Conditionals, Insignificant Relevance, and Inferentialism (PREPRINT)

Rolf Pfister[1]

[1]Munich Center for Mathematical Philosophy (MCMP), Ludwig-Maximilians-Universität München, Geschwister-Scholl-Platz 1, Munich, 80539, Germany.

Contributing authors: rolf.pfister@posteo.de;

Although there are a large number of approaches to conditionals, no consensus has yet been reached on the nature and the evaluation of conditionals. Among the more popular approaches are the suppositional account and a wider variety of relevance approaches. The latter require a relevance connection between the antecedent and the consequent to consider a conditional acceptable. In this article, the suppositional account and different approaches of relevance conditionals are analysed on a specific type of conditional: Conditionals whose antecedent and consequent have a relevance connection, but where the acceptability of the antecedent has no significance on the acceptability of the consequent. Such conditionals occur in cases of multiple implication of a consequent, as in overdetermination. When evaluating such conditionals, the approaches examined lead to different and partly incoherent results. It is argued that conditional approaches should consider such conditionals acceptable, which is a challenge for e.g. approaches based on statistical measures. Furthermore, it is argued that the probability of a conditional should only be evaluated according to the strength of the relevance connection between the antecedent and the consequent, but not according to other relevance connections. It is shown that only two approaches correctly evaluate such conditionals, one of which, inferentialism, may provide a basis for a coherent theory of conditionals.

1 Introduction

Conditionals play an important role in everyday language use as well as in scientific reasoning, e.g., to describe conditions under which a fact is acceptable. There are

many conditionals approaches, but many lead to unsatisfactory results or have theoretical shortcomings. For example, the material implication fits well in first-order logic but does not reflect how conditionals are used in everyday and scientific discussions (Skovgaard-Olsen, Singmann, & Klauer, 2016, p. 27). To solve these issues, a larger number of different conditional approaches have been developed, among which the suppositional account has become popular (cf. Evans & Over, 2004; Kaufmann, Over, & Sharma, 2023). In addition, a larger number of relevance approaches are in development, which have been increasingly discussed lately (cf. Rott, preprint).

One of the most important differences between the suppositional account and relevance approaches concerns the connection between the antecedent and the consequent. As an example, consider the following two conditionals:

(1) If the sun shines, the solar farm produces a lot of electricity.
(2) If food prices are high, the solar farm produces a lot of electricity.

While (1) seems intuitively acceptable, (2) sounds odd to many people. The reason is that there is no known relationship between the antecedent and the consequent of (2); hence, the acceptability of the consequent seems to be independent of the acceptability of the antecedent. However, in case both the antecedent and the consequent itself are acceptable, suppositional approaches consider not only (1), but also (2) to be acceptable. The strangeness of unconnected conditionals like (2) is explained by pragmatic circumstances, e.g., by a violation of conversational implicatures (Over & Cruz, 2023). In contrast, relevance approaches regard unconnectedness in conditionals not only as a pragmatic issue but also as a genuine defect (Skovgaard-Olsen, 2016, pp. 563-570)(Douven, Elqayam, & Krzyżanowska, 2023, ch. 1)(Skovgaard-Olsen, 2020, pp. 201-203). Therefore, they consider a conditional acceptable only in case there is a connection between the antecedent and the consequent.

Apart from this common basis, relevance approaches differ widely in their details. For example, they define the connection between the antecedent and the consequent in different ways, e.g., statistically, inferentially or causally. This can lead to divergent outcomes where a conditional is considered acceptable by one approach but not by another.

This article aims to examine how various relevance approaches as well as the suppositional account evaluate a certain type of conditionals: conditionals whose antecedent and consequent have a relevance connection, but where the acceptability of the antecedent has no significance on the acceptability of the consequent. This happens, for example, in the case of overdetermination, where the consequent is implied not only by the antecedent in question, but also by another antecedent. The article analyses and compares the various conditional approaches and evaluates whether some approaches can cover these cases better than others. It is hoped that this allows one to identify approaches that are more promising than others and whose further development may allow for a comprehensive and generally accepted theory of conditionals.

Unless otherwise stated, the considerations are limited to conditionals that are in the indicative mood and that are simple, i.e., whose antecedent and consequent are not themselves conditionals. The various relevance approaches differ in whether they rely on truth, belief, assertibility or acceptability of conditionals. Insofar as conditionals

are discussed in general in this article or several approaches are dealt with at once, the term 'acceptability' is used to refer to the specific meanings of the different approaches.

The article is structured as follows: Section 2 offers an overview of recently and widely discussed conditional approaches. Section 3 provides an analysis of the various approaches on conditionals whose consequents are implied by several mutually exclusive and exhaustive antecedents. Section 4 presents an analysis of the various approaches on conditionals whose consequents are implied by several non-exclusive antecedents. Section 5 discusses how the conditionals from section 3 and 4 are ideally evaluated and compares this with the actual results. Section 6 examines the most promising conditional approaches in this respect in more detail for their general applicability. Finally, section 7 draws a conclusion.

2 Overview of conditional approaches

This section provides an overview of various conditional approaches, in particular of the suppositional account and of recent and widely discussed relevance approaches. The aim is not to provide a complete description of each approach, but to present their core aspects that are relevant for the evaluation of the conditionals discussed in the following sections. Even though the various relevance approaches have partial overlaps and some follow the same principles, e.g., a statistical or causal considerations, no clear classification is possible; this is especially true as some approaches follow several principles, e.g., by combining statistical and causal considerations.

2.1 Suppositional Account

The suppositional account has many different interpretations, but all are based on the Ramsey test (cf. Over & Cruz, 2017, pp. 438-442). The Ramsey test allows one to determine the acceptability of a conditional by hypothetically assuming the antecedent to be true: The antecedent is added to one's stock of beliefs and when necessary, minimal changes are made to maintain consistency. Based on this, the acceptability of the consequent is evaluated, and in case the consequent is accepted, the conditional is also accepted, otherwise, it is not. Probabilistic interpretations of the suppositional account follow in general the conditional probability hypothesis (cf. Over & Cruz, 2017, p. 439)[1]

$$P(A \rightarrow C) = P(C \mid A) \tag{CPH}$$

As mentioned in section 1, suppositional approaches do not require any relevance connection between the antecedent and the consequent to consider a conditional acceptable, which distinguishes them from relevance approaches, which are presented next.

[1] In this article, the annotations in all formulae and citations are unified, with A for antecedent and C for consequent.

2.2 Douven, Krzyżanowska and Elqayam: Inferentialism

Douven et al. (2023) develop an approach of relevance conditionals called inferentialism. Building on the core idea that unconnected conditionals are genuinely defective, an inferential connection between the antecedent and the consequent is required to consider a conditional true (Douven et al., 2023, pp. 188f). The inferential connection is dependent on the given background knowledge and can be deductive, inductive or abductive, whereby abductive is understood in the sense that the consequent serves as an explanation for the antecedent (Douven et al., 2023, pp. 189f).[2] Alternatively, the inferential connection can be logical, statistical, causal, explanatory, metaphysical, epistemic, analogical, or a second-order functional property (Douven et al., 2023, p. 188f).

In case that for a conditional $A \rightarrow C$ exists a compelling argument from A to C, the conditional is considered true. In case there is a compelling argument from A to the negation of C, the conditional is considered false; and in case there is no compelling argument, the conditional is considered indeterminate. The probability assigned to a conditional indicates the extent to which it is considered true, i.e., that it will be realised.

2.3 Rott: Difference-making Conditionals

Rott (2022a) introduces a non-probabilistic approach of relevance conditionals, called difference-making conditionals, which is based on belief-revision semantics. A conditional is accepted in case two conditions are fulfilled, which are called the Relevant Ramsey Test: First, the consequent is accepted in case the agent's belief state is revised by the antecedent; and second, the consequent fails to be accepted in case the agent's belief state is revised by the antecedent's negation (Rott, 2022a, pp. 133, 139).[3]

Although Rott (2022a, p. 139) conceives the relevance condition not as a conjunction of two object-language sentences such as $(A > C) \wedge \neg(\neg A > C)$[4] but as an intrinsically contrastive connective, it does not have to be realised in terms of belief-revision semantics. Instead, it can also be used in standard conditional logics such as System P (cf. Rott, preprint, p. 4) to determine the trueness, acceptability, or assertability of conditionals (Rott, 2022a, p. 152).

2.4 Crupi and Iacona: Evidential Interpretation

Crupi and Iacona advocate an account called evidential interpretation. It is based on Chrysippus' idea that a conditional holds whenever the denial of its consequent is incompatible with its antecedent: In case the antecedent is true, the consequent cannot easily be false; and in case the consequent is false, the antecedent cannot easily be true (Crupi & Iacona, 2022a, pp. 2900f). This idea can be spelt out in a modal (Crupi &

[2] Abductive conditionals are also often called diagnostic or evidential conditionals. Abductive conditionals must not be confused with conditionals inferred by an abductive inference (cf. Pfister, 2022, p. 206).

[3] Rott (2022a, pp. 133, 149) also proposes a slightly different alternative, called the Dependent Ramsey Test. It differs from the Relevant Ramsey Test by the second condition, which requires that the consequent is rejected (i.e. its negation is accepted) in case the belief state is revised by the negation of the antecedent.

[4] The character '>' has thereby the meaning 'If A then plainly C.' (Rott, 2022a, p. 139)

Iacona, 2022a; Raidl, Iacona, & Crupi, 2022) and in a probabilistic approach (Crupi & Iacona, 2021, 2022b).[5]

In the modal approach, a conditional is considered true in case two requirements are fulfilled: (i) in the closest world in which the antecedent is true, the consequent must not be false, and (ii) in the closest world in which the consequent is false, the antecedent must not be true. While the first requirement expresses the commonly known Ramsey test, the second requirement is intended to capture the idea that the consequent holds in virtue of the antecedent (Crupi & Iacona, 2023, p. 121). In case an antecedent is always false or a consequent is always true, the conditional is considered true (Crupi & Iacona, 2022a, p. 2902).

In the probabilistic approach, the acceptability of a conditional $A \rightarrow C$ is equal to the degree of incompatibility between the antecedent and the negation of the consequent (Crupi & Iacona, 2023, p. 122)

$$A \uparrow C = 1 - \frac{P(A \wedge \neg C)}{P(A) * P(\neg C)} \tag{DI}$$

in case that $P(A \wedge \neg C) \leq P(A) * P(\neg C)$. In the case of $P(A) = 0$ or $P(C) = 1$, the degree of incompatibility is 1, and in all other cases, it is 0.

2.5 Skovgaard-Olsen: Statistical Relevance

Skovgaard-Olsen (2020, p. 206) emphasises the role of conditionals as arguments in reasoning and therefore considers unconnected conditionals as semantically defective (Skovgaard-Olsen, 2020, pp. 201-203). The relevance of conditionals can be measured by the measure of difference

$$\Delta P = P(C \mid A) - P(C \mid \neg A) \tag{MD}$$

whereby $\Delta P > 0$ indicates positive relevance, $\Delta P < 0$ negative relevance, and $\Delta P = 0$ irrelevance (Skovgaard-Olsen et al., 2016, pp. 27f).

Empirically, the evaluation of conditionals can be described by the default and penalty hypothesis: By default, people assume that the antecedent and consequent are positively connected and therefore directly evaluate the acceptability of a conditional by $Acc(A \rightarrow C) = P(C \mid A)$ (Skovgaard-Olsen et al., 2016, p. 28). However, once the assumption of a positive connection is violated, $\mathrm{Acc}(A \rightarrow C)$ is considered to be 0.

Besides theoretical considerations on the question of whether $\mathrm{P}(C \mid A)$ should be a measure of the probability or the acceptability of a conditional (cf. e.g. Skovgaard-Olsen, 2016, p. 558), there are also mixed empirical results. For example, the evaluation of P(if A, then C) and Acc(if A, then C) may differ depending on the type of inferential relation of the conditional, as a comparison with results from Douven and Verbrugge (2010) indicates (Skovgaard-Olsen et al., 2016, p. 34). In addition, experiments show a clear dissociation in the evaluation of truth, probability and acceptability (Skovgaard-Olsen, Kellen, Krahl, & Klauer, 2017, p. 474).[6]

[5] Rott (2022b, pp. 13f) shows that both approaches do not result in the same logic and that only the modal, but not the probabilistic, approach validates disjunctive rationality $((A_1 \vee A_2 \rightarrow C) \wedge (\neg A_1 \rightarrow C) \vdash (\neg A_2 \rightarrow C))$. However, these differences are not important for the analyses in this article.

[6] However, there are also contradictory empirical results, see (Douven et al., 2023, p. 189).

2.6 Van Rooij and Schulz: Causal Relative Difference

van Rooij and Schulz (2019) argue that the assertibility of a conditional can be determined by the measure of relative difference: A conditional is assertible iff

$$\Delta^* P_A^C = \frac{P(C \mid A) - P(C)}{P(\neg A \wedge \neg C)} \tag{MRD}$$

is high (van Rooij & Schulz, 2019, pp. 58f). Alternatively, it is suggested that $\Delta^* P_A^C$ does not need to be high but that $\Delta^* P_A^C >> \Delta^* P_a^C$, whereby a stands for all (or the disjunction of all) relevant alternative antecedents (van Rooij & Schulz, 2019, p. 59). In comparison to Skovgaard Olsen's measure of difference ΔP, the measure of relative difference $\Delta^* P_A^C$ allows for the consideration of two additional intuitions: First, with increasing $P(C \mid \neg A)$ the required difference between $P(C \mid A)$ and $P(C \mid \neg A)$ decreases. Second, the value $P(C \mid A)$ is more important than the value of $P(C \mid \neg A)$. The measure of relative difference represents an asymmetrical correlation that is due to a causal relationship between the antecedent and the consequent. This understanding allows the evaluation of the assertibility of conditionals expressing a causal relationship, such as

(3) If it rains, the street is wet.

It also permits the evaluation of diagnostic conditionals (van Rooij & Schulz, 2019, pp. 65-69). In such, one infers from the assertability of a cause to the assertability of its effect, e.g., as in

(4) If the street is wet, then it rains.

Furthermore, van Rooij and Schulz (2019, p. 69) consider conditionals to be assertible in case both the antecedent and the consequent are caused by a common cause. An example is the conditional

(5) If the barometer falls, there is a storm.

where both propositions are caused by low air pressure. Furthermore, conditionals are considered assertible in case the antecedent and the consequent have a deductive or semantic relationship or can be metaphysically grounded.

2.7 Günther: Causality

Günther (2022) proposes a conditional approach based on causal models, allowing for both causal and evidential conditionals. Conditionals are believed by an agent to be true in case they are true at each most plausible world. A world is the more plausible the more it corresponds to the agent's beliefs about which facts are true and, subordinately, the more the world corresponds to the agent's causal beliefs (Günther, 2022, p. 616).[7]

[7] The account does not require absolute certainty, but only relative certainty, i.e., the agent only has to be "most certain" about the state of a fact. This is the case when she is at least quite certain about the state of the fact and is not more certain about any other state of the fact (Günther, 2022, p. 624).

While causal conditionals represent causal relations in which the antecedent causes the consequent, evidential[8] conditionals represent causal relations in which the antecedent is caused by the consequent, such as e.g. in (4). In addition, the representation allows the evaluation of backtracking conditionals, where the non-occurrence of an effect indicates that some of its causes are not present (Günther, 2022, p. 622). For example, in case Tom is seen leaving an interview dissatisfied, one can conclude

(6) If Tom had left the interview smiling, the interview would have gone well.

In contrast, conditionals whose antecedent and consequent are based on a common cause, such as (5), are not considered true.

2.8 Berto and Özgün: Topicality

Berto and Özgün (2021, p. 3708) present an approach of relevance conditionals in which conditionals are considered acceptable in case the antecedent and the consequent are of the same topic. More precisely, the topic of the consequent has to be fully included in the topic contextually determined by its antecedent. The consequent can be either of the same topic as the antecedent or of a topic of some relevant background assumptions, which are determined by the antecedent and the context. For example,

(7) If we keep burning fossil fuel at this pace, the polar ice will melt.

is considered an acceptable conditional. Even though the antecedent and the consequent do not share the same topic, they are connected by the topics of background assumptions, such as "emission of CO_2" and "raising global temperature". "The criterion of relevance [...] aims at giving a catch-all condition, covering relevance of any kind, whether inferential or not" (Berto & Özgün, 2021, p. 3702). In case the antecedent and the consequent are topically connected, the acceptability of a conditional is equal to the conditional probability $P(C \mid A)$. In case they are not topically connected, the acceptability of the conditional is 0.

3 Evaluation of conditionals with several mutually exclusive antecedents

In this section, conditionals are to be analysed whose consequent is implied not only by one but by several antecedents. Moreover, the antecedents are together exhaustive, i.e., no other antecedent implies the consequent. While the next section examines cases where the antecedents are non-exclusive, this section considers mutually exclusive antecedents. The simplest case of mutually exclusive antecedents is when both a fact A and its negation $\neg A$ imply a consequent C; alternatively, this can occur when several mutually exclusive facts $A_1 ... A_n$ imply a consequent C.[9]

As an example, consider a case in which Alice expresses

(8) If the weather is good on the weekend, I go to the mountains.

[8] Evidential conditionals are often also called diagnostic or abductive conditionals.
[9] Since the antecedents are mutually exclusive, the two descriptions are equivalent in that the occurrence of any antecedent implies the negation of all other antecedents.

Alice states the conditional because she likes hiking and plans to go hiking in the mountains with Bob on the weekend. Since Alice normally does not go to the mountains, the conditional is considered acceptable by all conditional approaches presented in the previous section. This is because the antecedent and the consequent are causally connected, the acceptability of the antecedent has a statistically significant impact on the acceptability of the consequent, and the consequent is only acceptable in case the antecedent is accepted.

Now suppose Alice is also looking for plans in case the weather is bad on the weekend. Carol suggests that they go to a spa in the mountains together, since the spa is unusually empty on bad weather days. Alice agrees and therefore expresses

(9) If the weather is not good on the weekend, I go to the mountains.

In case Alice only states (9) but not (8), (9) is also considered acceptable by all relevance approaches mentioned in the previous section. However, in case both conditionals are stated together, the assessment of the conditionals differs among the various approaches, as shown next. For simplicity, the two conditionals are expressed with conditional variables, whereby A stands for 'the weather is good on the weekend' and C for 'I go to the mountains'.

(8') $A \rightarrow C$
(9') $\neg A \rightarrow C$

In the following, it is examined how the individual approaches evaluate the two conditionals when Alice expresses both, i.e., in case the weather is good, she goes hiking in the mountains, and in case the weather is bad, she goes to the spa in the mountains.

The suppositional account evaluates the probability of a conditional based on the formula $P(A \rightarrow C) = P(C \mid A)$. Since the consequent is certain for the occurrence of each antecedent, both (8) and (9) have a conditional probability of $P = 1$ and are therefore considered acceptable.

Douven, Krzyżanowska and Elqayam's inferentialism requires an inferential connection between the antecedent and the consequent. Such a connection is present in both conditionals, since both are based on strong causal relations. Consequently, both conditionals are evaluated as true.

Rott's approach of difference-making conditionals accepts a conditional in case the following two requirements are met: In case the antecedent is accepted, the consequent is accepted, and in case the negation of the antecedent is accepted, the consequent is not accepted. Thus, to accept (8'), it must be true that $A \rightarrow C$ and that $\neg A \rightarrow \neg C$; while to accept (9'), it must be true that $\neg A \rightarrow C$ and that $A \rightarrow \neg C$. Since these two sets of statements contradict each other, the acceptance of (8') and (9') together has to be negated. Moreover, Rott (2022a, pp. 145-148) considers Aristotle's second thesis (AST) to be valid:

$$\neg((A \rightarrow C) \wedge (\neg A \rightarrow C)) \qquad \text{(AST)}$$

AST allows one to conclude from the truth of (8') that (9') is false, and likewise from the truth of (9') that (8') is false. Consequently, it is not possible for (8) and (9) to

be considered true at the same time, which also speaks for their non-acceptance. In general, AST seems intuitively appealing, as an example from Crupi and Iacona (2023, p. 122) illustrates: "If the presence of white smoke is a reason for believing that a new pope has been elected, it is hard to see how the absence of white smoke can also be a reason for believing that a new pope has been elected." However, although AST is convincing for the given example, the example expresses a simple case with only one relevance connection. But, as shown above, there are also cases where both a fact and its negation are positively relevantly connected to the same outcome. Therefore, it seems that AST cannot be held.

Crupi and Iacona's evidential interpretation requires that the consequent cannot easily be false in case the antecedent is true, and that the antecedent cannot easily be true in case the consequent is false. Both conditions are fulfilled for each of the two conditionals above, and therefore, both conditionals are considered true. This is also underlined by the statement that conditionals are true in case the consequent is necessary, which is here the case (Crupi & Iacona, 2022a, p. 2913). Similar to Rott, Crupi and Iacona (2022a, p. 2913) consider AST appealing, but prefer a restricted version called Restricted Aristotle's Second Thesis (RAST):

$$\Diamond\neg C \models \neg((A \rightarrow C) \wedge (\neg A \rightarrow C)) \qquad \text{(RAST)}$$

RAST differs from AST in that an additional requirement must be fulfilled: Only in case the consequent is not necessarily true, it cannot be true that both an antecedent and the negation of the antecedent imply the same consequent. Since in the case of (8) and (9) the consequent is necessarily true – as the antecedents are exhaustive – RAST, unlike AST, does not apply and thus plays no role in their evaluation.

Skovgaard-Olsen's statistical relevance approach considers conditionals to be acceptable in case $\Delta P = P(C \mid A) - P(C \mid \neg A)$ is positive. This allows for two different cases: In the first case, both conditionals have the same probability[10] of the consequent being acceptable in case the antecedent is accepted. Then, $P(C \mid A)$ and $P(C \mid \neg A)$ have the same value, which leads to both $\Delta P = 0$. Consequently, both conditionals are considered irrelevant and thus unacceptable. In the second case, both antecedents have different probabilities[11] of the consequent being acceptable in case the antecedent is accepted. In that case, ΔP evaluates the more probable conditional as positively relevant and therefore acceptable. The less probable conditional is evaluated by ΔP as negatively relevant and therefore unacceptable.

Van Rooij and Schulz's approach of causal relative difference evaluates a conditional assertible in case it satisfies the measure of relative difference $\Delta^* P_A^C$. Although they define the measure differently from Skovgaard-Olsen's ΔP, the result is the same: In case both (8') and (9') are given and have the same probability, both $\Delta^* P_A^C = 0$ and they are considered not assertible. In case both conditionals have different probabilities, the more probable one has a positive $\Delta^* P_A^C$ value and is considered assertible, whereas the less probable one has a negative $\Delta^* P_A^C$ value and is considered not assertible. van Rooij and Schulz (2019, pp. 60-63) consider $\Delta^* P_A^C$ to be an accurate indicator of a causal relationship between the antecedent and the consequent. In this example

[10] Respectively the difference between the two probabilities is smaller than some significance factor ϵ.
[11] Respectively the difference between the two probabilities is larger than some significance factor ϵ.

case, however, this is not true, neither for both conditionals in case they have the same probability, nor for the less probable conditional in case they have different probabilities: Even though $\Delta^* P_A^C$ being not high indicates that there is no causal relation, there is one between the antecedent and the consequent in both conditionals.

Günther's causality approach considers conditionals to be believed as true in case they correspond most to the facts and causal model believed by an agent. In the case of (8) and (9), both conditionals correspond to the facts, and in both, the antecedent is a causal reason for the consequent. That the antecedents of the two conditionals are contradictory is not a problem with respect to the requirement that the most plausible world needs to correspond with the agent's belief about which facts are true. This, because the agent has no belief about which of the two mutually exclusive antecedents is true, i.e., what the weather will be like on the weekend. Thus, according to Günther's approach, the two conditionals together are believed to be true.

Berto and Özgün's topicality approach requires that the antecedent and the consequent are of the same topic or are topically connected by some background assumptions. Although the requirement is imprecise, it can be assumed that it is fulfilled for both (8) and (9) – in both cases, the antecedent and the consequent are connected by some background knowledge of Alice wanting to enjoy activities with her friends. Consequently, both conditionals are considered acceptable.

Overall, it becomes apparent that the various approaches evaluate conditionals whose consequent is fulfilled by several mutually exclusive and exhaustive antecedents differently. Table 1 summarises the results of the evaluation.

Table 1 Evaluation of conditionals in the case of the consequent being implied by several mutually exclusive and exhaustive antecedents

Approach	$(A \to C) \wedge (\neg A \to C)$
Suppositional account	acceptable
Douven, Krzyżanowska & Elqayam: inferentialism	acceptable
Rott: difference-making conditionals	unacceptable
Crupi & Iacona: evidential interpretation	acceptable
Skovgaard-Olsen: statistical relevance	unacceptable
Van Rooij & Schulz: causal relative difference	unacceptable
Günther: causality	acceptable
Berto & Özgün: topicality	acceptable

4 Evaluation of conditionals with several non-exclusive antecedents

As the section above, this section is about the evaluation of conditionals by the conditional approaches presented in section 2. The consequent of the conditionals is again implied not by only one but by several antecedents. Unlike in the last section, however, the antecedents are not mutually exclusive, but non-exclusive, i.e., several of them can occur simultaneously. Consequently, they need not be exhaustive and there may be other, unknown antecedents to the same consequent.

Consider the following example (cf. Pfister, 2022, p. 206): David has a neighbour who often throws parties that are so loud that David feels disturbed at night. More specifically, David cannot sleep well on four out of five nights in which the neighbour has a party. Therefore, David states

(10) If my neighbour throws a party, I cannot sleep well at night.

As such, the conditional is rated acceptable by all conditional approaches presented in section 2: The antecedent and the consequent are causally related, the acceptance of the antecedent has a statistically significant positive influence on the acceptance of the consequent, and the consequent would not occur without the antecedent.

Suppose now that David learns next that a new bar opens directly under his flat. He also learns that the bar will play very loud music and that the sound insulation of the house is very poor. Therefore, he states

(11) If the bar opens under my place, I cannot sleep well at night.

and he is certain of it. In case (11) is to be evaluated without (10), it is considered acceptable by all conditional approaches, as it fulfils all requirements. For simplicity, the two conditionals are expressed with conditional variables, whereby A_1 stands for 'my neighbour throws a party', A_2 for 'the bar opens under my place', and C for 'I cannot sleep well at night'.

(10') $A_1 \rightarrow C$
(11') $A_2 \rightarrow C$

In the following, it is examined how the two conditionals are evaluated in case both A_1 and A_2 are given as well as their relevance connections to the consequent C.

The suppositional account evaluates (10) and (11) by $P(A \rightarrow C) = P(C \mid A)$. Since the consequent is certainly fulfilled by A_2 (and in four out of five cases additionally by A_1), the consequent is certain. Thus, both (10) and (11) are assigned $P = 1$ and are considered acceptable.

Douven, Krzyżanowska and Elqayam's inferentialism evaluates both conditionals as true, since in both conditionals there exists an inferential connection between the antecedent and the consequent. Inferentialism determines the probability of a conditional by the inference heuristic: the probability that a conditional is true is "the likelihood that we can make a compelling case for the consequent, starting from the antecedent plus background knowledge" (Douven et al., 2023, p. 200). This heuristic is shown to be empirically much more accurate than the thesis of the suppositional account that probability ratings express conditional probability ratings, i.e., that $(A \rightarrow C) = P(C \mid A)$ (Douven, Elqayam, & Mirabile, 2022). Based on the inference heuristic, (10) is assigned a probability of $P = 0.8$, since four times out of five David does not sleep well at night when his neighbour throws a party. (11) is assigned a probability of $P = 1$ because it is certain that David cannot sleep well in case the bar opens under his place.

Rott's approach of difference-making conditionals accepts a conditional in case two conditions are met: First, the consequent is accepted in case the agent's belief state is revised by the antecedent; and second, the consequent fails to be accepted in case the

agent's belief state is revised by the antecedent's negation. For (10), the first but not the second condition is satisfied: The consequent is accepted due to its implication by A_2, regardless of whether the antecedent is believed to be true or false. Consequently, (10) is not considered acceptable. For (11), the first condition is always fulfilled and the second in the case that A_1 does not imply C, which occurs 20 % of the time. Since Rott offers a purely qualitative framework and does not propose any probabilistic version, a probabilistic interpretation can only be based on own assumptions. In case one follows the simplest interpretation – the acceptability of a conditional is equal to the probability that both conditions are fulfilled – then the acceptability of (11) would be 0.2.[12] Rott (2022b, p. 17) explicitly discusses a case where two different antecedents both imply the same consequent. In case only one of the antecedents is fulfilled, the corresponding conditional is considered acceptable, since the antecedent makes a difference to the outcome. In case both antecedents are fulfilled, each alone makes no difference. However, Rott considers the corresponding conditionals to be "rather unassertable than unacceptable". It is not entirely clear how this assessment relates to the evaluation results above, but since unassertability is relatively close to unacceptability, the results seem to be confirmed.

Crupi and Iacona's evidential interpretation offers not only a modal but also a probabilistic version(cf. sect. 2.4). The acceptability of (10) is determined by the degree of incompatibility (DI), since $P(A \wedge \neg C) \leq P(A) * P(\neg C)$, which leads to Acc(10) = 1. For (11), $P(C) = 1$ and therefore $Acc(11) = 1$.

Skovgaard-Olsen's statistical relevance approach evaluates the acceptability of conditionals by default by $Acc(A \rightarrow C) = P(C \mid A)$. Since the consequent is always fulfilled by A_2, both $P(C \mid A_1)$ and $P(C \mid A_2)$ are 1. Therefore, by default, $Acc(10) = 1$ and $Acc(11) = 1$. However, conditionals are only considered acceptable in case they also have a positive ΔP value, which is measured by the measure of difference (MD). Since the consequent is always fulfilled by A_2 but only in four out of five cases by A_1, $\Delta P(10) = 0$ and $\Delta P(11) = 0.2$. Consequently, only (11) but not (10) is considered acceptable since only A_2 but not A_1 increases the probability of the consequent being true.

Van Rooij and Schulz's approach of causal relative difference evaluates the assertibility of a conditional by the measure of relative difference (MRD). Although the approach relies on probabilities, van Rooij and Schulz (2019, pp. 58, 63) state that the assertibility of a conditional itself cannot be indicated by degree: A conditional is either assertible – iff $\Delta^* P_A^C$ is high – or not assertible. Independent of that, in case both A_1 and A_2 are taken to be true, the measure of relative difference leads to an invalid result, since one would need to divide by 0; an alternative method of calculation is not given for such cases. Nevertheless, van Rooij and Schulz discuss the occurrence of alternative causes, concluding for pragmatic reasons that alternative causes are complete causal explanations for the consequent and are therefore considered incompatible with each other. Based on these findings, and considering that the main idea of the approach is that conditionals must be causally relevant, at least (10), and arguably also (11), is considered non-assertible.

[12] Alternatively, for example, one could consider a conditional acceptable to the degree of $P(C \mid A)$ in case both conditions are satisfied, which would lead to an acceptability of 1.

Günther's causality approach does not provide a probabilistic interpretation. Nevertheless, it offers some indications of how an evaluation could be made. In general, a conditional is believed to be true in case it is true at each most plausible world, whereby a world is the more plausible the more it corresponds to the beliefs about which facts are true and, subordinately, the more the world corresponds to the causal beliefs. Taking A_1 and A_2 as given, the most plausible world is the one in which both antecedents and the consequent are true. Whether (10) and (11) are believed thus depends on whether their causal relationships are believed. Their belief can be affirmed not only because the consequent could otherwise not be true, but also because both conditionals are based on a strong causal connection. Due to the certain causal relationship in (11), it appears appropriate to set $Bel(11) = 1$. For (10), where the causal link is less strong and the antecedent implies the consequent only in four out of five cases, it seems appropriate to assign $Bel(10) = 0.8$.

Berto and Özgün's topicality approach considers a conditional acceptable to the degree of the conditional probability $P(C \mid A)$ in case the antecedent and the consequent are topically connected; otherwise, the conditional is unacceptable. The requirement of being topically connected is fulfilled by (10) as well as by (11). Since the consequent is always fulfilled by at least A_2, $Acc(10) = 1$ and $Acc(11) = 1$.

The analysis shows that the conditional approaches evaluate conditionals whose consequent is fulfilled by several non-exclusive antecedents quite diversely. The results are summarised in table 2.

Table 2 Evaluation of conditionals in the case of the consequent being implied by several non-exclusive antecedents

Approach	(10') $A_1 \rightarrow C$	(11') $A_2 \rightarrow C$
Suppositional account	1	1
Douven, Krzyżanowska & Elqayam: inferentialism	0.8	1
Rott: difference-making conditionals	0	* 0.2
Crupi & Iacona: evidential interpretation	1	1
Skovgaard-Olsen: statistical relevance	0	1
Van Rooij & Schulz: causal relative difference	0	0
Günther: causality	* 0.8	1
Berto & Özgün: topicality	1	1

* the value is based on an own interpretation, since the approach itself does not provide a probabilistic interpretation.

5 Interpretation of the evaluation results

In the last two sections, it was shown that the suppositional account and the presented relevance approaches evaluate certain types of conditionals quite differently. Whereby section 3 concerns conditionals whose consequent is implied by several mutually exclusive and exhaustive antecedents, section 4 concerns conditionals whose consequent is implied by several non-exclusive antecedents. Both types of conditionals share one

important aspect: the antecedent and the consequent of the conditionals have a relevance connection, but the acceptability of the antecedent has no significance on the acceptability of the consequent.

The differentiation between the two cases is important in that the statistical insignificance is realised in different ways. In the case of mutually exclusive antecedents, the consequent is implied either way, whereas, in the case of non-exclusive conditionals, the consequent is implied anyway. This differentiation allows one to express more precisely the two cases in which a conditional occurs that has a relevance connection, but where its antecedent is not statistically relevant for its consequent: In the first case, the consequent C is implied not only by the antecedent A_1 of the conditional in question but also by other antecedents $A_2 \ldots A_n$ that are mutually exclusive, exhaustive together with A_1, and have all the same or a higher probability P(x) of implying the consequent C as A_1.[13] In the second case, the consequent C is implied not only by the antecedent A_1 of the conditional in question but also by other antecedents $A_2 \ldots A_n$ that are non-exclusive and whose combined probability of implying the consequent C is 1[14].[15]

It could be argued that both cases are purely theoretical without practical relevance and therefore do not need to be covered by conditional approaches. However, not only are the above cases realistic – both Alice's and David's situations can occur in everyday life – but also the following examples show that such cases are common and therefore conditional approaches must be able to handle them.

In the case of mutually exclusive antecedents, imagine a discussion (in mid-2023) about the war between Russia and Ukraine in which the following two statements are uttered:

(12) If Russia loses the Russia-Ukraine war, there will be a new Cold War.
(13) If Russia wins the Russia-Ukraine war, there will be a new Cold War.

Both conditionals can be well justified: For instance, it can be reasoned that in case Russia loses the war, a new nationalistic Russian government is likely to come to power and increase its hostility towards Western countries; and in case Russia wins the war, Western countries will tighten their sanctions and try to isolate Russia to prevent it from invading another country. Both conditionals can be stated separately, but also together – both scenarios seem possible and plausible and as such acceptable. This applies regardless of how likely one considers each of the two antecedents to occur. Even in case one considers it much more likely that Russia will lose the war than that it will win the war, or conversely, both conditionals themselves remain plausible.

In case another scenario with a different outcome is also conceivable, e.g.,

(14) If Russia and Ukraine sign a peace treaty, there will be no new Cold War.

and it is assigned a probability which is greater than 0, (12) and (13) are considered acceptable by most relevance approaches, and the contradictory evaluation results

[13] In the case of a non-probabilistic interpretation, all conditionals ($A_2 \ldots n \rightarrow C$) are considered acceptable.
[14] Respectively larger than 1 minus some significance factor ϵ.
[15] In the case of a non-probabilistic interpretation, there is at least one conditional ($A_2 \ldots n \rightarrow C$) considered acceptable.

above would not occur.[16] However, at least at the time of writing in mid-2023, a peace treaty seems very unlikely, and the crucial point is not whether there could be other war outcomes in this particular case, but that there are realistic situations in which all potential scenarios are equally likely to lead to the same outcome.

Similarly, there are many situations in which occurs the case of several non-exclusive antecedents that all imply the same consequent. For example, one buys a plant in a nursery, whereupon the gardener, based on his experience that many customers fulfil one or both of the antecedents, says

(15) If the plant is placed in direct sun, it dies.
(16) If the plant is not watered regularly, it dies.

Again, it seems to be an everyday situation, and it seems appropriate to accept each conditional separately as well as both together.

Conditionals, which have a relevance connection, but where the antecedent is not statistically relevant for the consequent, can also not be expressed as concessive conditionals, i.e., as "even if" conditionals. As an example, for Alice's case, consider conditionals (8) and (9) in their concessive form

(17) Even if the weather is good on the weekend, I go to the mountains.
(18) Even if the weather is not good on the weekend, I go to the mountains.

Although both conditionals can be acceptable in certain circumstances, in Alice's situation, they do not express the underlying reasons: Alice is not going to the mountains despite the weather being good (or bad), but because the weather is good (or bad). In both cases, each conditional is based on a positive relevance relation in which the antecedent provides a reason for the consequent. Consequently, expressing such cases through concessive conditionals is not a solution.

The examples in this section already indicate that conditionals whose antecedent and consequent have a relevance connection without statistical significance are not only common, but also seem acceptable. This is because the conditionals fulfil the basic idea of relevance approaches: A conditional is considered acceptable in case there is a supportive relevance connection between the antecedent and the consequent. In the following, additional deliberations are made to determine whether such conditionals should be considered acceptable – as some of the conditional approaches claim – or unacceptable – as some other of the approaches claim.

Among the approaches that consider such conditionals unacceptable are those that use statistical measures such as Skovgaard-Olsen's measure of difference (MD) ΔP and Van Rooij and Schulz's measure of relative difference (MRD) $\Delta^* P_A^C$.[17] Both approaches are based on the idea that a relevance connection implies positive statistical relevance. However, as shown above, this is not true for the types of conditionals discussed in this article, which raises the question of which of the two aspects is more important. Although Skovgaard-Olsen does not explicitly address their relation, statistical relevance seems to be a means to measure the more fundamental relevance

[16] Except that AST remains a problem, since both A and $\neg A$ from (12) and (13) still can lead to the same consequent.
[17] A detailed analysis of different measures of evidential support can be found in Rott (preprint, pp. 23-38).

connection. For example, Skovgaard-Olsen (2020, pp. 201-203) argues that the relevance connection of conditionals plays a central role in argumentation and reasoning and makes it possible, for instance, to express arguments. As such, they play a vital role. Similarly, van Rooij and Schulz (2022, p. 366) argue that the semantic analysis of a conditional suffices and that the relevance measure turns out to be a pragmatic and cancellable implicature.

Consequently, statistical measures can be considered as a helpful but not completely reliable indicator of the existence of a relevance connection: A positive ΔP or $\Delta^* P_A^C$ value can be a sufficient, but not a necessary, indicator of a relevance condition.[18] On this basis, it seems that relevance approaches that rely on statistical measures consider conditionals with statistically insignificant relevance connections to be unacceptable not because the conditionals are genuinely erroneous, but because the statistical measure is incapable of correctly capturing the relevance connection. Consequently, the relevance connection and not a positive statistical measure is the main aspect, and since the former is present in the conditionals in question, it seems more appropriate to consider them acceptable rather than unacceptable.

A further possibility to determine the acceptability of conditionals whose antecedent and consequent have a relevance connection without statistical significance is offered by coherence.

Conditionals with mutually exclusive antecedents, such as (8) and (9), are individually considered acceptable because they obtain a relevance connection. A relevance connection is between the antecedent and the consequent and exists independently of other possible relevance connections. Consequently, in case a relevance connection is accepted when it is the only one present, it should also be accepted when others are present. This is especially true as, since the antecedents are mutually exclusive, and only one of the relevance connections implies the consequent. Not accepting a relevance connection just because the consequent can also be realised in the absence of the antecedent by another antecedent that has the same or a higher probability of implying the consequent seems incoherent. Similarly, incoherence occurs in the following way in case the conditionals in question, such as (12) and (13), are not accepted together: In case an additional conditional not leading to the same consequent is accepted, such as (14), (12) and (13) would be suddenly considered acceptable again by all conditional approaches. Yet, it is not clear, why their acceptability should depend on the acceptability of an additional conditional.

Additionally, incoherence would also occur in another way, in case conditionals such as (8) and (9) are accepted alone, but not both together: (8) would be acceptable for Bob, but not for Alice, and (9) would be acceptable for Carol, but again not for Alice. However, since the same relevance connection applies to Alice and Bob respectively Alice and Carol, it seems incoherent that the conditional is accepted once and once not. This applies equally to non-exclusive conditionals such as (10) and (11): Imagine David lives together with Eve. Unlike David, Eve can sleep well when music is played; hence (11) does not apply to her. However, like David, Eve feels heavily disturbed by voices from the neighbour's party; hence (10) does apply to her. This again would lead

[18] Whether a positive value is always a sufficient indicator or whether there are cases in which a conditional is to be considered as unacceptable despite a positive value has to be investigated separately and depends on additional theoretical assumptions.

to an incoherence in case (10) and (11) are accepted alone but not together: Then, David considers (10) as unacceptable, whereas Eve considers it acceptable – although for both applies the same relevance connection.

Accepting not conditionals, where the antecedent has a relevance connection but is not statistically significant for the consequent, also leads to another kind of incoherence: In case none of the conditionals gets accepted, none of them would consequently imply the consequent and hence the consequent would be considered as unacceptable. However, the consequent becomes a fact and should be as such accepted – for example, Alice does go to the mountains and David cannot sleep well at night.

Overall, all of the considerations above indicate that conditionals whose antecedent and consequent have a relevance connection without statistical significance should be considered acceptable. Consequently, conditionals with mutually exclusive antecedents such as (8) and (9) should be considered acceptable, individually as well as both together. The same is true for such conditionals with non-exclusive antecedents such as (10) and (11). A question that arises here is how probabilistic evaluations should be, for example in the case of (10), where the antecedent leads to the consequent in only four out of five cases. It is recommended to follow the probability of implication and assign the same probability to the conditional. This reflects how often the relevance connection actually leads to the implication of the consequent in case the antecedent is given. Hence, for example, $P(10) = 0.8$ and $P(11) = 1$.

Table 3 compares which of the conditional approaches examined in this article determine the correct evaluation based on these results and which do not.

Table 3 Evaluation of conditionals with statistically insignificant relevance connections

Approach	*with mutually exclusive antecedents*	*with non-exclusive antecedents*
Suppositional account	correct	incorrect
Douven, Krzyżanowska & Elqayam: inferentialism	correct	correct
Rott: difference-making conditionals	incorrect	incorrect
Crupi & Iacona: evidential interpretation	correct	incorrect
Skovgaard-Olsen: statistical relevance	incorrect	incorrect
Van Rooij & Schulz: causal relative difference	incorrect	incorrect
Günther: causality	correct	correct
Berto & Özgün: topicality	correct	incorrect

Table 3 shows that only two conditional approaches, Douven, Krzyżanowska & Elqayam's inferentialism and Günther's causality approach, correctly evaluate conditionals with statistically insignificant relevance connections. All other approaches fail either in the case of mutually exclusive antecedents, in the case of non-exclusive antecedents, or in both cases.

6 Examination of promising conditional approaches

In the last section, it was shown that only Douven, Krzyżanowska & Elqayam's inferentialism and Günther's causality approach correctly evaluate conditionals with

statistically insignificant relevance connections. The two approaches are now examined in more detail to what extent they are generally suitable for capturing the nature and evaluation of conditionals.

Douven, Krzyżanowska & Elqayam's inferentialism's main idea and conceptual outline is described in sect 2.2. There are several aspects that are salient and require closer examination.

First, the inferential connection can be not only deductive, inductive or abductive, but also logical, statistical, causal, explanatory, metaphysical, epistemic, analogical, or a second-order functional property (Douven et al., 2023, p. 191). Not only is this understanding very broad, but some of the concepts, such as abductive and explanatory connections, are not well-defined (cf. Pfister, 2022, ch. 2.4, 7). Consequently, the evaluation of conditionals and especially of the argumentative strength of the connection between antecedent and consequent are difficult to assess.

Second, inferentialism, at least at present, offers no logic that can be used to evaluate conditionals. Douven et al. (2023, p. 19) point out that inferentialism is still under development and that a logic may be developed at a later stage. Moreover, it may be that the principles that people follow in regard to conditionals cannot be expressed through logic – but nevertheless, inferentialism can help to better understand the role of conditionals (Douven et al., 2023, ch. 3.1). While both arguments are convincing, a logic would still be desirable, as it would support the formalisation of conditionals, which would be beneficial for scientific reasoning and especially for artificial intelligence. Douven et al. (2023, p. 204) argue that there are already two other relevance approaches with logics that appear promising, in particular Crupi & Iacona's evidential interpretation and Berto & Özgün's topicality approach. However, as shown above, both approaches incorrectly evaluate conditionals with statistically insignificant relevance connections, which not only shows that they are inappropriate in this respect, but also that they are different from inferentialism.

In addition, both approaches also face other problems. For example, Crupi & Iacona provide a logic for a modal interpretation as well as a logic for a probabilistic interpretation (cf. sect. 2.4). Rott (2022b, p. 13) shows not only that the two logics are not identical, but also that the satisfaction of contraposition, the main idea on which the approach is built, supports the relevance connection only to a limited extent (Rott, 2022b, pp. 6-11)(Rott, 2023).

Berto & Özgün require that the antecedent and the consequent are of the same topic or are connected by the topics of background assumptions (cf. sect. 2.8). Even though (Berto & Özgün, 2021, pp. 3606-3608) elaborate on the notion of topicality, it remains unclear how exactly to evaluate whether the antecedent and the consequent are topically connected or not. Based on the specifications provided, the requirement of topicality as an indicator of a relevance connection may be too permissive. For example, consider:

(20) If Alice likes sweets, Bob likes sweets.

The requirement of topicality seems to be fulfilled in the conditional – Alice and Bob are topically connected through their friendship, and in both cases, it is about liking sweets. However, assuming that their preferences for sweets are independent

of each other and did not play a role in their friendship, there does not seem to be a relevance connection between the antecedent and the consequent, insofar as the antecedent influences the consequent in any way. Therefore, the notion of topicality seems to be either under-defined or too permissive and is not a suitable indicator of relevant connections.

Third, the fact that inferentialism allows for inductive and statistical inference connections can be problematic. Since the concept of induction is not precisely defined, it may be too permissive and allow for assigning a relevance connection to unconnected conditionals.

As an example, consider the conditional

(19) If mankind uses electricity, Antarctica is covered in snow that year.

of which both the antecedent and the consequent have been true for many years. Since there are many positive occurrences and not a single negative one, an inductive or statistical argument is well supported, and consequently, the conditional can be considered acceptable. However, there is no relevance connection between the antecedent and the consequent such that the antecedent influences the consequent in any way.[19] It is therefore questionable whether a purely inductive or statistical connection is sufficient or whether this allows for the same criticism that the suppositional account faces (cf. sect. 1).

Fourth, unlike most other conditional approaches, inferentialism does not consider the closure Modus Ponens

$$A, A \to C \vdash C \tag{MP}$$

to be valid. Douven et al. (2023, ch. 2.2) argue that MP should be invalid because in everyday practice, we tend to rely much more on compelling but inconclusive, i.e., non-truth-preserving, arguments than on deductively valid ones.

As an example, Douven et al. (2023, p. 189) provide the conditional

(21) If John lives in Chelsea, he is rich.

which is compelling – as most people in Chelsea are rich – but not truth-preserving – as not all people in Chelsea are rich. Since it could be that John is one of the few people who live in Chelsea but are not rich, MP must be considered invalid (Douven et al., 2023, p. 190). However, it seems that the inconclusiveness is not due to MP but to the inductive argument on which the conditional is based. Yet, since the inductive inference is only true for most but not all cases, its argumentative strength is less than one.

Thus, in (21), the uncertainty in inferring from the truth of the antecedent to the truth of the consequent does not arise from MP itself, but from its non-maximum argumentative strength. For comparison, the deductive conditional

(22) If 2*x = 10, then x = 5.

[19] In fact, there may be a weak relevance connection due to climate change, but this would be a negative one.

has an argumentative strength of 1 and is truth-preserving. Consequently, it seems advisable to accept MP as a valid conclusion and instead consider the argumentative strength of a conditional for its uncertainty. In case the argumentative strength is less than 1, the inference from the truth of the antecedent and the truth of the conditional to the truth of the consequent may be false – but not because MP is invalid, but because the conditional, i.e., one of the premises, may not be true in this specific instance. This also fits well with Douven et al. (2023, p. 200)'s inference heuristic, which states that the probability that a conditional is true is "the likelihood that we can make a compelling case for the consequent, starting from the antecedent plus background knowledge".

Moreover, this understanding also fits well with the previously discussed aspect of inferentialism, the problem that inductive or statistical connections can be too permissive. Understanding it in this way not only allows MP to be considered valid, but also strengthens the inductive relationship to the point where unrelated correlations are no longer sufficient for a condition to be considered true. Specifically, inductive arguments could be understood as those that have the same form as deductive arguments but are inconclusive for some reason, e.g., because there are exceptions or possible preventions. For example, (21) can be supported by an argument whose premises state that owning a home in Chelsea is expensive and that only rich people can afford expensive housing. Nevertheless, exceptions are possible, e.g., one can live with a friend or has only recently become poor. Consequently, a conditional has a deductive relevance connection if there is a compelling and conclusive argument from the antecedent to the consequent. In case the argument is compelling but inconclusive, the conditional has an inductive relevance connection. In case there is no compelling argument, but only an unrelated correlation, as in (19), a conditional has no relevance context and is not considered acceptable despite its inductive or statistical generalisability.

Overall, none of the four aspects examined opposes inferentialism in its entirety, and it seems that they can be at least partially resolved. Nonetheless, they pose a challenge to inferentialism and must be addressed in case inferentialism is to be used to evaluate the truth of various types of conditionals. This is especially true for the exact specification of the different types of relevance connections – what types there are, how they are exactly defined, and how they can be formalised. This being the case, it has to be agreed with Douven et al. (2023, ch. 3.1) that inferentialism is still under development, and it is to be hoped that the open questions can be solved soon.

Günther's causality approach's main idea is described in sect 2.7. One aspect that requires a more thorough consideration is the acceptance of indicative conditionals compared to subjunctive conditionals. For this, Günther (2022, p. 620) provides an example in which one supposes that on a Sunday night, one approaches a small town that has exactly two snackbars. Seeing a person eating a hamburger shortly before entering the town, Günther argues that one has good reason to accept

(23) If snackbar A is closed, then snackbar C is open.

After entering town, one sees that snackbar A is in fact open. Günther (2022, pp. 620-622) shows that under these circumstances

(24) If snackbar A were closed, then snackbar C would be open.

is not accepted by the approach and argues that this is desired for the following reason: Indicative conditionals such as (23) are understood epistemically and show how one revises one's belief on learning the antecedent. In contrast, subjunctive conditionals scuh as (24) tell how the world would be in case the antecedent were true. From this, Günther (2022, p. 620) concludes that (24) must be rejected because there is no causal connection between the antecedent and the consequent.

While Günther's reason is correct in itself – the antecedent and the consequent are not causally connected – his conclusion not to accept (24) seems problematic for the following reason: By seeing a person eating a hamburger when entering the city lets one conclude

(25) Snackbar A is open or snackbar C is open (or both).

Learning at a later time that snackbar A is open does not object to accepting (25) from now on; in fact, it supports it further. However, in case one accepts (25), one also has to accept (24), since (25) provides a relevance connection for (24). More precisely, (25) provides a deductive connection for (24): from (25) $A \vee C$ and (24)'s antecedent $\neg A$ necessarily follows (24)'s consequent C. As a result, (24) should be considered acceptable in the example.

The fact that the conditional is considered unacceptable by Günther's causality approach shows that the approach is too limited in that it can only analyse causal and evidential conditionals, but not non-causal conditionals such as deductive ones. Equally, it does not allow the evaluation of other types of relevance connections, such as inductive ones such as (21), mathematical ones such as (22) or analogical ones such as

(26) If Jim's son likes ice skating, he will like ice hockey.

Apart from the fact that the approach can only evaluate causal but not all types of conditionals, the other types are not simply classified as unevaluable but as false; hence, it is not clear when the limits of the approach are exceeded.[20]

Both aspects – the limitation to causally connected conditionals and the impossibility of distinguishing between evaluable and unevaluable conditionals – pose serious challenges to Günther's causality approach. While other challenges appear to be solvable, such as considering uncertainty, at least for the moment, it is not foreseeable how these two main challenges can be solved.

7 Conclusion

The article shows that most relevance approaches as well as the suppositional account fail to correctly evaluate conditionals with statistically insignificant relevance connections. This applies to cases of mutually exclusive, exhaustive antecedents or cases of non-exclusive antecedents, or both. Among others, the evaluation of conditional approaches on these cases shows that approaches relying on statistical measures such

[20] This is because in the absence of a causal connection, the approach cannot distinguish whether there is no relevance connection at all or a non-causal, e.g., deductive, one.

as ΔP to determine whether a relevance connection exists fail. This is because statistical measures do not measure the strength of the relevance connection ($P(A \vDash C)$), but only the influence the acceptance of the antecedent has on the acceptance of the consequent ($P(C \mid A)$). Furthermore, it is shown that the relevance connection should be evaluated independently of the presence or absence of other relevance connections. This is because a relevance connection exists independently of others and, in contrast to the acceptance of the consequence, is not influenced by other relevance connections. Besides that, incoherences would arise in case relevance connections are not evaluated independently of others.

Only two approaches, Douven, Krzyżanowska & Elqayam's inferentialism and Günther's causality approach, can correctly capture conditionals with statistically insignificant relevance connections. An examination of both approaches in detail shows that the causality approach is too restrictive due to its exclusive focus on causal relationships and cannot successfully evaluate all types of conditional relevance connections, at least at present. Inferentialism, in contrast, is very permissive and requires further specification, especially regarding how the different types of relevance connections can be defined and evaluated or even formalised. Nevertheless, inferentialism constitutes a promising approach, and its further development could form the basis for a coherent theory of conditionals that meets our expectations for more complex example cases. It is hoped that this article contributes to this development and points out directions that may be more promising than others.

References

Berto, F., & Özgün, A. (2021). Indicative conditionals: Probabilities and relevance. *Philosophical Studies*, *178*(11), 3697–3730,

Crupi, V., & Iacona, A. (2021). Probability, evidential support, and the logic of conditionals. *Argumenta*, *6*, 211–222,

Crupi, V., & Iacona, A. (2022a). The evidential conditional. *Erkenntnis*, *87*(6), 2897–2921,

Crupi, V., & Iacona, A. (2022b). Three ways of being non-material. *Studia Logica*, 1–47,

Crupi, V., & Iacona, A. (2023). Outline of a theory of reasons. *The Philosophical Quarterly*, *73*(1), 117–142,

Douven, I., Elqayam, S., Krzyżanowska, K. (2023). Inferentialism: A manifesto. *Conditionals: Logic, linguistics and psychology* (pp. 175–221). Springer.

Douven, I., Elqayam, S., Mirabile, P. (2022). Inference strength predicts the probability of conditionals better than conditional probability does. *Journal of Memory and Language*, *123*, 104302,

Douven, I., & Verbrugge, S. (2010). The adams family. *Cognition*, *117*(3), 302–318,

Evans, J.S.B., & Over, D.E. (2004). *If: Supposition, pragmatics, and dual processes.* Oxford University Press, USA.

Günther, M. (2022). Causal and evidential conditionals. *Minds and Machines*, *32*(4), 613–626,

Kaufmann, S., Over, D.E., Sharma, G. (2023). *Conditionals: Logic, linguistics and psychology.* Springer Nature.

Over, D.E., & Cruz, N. (2017). Probabilistic accounts of conditional reasoning. *The routledge international handbook of thinking and reasoning* (pp. 434–450). Routledge.

Over, D.E., & Cruz, N. (2023). Indicative and counterfactual conditionals in the psychology of reasoning. *Conditionals: Logic, linguistics and psychology* (pp. 139–173). Springer.

Pfister, R. (2022). Towards a theory of abduction based on conditionals. *Synthese*, *200*(3), 206,

Raidl, E., Iacona, A., Crupi, V. (2022). The logic of the evidential conditional. *The Review of Symbolic Logic*, *15*(3), 758–770,

Rott, H. (2022a). Difference-making conditionals and the relevant ramsey test. *The Review of Symbolic Logic*, *15*(1), 133–164,

Rott, H. (2022b). Evidential support and contraposition. *Erkenntnis*, 1–19,

Rott, H. (2023). On the logical form of evidential conditionals. *Logic and Logical Philosophy*, 1–18,

Rott, H. (preprint). Conditionals, support and connexivity. *preprint*, ,

Skovgaard-Olsen, N. (2016). Motivating the relevance approach to conditionals. *Mind & Language*, *31*(5), 555–579,

Skovgaard-Olsen, N. (2020). Relevance and conditionals: A synopsis of open pragmatic and semantic issues. *Logic and uncertainty in the human mind* (pp. 192–206). Routledge.

Skovgaard-Olsen, N., Kellen, D., Krahl, H., Klauer, K.C. (2017). Relevance differently affects the truth, acceptability, and probability evaluations of "and","but","therefore", and "if–then". *Thinking & Reasoning*, *23*(4), 449–482,

Skovgaard-Olsen, N., Singmann, H., Klauer, K.C. (2016). The relevance effect and conditionals. *Cognition*, *150*, 26–36,

van Rooij, R., & Schulz, K. (2019). Conditionals, causality and conditional probability. *Journal of Logic, Language and Information*, *28*, 55–71,

van Rooij, R., & Schulz, K. (2022). Causal relevance of conditionals: semantics or pragmatics? *Linguistics Vanguard*, *8*(s4), 363–370,