Relativity Theory as a Theory of Principles. A Reading of Cassirer's Zur Einstein'schen Relativitätstheorie

In his Zur Einstein'schen Relativitätstheorie Cassirer presents relativity theory as the last manifestation of the tradition of the 'physics of principles' that, starting from the nineteenth century, has progressively prevailed over that of the 'physics of models.' In particular, according to Cassirer, the relativity principle plays a similar role as the energy principle in previous physics. The paper argues that this comparison represents the core of Cassirer's neo-Kantian interpretation of relativity. Unlike the individual physical laws, these principles do not pretend to provide models of any specific physical system, but they do impose constraints on the law-like statements that describe them. The latter do not qualify as proper laws unless they satisfy such constraints. Cassirer pointed out that before and after Kant, the history of physics presents significant instances in which the search for formal conditions that the laws of nature must satisfy preceded and made possible the direct search for such laws. In his earlier years, Cassirer seems to have regarded principles like the energy principle, the relativity principle, the principle of least action, etc., as a constitutive but provisional form of a priori, imposing specific limitations on the form of the allowable laws of nature. Only in his later years, by attributing an autonomous status to these statements of principle, did Cassirer attribute a definitive but merely regulative meaning to the *a priori*. This does not impose specific requirements on natural laws but only a motivation to search for them.

Keywords: Ernst Cassirer • Marburg neo-Kantianism • relativized *a priori* • principles in physics • energy principle • relativity principle

Introduction

Shortly after the Eddington-Dyson eclipse expeditions 'confirmed' general relativity, Einstein became an international celebrity (Kennefick, 2009), hailed, especially by the British and American press, as the physicist who had 'dethroned Newton.' In the context of continued national tensions after the First World War, in November of 1919 Einstein attempted to present relativity theory in plain terms to the English-speaking public with a brief article for the *Times*, the prestigious London daily newspaper. He famously characterized relativity theory as a 'principle theory' rather than a 'constructive theory' [citation removed]. Constructive theories, like the kinetic theory of gases, try to "construct a model [*ein Bild*]" of the phenomena (Einstein, 1919a) that behaves according to given laws of nature (a mechanical model of a gas). Principle theories, like thermodynamics, starting from universally recognized empirical facts (there is *perpetuum mobile* of the second kind), search for "mathematically formulated criteria" that any possible laws of nature must satisfy if those facts are to hold (the second principle of thermodynamics) (Einstein, 1919a).

In the last decade, this distinction has acquired a life of its own and attracted considerable attention among philosophers of physics (Brown, 2005, Janssen, 2009). However, Einstein's principle/constructive theory opposition surprisingly played no role in the debate about relativity that engaged the philosophical community in the 1920s, especially in Germany (Hentschel, 1990a, 1990b). Neither Moritz Schlick nor Hans Reichenbach—Einstein's two primary philosophical interlocutors—mentioned the article in the *Times* or referred to relativity theory as a 'theory of principles.' As this paper will show, it has seldom been noticed that Ernst Cassirer was, to a certain extent, an exception to the rule. There is no evidence that Cassirer ever read Einstein's short article for the *Times* either. However, in his 1921 book on relativity, *Zur Einstein'schen Relativitätstheorie* (Cassirer, 1921b), he arrived at a strikingly similar conclusion. In Cassirer's view, relativity theory was nothing but an example of the prevalence of the 'physics of principles' over the 'physics of models' (*Bilder* or *Modelle*)—a trend that had started to emerge at least from the second half of the nineteenth century. The convergence between Einstein and Cassirer on this issue is less surprising than one might think. At that time, the opposition between two styles of doing physics, one based on general principles and the other on detailed models, was a common trope (see e.g. Sommerfeld, 1915). Most of all, most early relativists agreed that special relativity did not introduce a new theory in the usual sense but a new "principle" constraining first-level theories (M. Laue, 1911c, 185f. Minkowski, 1908, 55).

However, Cassirer was possibly the only philosopher who not only appreciated and emphasized this point but made it the core of his neo-Kantian interpretation of the theory of relativity. In Cassirer's eyes, the "physics of principles" had unmistakable Kantian overtones, at least in light of the 'Marburg' interpretation of Kant, which was part of his philosophical training.¹ Indeed, throughout his career, Cassirer repeatedly insisted that certain universal principles can be singled out in the structure of all physical theories (the virtual work principle, the energy principle, the least action principle, etc.) and remain unaltered despite the changes undergone by physical laws. These principles, unlike the usual physical laws, do not directly say anything about the properties of any specific physical system; rather, they impose general constraints on law-like statements concerning them. The latter do not qualify as proper physical laws unless they satisfy these conditions. In insisting on the importance of 'principles' in physics, Cassirer did not make any claim to originality. He simply meant to report on a 'practice of principles' (Seth, 2010) that was widespread in nineteenth- and early twentieth-century physics.² The compatibility of candidate physical laws³ with abstract principles had progressively become more important than constructing detailed intuitive pictures (*Bilder*) according to such laws.⁴ This strategy allowed for empirical predictions but avoided speculations about the nitty-gritty of the physical systems at stake (Wilson, 2017).

In Cassirer's view, despite its revolutionary nature, relativity theory was ultimately nothing but the last manifestation of the well-established tradition of the 'physics of principles' that has progressively prevailed over that of the 'physics of models.' Although Cassirer's interpretation of relativity has attracted considerable scholarly attention in the last decades (Ferrari, 1994, 1996, ch. 4, Ryckman, 2003, 2005, ch. 2), Cassirer's characterization of relativity theory as a 'theory of principles' has not been fully appreciated. Indeed, it is undeniable that this motif is not always easy to disentangle from other lines of argument that run through Cassirer's book. As Hermann Weyl perceptively pointed out, although Cassirer was a "mind of rare universality," his writings resemble "more a suite of bourrées, sarabands, minuets, and gigues than variations on a single theme" (Weyl, 1954, 624). If one pushes Weyl's comparison further, one might claim that Cassirer's prose resembles more the work of a post-Romantic composer rather than works of the Classical Age. Cassirer seemed to focus, so to speak, on texture over structure, often proceeding by local associative transitions rather than global motivic development. For this reason, in reading Cassirer's book on relativity, one might be persuaded by this or that line of argument; however, one is often left struggling to identify the overall message that he is attempting to convey.

By situating Cassirer's work on relativity in the context of his early philosophy of physics, this paper hopes to show that the characterization of relativity theory as a 'theory of principles' can be considered the book's central message. In particular, Cassirer attributed to the relativity principle the same role he had attributed to the energy principle in previous works, most prominently in his first monograph on Leibniz (Cassirer, 1902): The relativity principle, like the energy principle, is not one law of nature among others; it is a second-order constraint that we impose on the laws of nature. To allow the reader

¹Cassirer often emphasized that it was one of Cohen's (1885) fundamental merits to have shown that the 'barycenter' of Kant's first *Critique* was not the intuitions of space and time or the categories but the 'synthetic principles' (*Grundsätze*; Cassirer, 1907, 624; fn. 2, 1912, 1918a, 187). From this perspective, according to Cassirer (1920a, 11ff.), the dualisms that plague Kant's philosophy—between subjective and objective, form and content, intellect and sensibility, and so on—are ultimately resolved in the correlation between the universal and the particular, which is dealt with in the *Kritik der Urtheilskraft*. Special empirical laws are all special cases of the synthetic principles because the latter are the conditions that *any* possible laws of nature must satisfy to be recognized as such. We know *a priori* that *all* empirical laws satisfy such conditions because otherwise they would have been rejected from the outset. The synthetic principles (*Grundsätze*) are said to be *constitutive* in that they impose specific formal constraints on the *possible* candidate laws of nature. The search for the actual empirical laws of nature is guided by the expectation that the latter are organized in a progressively more coherent system—that is, the principle (*Prinzip*) of the 'formal finality of nature.' The latter is merely a *regulative* principle because it only provides a guide for the search of the empirical laws without imposing any specific constraints on them (Stadler, 1874). Thus, I disagree with the widespread interpretation that the Marburg School defended a regulative meaning of the *a priori*.

²The importance of the role of principles in physics goes back at least to the work of Hermann von Helmholtz (Bevilacqua, 1993). At the turn of the century, Hendrik H. Lorentz (Lorentz, 1900, 1905, see Frisch, 2005) and Henry Poincaré (Poincaré, 1904, 1905, see Darrigol, 1995, Giedymin, 1982) presented the opposition between the 'physics of principles' and the 'physics of models' as commonplace (Bordoni, 2017) for other examples.

³Laws of mechanics, electrodynamics, etc.

⁴For instance, macroscopic thermodynamic properties of a system might be reduced to the motion of its microscopic constituents according to the laws of mechanics (mechanical models of gases). Mechanical properties, such as inertial mass, might be reducible to electromagnetic effects implied by Maxwell's equations (electromagnetic models of the electron).

to appreciate this point, this paper divides the development of Cassirer's philosophy of physics up to the 1920s into two phases, in each of which Cassirer focuses mainly, although non-exclusively, on a single 'principle':

- energy principle (section 1): in his oft-neglected first monograph on Leibniz, Leibniz' System in seinen wissenschaftlichen Grundlagen (Cassirer, 1902), the young Cassirer contends that Leibniz treated the principle of conservation of mechanical work not as a consequence of the laws of mechanics but as a general principle, a requirement that any law of nature has to satisfy. By taking the form of a 'condition' on first-order laws (e.g., the collision rules), the principle does not postulate the existence of mechanical energy as a separate 'thing.' In the immediately following years, Cassirer generalized this conclusion in two directions: in his historically orientated work, he presented conservation principles as an instantiation of those 'invariants of experience' that serve as possible candidates for a priori conditions that any good physical theory in general must meet (Cassirer, 1906a), while in his theoretical work, the energy principle was presented as a significant example of the transition from 'substance-concepts' to 'function-concepts' (Cassirer, 1910).
- relativity principle (section 2): in his monograph on relativity, Zur Einstein'schen Relativitätstheorie (Cassirer, 1921b), Cassirer returned to a certain extent more explicitly to the approach of his first book. The special and general relativity principles are compared explicitly to the energy principle. Just as the energy principle does not imply the reduction of physics to mechanics, the relativity principle does not imply its reduction to electrodynamics. Both principles impose constraints on laws of nature rather than being mere accidental byproducts of those laws. The relativity theory, like the energy principle, can be considered an instance of the general tendency of modern physics to move from the 'physics of models' to the 'physics of principles.' Cassirer soon integrated this line of argument into his growing body of work on the role of 'symbols' in physics, showing how, in the history of physics, the 'symbolism' of principles has prevailed over the 'schematism' of pictures and models (Cassirer, 1927a, 1929).

The paper concludes that Cassirer's book on relativity has the merit of grasping quite clearly what Eugen Paul Wigner, two decades later, on the occasion of Einstein's 70th birthday (a few years after Cassirer's death), called the 'reversal of the trend' (Wigner, 1949, 5). Instead of deriving the principles from what are believed to be the laws of nature, physicists test the acceptability of the laws of nature through certain general principles. Historically, Wigner pointed out, the power of these principles was established "so firmly that we have to be reminded that they are based only on experience" (5). Einstein had certainly made extensive use of this heuristic technique, but Cassirer could argue with good reason that this 'principle strategy' has always been practiced in the history of physics from Leibniz to Poincaré.

In Cassirer's assessment, it was Kant's merit to have articulated the philosophical importance of the search for criteria, separating the wheat from the chaff, the law-like statements that can be taken as 'laws of physics'⁵ from those that cannot. However, as Cassirer conceded, Kant mistakenly believed that a set of selection criteria could be fixed once and for all. Cassirer's historical-critical analysis of the role of 'principles' in physics can ultimately be considered his life-long attempt to avoid the shortcomings of Kant's original program by preserving its key insight. In doing so, Cassirer was forced to unravel a more complex dialectic between *a priori* and *a posteriori*, constitutive and regulative. This dialectic marks the main stages of Cassirer's work in the philosophy of physics and in particular of his conception of the *a priori*. It is often claimed that, whereas the young Reichenbach (1920) promoted a constitutive but relativized *a priori* as the limit of a process of progressive discovery (Ryckman, 2005, 245ff. Friedman, 2008, 2005). However, I hope to offer evidence that shows that the shift towards a regulative conception of the *a priori* happened only in Cassirer's (1936b) later work [citation removed].

Up to the 1920s, Cassirer seems to have considered the surprising 'resilience' of some principles, despite the chaotic rise and fall of individual theories, as a sign that they could be considered, at least provisionally, as candidates for *constitutive*, *a priori* conditions of what counts as a law of nature *in general*. Cassirer conceded that the development of science can always force us to search for 'better' *constitutive* principles in an infinite convergent process. After relativity theory, the refutation of principles previously held as *a priori* principles had become more than a theoretical possibility. By the 1920s, Cassirer, nearly imperceptibly, started to suggest that only the overall process of searching for progressively more adequate

 $^{^{5}}$ See fn. 1.

constitutive principles should be considered *a priori*. As has been shown [citation removed] Cassirer did not attempt to resolve this ambiguity until the 1930s: (1) he attributed explicitly to the 'statements of principle' an autonomous role in the structure of physical theories (as opposed to the statements of law and statements of measurement), while denying them, even provisionally, *a priori* status (Cassirer, 1936b, 66; tr. 1956, 52–53); (2) he shifted the position of the *statements a priori*—like the causality principle—to a deeper level, by attributing to them an absolute but *regulative* meaning (Cassirer, 1936b, 75; tr. 1956, 60). The *a priori* motivates and guides the search for the laws of nature, without providing any particular insight into their structure [citation removed].

1 The Principle of Conservation of Energy in Cassirer's Early Marburg Phase

After having studied in Berlin under the guidance of Georg Simmel, Cassirer moved to the small university of Marburg in 1896. At that time, the so-called Marburg School of neo-Kantianism was perhaps at the beginning of its golden age (Ferrari, 1988, Sieg, 1994). Over the years, Hermann Cohen and Paul Natorp had used a series of philosophical prizes (*Preisaufgaben*) to support Marburg's doctoral students and develop some of the core insights of a "little school" that had started to gather in Marburg (Cohen to Natorp, Apr. 19, 1897; Holzhey, 1986, Vol. 2, Doc. 42). For the years 1898–1899, the argument proposed by Natorp required an examination of Leibniz's philosophy of the foundation of mathematics and mechanics (1:382). The winner of the competition was the young Cassirer, who soon emerged as the 'rising star' of the Marburg group (Natorp to Görland, Nov. 21, 1898; Vol. 2, Doc. 45). Cassirer further worked on the manuscript, using part of it as the basis for his dissertation on Descartes (Cassirer, 1899), which became the first chapter of a book on Leibniz by the end of 1901 (Cassirer, 1902). When sending Paul Natorp the first drafts of his *Leibniz' System*, Cassirer (1902) invited him to consider it as part of a larger "study on the prehistory of criticism" (Cassirer to Natorp, Nov. 26, 1901; Cassirer, 1995–, Vol. 18, Doc. 43) that he was working on.⁶

The 'whiggish' project of a 'prehistory of criticism' had been introduced by Natorp (1882a, 1882c, 1882b) himself two decades earlier, and it became one of the tenets of Marburg historiography (Cohen, 1883, 1885). In particular, the second part of Cassirer's book dedicated to the 'fundamental concepts of mechanics' (Cassirer, 1902, part II) can be considered as a prototypical example of this sort of 'typological' interpretation of the history of science and philosophy.⁷ On the one hand, Cassirer emphasized that Leibniz treated principles such as vis viva conservation, the continuity principle, etc., not as individual laws of nature among others—such as specific rules about impact and collisions among bodies—but as second-order conditions that we impose on any dynamical law. On the other hand, Cassirer argued that by acknowledging the role of such principles in physics, Leibniz 'anticipated' the Kantian concept of the a priori, even if he could not free himself from its metaphysical underpinnings. By regarding Leibniz's scientific work as part of the pre-history of Kant's a priori, Cassirer, ultimately following Cohen (1885), could clarify the latter concept. Certain statements are not considered a priori because of their origin (because they are part of the structure of the mind or derived from the table of categories); they are a priori because of their role (Leistung) as a condition of possibility of the mathematical science of nature. In this form, Cassirer argued, more or less explicitly, that Kant's combination of the *a priori* and the transcendental maintains its validity for the philosophical assessment of modern science.

1.1 Leibniz and the Conservation of Mechanical Work: Cassirer's Leibniz' System

Some of the details of Cassirer's reconstruction of Leibniz's role in the history of the discovery of the energy principle can be used to make Cassirer's interpretative strategy clear. As Cassirer pointed out, in the historical literature of his time (Mach, 1872, Dühring, 1873, Helm, 1887, Planck, 1887), Leibniz's contribution to the discovery of energy conservation was usually downplayed and limited to the establishment of the conservation of kinetic energy mv^2 in the collision of elastic bodies (Cassirer, 1902, 308ff). Nevertheless, according to Cassirer, the vis viva controversy was for Leibniz only a polemical opportunity. Leibniz's contribution to the history of energy conservation indeed appears minor if one

 $^{^{6}}$ The study was the first outline of *Das Erkenntnisproblem*, which would be concluded only a few years later (Cassirer, 1906a).

 $^{^{7}}$ Typology is a method of biblical interpretation in which events, persons and so on in the Old Testament are seen as 'types' that prefigure the corresponding antitypes of Christ found in the New Testament.

only considers his role in the *discovery* of the energy principle.⁸ However, in Cassirer's view, Leibniz's contribution is of fundamental importance if one considers Leibniz's *justification* of his conservation principle. According to Cassirer, Leibniz realized that the principle of conservation of *vis viva* was not a single empirical law among others describing a certain class of phenomena but a "general principle", a constraint that *all* particular laws of nature have to satisfy to be recognized as such: "From here Leibniz first acquires the general version of the principle of *conservation of energy*, which he conceives not only as an *individual theorem* of analytical mechanics but as a *fundamental rule* of all physics" (Cassirer, 1904b, 117; last emphasis mine).

According to Leibniz, for qualitatively different phenomena (e.g., elasticity, gravitational free fall, hydrostatic pressure, etc.) to be quantitatively compared, the "general definition of an abstract unity" was needed (Cassirer, 1902, 304). In principle, the choice of measuring standard is arbitrary. However, it must be assumed that the measurement yields identical results in the chosen units (304). In this assumption, Cassirer argues, "the essential content of the principle of conservation is already implicit" (306). For any quantity that arises *ex nihilo* and disappears *ad nihilum* without being compensated for, the invariability of the chosen unit would not be granted. In this sense, "Leibniz's conservation principle is regarded as the *necessary condition for the application of pure mathematics to reality*" (306; my emphasis), that is, for establishing the quantitative equivalence of qualitatively different phenomena.

According to Cassirer, Leibniz's contribution to the history of the energy principle was the choice of 'mechanical work' as a common denominator.⁹ Leibniz's choice of mechanical work as a unit of measure did not imply the reduction of all phenomena to mechanics. It was grounded on the fact that mechanical effects are more familiar and easily measurable than other effects. 'Causes' are to be called equal when they produce equal 'effects,' that is, if they are able to perform an equal amount of mechanical work as measured in work units—if they produce an equal degree of tension in an equal number of elastic springs, raise an equal weight to the same height, communicate to an equal number of bodies the same amount of velocity, etc. (Cassirer, 1902, 305). The postulation of such one-to-one coordination exhausts the essential content of the energy principle without any need to introduce mechanical energy as a separate reality.¹⁰

On the one hand, Leibniz, like many of his contemporaries, took for granted that "all happening can be traced back to mechanical processes and can only be fully explained by reference to them" (Cassirer, 1902, 318). On the other hand, according to Cassirer, Leibniz recognized that "the value and validity of the concept of conservation" does not depend "on special ideas about the nature of physical forces" (319); on the contrary, the forces of nature of whatever kind must satisfy the conservation of mechanical work. Thus, Leibniz did not regard his conservation principle as the *consequence* of the "mechanical interpretation of the phenomena" (319); the latter is only an *instance* of a worldview that satisfies that principle: "Such a reversal [*Umkehrung*] does not change the outlined worldview, but it does change the doctrine of principles of scientific knowledge" (319). Cassirer could then project more or less explicitly onto Leibniz a debate on the use of models or *Bilder* that was fashionable among physicists at the turn of the century (Deltete, 1999). In general, according to Cassirer, Leibniz's physics can avoid the construction of "hypothetical models, in which one tries to grasp the essence of the phenomena" (Cassirer, 1902, 319), and relies only on general principles that any such models, of whatever nature, have to satisfy.

According to Cassirer, this showed how the conservation of mechanical work "relates to *experience*", that is, "in which proportion *a priori* and empirical elements contribute to its justification" (320). In Leibniz's work, Cassirer pointed out that the "equivalence of cause and effect and the principle of the impossibility of the *perpetuum mobile* resulted in two different basic motifs of the conservation law" (319–320). If the equality of cause and effect were not satisfied, and if it were possible to create work out of nothing, then the absurdity of a *perpetuum mobile* would ensue. On the one hand, Leibniz's conservation principle appears to be *a posteriori*, being based on the repeated experience of failure in constructing a perpetual motion machine. On the other hand, it is *a priori* because the equivalence of cause and effect serves as "a criterion for assessing and differentiating the value of given experiences" (321). Cassirer explains his stance clearly in the following passage by referring to his derivation of rules for the collision of bodies:

The relationship between the conservation law and experience cannot be described more clearly than in this case. The value of the law lies in its fertility as a *principle* for the exact investigation of the phenomena. For

 $^{^8 {\}rm See}$ Cassirer, 1904b, 267; fn. 205; 315; fn. 256 for more detail.

 $^{^{9}}$ Cassirer provides some additional details in his commentaries on the German translation of some of Leibniz's *Haupt-schriften*, which he was working on at that time (Cassirer, 1904a, 1:249f.; fn. 184).

¹⁰On Cassirer's notion of coordination, see Ryckman, 1991.

this very reason, it is independent of 'experience' in the trivial sense of the latter word, which only denotes a disordered set of random observations. Compared to this indefinite chaos of perceptual content, the equivalence [of cause and effect] contains a rule of judgment, through which only scientific experience can emerge from the lawless combination of the immediate content of the mind. In this sense, we can call the energy law an *a priori* law—provided that we use the expression of the *a priori* as the fundamental tool for gaining knowledge from the mere descriptions of existing facts. The attribution of such *status* to the principle could still be considered a daring philosophical stance in Leibniz's time; however, it appears to modern development as a sober expression of a historical fact: [...] [Leibniz's approach] has become a maxim of scientific research. (Cassirer, 1902, 321)

Leibniz expressly conceded that the exceptions to vis viva conservation appear at first sight to be overwhelming. Indeed, unlike momentum, which is always conserved, the quantity of vis viva appears to be conserved only in elastic collisions. In inelastic collisions—such as when a ball of soft clay strikes another ball of soft clay—momentum $m\mathbf{v}$ is conserved but vis viva mv^2 is lost. Because macroscopic collisions are at least partially inelastic, "[t]he entire material of observations therefore forms a single major contradiction to the principle" (321). However, rather than abandoning the universality of his principle in the face of the empirical evidence, Leibniz considered it to be more fundamental than the latter. It is not that vis viva is conserved because bodies are elastic but rather the other way around: because vis viva is conserved, bodies must always have some degree of elasticity (308).

In this sense, the *a posteriori-a priori* opposition does not fully grasp Leibniz's contribution to the justification of the energy principle (318). According to Cassirer, the apparent contradiction can be resolved if one does not look at the 'origin' (*Ursprung*) of the principle but at its 'function' (*Leistung*) in the overall system of physics. If mechanical work were not conserved, then causes would produce different effects depending on the unit of measure chosen, and nature would be without laws; the whole science of dynamics would become something indeterminate and contradictory, *quiddam vagum et absonum* (Leibniz, 1850, 3:210). "As Leibniz says against Johann Bernoulli, this requirement means nothing less than a *condition of the possibility of dynamics as a science*" (Cassirer, 1902, 402; my emphasis). In attributing to Leibniz this sort of proto-transcendental argument, Cassirer aimed to emphasize what he saw as Leibniz's fundamental contribution to the history of the energy principle. For Leibniz, the conservation of mechanical work was not some mathematical *equation* among others that describes a certain group of phenomena. Something much more essential was at stake, namely, the very possibility of dynamics as a science. Thus, Leibniz treated the conservation of mechanical work as a fundamental *principle* that lurks behind all equations of dynamics.

In Cassirer's judgment, Leibniz's role in the *discovery* of the energy principle in its generality was indeed minor; the modern concept of 'energy' was acquired only much later in the nineteenth century by taking into account not only mechanical energy but other forms of energy (e.g., thermal, electromagnetic, etc.). Nevertheless, Leibniz's role in defining the strategy for the *justification* of the principle has been historically relevant: "In the *proof* of the fundamental law, Leibniz can now clearly distinguish between the two lines of thought, which have also been expressed separately in the development of modern theory" (317). Indeed, in justifying the energy principle, its discoverers in the nineteenth century, Mayer (1842) and Helmholtz (1847), were confronted with a choice between the *a posteriori* or bottom-up approach vs. the *a priori* or bottom-down approach. "While Robert Mayer started from the equality of cause and effect" as a *metaphysical principle*, Cassirer points out that "Helmholtz based his investigation on the principle of the excluded perpetual motion machine", which he considered as an *empirical generalization*.

In mechanics, the impossibility of perpetual motion can easily be demonstrated. Thus, Helmholtz¹¹ started from the assumption that all processes in nature were mechanical, for example that heat was nothing but motion. The energy principle was "identical to the assumption that all effects in nature can be traced back to attractive and repulsive forces, the intensity of which depends only on the distance between the points acting on one another" (Cassirer, 1902, 318). On the contrary, Mayer thought of the energy principle in terms of the correlation of numerical values—a given number of work units always corresponds to a fixed number of heat units. Thus, Mayer could ignore the question of the nature of heat for the purpose of relating it to a mechanical equivalent. Following Mayer, "the law of conservation is obtained and carried out independently of every special conception of nature, in particular every special mechanical interpretation of individual physical processes" (318). However, in Cassirer's view, the alternative between Mayer's top-down approach, which builds on a self-evident principle, and Helmholtz's bottom-up approach, which relies on a broad empirical generalization, is not exhaustive. As Leibniz had

¹¹For the importance of Helmholtz in neo-Kantianism, see Biagioli (2016).

already sensed, the issue is ultimately irrelevant. The justification of the conservation principle resides entirely in its capacity to serve as an effective selection criterion for the acceptability of individual laws.

1.2 Beyond Leibniz. From Das Erkenntnisproblem to Substanzbegriff und Funktionsbegriff

It is not the goal of this paper to systematically assess the historical accuracy of Cassirer's reconstruction, which certainly raises the eyebrows of many of today's scholars. What is relevant for the present paper is the extent to which Cassirer's *historical* work reveals his *philosophical* agenda. In Cassirer's first book, one can recognize the emergence of themes that would soon become trademarks of his philosophy of physics. It is not by chance that when the book was published, Cassirer, now back in Berlin, was already working on the manuscripts of some of his later major works (Cassirer to Natorp, Dec. 13, 1902; Cassirer, 1995–, Vol. 18, Doc. 55). When he sent a copy of the book to Natorp, Cassirer wrote that, in addition to preparing a German anthology of Leibniz's writings with Arthur Buchenau (Leibniz, 1904), he was still dealing with his "work on the prehistory of the critique of reason" (Cassirer to Natorp, Dec. 13, 1902; Cassirer to Natorp, Dec. 13, 1902; Vol. 18, Doc. 55). However, he soon realized that the sections on Kant required further work (Cassirer to Natorp, Dec. 13, 1902; Vol. 18, Doc. 55).

After moving back to Berlin, Cassirer (1906b) maintained strong ties with the Marburg group, which is testified by his strenuous defense of Cohen (1902) against the attacks of Leonard Nelson (1905). By 1905, the first volume of *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* was finished, and Cassirer already planned to add a second volume. However, he was not fully satisfied with the title. In reality, Cassirer wanted not only to describe a series of theories of knowledge but also to capture the logic of its development.¹² For this reason, already at that time, he realized that a third systematic volume was required (Cassirer to Natorp, Jul. 31, 1905; Cassirer, 1995–, Vol. 18, Doc. 70). *Das Erkenntnisproblem* (Cassirer, 1906a) was presented as a habilitation thesis at the University of Berlin in April. In July 1906, Cassirer held his *Probevorlesung* and obtained the *venia legendi* for philosophy. The title of the lecture, "Substanzbegriff und Funktionsbegriff" (Cassirer, 1906c), reveals that the core idea of what would become the theoretical counterpart of *Das Erkenntnisproblem* was already fully formed (Cassirer, 1910). In particular, in the physics part of the *Probevorlesung*, Cassirer used the historical development of the concept of 'energy' as a prime example of how a substance-concept could be transformed into a function-concept. In this way, Cassirer developed an idea that was already present in his book on Leibniz but without indulging in the somewhat cryptic jargon of the Marburg School.

As Cassirer pointed out, it was initially natural to assume that energy is "a constant thing 'behind' the phenomena" (Cassirer, 1906c, 11). However, it was progressively realized that the function of the concept of energy is only to establish quantitative equivalences among different phenomena (e.g., heat, motion, or electricity): "All reality of energy resolves itself, from the point of view of knowledge, in the assessment of equivalence relations" (Cassirer, 1906c). For this purpose, a common numerical scale is constructed, the unit of which as the unit of energy serves as a common denominator for comparison. It is at this point that energy as a concept of substance has been replaced by a concept of function (11ff.). The "substantiality" which is ascribed to energy means "nothing other than a constancy of pure numerical relationships" (12). A constant numerical coefficient is assigned to each form of energy, called the 'mechanical equivalent' of that form of energy. This coefficient serves to convert any quantity of that energy into a corresponding amount of mechanical work, just as one can convert feet into meters: "the 'essense' of the individual types of energy is sufficiently known and clarified if one specifies the fixed equivalence values that univocally regulate the connection and the transition between the different areas can be specified" (Cassirer, 1909, 93)

At the time the lecture was held, Cassirer was probably already working on the corresponding chapters of his monumental monograph of the same title, which he finished in July 1910 (Cassirer, 1910). In the physics sections of the book, the relational conception of energy (Hiebert, 1962) that, not without some arbitrariness, Cassirer attributed to Leibniz in his historical work, was now developed systematically, relying on a tradition that, from Mayer (1842) and Rankine (1855), leads to Helm's (1898) and Ostwald's (1902) energetics. It suffices to quote a significant passage:

 $^{^{12}}$ As Cassirer pointed in the 'Introduction' of the first volume of *Das Erkenntnisproblem*, the 'Apriori,' is not a fixed "psychological or physiological 'disposition' " (Cassirer, 1906a, 6). It must be discovered in the history of science as "the conservation of a general logical structure in all consecutive conceptual systems" (17f.).

The law of energy directs us to coordinate every member of a manifold with one and only one member of any other manifold, in so far as to any *quantum* of motion there corresponds one *quantum* of heat, to any *quantum* of electricity, one quantum of chemical attraction, and so on. In the concept of work, all these determinations of magnitude are related to a common denominator. If such a connection is once established, then every numerical difference that we find within one series can be completely expressed and reproduced in the appropriate values of any other series. The unit of comparison, which we take as a basis, can arbitrarily vary without the result being affected. If two elements of any field are equal when the same amount of work corresponds to them in any series of physical qualities, then this equality must be maintained, even when we go over to any other series for the purpose of their numerical comparison. In this postulate, the essential content of the principle of conservation is already exhausted; for any quantity of work, which arose 'from nothing' would violate the principle of the mutual one to one coordination of all series [...] In any case, it appears that energy in this form of deduction is never a new thing, but is a unitary system of reference on which we base measurement. All that can be said of it on scientific grounds is exhausted in the quantitative relations of equivalence, that prevail between the different fields of physics. (Cassirer, 1910, 253f.; tr. 1923b, 190f.)

According to Cassirer, the energy principle opened up the possibility of turning "nature into a system, without our being obliged to require representation of this system in a unitary, intuitive picture [Bilde], like the one offered by mechanism" (Cassirer, 1910, 266; tr. 1923b, 200). 'Mechanism' was the attempt to unify the phenomena by reducing everything to one class, mechanical motions, by providing mechanical models of thermodynamic or electromagnetic phenomena. Modern energetics, if properly understood, indicates the possibility of unifying different classes of phenomena under a common principle: the assumption that energy, whether it is derived from motion, heat, or electricity, is always equivalent to a proportional amount of mechanical work. Against this conception, Cassirer pointed out, the objection has been raised that the qualitative difference between separate classes of phenomena has not been abolished but glorified into a sort of neo-Aristotelian qualitative physics (see Duhem, 1903, 197ff.). However, Cassirer replied that this criticism is unjustified. The quantitative equivalence between different phenomena "gives a no less definite logical connection than reduction to a common mechanical model" (Cassirer, 1910, 202; tr. 1923b, 269).

Whereas all attempts to reduce all phenomena to mechanics have repeatedly failed, the principle of conservation of energy has survived the rise and fall of the most disparate theories and models. This surprising resilience cannot be a coincidence. According to Cassirer, the provisional hypothesis could be made that principles of this kind are constitutive conditions *a priori* for the very acceptability of particular physical theories *in general* (see Cassirer, 1936b, 207; tr. 1956, 117). A theory that does not comply with such principles would probably be rejected from the outset. Using a geometrical metaphor, *Substanzbegriff und Funktionsbegriff* famously defined transcendental philosophy as a 'theory of the invariants of experience.' It singles out the stable elements in the synchronic and diachronic succession of different if not rival theories as possible candidates for *a priori* principles.¹³ By using this analogy, Cassirer attempted to spread Marburg's brand of historicized Kantianism beyond the Marburg inner circle. Despite Cohen's lukewarm reaction (Cohen to Cassirer, Aug. 10, 1910; Cassirer, 1995–, Vol. 18, Doc. 45), Cassirer's endeavor turned out to be very successful.

Substanzbegriff und Funktionsbegriff (Cassirer, 1910) rapidly became the most respected and widely read work outside of Marburg, followed by Natorp's *Die logischen Grundlagen der exakten Wissenschaften* (Natorp, 1910), which was published at around the same time. The Marburg School had reached its peak. However, when the 70-year-old Cohen retired in 1912, the school (Natorp, 1912, Cassirer, 1912) was already starting to dissolve (Natorp to Görland, Jun. 6, 1912; Holzhey, 1986, Vol. 2, Doc. 131). Cassirer's intense personal and philosophical relationship with Cohen remained unaffected (Cassirer, 1918b, 1920b, 1926, 1943). In a letter to Natorp from the beginning of 1914, written on the occasion of the latter's 60th birthday, Cassirer continued to emphasize his gratitude for his "Marburg apprenticeship" (Ernst and Toni Cassirer to Natorp, Jan. 23, 1914; Cassirer, 1995–, Vol. 18, Doc. 208). However, the publication of Cassirer's 'studies on the history of the German spirit' collected in 1916 in *Freiheit und*

¹³Substanzbegriff und Funktionsbegriff expresses the same conception of the *a priori* as Das Erkenntnisproblem (see fn. 12) by using an analogy with Klein's Erlangen program (Klein, 1872). The historical development of science, Cassirer claims, "must leave a certain body of principles [*Prinzipien*] unaffected" (Cassirer, 1910, 355; tr. 1923b, 268). It is surmised that this invariance is not a coincidence: "those moments, which persist in the advance from theory to theory *because* they are the conditions of *any* theory" (Cassirer, 1910, 357; tr. 1923b, 269; first emphasis mine). At no given stage can the goal of finding these conditions be fully achieved; "nevertheless it remains as a *demand*" to search for them. Strictly speaking, only "the ultimate *logical invariants*" should be considered a *priori* (Cassirer, 1910, 357; tr. 1923b, 269). However, we can never be sure to have them. In my view, at this point, Cassirer seems to consider *each* invariant as constitutive, although provisional *a priori*. Thus, I do not agree with those who attribute to Cassirer a 'regulative' conception of the *a priori* already in the 1910s. Cf. e.g. Friedman, 2008, Ryckman, 2005, Heis, 2014a.

Form (Cassirer, 1916) could be seen as the transition from an early Cassirer, the neo-Kantian 'philosopher of science,' to a mature Cassirer, the well-rounded 'philosopher of culture.' As Cassirer conceded in a letter to Natorp, "[t]he actual external connection to the school [Schulzusammenhang] is loosening more and more" (Cassirer to Natorp, Jan. 1, 1917; Cassirer, 1995–, Vol. 18, Doc. 226). Nevertheless, Cassirer also pointed out that "the more each of us tries to continue on our own path, the closer we ultimately get to the same problems and tasks. And this is basically the best and safest proof that we can hope for our intimate relationship" (Cassirer to Natorp, Jan. 1, 1917; Vol. 18, Doc. 226).

2 From the Physics of Models to the Physics of Principles: Cassirer and the Principle of Relativity

After over a decade as a *Privatdozent* in Berlin, in 1919 Cassirer became an ordinary professor at the recently founded University of Hamburg. At about the same time, the success of the Eddington-Dyson eclipse rushed Cassirer back to the philosophy of the natural sciences.¹⁴ Towards the end of the year, the *New York Times* announced the confirmation of general relativity, and Einstein (1919b) himself wrote a celebrated article in the London *Times* on November 28, 1919 [citation removed]. In it, he famously declared relativity theory to be a theory of principle (like classical thermodynamics) rather than a constructive theory (like the kinetic theory of gases), a theory based on empirically based abstract principles rather than on the construction of detailed models, or *Bilder* (Einstein, 1919b). On December 14, 1919, the front page of the *Berliner Illustrierte Zeitung* printed a portrait of Einstein where he was described as a 'new great figure in world history,' comparable to Copernicus, Kepler, and Newton.¹⁵

It is not clear whether Cassirer read Einstein's article for the London *Times*. As far as I can see, it is never mentioned in his writings. However, Cassirer reached a surprisingly similar conclusion, probably relying on common sources. Relativity theory appeared to Cassirer as an example of the progressive prevailing of the 'physics of principles' over the 'physics of models,' which was still dominant in the nineteenth century. In the following months, he was able to rapidly finish a short but insightful philosophical book on relativity (Cassirer, 1921b). He sent the first draft of the manuscript to Einstein for feedback in May 1920 (Cassirer to Einstein, May 10, 1920; Einstein, 1987–, Vol. 10, Doc. 11).¹⁶ Einstein's reaction was respectful but somewhat lackadaisical (Einstein to Cassirer, Jun. 5, 1920; Vol. 9, Doc. 44), reflecting Einstein's mild hostility towards Kant's philosophy at that time (Beller, 2000).¹⁷

Nevertheless, their relationship remained friendly (Cassirer to Einstein, Jul. 15, 1920; *The Albert Einstein Archives at the Hebrew University of Jerusalem*, 3-393).¹⁸ In August, Cassirer announced to Einstein that the revised manuscript (Cassirer, 1921b) was ready for publication (Cassirer to Einstein, Aug. 28, 1920; Einstein, 1987–, Vol. 10, Doc. 112). Amid numerous political and antisemitic attacks against Einstein (1920b), Cassirer hoped the book would help to dispel "the spiritual confusion that still persists in so many minds about these things and which seems to be deliberately exploited from some quarters" (Cassirer to Einstein, Aug. 28, 1920; Einstein, 1987–, Vol. 10, Doc. 112). Indeed, the book

¹⁴The Marburg School had developed a philosophical interest in relativity theory early on. Natorp wrote an often-criticized but, after all, well-informed section on relativity in his 1910 monograph (Natorp, 1910, 392ff.). Just afterwards, Cassirer wrote to Natorp that a student of Cohen's, Otto Buek (1904, 1912), planned to "work on the relativity principle" (Cassirer to Natorp, May 16, 1911; UB Marburg; Hs 83:643). Some years later, in 1914, Cohen wrote to Natorp that Buek was attending Einstein's lectures in Berlin (Cohen to Natorp, Nov. 28, 1914; Holzhey, 1986, Vol. 2, Doc. 145). Curiously, Buek became a good friend of Einstein's and signed a pacifist manifesto with him in 1918 (Einstein, 1987–, Vol. 6, Doc. 8).

¹⁵Berliner Illustrierte Zeitung, Vol. 28, no. 50, December 14th, 1919.

¹⁶Einstein read the book while traveling to the Netherlands to meet his friend Paul Ehrenfest (Einstein to Elsa Einstein, May 19, 1920; Vol. 10, Doc. 19; see also Einstein to Vahinger, Jun. 3, 1920; Vol. 9, Doc. 41).
¹⁷As usual, Einstein conceded to the Kantians that "one must approach experiences with some sort of conceptual functions

¹⁷As usual, Einstein conceded to the Kantians that "one must approach experiences with some sort of conceptual functions in order for science to be possible". However, he insisted that he did not believe "that our choice of these tools is constrained by virtue of the nature of our intellect" (Einstein to Cassirer, Jun. 5, 1920; Einstein, 1987–, Vol. 9, Doc. 44). Although Mach failed to appreciate the non-empirical origin of some physicists' conceptual tools, he was right in recognizing that the latter must be 'replaceable.' Einstein praised Cassirer for having grasped the spirit of the theory. Nevertheless, he complained that the manuscript did not sufficiently emphasize the empirical origins of both the special and the general principle of relativity (Einstein to Cassirer, Jun. 5, 1920; Vol. 9, Doc. 44). This issue was of great concern to Einstein at that time. He feared that both of his theories were perceived as overly speculative and abstract (see Einstein, 1920a, [p. 1]). Cassirer promised a substantial revision of the manuscript in light of Einstein's commentaries (Cassirer to Einstein, Jun. 16, 1920; Einstein, 1987–, Vol. 9, Doc. 58).

¹⁸Cassirer invited Einstein to be his personal guest and stay at his house during his visit to Hamburg to deliver a lecture (Cassirer to Einstein, Jul. 15, 1920; 3-393), which was held on July 17, 1920 (Reich, 2000). After dinner, a discussion session was organized at Cassirer's home at 26 Blumenstrasse. The encounter is vividly recollected by Cassirer's wife Toni Bondy in her biography of Cassirer (Cassirer Bondy, 1981, 134–135), giving the erroneous date of January 1921 (Reich, 2000).

was also written to rebuke the charges of 'relativism,' 'abstractness' and 'unintuitiveness' made against 'Einsteinery' in some philosophical circles (see also Cassirer, 1920c). Cassirer, faithful to his continuist approach to the history of science, meant to show that the allegedly unsettling features of the new theory simply brought to light tendencies that were all clearly recognizable in previous theories.

2.1 Cassirer's Zur Einstein'schen Relativitätstheorie and his 1920/1921 Lectures on Relativity

Cassirer's book was published at the beginning of 1921 (Ryckman, 2005). In the 1920/1921 winter term, from early October to the end of January, Cassirer also gave a series of 13 lectures on relativity (Cassirer, 1920–21), the notes for which constitute a neglected source for understanding the core argument of Cassirer's book. In particular, the lectures seem to offer an additional glimpse into Cassirer's sources. Besides Einstein's popular work on relativity (Einstein, 1917), Cassirer had clearly been influenced by Max Planck's lectures from the 1910s (Planck, 1909, 1910, 1915b) and, most of all, by the writings of Planck's student, Max von Laue. Laue not only authored the first textbook on relativity (M. Laue, 1911b) but also published some papers with philosophical content on the topic (M. Laue, 1913, M. v. Laue, 1921). In particular, I will argue that von Laue's reading of relativity offered Cassirer a *trait d'union* between his old writings on the pre-history of energy conservation and his philosophical analysis of Einstein's new theories. Indeed, Cassirer returned more directly to the argumentative structure of his book on Leibniz (Cassirer, 1902) by giving the relativity principle the same *status* as the principle of conservation of energy. Both of these principles are *constraints* that we impose on the laws of nature, rather than a simple accidental *byproduct* of these laws. This comparison is indeed prominent in the 'conclusion' of Laue's textbook (M. Laue, 1911c, 184ff.) and in his more popular writings (M. Laue, 1913).

Possibly following Einstein's suggestion, these lectures emphasize the empirical origin of the theory. On this point, Cassirer refers to Laue: "For the justification of the principle of relativity—says M. Laue (1911a, 104)—this series of experiments plays the same role as the attempts to build a *perpetuum mobile* played in the justification xof the energy principle" (Cassirer, 1920–21). The continuing series of failures to construct a *perpetuum mobile* finally cemented the belief that they could not be an accident. The question was then turned upside down. It has been repeatedly verified that it is in no way possible, either by mechanical, thermal, chemical, or other processes, to obtain a *perpetuum mobile* is to be impossible? The energy principle was the answer to this question. This principle plays the role of a *constraint* that all dynamical laws governing those processes have to satisfy if the *perpetuum mobile* of the first kind is to be impossible.

The special theory of relativity is the result of the very same strategy. As Cassirer put it, using one of his favorite quotes from Goethe: "The highest art in intellectual life and worldly life—Goethe once wrote to Zelter—consists *in turning the problem into a postulate*¹⁹ that enables us to get through" (Cassirer, 1920–21, 101). Like the discoverers of the energy principle, Einstein "made a virtue out of necessity" (101). Experience has increasingly shown that a privileged ether system cannot be found by experiments that rely on either mechanical, optical, or electromagnetic processes. The theory of relativity surmised that this failure could not be an accident, and it raised this conjecture to the *status* of a postulate (Einstein, 1905). It introduced as a "heuristic maxim the most general assumption that such a system *cannot* and *must* not exist" (Cassirer, 1920–21, 121–122). The apparently coincidental failure of all ether drift experiments was raised to the level of the "*principle* [*Prinzip*]" that for the physical description of natural processes no particular inertial reference body should be singled out (121-122). In this way, the "true philosophical import" of relativity theory was to have transformed "a mere negative expression into a positive expression", "a *limitation* of physical knowledge into a *principle* of such knowledge" (Cassirer, 1921b, 74; tr. 1923b, 408). As Cassirer put it in his lecture notes:

The principle of the constancy of the speed of light combined with the principle of relativity proves to be suitable to give a complete account not only of all mechanical but also of all optical-electrical phenomena. In their connection, the two principles no longer represent a unified thing that exists in nature; but they formulate a uniform basic relation which all special laws of nature obey: a most general moment of our knowledge of nature. They create the framework for the form of natural law in general, and this form is truly uniform, although it is precisely through it that it is required that the particular space and time values that are determined within different systems do not agree, but only stay in a determinate functional relationship, in mutual coordination [Zurordung]. This methodical state of affairs [methodische Sachverhalt] is expressed in the proposition that forms the actual core of the theory of relativity: [...] 'the general laws of nature are covariant with respect to Lorentz transformations.' (Cassirer, 1920–21, 81)

 $^{^{19}\}mbox{Goethe}$ to Zelter, Aug. 9, 1828; Goethe and Zelter, 1832, 61.

Thus, the content of special relativity takes (so to speak) the form of an imperative, a demand that *all* laws of nature have to satisfy if the relativity and light postulates are to hold. *Every* fundamental law of nature *must* be so constituted that it is transformed into a law of exactly the same form when, instead of the space-time variables x, y, z, t of the coordinate system K, we introduce new spacetime variables x', y', z', t' of a coordinate system K'. The relationship between the unprimed and the primed variables is given by the Lorentz transformation. The latter are not laws of nature but serve as criteria for selecting the allowable law of nature. If we express a law-like statement mathematically in a system K by applying the Lorentz transformation, then we obtain its expression with respect to K'. If the two expressions are identical, then the candidate law is well formulated; if not, then it has to be rejected or modified so that it complies with the relativity principle.

To further support this interpretation, Cassirer could quote Einstein's repeated characterization of the relativity principle as "a general maxim $[\ldots]$, which should serve as a 'heuristic aid when searching for the general laws of nature²⁰" (Cassirer, 1921b, 38; tr. 1923b, 377). Thus, special relativity is not a theory in the proper sense of the word (like mechanics or electrodynamics) that entails the individual laws of nature (Maxwell's equations, Newton's particle dynamics, hydrodynamics, elasticity theory, etc.). It is rather the expression of a "law of lawlikeliness [Gesetz der Gesetzlichkeit]" (Cassirer, 1920–21, 88), "a criterion of their validity and admissibility for all special physical areas and for all special physical theories" (Cassirer, 1921b, 33; tr. 1923b, 359). In this "formal limitation, which is placed on natural laws by this maxim, lies—as Einstein himself urged—the characteristic 'sagacity' $[Sp\"{u}rkraft]^{21}$ of the principle of relativity" (Cassirer, 1921b, 38; tr. 1923b, 377f). Indeed, the theory does not simply assert that the actual laws of nature that we know happen to be Lorentz invariant—the theory makes the bolder claim that all possible laws of nature must be Lorentz invariant. In this form, a merely analytic principle describing the formal properties of certain equations is transformed into a synthetic one that allows for new results. Maxwell's equations turned out to already be Lorentz invariant, while on the contrary, Newton's equations of motion were not. Thus, they had to be modified in conformity with the new kinematics. From this modification, Einstein obtained testable consequences—the velocity-dependence of mass that could be observed in fast-moving electrons.²²

The *status* that Cassirer attributes to the relativity principle is thus similar to the status that he attributed to the energy principle in his book on Leibniz (Cassirer, 1902). Both principles, like any other physical statement, are expressed in the form of functional equations. However, they are not used as *equations* expressing this or that individual law of nature, but rather as *principles* that impose a formal requirement on the analytical formulation of all possible laws. As Cassirer points out, explicitly referencing his earlier monograph, it was Leibniz himself who "referred to this logical moment in it" (Cassirer, 1921b, 45; tr. 1923b, 384). As we have seen, the young Cassirer had already pointed out that, in the nineteenth century, "the same process of thought" introduced by Leibniz was "repeated on broader physical lines in the discovery and grounding of the modern principle of energy" (Cassirer, 1921b, 46; tr. 1923b, 384).

Helmholtz "tried to express the principle of conservation of energy in the language of classical mechanics and tried to prove it with the formulas of this mechanics" (Cassirer, 1921a, 288). For the young Helmholtz, the energy principle was the consequence of the fact that all forces in nature are mechanical, *i.e.*, central forces whose intensity depends only on distance (Cassirer, 1920–21, 64). On the contrary, Cassirer argues, "Mayer's idea took a different turn from the start" (Cassirer, 1921a, 288). Whether, for example, heat is "in its essence nothing other than molecular motion" remains open; "it is sufficient that the constant

²⁰Einstein, 1917, 29.

 $^{^{21}}$ The expression is borrowed from Einstein (1917, 67).

²²As Cassirer rightly noted, this result was not new; Lorentz (1903–4) had obtained the same law of the velocity-dependence of the mass of the electron: "In the modern electron theory, it follows from the well-known investigation of Kaufmann [1906] that the 'mass' of an electron is not unchangeable, but that it rapidly increases with the velocity of the electron as soon as the latter approaches the velocity of light" (Cassirer, 1921b, 66; tr. 1923b, 400–401). By assuming that the particles constituting the electron experience a contraction in the direction of motion, Lorentz arrived at the same law of motion as Einstein, which was later confirmed (Bucherer, 1908). As Cassirer points out, the theory of relativity leads to the same result, "but it reveals in this too its peculiar nuance and character" (Cassirer, 1921b, 66; tr. 1923b, 400–401). Lorentz (1903–4) maintained Newton's second law and shifted the explanatory burden to electrodynamics. The mass, which is supposed to be constant according to the old mechanics, varies because the electron self-field (i.e., the magnetic field generated by the electron) resists its motion. On the contrary, Einstein *modified* Newton's point dynamics so that it satisfies the relativity principle. Thereby, he obtained the velocity dependency of masses of any kind (Einstein, 1905, §10). No particular model of the electron was required. Electrons only served to test the theory because electrons in β -rays move with a velocity close to that of light.

exchange ratio between the two is established and brought to an exact expression" (288). Instead of investigating the peculiar qualitative *nature* of heat or mechanical motion, Mayer only postulated the constant quantitative *proportion* between work and heat units. The latter imposes a constraint on the fundamental forces that there could be, without addressing the question of the metaphysical nature of the individual forces that happen to be (e.g., electromagnetic, mechanical, etc.).

We are confronted with the same methodological approach in the foundation of the special relativity principle. Einstein's formulation of the relativity principle did not imply the reduction of all physics to electrodynamics, just as Mayer's formulation of the energy principle does not imply the reduction of all physics to mechanics. The initial contradiction between mechanics and electrodynamics that is revealed by the negative result of ether drift experiments was overcome not "by using the electrodynamic processes as a key to the mechanical" but by establishing "a far more perfect and deeper unity between the two than previously existed" (Cassirer, 1921b, 33; tr. 1923b, 373; my emphasis). The unification of the two separate fields of theoretical physics—electrodynamics and mechanics—is not obtained through a process of horizontal integration, a reduction of the one to the other, but through a vertical integration, a subsumption of both theories under a higher principle. Rather than the reduction of mechanics to electrodynamics and of electrodynamics to mechanics, in relativity theory "a truly universal principle, a heuristic maxim of investigation in general, is established, which claims to contain a *criterion* of the validity and permissibility of *all* particular physical fields and theories" (Cassirer, 1921b, 32f; tr. 1923b, 373; my emphasis). This result was not reached simply by heaping up experimental data but rather "rests on a critical transformation of the system of fundamental physical concepts" (Cassirer, 1921b, 33; tr. 1923b, 373).²³

Cassirer could then point out that, despite its apparent radical novely, the relativity principle, like the energy principle, is ultimately an instance of "that general direction of physical thought, which has been called the 'physics of principles' in contrast to the physics of pictures and mechanical models" (Cassirer, 1921b, 16; tr. 1923b, 359). In Cassirer's view, the history of the energy principle shows that "[p]hysics in the nineteenth century more and more ceased to be a physics of models [Bilder] in order to transform itself instead into a physics of *principles*" (Cassirer, 1920–21, 63). Physics previously devised "a separate model for each of the different areas, and it was often satisfied with simply placing these often very different models side by side and lining them up" (123). It is true that when we observe the evolution of physics, at first we are facing an ever-changing "chaos of pictures and opinions". We might therefore ask, "which is the constant component [Bestandteil] of the physical theories and hypotheses, if their model-like [bildlicher] component does not play this role? What established some sort of objective connection among such models?" (64). The energy principle, the least action principle, etc., have survived despite the demise of the particular models that attempted to describe a certain class of natural processes directly. The reason for this stability is that the formulation of "a 'principle' [...] never refers directly to things", but it "sets up a general rule for complex functional dependencies and their mutual connection" (Cassirer, 1921b, 17; tr. 1923b, 359). Principles show an astounding resilience, despite the rapid succession of triumphs and collapses of the individual theories, because they are the very criteria for selecting what can be considered to be an acceptable physical theory.

According to Cassirer, the passage from special to general relativity shows the same epistemological tendency. At the same time, it testifies to the fact that physicists must be ready to abandon some previously selected principles in favor of more general ones. Einstein's initial goal was to formulate a special-relativistic theory of gravitation starting from Newton's action-at-a-distance theory. As in any other field of physics (e.g., classical point dynamics, elasticity, hydrodynamics, etc.), Einstein attempted to modify Newton's gravitational laws so that they conformed to the new relativistic kinematics. However,

²³As Natorp (1910, 392ff.) had already realized a decade earlier, it was in Minkowski's work that this radical change in the concept of space and time found its most fruitful expression (Minkowski, 1909). However, Cassirer, in contrast to Natorp, did not attempt to 'immunize' but to 'revise' Kant, or at least to locate the spirit of Kantian philosophy at a deeper level (Hentschel, 1990b). In Cassirer's view, the main result of Kant's philosophy of space and time was the recognition that "the reality assigned to space and time is not that of things [*Dingen*], but of conditions [*Bedingungen*]" (Cassirer, 1920–21, 112). They are not 'things' that exist in *natura rerum*, but rather indispensable 'conditions' of all our empirical knowledge. From the point of view of critical idealism, their empirical reality resolves in their objective validity (112). The passage from classic to Minkowski spacetime does not change this fundamental result. Minkowski's spacetime is nothing but a symbolic representation of certain analytical relations between variables that appears in the laws of nature. "The 'postulate of the absolute world'", as Minkowski defines this requirement, is ultimately "a postulate of absolute method" (Cassirer, 1921b, 117; tr. 1923b, 445f). The opposition of the electromagnetic worldview and the mechanistic worldview was overcome not ontologically, by reducing one set of phenomena to the other, but by methodologically conforming both sets of laws to a common principle; cf. Corry, 2010.

he realized early on that the modification did not proceed successfully, which revealed the limitations of special relativity. Meanwhile, the method of special relativity pointed beyond itself, leading to an extension of the relativity principle to accelerated motions. Einstein understood that the notion of a class of privileged non-accelerated frames that still hold in the special theory could not be empirically singled out because of the identity of inertial and gravitational mass. Again, the *problem* was transformed into a *postulate*. The general principle of relativity, according to which every coordinate system is just as good as any other, was elevated to a condition that we impose on the formulation of the laws of nature, a powerful selection principle that restricts the range of possible laws. Cassirer makes this point very clearly in his lecture note:

We can only call those law-like statements laws of nature (*i.e.*, give them objective universality) if their form is independent of the peculiarity of our empirical measurements, of the special choice of the variables x_1, x_2, x_3, x_4 , which express the space and time parameters. In this sense, the principle of general relativity—that the general laws of nature do not change form in any transformation of space-time variables—could be considered an *analytical* assertion: a definition of a 'general' law of nature. However, one can call *synthetic* the requirement that there must be such last invariants. To justify this demand, physicists can ultimately only rely on a '*transcendental principle*,' on a principle of the 'possibility of experience.' One cannot prove the invariance that they claim as a metaphysical absolute; one can only show that physics as a science is and remains dependent on such an assumption. (Cassirer, 1920–21, 103)

The reference to the notion of 'transcendental' was probably too 'on the nose,' and it did not appear in the corresponding passage of the book (Cassirer, 1921b, 45). However, it is revealing of Cassirer's agenda. Like the special relativity principle, the general relativity principle ultimately serves as a "formal constraint that is placed on the laws of nature" (38; tr. 1923b, 377-378). Cassirer could not avoid emphasizing the Kantian overtones of Einstein's insistence on the general relativity principle as a requirement, a constraint, a demand that one imposes on the laws of nature.²⁴ "In fact, it can be shown that the general doctrine of the invariance and univocality of certain values, which is given in the first place by the theory of relativity, must recur in some form in any theory of nature" (Cassirer, 1921b, 45; tr. 1923b, 384). At first sight, the general principle of relativity seems merely to be an empty analytic description of what a law of nature happens to be. Indeed, all laws of nature can trivially be written in a generally covariant form (Kretschmann, 1917). However, as Einstein (1918b) argued, they acquire a synthetic meaning once they are combined with the principle of simplicity. One needs to search for the laws of nature, which cannot be simplified through the choice of a particular coordinate system. In this way, the principle of general relativity acquires its characteristic Spürkraft, a strong restricting power. Together with the assumption that the field equations are of the second order, this requirement was sufficient to limit the number of possible generalizations of the Poisson equation for gravity to one possibility, which led to Einstein's 1915 theory of gravitation.

2.2 Principles and Symbols

There are, of course, many other themes and lines of argument intertwined in Cassirer's book. However, most of them pertain, so to speak, to its *pars destruens*. Cassirer aims to show that the new relativity theory is opposed to both naive empiricism and naive realism (Cassirer, 1921b, 47; tr. 1923b, 386); it is better framed within Cassirer's own 'critical idealism.' However, as far as I can see, most interpreters have missed the main line of the argument, which constitutes the *pars construens* of the book: the characterization of relativity theory as an example of the 'physics of principles' as opposed to the 'physics of models.' The relativity principle, like the energy principle, is an abstract requirement that we impose on the formulation of the laws of nature, rather than an intuitive model-like description of the properties of some physical system. In this way, Cassirer arrived at a conclusion that is surprisingly similar to Einstein's 1919 article for the London *Times* (Einstein, 1919b), where special and general relativity are classified as 'principle theories' (like thermodynamics), as opposed to 'constructive theories' based on *Bilder* (like the kinetic theory of gases). As we have surmised, there is no evidence that Cassirer read that article. Nevertheless, the convergence of Einstein and Cassirer on this issue is less surprising than one might judge at first sight. On the contrary, what is surprising is that no other contemporary philosopher has emphasized this point.

 $^{^{24}}$ I disagree with the otherwise excellent Ryckman, 2005, who claims that Cassirer regarded the principle of general covariance as a 'regulative' principle, but still 'constitutive' principle. On my reading, the principle of general covariance must be regarded as 'constitutive,' since it imposes a rather strict condition on the formulation of the laws of nature. Cassirer would embrace a 'regulative' conception of the *a priori* only in the 1930s. See [citation removed]. For a systematic overview of Cassirer's conception of the *a priori*, see Heis, 2014b.

The opposition of a 'physics of principles' and a 'physics of models' was widespread in the scientific and philosophical debate, to the point of having become platitudinous (Lorentz, 1900, 1905, Poincaré, 1904, Sommerfeld, 1915). Cassirer referred to it more or less explicitly in his previous writings (Cassirer, 1909, 1910). Moreover, the idea that relativity theory was ultimately a second-order theory that imposes constraints on other, first-level dynamical laws was shared by virtually all early relativists (although with different nuances): not only by Einstein²⁵ but also by Minkowski (1908),²⁶ Planck (1915b)²⁷ and most of all M. Laue (1911c),²⁸ who was one of Cassirer's main sources. The temptation to give a Kantian (or better, neo-Kantian) reading of the role of those 'principles' with which all laws of nature seem to comply was hard to resist. At this point, Cassirer seems somewhat ambivalent in identifying each of these principles (at least provisionally) as constitutive *a priori*, relative to the historically given fact of science,²⁹ or to consider *a priori* only the general tendency to search for progressively more general principles.³⁰

This vacillation was not fully resolved, which left him open to attack by his critics,³¹who viewed Cassirer's conception of the *a priori* as either to narrow or too vague (Schlick, 1921, 102). Objections of this kind might have been one of the reasons why Einstein's reaction to Cassirer's manuscript was less positive than one might expect (Einstein to Schlick, Aug. 10, 1921; Einstein, 1987–, Vol. 12, Doc. 202).³² In Einstein's view, 'principles' were based on empirical generalization and were certainly not a *a priori*.³³ On the contrary, they were obtained by breaking the alleged *a priori* validity of certain previously accepted concepts.³⁴ Moreover, the goal of physics, according to Einstein, ultimately remained that of constructing 'models' of natural phenomena (we want to know whether matter is made of particles or fields, which is the mathematical structure of the latter, etc.; the search for 'principles' is ultimately a provisional strategy that helps physicists to restrict the range of available models when they are confronted with an *embarras de richesses* (Einstein, 1918a, 701; see [citation removed]).

Cassirer, on the contrary, saw the history of physics as a one-directional progression from the physics of models to the physics of principles, a progressive liberation from particular images of the world towards a general abstract unity: "Latest conception: physics of principles. Planck: detachment from the peculiarity of the individual images. Removal of everything that is only a sensual and descriptive element. Search for unity. He wants to embrace the whole of unity. Sens[itive] and [I]tuit[ive] are only accidents, anthropomorphic features. [...] such a development of relativity theory can be characterized in this manner. Level of pure symbolic expression" (Cassirer, 1998–, Vol. 4, Doc. I; fragment of 13/6/1922). The use of the word 'symbol' in this unpublished fragment shows how the opposition between the 'physics of

 34 See, *e.g.*, Einstein, 1916, 120.

 $^{^{25}}$ Among the many possible sources, see in particular Einstein (1914), in which the relativity principle is compared with the energy principle, both being requirements imposed on first-order laws.

 $^{^{26}}$ Minkowski (1915, Draft RP A, Lp. 7) explicitly compares the relativity principle to the energy principle; both are not customary laws of nature but demands that we impose on possible laws of nature; see also (Minkowski, 1908).

²⁷Planck used the principle of least action and the relativity principle to construct 'relativistic general dynamics,' encompassing mechanics and electrodynamics as special cases; see Liu (1997).

 $^{^{28}}$ Laue also resorted to a comparison between the relativity principle and the energy principle, emphasizing that both are "criteria for the admissibility" of possible theories (185f.).

²⁹In my view, contrary to what is usually claimed, in this period, Cassirer's stance does not seem to differ significantly from the young Reichenbach's 'method of successive approximations' (Reichenbach, 1920, 66). In other words, both claim that it is always possible to find 'better' *constitutive* principles in an infinite convergent process (Cassirer to Reichenbach, Jul. 7, 1920). The difference between Cassirer and Reichenbach lies in the nature of such principles: for Cassirer they are principles that *constrain* the *form* of the laws of nature; for Reichenbach they are principles that *coordinate* the form of the laws of nature with their empirical *content* (see Padovani, 2009).

 $^{^{30}}$ Cassirer seems to come close to defending this position in a letter to Schlick (Cassirer to Schlick, Oct. 23, 1920; Cassirer, 1995–, Vol. 18, Doc. 88). *Stricto sensu*, he claimed, only the unity or legality of nature in general is a priori. However, the requirement of the unity of nature, as Schlick (1921, 102) pointed out, does not impose any specific constraint on physical laws. In other words, the *a priori* thus conceived is not *constitutive* anymore. However, in my view, only in *Determinismus und Indeterminismus* did Cassirer (1936b) explicitly concede that the *a priori* has only a *regulative* meaning.

 $^{^{31}}$ A few years later, Einstein (1924) made this point quite clearly in his review of Elsbach (1924). Referring explicitly to Cassirer, Einstein points out that when confronted with scientific theories that do not satisfy the previously recognized *a priori* principles, transcendental philosophers are at a crossroads: they can (a) concede that they were mistaken (what they thought to be an 'invariant' of experience turned out not to be); or they can (b) consider the proper constitutive *a priori* only as the never reachable limit of an approximation process (Einstein, 1924, 1688f.). Ultimately, as Einstein put it, neo-Kantians are neither with Mohamed nor with the Prophet (1688). In both cases, nothing in their system is ever really a *a priori* (*i.e.*, a condition of the possibility of physical sciences *in general*). Indeed, according to Einstein, philosophy should concede that it should at most describe how science happens to work, not dictate how it must work (Einstein, 1928, 162).

 $^{^{32}}$ Cassirer's approach is more akin to the 'Kantian' stance of Hilbert; see Hilbert, 1921, 1923; see Ryckman, 2008 for more details.

³³See Einstein to Besso, Aug. 28, 1918; Einstein, 1987–, Vol. 8, Doc. 607; cf. also Einstein to Besso, Sep. 8, 1918; Vol. 8, Doc. 612.

principles' and the 'physics of models' became entangled with the 'symbolic turn' in Cassirer's philosophy that was announced in the last pages of *Zur Einstein'schen Relativitätstheorie* (Cassirer, 1921b, 126ff; tr. 1923b, 454ff), in which the notion of 'symbolic form' is introduced. In a talk that was given in Hamburg in July 1921 titled *Die Begriffsform im mythischen Denken*—which was published in an expanded form the following year (Cassirer, 1922)—Cassirer started to apply the 'transcendental method' to myth, language, and the entire foundation of the philosophy of culture. Whereas the emerging logical empiricism aimed to proceed 'analytically' by investigating the details of single sciences or even physical theories, Cassirer proceeded 'synthetically' by considering natural sciences as cultural 'facts' among others. This approach of course required an enormous undertaking that culminated in the first two volumes of *Philosophie der symbolischen Formen*, on language and myth, published in 1923 and 1924 (Cassirer, 1923a, 1925).

Conclusion

Such a 'symbolic turn' soon made its way into Cassirer's philosophy of natural science (see Cassirer, 1927b, 35). In the sections dedicated to physics in the third 'epistemological' volume of the *Philosophie der* symbolischen Formen, which was finished in 1927, Cassirer (1929) incorporated the opposition between the 'physics of principles' and the 'physics of models' into his reflections about the symbolic nature of physical knowledge. The history of nineteenth-century physics is described as "the progress from the 'model' to the 'principle'" (538; tr. 1957, 461). Cassirer returned in some detail to the case of the discovery and justification of the energy principle and the Helmholtz-Meyer controversy. He located in Planck's (1910) talk in Königsberg a "decisive methodological conclusion," where "the primacy of principles over models is recognized and carried out in every particular" (Cassirer, 1929, 540; tr. 1957, 463). The key methodological issue was not one of images but of principles, an attempt to encompass different and even conflicting natural laws in one supreme, all-embracing rule. In this respect, Cassirer saw that "a definite and unmistakable line runs from the principle of the conservation of energy to the general principle of relativity" (Cassirer, 1929, 537; tr. 1957, 460). In modern physics (Cassirer, 1927a), the tendency toward unification has triumphed over the tendency toward representation: "The schematism of images has given way to the symbolism of principles" (Cassirer, 1929, 547; tr. 1957, 467; my emphasis).

In a series of lectures titled 'Die Einheit der Wissenschaft', given for the new cultural program Hochschulfunk on the national radio station $Deutsche Welle^{35}$ in 1931, Cassirer described this tendency by relying again on Planck's authority:

According to Planck's account, the entire history of physics appears to be nothing other than the constant, more or less conscious, struggle for this goal; as a single coherent process of progressive unification [...] physicists are not satisfied with the immediate sensory experience, but instead design their own model, an ideal schema of the knowledge of reality. [...] Older physics devised a separate model for each of the different areas, and it was often satisfied with simply placing these often very different models side by side and lining them up. However, the further science progressed, the more this mere juxtaposition of images had to be abandoned. Instead of a physics of models, modern natural science became a physics of principles. Moreover, even these latter could not stand next to each other as an unrelated multiplicity. An attempt had to be made to understand them as emanations from a basic principle and to interpret them as its applications [...] The content of this highest physical principle of unity has, of course, been understood and determined differently in different epochs of natural science, depending on the state of empirical research. (Cassirer, 1931, 125f.; my emphasis)

Cassirer appreciated the fact that, in Planck's view, the unity of the *physical worldview* was not to be understood as a reduction of different branches of physics to one another, as their integration under common 'general principles.' The unifying principle has changed over time, but the general tendency of searching for progressively more general principles has remained constant (125). While the energy principle had dominated nineteenth-century physics, by 1915 Planck (1915a) had indicated the least action principle as a fundamental unifying principle that contained the energy principle as a special case. The least action principle would become Cassirer's go-to example of a 'principle' in the next decade. However, by that time Cassirer's attitude towards the role of 'principles' in physics had changed.

Up to the 1920s, Cassirer considered each of these unifying principles (*e.g.*, the energy principle) as a provisional candidate for a constitutive *a priori*, imposing a specific constraint on the possible laws of nature. By the end of the 1920s, Cassirer suggested—mostly in private correspondence—that only

³⁵The *Deutsche Welle* was intended to serve as the central German radio station, as opposed to the regional networks. The program *Hochschulfunk*, or radio university, was introduced in 1930 as a cultural program.

the quest for the unity of nature in general (rather than for a specific unifying principle) should be appropriately considered *a priori* (Cassirer to Schlick, Oct. 23, 1920; Cassirer, 1995–, Vol. 18, Doc. 88). However, he did not fully embrace this position. As I have shown in a separate paper [citation removed], Cassirer attempted to solve this ambiguity in the 1930s, in his last major book on the philosophy of science, which was dedicated to the new quantum theory (Cassirer, 1936b). On the one hand, Cassirer for the first time introduced 'statements of principle' as a separate class of statements but deprived them of their *a priori* status (60; tr. 1956, 52f.). On the other hand, Cassirer transformed the *a priori* into a *regulative* principle that motivates the search for the laws of nature without imposing any specific constraints on them (Cassirer, 1936b, 75; tr. 1956, 60).

In this way, Cassirer seems to have settled for a sort of *motivational Kantianism*, which does not purport to search for the conditions without which physics would be impossible but only for the conditions without which physics would not be worth pursuing. One might rightly wonder whether Cassirer's mature stance still deserves to be called a form of 'Kantianism' (Ferrari, 2009). However, infamous 'isms' aside, Cassirer (quite isolated among twentieth-century professional philosophers) has the merit of having perceived the importance of the 'meta-character' of some statements in physics—an issue that has recently gained new momentum in contemporary philosophy of science (Lange, 2009, 2016). In particular, as this paper has tried to show, Cassirer was the only philosopher who realized that relativity theory was not a theory in the usual sense of the word, entailing individual laws of nature; it was a second-order theory that provided general constraints on all possible laws as long as space-time variables entered into their formulation.

References

Beller, Mara. 2000. "Kant's Impact on Einstein's Thought." In *Einstein: The Formative Years 1879–1909*, edited by Don Howard and John Stachel, 83–106. Boston: Birkhäuser.

Bevilacqua, Fabio. 1993. "Helmholtz's Ueber die Erhaltung der Kraft: The Emergence of a Theoretical Physicist." In Hermann Von Helmholtz and the Foundations of Nineteenth-century Science, edited by David Cahan, 291–333. Berkeley: University of California Press.

Biagioli, Francesca. 2016. Space, Number, and Geometry from Helmholtz to Cassirer. Berlin: Springer.

Bordoni, Stefano. 2017. When Historiography Met Epistemology: Sophisticated Histories and Philosophies of Science in French-speaking Countries in the Second Half of the Nineteenth Century. Leiden/Boston: Brill.

Born, Max. 1915. Dynamik der Kristallgitter. Leipzig: Teubner.

Brown, Harvey R. 2005. Physical Relativity: Space-time Structure from a Dynamical Perspective. Oxford: Clarendon.

Bucherer, Alfred Heinrich. 1908. "Messungen an Becquerelstrahlen: Die experimentelle Bestätigung der Lorentz-Einsteinschen Theorie." Physikalische Zeitschrift 9:755–762.

Buek, Otto. 1904. "Die Atomistik und Faradays Begriff der Materie." PhD diss., Universität Marburg.

_____. 1912. Michael Faradays System der Natur und seine begrifflichen Grundlagen. Berlin: B. Cassirer.

Cassirer, Ernst. 1899. "Descartes' Kritik der mathematischen und naturwissenschaftlichen Erkenntnis." PhD diss., Universität Marburg.

——. 1902. Leibniz' System in seinen wissenschaftlichen Grundlagen. Marburg: Elwert. Repr. in Cassirer, 1998–, Vol. 1. ——. 1904a. "[Footnontes and commentaries]." In Leibniz, 1904.

——. 1904b. "[Preface and Introductions]." In Leibniz, 1904.

_____. 1906a. Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit. Vol. 1. Berlin: B. Cassirer. Repr. in Cassirer, 1998-, Vol. 2.

_____. 1906b. Der kritische Idealismus und die Philosophie des 'gesunden Menschenverstandes'. Giessen: Töpelmann.

— 1906c. "Substanzbegriff und Funktionsbegriff." Probevorlesung an der Königlichen Friedrich-Wilhelms-Universität in Berlin (26. Juli 1906). Repr. in Cassirer, 1995–, Vol. 8, 3–16.

Cassirer, Ernst. 1907. Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit. Vol. 2. Berlin: B. Cassirer. Repr. in Cassirer, 1998–, Vol. 3.

———. 1909. "Review of Hönigswald, Beitraege zur Erkenntnistheorie und Methodenlehre [Hönigswald, 1906]." Kant-Studien 14:91–98. Repr. in Cassirer, 1998–, Vol. 9, 447-459.

——. 1910. Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik. Berlin: Bruno Cassirer. Repr. in Vol. 6.

— 1912. "Hermann Cohen und die Erneuerung der Kantischen Philosophie." Kant-Studien 17:252–273. Repr. in Vol. 9, 119–138.

——. 1916. Freiheit und Form: Studien zur Deutschen Geistesgeschichte. Berlin: B. Cassirer. Repr. in Vol. 7.

. 1918a. Kants Leben und Lehre. Berlin: B. Cassirer. Repr. in Vol. 8.

———. "Zur Lehre Hermann Cohens." 4. April 1918 Berliner Tageblatt und Handels-Zeitung. April. 11, 1918. Repr. in Vol. 9, 494–497.

— 1920–21. "[Die philosophischen Probleme der Relativitätstheorie]." 13 lectures at the University of Hamburg October 13, 1920 to January 26, 1921. Publ. in Cassirer, 1995–, Vol. 8, 29–116.

—. 1920a. Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit: Die nachkantischen Systeme. Vol. 3. Berlin: B. Cassirer. Repr. in Cassirer, 1998–, Vol. 3.

—. 1920b. "Hermann Cohen." Korrespondenzblatt des Vereins zur Gründung und Erhaltung einer Akademie für die Wissenschaft des Judentums 1:1–10. Repr. in 9, 498–509.

____. 1920c. "Philosophische Probleme der Relativitätstheorie." Die Neue Rundschau 31:1337–1357. Repr. in Vol. 9.

——. 1921a. Idee und Gestalt: Goethe/Schiller/Hölderlin/Kleist. Fünf Aufsätze. Berlin: B. Cassirer. Repr. in Vol. 9, 241-435.

——. 1921b. Zur Einstein'schen Relativitätstheorie: Erkenntnistheoretische Betrachtungen. Berlin: B. Cassirer. Repr. in Vol. 10.

—. 1922. "Die Begriffsform im mythischen Denken." In *Studien der Bibliothek Warburg*, vol. 1. Leipzig: Teubner. Repr. in Vol. 16, 2–73.

—. 1923a. Philosophie der symbolischen Formen. Erster Teil: Die Sprache. Berlin: B. Cassirer. Repr. in 11.

——. 1923b. Substance and Function, and Einstein's Theory of Relativity. Edited by Marie Taylor Swabey. Chicago, London: Open Court.

—. 1925. Philosophie der symbolischen Formen. Zweiter Teil: Das mythische Denken. Berlin: B. Cassirer. Repr. in 12.

——. "Von Hermann Cohens geistigem Erbe." Almanach des Verlages Bruno Cassirer. 1926. Repr. in Vol. 16, 480–486.
 ——. 1927a. "Das Symbolproblem und seine Stellung im System der Philosophie." Zeitschrift für Ästhetik und allgemeine Kunstwissenschaft 21:295–322. Repr. in Vol. 17, 253–282.

——. 1927b. "Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie." In Jahrbücher der Philosophie. Eine kritische Übersicht der Philosophie der Gegenwart (3. Jahrgang), edited by Gemeinschaft mit zahlreichen Fachgenossen v. Willy Moog, 31–92.

—. 1929. Philosophie der symbolischen Formen. Dritter Teil: Phänomenologie der Erkenntnis. Berlin: B. Cassirer. Repr. in Vol. 13.

——. 1931. "Die Einheit der Wissenschaft." Two lectures given on October 21 and October 28, 1931 for the program *Hochsculfunk* aired by the radio station *Deutsche Welle*. Pub. in Cassirer, 1995–, Vol. 8, 117–134.

— 1936b. "Determinismus und Indeterminismus in der modernen Physik: Historische und systematische Studien zum Kausalproblem." *Göteborgs Högskolas Årsskrift* 42. Repr. in Cassirer, 1998–, Vol. 19.

——. 1943. "Hermann Cohen, 1842-1918." Social Research: An International Quarterly of Political and Social Science 10:219–232. Repr. in Vol. 24, 161–173.

——. 1956. Determinism and Indeterminism in Modern physics: Historical and Systematic Studies of the Problem of Causality. New Haven: Yale University Press.

———. 1957. The Philosophy of Symbolic Forms: Volume Three: The Phenomenology of Knowledge. With an introduction by Charles W. Hendel. New York: Yale University Press.

_____. 1995–. Nachgelassene Manuskripte und Texte. Edited by John Michael Krois. 18 vols. Hamburg: Meiner.

------. 1998-. Gesammelte Werke: Hamburger Ausgabe. Edited by Birgit Recki. 26 vols. Hamburg: Meiner.

Cassirer Bondy, Toni. 1981. Mein Leben mit Ernst Cassirer. Hildesheim: Gerstenberg.

Cohen, Hermann. 1883. Das Princip der Infinitesimal-Methode und seine Geschichte: Ein Kapitel zur Grundlegung der Erkenntnisskritik. Berlin: Dümmler. Repr. in Cohen, 1977–, Vol. 5/I.

. 1885. Kants Theorie der Erfahrung. 2nd ed. Berlin: Dümmler. Repr. in Vol. 1/I.

. 1902. Logik der reinen Erkenntnis. Berlin: B. Cassirer. Repr. in Vol. 6.

———. 1977–. Werke. Edited by Helmut Holzhey. 15 vols. Hildesheim: Olms.

Corry, Leo. 2010. "Hermann Minkowski, Relativity and the Axiomatic." In *Minkowski Spacetime: A Hundred Years Later*, edited by Vesselin Petkov, 3–41. Dordrecht et al.: Springer.

Darrigol, Olivier. 1995. "Henri Poincaré's Criticism of fin De Siécle Electrodynamics." Studies in History and Philosophy of Science. Part B: Studies in History and Philosophy of Modern Physics 26:1–44.

- Deltete, Robert J. 1999. "Helm and Boltzmann: Energetics at the Lübeck Naturforscherversammlung." Synthese 119:45.
- Duhem, Pierre. 1903. L'évolution de la mécanique. Paris: Joanin.

Dühring, Eugen. 1873. Kritische Geschichte der allgemeinen Principien der Mechanik. Berlin: Theobald Grieben.

Einstein, Albert. 1905. "Zur Elektrodynamik bewegter Körper." Annalen der Physik, 4th ser., 17:891–921. Repr. in Einstein, 1987–, Vol. 2, Doc. 23.

. 1914. "Zum Relativitäts-Problem." Scientia 15:337–348. Repr. in Vol. 4, Doc. 31.

Einstein, Albert. 1916. "Ernst Mach." Physikalische Zeitschrift 17:101–104. Repr. in Vol. 6, Doc. 29.

. 1917. Über die spezielle und die allgemeine Relativitätstheorie (gemeinverständlich). Braunschweig: Vieweg. Repr. in Vol. 5, Doc. 42.

— 1918a. "Dialog über Einwände gegen die Relativitätstheorie." Die Naturwissenschaften 6:697–702. Repr. in Vol. 7, Doc. 13.

_____. 1918b. "Prinzipielles zur allgemeinen Relativitätstheorie." Annalen der Physik, 4th ser., 55:241–244. Repr. in Vol. 7, Doc. 4.

——. 1919a. "Was ist Relativitäts-Theorie." November 28, 1919. The original German manuscript is transcribed in Vol. 7, Doc. 25.

. "What is the theory of relativity?" The Times. November. 28, 1919. Repr. in Vol. 7, Doc. 25.

— . 1920a. "Grundgedanken und Methoden der Relativitätstheorie, in ihrer Entwicklung dargestellt." January 22, 1920. Unpublished 35-page draft of an article for *Nature*. Published in Einstein, 1987–, Vol. 7, Doc. 31.

. "Meine Antwort: Über die anti-relativitätstheoretische G.m.b.H." *Berliner Tageblatt.* August. 7, 1920. Repr. in Vol. 7, Doc. 45.

. 1924. "Review of Elsbach, Kant und Einstein [Elsbach, 1924]." Deutsche Literaturzeitung 45:1685–1692. Repr. in Einstein, 1987–, Vol. 14, Doc. 321.

—. 1928. "A propos de 'La Déduction Relativiste' de M. Émile Meyerson [Meyerson, 1925]." Revue philosophique de la France et de l'étranger 105:161–166. Repr. in Einstein, 1987–, Vol. 16, Doc. 152.

—. 1987–. The Collected Papers of Albert Einstein. Edited by John Stachel et al. 15 vols. Princeton: Princeton University Press.

- Elsbach, Alfred Coppel. 1924. Kant und Einstein: Untersuchungen über das Verhältnis der modernen Erkenntnistheorie zur Relativitätstheorie. Berlin: de Gruyter.
- Ferrari, Massimo. 1988. Il giovane Cassirer e la scuola di Marburgo. Milan: Angeli.
- ——. 1994. Cassirer, Schlick und die Relativitätstheorie: Ein Beitrag zur Analyse des Verhältnisses von Neukantianismus und Neopositivismus. In Neukantianismus. Perspektiven und Probleme, edited by Ernst Wolfgang Orth and Helmhut Holzhey, 418–441. Würzburg: Königshausen & Neumann.

—. 1996. Ernst Cassirer: Dalla scuola di Marburgo alla filosofia della cultura. Florence: Olschki.

- 2009. "Is Cassirer a Neo-Kantian Methodologically Speaking?" In *Neo-Kantianism in Contemporary Philosophy*, edited by Rudolf A. Makkreel and Sebastian Luft, 293–312. Bloomington: Indiana University Press.
- Friedman, Michael. 2001. Dynamics of Reason: The 1999 Kant Lectures at Stanford University. Stanford, Calif.: CSLI Publ.
 2005. "Ernst Cassirer and the Philosophy of Science." In Continental Philosophy of Science, edited by Gary Gutting, 71–83. Oxford: Oxford University Press.
 - ——. 2008. "Ernst Cassirer and Thomas Kuhn: The neo-Kantian Tradition in History and Philosophy of Science." Philosophical Forum 39:239–252.
- Frisch, Mathias. 2005. "Mechanisms, Principles, and Lorentz's Cautious Realism." Studies in History and Philosophy of Science. Part B: Studies in History and Philosophy of Modern Physics 36:659–679.
- Giedymin, Jerzy. 1982. "The Physics of the Principles and Its Philosophy: Hamilton, Poincaré and Ramsey." In Science and Convention: Essay on Henri Poincaré's Philosophy of Science and the Conventionalist Tradition, 42–89. Oxford: Pergamon Press.
- Giovanelli, Marco. 2020. "Like Thermodynamics before Boltzmann': On Einstein's Distinction between Constructive and Principle Theories." Studies in History and Philosophy of Science. Part B: Studies in History and Philosophy of Modern Physics 71:118–157.
 - ——. 2022. "Motivational Kantianism: Cassirer's Late Shift Towards a Regulative Conception of the a Priori." Studies in History and Philosophy of Science 95:118–125.
- Goethe, Johann Wolfgang von, and Carl Friedrich Zelter. 1832. Briefwechsel zwischen Goethe und Zelter in den Jahren 1799 bis 1832. Edited by Ludwig Geiger. Leipzig: Reclam.
- Heis, Jeremy. 2014a. "Ernst Cassirer's Substanzbegriff und Funktionsbegriff." HOPOS: The Journal of the International Society for the History of Philosophy of Science 4:241–270.
- ——. 2014b. "Realism, Functions, and the a priori: Ernst Cassirer's Philosophy of Science." Studies in History and Philosophy of Science 48:10–19.
- Helm, Georg Ferdinand. 1887. Lehre von der Energie: Historisch-kritisch entwickelt. Nebst Beiträgen zu einer allgemeinen Energetik. Felix.
 - ——. 1898. Die Energetik nach ihrer geschichtlichen Entwickelung. Leipzig: Veit.
- Helmholtz, Hermann von. 1847. Über die Erhaltung der Kraft: Eine physikalische Abhandlung. Berlin: Reimer.
- Hentschel, Klaus. 1990a. Interpretationen und Fehlinterpretationen der speziellen und der allgemeinen Relativitätstheorie durch Zeitgenossen Albert Einsteins. Basel et al.: Birkhäuser.
- ——. 1990b. "Philosophical Interpretations of Relativity Theory: 1910-1930." PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association 2:169–179.
- Hiebert, Erwin N. 1962. *Historical Roots of the Principle of Conservation of Energy*. University of Wisconsin, Madison: The State Historical Society of Wisconsin for The Department of History.
- Hilbert, David. 1921. "Natur und mathematisches Erkennen." Vortrag März 1921, Kopenhagen. In Hilbert, 2009, 382–392. ______. 1923. "Grundsätzliche Fragen der modernen Physik." In Hilbert, 2009, 396–432.
- ———. 2009. David Hilbert's Lectures on the Foundations of Physics 1915-1927. Edited by Tilman Sauer and Ulrich Majer. Berlin/Heidelberg: Springer.
- Holzhey, Helmut. 1986. Cohen und Natorp. Vol. 2. Basel: Schwabe.
- Hönigswald, Richard. 1906. Beitraege zur Erkenntnistheorie und Methodenlehre. Leipzig: Fock.
- Janssen, Michel. 2009. "Drawing the Line between Kinematics and Dynamics in Special Relativity." Studies in History and Philosophy of Science. Part B: Studies in History and Philosophy of Modern Physics 40:26–52.
- Kaufmann, Walter. 1906. "Über die Konstitution des Elektrons." Annalen der Physik, 4th ser., 18:487–553.
- Kennefick, Daniel. 2009. "Testing Relativity from the 1919 Eclipse—a Question of Bias." Physics Today 62:37–42.
- Klein, Felix. 1872. Vergleichende Betrachtungen über neuere geometrische Forschungen. Erlangen: Verlag von Andreas
- Deichert. Repr. in Klein, 1921, , I, 460–497. Republished as: "Vergleichende Betrachtungen über neuere geometrische Forschungen." Mathematische Annalen 43 (1893): 63–100.
- . 1921. Gesammelte mathematische Abhandlungen. Springer, Berlin.
- Kretschmann, Erich. 1917. "Über den physikalischen Sinn der Relativitätspostulate: A. Einsteins neue und seine ursprüngliche Relativitätstheorie." Annalen der Physik, 4th ser., 53:575–614.
- Lange, Marc. 2009. Laws and Lawmakers: Science, Metaphysics, and the Laws of Nature. Oxford/New York: Oxford University Press.
- ——. 2016. Because without Cause: Non-causal Explanations in Science and Mathematics. New York: Oxford University Press.
- Laue, Max. 1911a. "Bemerkungen zum Hebelgesetz in der Relativitätstheorie." Physikalische Zeitschrift 12:1008–1010. Repr. in M. v. Laue, 1961, Vol. 1, Doc. 12.
 - —. 1911b. Das Relativitätsprinzip. Braunschweig: Vieweg.
- ——. 1911c. "Zur Diskussion über den starren Körper in der Relativitätstheorie." *Physikalische Zeitschrift* 12:85–87. Repr. in Vol. 1, Doc. 9.
- —. 1913. "Das Relativitätsprinzip." Jahrbücher der Philosophie 1:99–128. Repr. in Vol. 1, Doc. 1.
- Laue, Max von. 1921. Kant-Studien 26.
- _____. 1961. Gesammelte Schriften und Vorträge. 3 vols. Braunschweig: Vieweg.
- Leibniz, Gottfried Wilhelm. 1850. Leibnizens mathematische Schriften. Edited by Carl Immanuel Gerhardt. 7 vols. Halle: Schmidt.

——. 1904. Hauptschriften zur Grundlegung der Philosophie. Edited by Ernst Cassirer. Translated by Arthur Buchenau. Vol. I: Hamburg: Meiner.

Liu, Chuang. 1997. Planck and the Special Theory of Relativity. Proceedings of a conference commemorating a century of physics, berlin, 22 - 24 march 1995 - università degli Studi di Pavia. In The Emergence of Modern Physics, edited by Roger H. Stuwer, Fabio Bevilacqua, and Dieter Hoffmann, 287–296. Pavia: La Goliardica Pavese.

Lorentz, Hendrik Antoon. 1900. "Elektromagnetische Theorien physikalischer Erscheinungen." Rektoratsrege, gehalten zur Feier des 325. Jahrestages der Universität Leyden am 8. Februar 1900. Physikalische Zeitschrift 1:498–501, 514–519.
 ——. 1903–4. "Electromagnetic Phenomena in a System Moving with Any Velocity Smaller Than That of Light."

Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam 6:809–831. ——. "Ergebnisse und Probleme der Elektronentheorie." Vortrag, gehalten am 20. Dezember 1904 im Elektrotechnischen

Verein zu Berlin. Elektrotechnische Zeitschrift, 555–560. Repr. in Lorentz, 1935–39, Vol. 8, Doc. 76–124.

—. 1935–39. Collected Papers. Edited by Pieter Zeeman and Adriaan Daniël Fokker. 9 vols. The Hague: Nijhoff.

Mach, Ernst. 1872. Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit [in German]. Prag: J.G. Calve. Mayer, Robert Julius von. 1842. "Bemerkungen über die Kräfte der unbelebten Natur." Annalen der Chemie und Pharmacie 43:233–240.

Meyerson, Émile. 1925. La déduction relativiste. Paris: Payot.

Minkowski, Hermann. "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern." Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse, 53–111.

— 1909. "Raum und Zeit." Jahresberichte der Deutschen Mathematiker-Vereinigung 18:75–88. Repr. in Minkowski, 1911, Vol. 2, 431–446. Also pub. in: Physikalische Zeitschrift 10, 1909, 104–111.

_____. 1911. Gesammelte Abhandlungen. Edited by David Hilbert, Andreas Speiser, and Hermann Weyl. Leipzig: Teubner. _____. 1915. "Das Relativitätsprinzip." Annalen der Physik, 4th ser., 47:927–938.

Natorp, Paul. 1882a. Descartes' Erkenntnisstheorie: Eine Studie zur Vorgeschichte des Kriticismus. Marburg: Elwert.

- ——. 1882b. "Die kosmologische Reform des Kopernikus in ihrer Bedeutung für die Philosophie." *Preussische Jahrbücher* 71:355–375.
- . 1882c. "Galilei als Philosoph: Eine Skizze." *Philosophische Monatshefte* 18:193–229.

—. 1910. Die logischen Grundlagen der exakten Wissenschaften. Leipzig Berlin: Teubner.

_____. 1912. "Kant und die Marburger Schule." Kant-Studien 17:193–221.

Nelson, Leonard. 1905. "Review of Cohen, Logik der reinen Erkenntnis [Cohen, 1902]." Göttingische gelehrte Anzeigen 8:610–630.

Ostwald, Wilhelm. 1902. Vorlesungen über Naturphilosophie: Gehalten im Sommer 1901 an der Univ. Leipzig. Leipzig: Veit. Padovani, Flavia. 2009. "Relativizing the Relativized a Priori: Reichenbach's Axioms of Coordination Divided." Synthese 181:41–62.

Planck, Max. 1887. Das Princip der Erhaltung der Energie. Leipzig: Teubner.

. 1909. "Die Einheit des Physikalischen Weltbildes." Vortrag gehalten am 9.12 .1908 in Leiden. *Physikalische Zeitschrift* 10:62–75. *Die Einheit des Physikalischen Weltbildes.* Vortrag gehalten am 9.12 .1908 in Leiden. Leipzig: Hirzel, 1909.

———. 1910. Die Stellung der neueren Physik zur mechanischen Naturanschauung. Vortrag gehalten am 23. September 1910 auf der 82. Versammlung deutscher Naturforscher und Ärzte in Königsberg. Leibzig: Hirzel. Physikalische Rundblicke. Leipzig: Hirzel, 1922.

——. 1915b. Das Verhältnis der Theorien zueinander. In Die Kultur der Gegenwart: Ihre Entwicklung und ihre Ziele, edited by Paul Hinneberg, Part 3, Sec. 1, Vol. 3: Physik. Ed. by Emil Warburg, 714–731. Leipzig: Teubner.

Poincaré, Henri. 1904. "L'état actuel et l'avenir de la physique mathématique." Conférence lue le 24 Septembre 1904 au Congrés d'art et de Science de Saint-Louis. Bulletin des sciences mathématiques 28:302–324. Repr. in: La valeur de la science. Paris: Flammarion, 1905.

—. 1905. The principles of mathematical physics. In Congress of Arts and Science, Universal Exposition, St. Louis, 1904, edited by Howard J. Rogers, vol. 1: Philosophy and Mathematics, 604–622. Houghton: Mifflin & Co.

Rankine, William John Macquorn. 1855. "Outlines of the Science of Energetics." Proceedings of the Royal Philosophical Society of Glasgow 3:381–399.

Reich, Karin. 2000. "Einsteins Vortrag über Relativitätstheorie an der Universität Hamburg am 17.7.1920. Vorgeschichte, Folgen." Mitteilungen der Mathematischen Gesellschaft in Hamburg 19:51–68.

Reichenbach, Hans. 1920. "Die Einsteinsche Raumlehre." Die Umschau 24:402-405.

Ryckman, Thomas. 1991. "Conditio Sine Qua Non? Zuordnung in the Early Epistemologies of Cassirer and Schlick." Synthese 88:57–95.

——. 2003. Two Roads from Kant: Cassirer, Reichenbach and General Relativity. In Logical Empiricism: Historical & Contemporary Perspectives, edited by Paolo Parrini, Wesley C. Salmon, and Merrilee H. Salmon, 159–193. Pittsburgh, Pa.: University of Pittsburgh Press.

—. 2005. The Reign of Relativity: Philosophy in Physics 1915-1925. Oxford/New York: Oxford University Press.

. 2008. "Invariance Principles As Regulative Ideals: From Wigner to Hilbert." *Royal Institute of Philosophy Supplements* 63:63–80.

Schlick, Moritz. 1921. "Kritizistische oder empiristische Deutung der neuen Physik? Bemerkungen zu Ernst Cassirers Buch: Zur Einsteinschen Relativitätstheorie." Kant-Studien 26:96–111.

Seth, Suman. 2010. Crafting the Quantum: Arnold Sommerfeld and the Practice of Theory, 1890–1926. Cambridge: MIT Press.

Sieg, Ulrich. 1994. Aufstieg und Niedergang des Marburger Neukantianismus: Die Geschichte einer philosophischen Schulgemeinschaft. Würzburg: Königshausen und Neumann.

Sommerfeld, Arnold. 1915. "Review of Born, Dynamik der Kristallgitter [Born, 1915]." Die Naturwissenschaften 50:669–670. Stadler, August. 1874. Kants Teleologie und ihre erkenntnisstheoretische Bedeutung: Eine Untersuchung. Berlin: Dümmler. The Albert Einstein Archives at the Hebrew University of Jerusalem.

- Weyl, Hermann. 1954. "Address on the Unity of Knowledge delivered at the Bicentennial Conference of Columbia University." Repr. in Weyl, 1968, Vol. 4, Doc. 165.
 - _____. 1968. Gesammelte Abhandlungen. Edited by Komaravolu Chandrasekharan. 4 vols. Berlin: Springer.

Wigner, Eugene Paul. 1949. "Invariance in Physical Theory." In "Theory of Relativity in Contemporary Science. Papers Read at the Celebration of the Seventieth Birthday of Professor Albert Einstein in Princeton." Proceedings of the American Philosophical Society 93:521–526.

Wilson, Mark. 2017. Physics Avoidance: And Other Essays in Conceptual Strategy. Oxford: Oxford University Press.