{X}=\vec{x}\right)} = \Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}} \mid \mathcal{W}_0\right)_{do\left(\vec{X}=\vec{x}\right)}$$

and

$$\Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}} \mid \mathcal{W}_0\right)_{do\left(\vec{X}=\vec{x}\right)} = \Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}} \mid \mathcal{W}_0\right)_{do\left(\vec{Y}=\vec{y}\right)} = \Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}}\right)$$

for any two interventions on endogenous variables $\vec{X}$ and $\vec{Y}$. This means that the post-intervention probability $\Pr\left(\cdot \mid \mathcal{W}_0\right)_{do\left(\vec{X}=\vec{x}\right)}$ re-allocates the probability mass $\Pr\left([\![\vec{U} = \vec{u}]\!]_{\mathcal{W}}\right)$ away from the possible world $w_{\vec{u}}$ that is legal in $\mathcal{M}$ to the possible world $w_{\vec{u}}^{\vec{X}=\vec{x}}$ that is legal in $\mathcal{M}_{\vec{X}=\vec{x}}$. This in turn means that the post-intervention probability $\Pr\left(\cdot \mid \mathcal{W}_0\right)_{do\left(\vec{X}=\vec{x}\right)}$ is what Lewis (1976: 310) calls *the image of* the pre-intervention probability $\Pr\left(\cdot \mid \mathcal{W}_0\right)$ (not Pr) *on* $[\![\vec{X} = \vec{x}]\!]_{\mathcal{W}}$, which is the pre-intervention probability of counterfactuals with antecedent $\vec{X} = \vec{x}$,

$$\Pr\left([\![\vec{X} = \vec{x} \mathrel{\Box\!\!\rightarrow} \cdot]\!]_{\mathcal{W}} \mid \mathcal{W}_0\right).$$

My claims follow by noting that both of them are identical to the conditional probability $\Pr\left(\cdot \mid [\![IT\left(\vec{X} = \vec{x}\right)]\!]_{\mathcal{W}}\right) = \Pr\left(\cdot \mid \mathcal{W}_0^{\vec{X}=\vec{x}}\right)$.

These results remain true if the intervention on the endogenous variables $\vec{X}$ does not set their values to $\vec{x}$ but imposes a probability distribution on them so that the intervention-function $\Pr^*\left([\![\vec{X} = \vec{X}(w)]\!]_{\mathcal{W}}\right)$ takes on not just the values 1 and 0, but values between 1 and 0 that sum to 1. In this case we are conditioning in a more generalized way (Jeffrey 1965/1983):

$$\Pr(\cdot \mid \mathcal{W}_0)_{do(\vec{X}=\vec{x})} = \sum_{\vec{x}} \Pr\left(\cdot \mid [\![IT\left(\vec{X} = \vec{x}\right)]\!]_{\mathcal{W}}\right) \times \Pr{}^*\left([\![\vec{X} = \vec{x}]\!]_{\mathcal{W}}\right).$$

Intervening is imaging is conditioning.

3 Conclusion

The mathematics establishing them is entirely trivial, but that does not mean that my claims are trivial also philosophically. They show that, for an important class of conditionals, probabilities of conditionals are conditional probabilities. They show that, on at least one version of it (Meek & Glymour 1994), causal decision theory is a species of evidential decision theory (Jeffrey 1965/1983) – specifically, one that respects Carnap (1947)'s "principal of total evidence": expected utility is calculated with respect to the probability conditional on not just the evidence that an act is taken, but the decision maker's total evidence. Normally, this will include the information that the decision maker herself brings about this act all by herself. And they reinforce a message that is at least implicit in the interventionist approach to causation (Spirtes et al. 1993/2000, Pearl 2000/2009, Woodward 2003): causation is correlation – correlation not between what is observed and observed, but between what is done and observed.[1]

[1] As a postscriptum, let me briefly address an issue I discuss in detail elsewhere. $IT\left(\vec{X} = \vec{x}\right)$ is defined only relative to an acyclic causal model $\mathcal{M}$. In the context of decision theory (Meek & Glymour 1994, Hitchcock 2016) one may want to allow for uncertainty over which acyclic causal model $\mathcal{M}$ is true. Stern (2017) offers one way of doing so by assigning degrees of certainty to pairs of directed acyclic graphs D (possibly determined by acyclic causal models $\mathcal{M}$) and probabilities Pr such that Pr satisfies the causal Markov condition for D. Like Savage (1954)'s classical, as well as Lewis (1981)'s and Skyrms (1980, 1982)'s causal (Weirich 2020), the resulting interventionist decision theory fails to be partition-invariant: the recommendations of the theory depend on which set of mutually exclusive possible states of the world the decision maker considers.

An alternative is to generalize acyclic causal models to acyclic models of causality (Huber ms). Unlike in causal models, in acyclic models of causality each possible world has its own "causal laws" (possibly, but not necessarily an acyclic causal model) and directed acyclic graph.

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Consequently, $IT\left(\vec{X} = \vec{x}\right)$ now says that what would be the case if $\vec{X} = \vec{x}$ were made true by an intervention that sets the values of $\vec{X}$ to $\vec{x}$ is the case – without assuming any acyclic causal model or possible world to be true. Since $IT\left(\vec{X} = \vec{x}\right)$ still implies $\vec{X} = \vec{x}$,

$$\Pr\left(\cdot \mid [\![IT\left(\vec{X} = \vec{x}\right)]\!]_{\mathcal{W}}\right) = \Pr\left(\cdot \mid [\![IT\left(\vec{X} = \vec{x}\right)]\!]_{\mathcal{W}} \cap [\![\vec{X} = \vec{x}]\!]_{\mathcal{W}}\right).$$

This has the desirable consequence that, as in Jeffrey (1965/1983)'s evidential decision theory, one can arrive at a formula for calculating expected utility that is partition-invariant (Joyce 1999, 2000).

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