Infinite Scale Scepticism:
Probing the Epistemology of the Limit of Infinite Degrees of Freedom and Hilbert Space Non-Uniqueness

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Abstract

There has been much debate about how the semantic content of quantum theories with infinitely many degrees of freedom ought to be understood. This paper focuses instead on the epistemological status of the limit of infinite degrees of freedom. Do we have strong grounds to believe that the actual world is an infinite quantum system? I consider two types of argument for thinking that we do: indispensability arguments, which appeal to the role that unitarily inequivalent Hilbert space representations play in accounting for physical phenomena like spontaneous symmetry breaking, and what I call extrapolation arguments, which support the physical significance of the limit of infinite degrees of freedom by pointing to infinite volume cosmological models. Neither of these strategies turn out to be conclusive as they stand leaving the relevance of novel features of infinite quantum systems to real-world physics murky. I go on to discuss ways of motivating foundational work on infinite quantum systems which do not turn on the representational veracity of these features, and point out that when it comes to the question of the epistemological status of unitarily inequivalent Hilbert space representations other sources of non-uniqueness than the limit of infinite degrees of freedom must also be considered.

1 Introduction

The appearance of unitarily inequivalent Hilbert space representations in the limit of infinite degrees of freedom has sometimes been characterized as the central novel philosophical problem raised by quantum field theory (QFT).[1] In non-relativistic quantum mechanics, we are used to the idea that we can use different Hilbert spaces to represent the same quantum system: we can work with a Hilbert space of complex vectors, as in Heisenberg's matrix formulation of the theory, for instance, or a Hilbert space of complex-valued functions, as in Schrödinger's wavefunction formulation. These mathematical structures are typically taken to express the same physical content because there is a unitary transformation that takes operators in one Hilbert space to operators in the other. A result known as the

[1] (Baker [2016]) and (Ruetsche [2012a], [2012b]) are reviews of the philosophy of QFT that put this issue front and centre.

Stone-von Neumann theorem assures us that, for a wide class of salient systems, all sets of Hilbert space operators satisfying the relevant canonical commutation relations are unitarily equivalent in this sense.

This theorem fails to apply in the limit of infinite degrees of freedom, however. The algebraic approach to quantum theory allows for a smooth generalization to this more exotic class of systems. In this framework, a quantum system is associated with an abstract C^*-algebra of observables which admits Hilbert space representations—maps from elements of the algebra to Hilbert space operators. For a system covered by the Stone-von Neumann theorem, all representations of its C^*-algebra are unitarily equivalent, but systems with infinitely many degrees of freedom (henceforth, infinite systems for short) generically have C^*-algebras which admit multiple unitarily inequivalent representations (henceforth, inequivalent representations for short).

Standard axiomatic formulations of QFT attribute infinitely many degrees of freedom to both the short-distance (or ultraviolet) structure of the theory, since QFTs are taken to have infinitely many degrees of freedom in any finite space-time region, and its long-distance (or infrared) structure, as Minkowski space-time is infinite in volume. This gives rise to an uncountable infinity of inequivalent representations, raising a gamut of interpretive questions about the physical content of QFTs thus characterized. Do these inequivalent representations correspond to distinct statements of a theory's physical content and if so how do we choose between them? Should we instead identify the physical content of a theory with its abstract C^*-algebra? Or is an entirely new approach to theory semantics needed here? A growing philosophical literature grapples with such questions—some prominent entries being (Ruetsche [2011]), (Feintzeig [2018]) and (Lupher [2018]). Inequivalent representations afforded by the limit of infinite degrees of freedom have also been used to draw substantive conclusions about the structure and interpretation of QFT: they feature centrally in Doreen Fraser's ([2008]) arguments against a particle interpretation of the theory, Baker's ([2009]) arguments against a field configuration interpretation, and Baker and Halvorson's ([2010]) analysis of the nature of anti-matter (again, to name a few prominent examples).

Much of this work has focused on what I would label semantic questions: How should the physical content of QFT—as characterized by standard algebraic axiomatizations of the theory—be understood? What would the world be like if such a theory were true? This paper is instead concerned with epistemological questions: What credence should we assign to the claim that the actual world is a quantum system with infinitely many degrees of freedom? Can we claim to know that this is the case? How are the projects of figuring out the physical content of theories which exhibit inequivalent representations and figuring out what the actual world is like related to each other?

The discussion proceeds as follows. I start, in section 2, by giving some prima facia reasons to worry about the evidential support for QFT's theoretical claims about arbitrarily small and arbitrarily large length scales, which I collectively dub *infinite scales*. This motivates the need for additional arguments for believing that the world is in fact an infinite system, and sections that follow analyse two such arguments: indispensability arguments, which appeal to the role that inequivalent representations afforded by the limit play in accounting for observed physical phenomena (section 3); and extrapolation arguments, which support the existence of infrared inequivalent representations by pointing to infinite volume cosmological models (section 4). I find both of these strategies to be inconclusive as they stand (while flagging avenues for potential further development). My own view, therefore, is that we are not presently justified in assigning very high credence to theoretical claims about infinite scales, and specifically to the claim that the world has infinitely many quantum degrees of freedom. I call the view that we presently lack knowledge of the world's infinite scale structure, *Weak Infinite Scale Scepticism*.[2]

Section 5 consists of two independent lines of reflection on what this position implies, or rather does not imply. 5.1 stresses that it does not follow that philosophical engagement with infinite quantum systems has no epistemic value. I consider a number of ways that foundational work on infinite systems can be understood as epistemically fruitful which do not turn on the representational status of their novel features. 5.2 highlights an important limitation of my discussion, partially excused by the fact that it is also a lacuna of much of the extant philosophical literature on inequivalent representations. While the questions of whether the fundamental theory of our world has infinitely many quantum degrees of freedom and whether it admits of unitarily inequivalent Hilbert space representations are closely related, they are not, in fact, identical. I point out that inequivalent representations can arise in systems with finitely many degrees of freedom and suggest that these alternative sources of non-uniqueness need to be examined more carefully before the bearing of Hilbert space non-uniqueness on the physical structure of the actual world can be fully assessed.

The overarching goal of the paper is to map out the key epistemological issues surrounding the limit of infinite degrees of freedom and the breakdown of Hilbert space uniqueness. Section 7 concludes by surveying this territory, emphasising the relevance of these epistemological issues to methodological questions about how philosophical and foundational work in this area ought to proceed.

2 Epistemological Worries about Infinite Scales

I should begin by distinguishing the topic of this paper from another epistemological issue which has been discussed in the philosophical literature on inequivalent representations. The fact that quantum systems with infinitely many degrees of freedom lack a unique Hilbert space quantization has sometimes been taken to

[2] In this paper I speak interchangeably of knowing P and having epistemic justification to assign very high credence to P (along with other looser constructions). I take it that finer distinctions which can be drawn between these notions are not crucial in the present context.

pose a problem for scientific realism.[3] After all, if inequivalent Hilbert space representations are taken to correspond to distinct quantum theories then we seem to have a rather severe case of underdetermination on our hands. Ruetsche's ([2011]) book on interpretative issues raised by infinite quantum systems develops a more sophisticated version of this sort of challenge. She distinguishes a spectrum of interpretive options opened up by the non-uniqueness of a C*-algebra's Hilbert space representation and argues that these conflicting interpretations are implicated in different explanatory applications of infinite quantum theories. Consequently, according to Ruetsche, we are not in a position to believe what any single interpretation says about the world, problematizing many formulations of scientific realism.

Do we have good reasons to believe that the world is described by a theory which exhibits inequivalent representations in the first place, however? We need to address this question, it seems, before asking whether the breakdown of Hilbert space uniqueness has further epistemological implications. Since the philosophical literature on inequivalent representations has focused on the limit of infinite degrees of freedom we are naturally led to investigate the epistemological status of QFT's claims about infinite scales: arbitrarily small length scales (the extreme ultraviolet) and arbitrarily large length scales (the extreme infrared). In section 5.2 I circle back to the fact that inequivalent representations can arise in finite systems, problematising an exclusive focus on the limit of infinite degrees of freedom. Still, the epistemology of infinite scales is clearly an important part of the inequivalent representations puzzle, as well as having its own independent significance. Many interpretative issues in QFT depend on our understanding of the theories extreme ultraviolet and infrared claims, and big questions about the possibility of scientific knowledge of "the fundamental" also loom large here.

Do we have good reason to believe that the world has infinitely many quantum degrees of freedom then? There might seem to be a straightforward route to an affirmative answer: our best theories of matter are QFTs, QFTs are infinite quantum systems, therefore we have strong grounds to believe that the world is an infinite quantum system. My primary goal in this section is to discredit this naive argument and thus motivate the need for more careful attempts.

2.1 Lack of Positive Support: Scale Separation

The naive argument fails because the empirical success and community acceptance of a scientific theory does not imply that everything it says about the world is evidentially supported. This is most obvious in the case of deliberate idealizations. In the context of statistical mechanics, there has been much debate about the limit of infinite degrees of freedom owing to its (apparent) status as an idealization (a point we will discuss further in section 3). This is sometimes contrasted with the QFT context—while a tabletop ferromagnet appears to be finite, quantum fields

[3] See (Wallace [2006]) and (French [2011]) for discussions of inequivalent representations which focus on the apparent challenge posed to scientific (and specifically structural) realism.

actually are infinite systems so there is no reason to question a veridical reading of the limit of infinite degrees of freedom, or so the thought goes. As I pointed out in the introduction, however, infinitely many degrees of freedom arise from assumptions about infinite scales, and the epistemic status of these assumptions can be problematized in a number of ways.[4]

On the one hand, external conflicts with other theories, and internal problems within the formalism itself, have been cited as reasons to reduce our confidence in QFT's claims about infinite scales—or, at least, the extreme ultraviolet. I will return to these points in 2.2 and 2.3 (highlighting the existence of neglected infrared problems as well). I want to start, however, by considering what I take to be an even more pressing issue: the difficulty of providing positive support for claims about infinite scales arising from the phenomenon of scale separation.

Do claims about the extreme ultraviolet and extreme infrared actually receive positive support from the predictive successes of the standard model? The first point to acknowledge here is that such statements cannot be directly verified or falsified by experiments. Infinite scales are not just beyond the reach of current technology, they cannot be directly probed empirically. The smallest length scales we currently have experimental access to, via the large hadron collider, are of the order of 10^{-20}m. Probing arbitrarily small length scales would seem to require observing scattering events of arbitrarily high energy, per impossible. Meanwhile, the Hubble and Webb telescopes let us see out to length scales of the order 10^{26}m. In addition to the finite diameter (and therefore angular resolution) of physically possible telescopes, the finite size of the cosmological horizon is taken to put a cap on the largest length scales which we can ever directly observe. Infinite scales are infinitely far away from the finite scales on which any possible observations can take place and are thus unobservable in a rather radical sense.

Could claims about the extreme ultraviolet and infrared be confirmed indirectly? Considerations of the separation of scales—the formation of quasi-autonomous regimes which are highly insensitive to the details of much smaller and much larger length-scale physics—seem to render the prospects bleak. Renormalization group arguments demonstrate that QFTs with very different ultraviolet structures can display very similar lower energy, long-distance, behaviour. More specifically, we can impose an ultraviolet cutoff on the momentum modes of a QFT model without undermining its predictions about scattering on much lower momentum scales. One way to achieve this is to work on a discrete lattice so that QFT degrees of freedom are not assigned to regions smaller than the lattice spacing, l. At some length scale, $x \gg l$, the detectable effects of the ultraviolet cutoff are expected to die off faster than l/x. While not as systematically supported by renormalization group treatments, similar statements are believed to hold for

[4]While scepticism about the extreme ultraviolet has been advanced in the philosophical literature on QFT for some time (Wallace [2006, 2011]; Williams [2019]; James Fraser [2018, 2020]) the infrared case is comparatively undertheorised. I attempt to redress this imbalance here, emphasising some commonalities in the epistemological issues associated with the extreme ultraviolet and extreme infrared, while also acknowledging important differences.

the infrared. Imposing an infrared cutoff, by putting a QFT in a suitably large box of length L, is not expected to make a detectable difference to the theory's predictions about high energy particle scattering. Analogously to the case of the ultraviolet cutoff, finite size effects are typically expected to die off faster than x/L, for $x \ll L$.[5]

In truth, these points are rather preliminary and a great deal more remains to be said about scale separation in QFT (and beyond). Still, these statements about the insensitivity of the mid-scale structure of QFT models to the imposition of ultraviolet and infrared cutoffs are sufficient to raise serious doubts about the positive support for claims about infinite scales. In recent decades philosophers of science have moved towards selective theories of confirmation, according to which only some parts of a theory's semantic content gain support from its predictive successes.[6] This allows us to accommodate the complex attitudes that scientists typically take towards successful models, viewing some of their content as speculative or flagrantly idealized. The basic idea driving selective confirmation theories is that it is only the parts of a model which contribute to, or make a difference to, its experimentally confirmed predictions which receive support from them. The worry then is that a QFT's claims about the smallest and largest length scale structure of the world do not appear to be connected up with its empirical successes in the required way. We can vary a QFT model's structure on these scales—by imposing cutoffs, and by varying the fundamental dynamics—without affecting their agreement with observations made at finite scales with finite levels of precision. We are thus led towards an agnostic attitude towards QFT's claims about infinite scales, and therefore an agnostic response to the question of whether or not the world has finitely many or infinitely many degrees of freedom.

2.2 External Conflict: Gravity Problems

This scale separation argument, as vaguely sketched as it may be, seems to me to be the most important (and philosophically interesting) reason to worry about the epistemic status of the limit of infinite degrees of freedom. As I flagged above, however, there are other considerations which might lead us to reduce our confidence in QFT's extreme ultraviolet and infrared assumptions. The situation is much more complex than the statistical mechanics case, but there arguably is counterevidence of a kind against a naive representational reading of claims about infinite scales here too. I will not try to approximate a complete overview of the external and internal conflicts associated with the ultraviolet and infrared structure

[5] These statements about cutoff insensitivity are a key part of the background of Doreen Fraser's and David Wallace's debate about the formulation of QFT. See (Wallace [2006]) for a discussion of the properties of ultraviolet cutoff models and (Doreen Fraser [2009]) for a discussion of infrared cutoffs and finite-size effects. See, also, discussion in (James Fraser [2018] and Williams [2019]).

[6] This tendency originates in Kitcher's [1993] "working posits" and Psillos's [1999] "divide et impera" strategies in the context of the scientific realism debate. For an array of recent perspectives on selective confirmation see (Lyon and Vickers [2021]), and for application of these ideas to QFT see (James Fraser [2018]) and (Williams [2019]).

of QFT, instead emphasizing points which will become relevant later on.

The first cluster of issues to consider are external conflicts between QFT and other theories, especially gravitational theory. Here we should distinguish: i) considerations which license decreasing our credence in the extreme ultraviolet or infrared claims of particular QFT models, and ii) considerations which license decreasing our credence in the extreme ultraviolet or infrared assumptions of the QFT framework in general (encapsulated, at least provisionally, by the Haag-Kastler axioms).

When it comes to particular QFTs, the situation is fairly straightforward. The standard model of particle physics misses out gravitational interactions entirely, which are believed to become important at extremely short length scales as the Planck scale is approached.[7] The standard model also gets the long length scale physics of our world wrong. The Minkowski space-time structure assumed in conventional high energy physics models (and, indeed, by the Haag-Kastler axioms) is understood to be a local approximation to the curved space-time solutions of general relativity, which breaks down on large length scales. Consequently, the details of what empirically successful models like quantum electrodynamics (QED) and quantum chromodynamics (QCD), say about the extreme ultraviolet and infrared is not just unsupported but almost certainly false.

Whether intertheoretic conflict indicates that the QFT framework as a whole will break down, and in what regimes, is a trickier question. A plethora of theoretical arguments have motivated a variety of Planck scale theories that depart quite radically from the principles of QFT,[8] with at least some approaches doing away with the assumption of continuously many ultraviolet degrees of freedom.[9] The projected breakdown of QFT at the Planck scale has been taken quite seriously in the philosophical literature as a reason to question the physical significance of ultraviolet inequivalent representations (Wallace [2006]; Baker [2016]). The infrared fate of QFT has been less scrutinized—(Koberinski and Smeenk [2022]) being a recent exception. While we know that standard axiomatizations of QFT need to be modified in the infrared, relatively conservative generalizations of the Haag-Kastler axioms to curved space-times have been developed (Holland and Wald [2010]). Furthermore, we do not find analogous theoretical arguments for thinking that QFT, broadly construed to include curved space-time models, cannot

[7]There are also non-gravity related arguments for thinking that the standard model will break down in the ultraviolet long before the Planck scale. The most prominent, though nowadays also highly contentious, are Higgs mass naturalness arguments—see (Rosaler and Harlander [2019]) and (Wallace [2019]) for contrasting perspectives.

[8](Rickles [2008]) gives a useful overview of the main arguments for thinking that gravity cannot be completely described within the QFT framework. Categorising these arguments, and investigating their strength, seems to me to be an obvious project for philosophers of physics, which could further sophisticate our understanding of how the gravity problem bears on the ultraviolet and infrared reliability of QFT.

[9]This is most explicit in discrete theories, such as causal set theory (Surya [2019]). See (Hagar [2014]) for discussion of the notion of a "fundamental length" in various approaches to quantum gravity.

possibly describe the extreme infrared. Note, however, that there are also major anomalies associated with cosmological scales: the observed dark matter and dark energy densities being the most prominent. It is possible that these problems cannot be solved within the QFT framework, giving us reason to at least reduce our confidence in the infrared assumptions of QFT as well. I will argue for this last point in more detail in section 4 as it will end up being important in my evaluation of extrapolation arguments.

2.3 Internal Conflict: Divergence Problems

Another cluster of issues which should be touched on here are the internal problems associated with QFT's treatment of both the extreme ultraviolet and extreme infrared. The appearance of ultraviolet divergences in QFT perturbation series have long been a source of worry about the theory's consistency on arbitrarily short length scales. Infrared divergences also appear in the perturbative expansions of theories with massless fields—see (Miller [2021]) for a recent philosophical discussion. By focusing on the entirely non-perturbative Haag-Kastler axiomatization of QFT, the philosophical literature on inequivalent representations has largely sidestepped these issues with the series expansion. However, a major limitation of this approach is that neither the standard model nor any other empirically successful QFT has been constructed in this framework. What constructive field theorists, who work on this problem, typically do is start with a QFT model with an ultraviolet and infrared cutoff and attempt to take the limits in which these cutoffs are removed: the continuum and infinite volume limits. This has been carried out successfully in the case of some toy models, but for empirically successful QFTs, a gamut of difficulties arise when attempting to construct both limits.

Do these difficulties give us reason to worry that either of these limits do not exist, or are they simply technical obstacles to their construction? A range of opinions exist here. My own view is that the most compelling reasons to worry about the well-definedness of these limits are model dependent. In the language of the renormalization group, the existence of the continuum limit of a particular model is nowadays taken to depend on the ultraviolet renormalization group flow terminating in a fixed point (Hancox-Li [2015]). Some empirically successful models, such as QED, are believed (though not rigorously proven) to lack an ultraviolet fixed point—the infamous Landau pole problem—suggesting that a sensible continuum limit does not exist. In the case of other models, like QCD, the ultraviolet behaviour is believed (though again not rigorously proven) to be under control, and the infrared structure of the theory poses the most serious obstacle to constructive efforts.[10] While this is not generally believed to be the case for QCD,[11] it is also possible for the renormalization group flow of a QFT model to

[10] A caveat here that, while the infrared superselection structure of QCD is plausibly described by the Doplicher-Haag-Roberts (DHR) theory (mentioned in 5.2), long-ranged forces problematise the application of this approach to QED—see (Buchholz and Roberts [2014]). Thus, the infrared behaviour of QED may be in some respects more challenging than QCD's. Thanks to an anonymous referee for pointing this out.

[11] Within perturbation theory QCD appears to have an infrared Landau pole, however, this is

lack an infrared fixed point, problematizing the construction of the infinite volume limit.[12]

Again though, these problems are model dependent: they do not show that no empirically viable models of the Haag-Kastler (or Holland-Wald) axioms exist. Still, if cutoff versions of QED and QCD can underwrite their successful predictions, and their recovery from a continuum infinite volume model remains hypothetical, one might ask whether we ought to take QFT to make claims about infinite scales in the first place. Some of the recent philosophical literature on QFT has moved in this direction. Wallace ([2006, 2011]) has argued that it is ultraviolet cutoff QFTs that philosophers of physics ought to be focusing on. He takes a less dismissive attitude towards the infinite volume limit (in part due to questions about the status of this limit in cosmology, discussed in section 4).[13] The more radical view that QFT models with both an ultraviolet and infrared cutoff might have sufficient content to make sense of the practice of high energy physics is considered in James Fraser (2016a), however. Recent philosophical engagement with the effective field theory framework has also broached the semantic status of claims about infinite scales. Williams (2019) argues that we ought not to interpret QFT models as making claims about fundamental ontology at all. On this effective field theory inspired approach to the interpretation of QFT, claims about infinite scales might not even be viewed as part of the theory's semantic content in the first place.

While I take these interpretive ideas to be extremely interesting and important, this paper focuses on epistemic rather than semantic questions about infinite scales. One reason is that I take these interpretative proposals to be motivated by epistemological considerations, and indeed to presuppose some form of *Infinite Scale Scepticism*—on which more shortly. If the indispensability or extrapolation strategies examined in sections 3 and 4 succeeded in establishing warrant for claims about infinite scales this would problematize an interpretive stance which erases them from QFT's semantic content, for instance. Similarly, if we can really claim to know that the world is an infinite quantum system, the difficulties with QFT's ultraviolet and infrared structure must find some resolution, either in future physics or future mathematics. It thus makes sense to assume, for the purposes of the present investigation, that QFT does indeed posit infinitely many

generally taken to be an artefact of the approximation scheme. The infrared behaviour of the QCD coupling is not well understood theoretically—see, (Deur et al [2016]) for a review.

[12]See (Brydges et al [1996, 1998]) for discussions of infrared fixed points and the infinite volume limit.

[13]Wallace [2006] ends up arguing that infrared inequivalent representations can be associated with our ignorance of the boundary conditions of quantities like the global mass and charge at infinity. Note that, while Wallace appears to draw a deflationary conclusion from this analysis, other authors more convinced of the interpretative significance of the breakdown of Hilbert space uniqueness have also endorsed the connection between infrared inequivalent representations and boundary conditions at infinity (see (Halvorson and Müger [2007]; Baker and Halvorson [2010]; Swanson [2017]) for relevant discussions). A closer comparison of these perspectives would be useful, especially since Baker's extrapolation argument (considered in section 4) specifically targets the warrant for infrared inequivalent representations.

ultraviolet and infrared degrees of freedom. Once the epistemic status of infinite scales has been clarified one can turn to semantic questions (though this project will not be taken up in this paper).

2.4 Infinite Scale Scepticisms

I take it that a naive inference from the empirical successes of QFT to the reality of infinitely many degrees of freedom has been thoroughly discredited at this point. Empirically successful QFTs surely get something right about the world, but there are special reasons to worry about the reliability of their claims about infinite scales, including the positing of infinitely many degrees of freedom. Indeed, the considerations of this section, taken by themselves, seem to motivate some form of selective scepticism about this part of the theory.

It is instructive to distinguish two strengths of scepticism that one might conceivably adopt at this juncture. A more extreme reaction to the issues discussed in this section is a view I call *Strong Infinite Scale Scepticism*.

> *Strong Infinite Scale Scepticism:* theoretical claims about infinite scales, and therefore about whether the world has finitely many or infinitely many degrees of freedom, are in principle unknowable.

While I must confess to having sporadic sympathies for this sort of position, I suspect it would be exceedingly difficult to convincingly defend and define it here in order to ensure that I am not misread as attempting to do so in this paper.

One might suspect that the scale separation considerations discussed earlier are not special to QFT and will simply recur in a successor theory. After all, even if we had a well-supported theory of the Planck scale, we would still have infinitely far to go before we reached infinitesimal length scales; will it likely not still be the case that observations made at finite scales leave the extreme ultraviolet structure of this future theory wildly underdetermined?[14] The Strong Infinite Scale Sceptic is skating on thin ice here, however. For one thing, there are really multiple notions of scale separation, and the question of which of them is epistemically supported, and in which contexts, is fraught with difficulty (See (Williams [2015]; Franklin [2020]; Wallace [2019]; and Koberinski and Smeenk [2022]) for relevant discussions). It is thus debatable whether these expectations about the scaling behaviour of future theories are reasonable. More importantly for the dialectic of this paper, there are ways that one might try to support claims about infinite scales which have not yet been considered. While I will end up finding current versions of the indispensability and extrapolation arguments wanting, they fail in their details rather than in principle. The Strong Infinite Scale sceptic thus appears to be guilty of unwarranted speculation about the limits of future science.

[14] One way that this might turn out not to be the case is if consistency constraints single out a unique theory which is compatible with observations made at some finite scale. One can understand this scenario as the ideal realization of Dawid's [2013] no-alternatives argument. That no-alternatives arguments emerge as the most promising way out of the epistemic challenges associated with infinite scales is a recurring theme of this paper.

There is a weaker form of Infinite Scale Scepticism which appears to be more straightforwardly supported by the considerations of this section, however.

> *Weak Infinite Scale Scepticism:* theoretical claims about infinite scales, and therefore about whether the world has finitely many or infinitely many degrees of freedom, are not currently items of scientific knowledge.

The Weak Infinite Scale Sceptic does not claim that knowledge of arbitrarily small and arbitrarily large length scales is impossible, they merely hold that we do not have such knowledge now. This seems prima facie reasonable given the reasons to selectively reduce our credence in the extreme ultraviolet and infrared claims of our most current most fundamental theories discussed above.

Still, we are not yet ready to reach a verdict on this matter. The literature on inequivalent representations contains hints at more sophisticated strategies for establishing epistemic warrant for the limit of infinite degrees of freedom, however, which need to be analysed in detail. The following two sections examine two styles of argument: indispensability arguments (section 3), inspired by (Ruetsche [2011]), and extrapolation arguments (section 4), inspired by (Baker [2016]).

3 Indispensability Arguments

One strategy for establishing the epistemological credentials of the limit of infinite degrees of freedom would be to make a case that novel features of infinite systems play an indispensable role in accounting for empirically confirmed phenomena. Laura Ruetsche makes this sort of indispensability claim in her works on inequivalent representations (Ruetsche [2003, 2006, 2011]).[15] As was mentioned above, Ruetsche is primarily interested in the semantic implications of the breakdown of Hilbert space uniqueness in the limit of infinite degrees of freedom. While I use Ruetsche's works as a starting point then, our projects are different. This section focuses on whether a persuasive indispensability argument for believing that the world is an infinite quantum system can be constructed, glossing over many subtleties of Ruetsche's philosophy of QFT.

3.1 Spontaneous Symmetry Breaking

Ruetsche's main example of a phenomenon which inequivalent representations are put to work in modelling is spontaneous symmetry breaking (SSB), and since there has been a spate of recent philosophical work on SSB, this will also be our focus here. SSB is standardly defined as a situation in which a model has multiple ground states that are mapped into each other under the action of an exact symmetry of the dynamics. In the context of classical mechanics, a particle moving in a double potential well, of the form $V(x) = (x^2 - a^2)^2$, is a standard example. The potential is invariant under the reflection transformation $x \to -x$,

[15] See also discussion in (Doreen Fraser [2009], 560).

but the two minima at $x = \pm a$ are not. We can thus expect the system to display behaviour that violates the exact reflection symmetry of the dynamics when its total energy is close to the ground state energy.

Defined in this way, SSB cannot occur in finite quantum systems. Given physically reasonable assumptions, they have a unique ground state. Discounting the possibility of infinite potential barriers, for instance, a quantum state which is initially localized in one of a number of minima in a potential will not stay there forever due to quantum tunnelling. The true stationary ground state will thus be a superposition of quantum states associated with all of the minima in the potential. With the failure of the Stone-von Neumann theorem in the limit of infinite degrees of freedom, however, degenerate ground states become possible. Distinct ground states associated with distinct inequivalent representations can arise since superpositions of states associated with unitarily inequivalent Hilbert spaces cannot be formed and tunnelling between such sectors cannot occur. For this reason, in the algebraic approach to quantum theory, a spontaneously broken symmetry is often associated with a symmetry transformation of a model which is not unitarily implementable—meaning, roughly, that it relates states housed in inequivalent Hilbert space representations. If we have strong evidence for the occurrence of SSB phenomena it, therefore, seems that we have strong evidence that the world must be an infinite system.

Difficulties arise when we try to fill in the details of this argument, however. While SSB is certainly a crucially important concept in high energy physics modelling practice, a number of factors make developing an indispensability argument in this context awkward. Our inability to construct realistic models in standard axiomatic frameworks means that there is inevitably some speculation involved in statements about the role played by inequivalent representations in high energy physics. Furthermore, uncontroversial instances of SSB phenomena in standard model physics are difficult to come by. The textbook claim that the Higgs mechanism turns on the spontaneous breaking of electroweak gauge symmetry has been contested in the foundational literature for both philosophical reasons, as this is difficult to square with standard interpretations of gauge as a kind of representational redundancy (Earman [2004]; Smeenk [2006]), and on technical grounds, as a result known as Elitzur's theorem suggests that local gauge symmetries cannot be spontaneously broken in QFT (Friederich [2013]).[16] Chiral symmetry breaking in QCD is another prominent context in which the notion of SSB is appealed to, but chiral symmetry is only an approximate symmetry to begin with, so a degenerate vacuum structure does not actually obtain in this case.

Partly in response to these complications, Ruetsche initially focuses on the thermodynamic limit of quantum statistical mechanics, a setting where an indispens-

[16]There is, of course, much more to be said on this topic. I am not claiming here that the Higgs mechanism cannot be used as the basis of an indispensability argument here, rather, my point is that the controversy surrounding the interpretation of the Higgs mechanism is one of a number of factors which push advocates of an SSB based indispensability argument towards better-understood cases in statistical mechanics.

ability argument for inequivalent representations can apparently be made more forcefully. In statistical mechanics, SSB is associated with phase transitions. A ferromagnetic phase transition, in which spins inside a crystal align in a particular direction when the system is cooled to some critical temperature, is modelled as a bifurcation of the system's equilibrium states into a degenerate family related by an exact rotational symmetry of the dynamics. Again, this is not possible when the number of degrees of freedom is finite. Consequently, the standard approach to modelling SSB in statistical mechanics employs the thermodynamic limit, in which the volume and degrees of freedom of a model are taken to infinity. In this limit multiple equilibrium states associated with distinct inequivalent representations can occur, signalling the onset of SSB. In statistical mechanics then, we have an uncontroversial instance of an SSB phenomenon and the ability to rigorously construct the thermodynamic limit of at least some empirically successful models in the algebraic framework. An indispensability argument for the presence of infinitely many quantum degrees of freedom appears to be on the cards.

Here we run into a different problem. Real ferromagnets are clearly finite in volume and have spin degrees of freedom associated with a finite number of lattice sites. The limit of infinite degrees of freedom thus appears to be an idealization in this modelling context. That fact that SSB and phase transition concepts can be fruitfully applied to systems which are at least ostensibly finite has led to considerable debate in the literature about the representational status of the thermodynamic limit (Batterman [2005]; Butterfield [2011]; Calendar and Menon [2013]; Landsman [2013]; James Fraser [2016b]; Wallace [2018]). The deflationary readings of the thermodynamic limit which have been developed in this debate also open up a deflationary response to indispensability arguments in the QFT context, or so I will argue.

3.2 Deflating Indispensability Arguments

There are two paths that a proponent of the indispensability strategy might take at this juncture. First, they might ally themselves with the view, often associated with (Batterman [2005]), that the thermodynamic limit plays an indispensable explanatory role in statistical mechanics despite its status as an idealization. We need to be precise about the kind of indispensability claim that is being made here, however. Taking the thermodynamic limit to be indispensable in an auxiliary sense—because it plays an essential role in facilitating understanding, or deriving facts about, phase phenomena, for instance—would not necessarily support a veridical reading of the novel properties it affords.[17] Rather, the proponent of the indispensability argument needs to say that the novel features afforded by the thermodynamic limit represent physical features of real ferromagnets that cannot be captured without it. It is not clear that Batterman ever made this stronger representational indispensability claim and, in any case, it is not endorsed in his

[17] The possibility that the limit of infinite degrees of freedom plays an indispensable supporting role in derivations without the novel properties it affords being evidentially supported is discussed further in section 5.1.

later work on idealization and explanation (for instance, Batterman and Rice [2014]).

Furthermore, dubbing the thermodynamic limit an essential idealization does little to ameliorate the puzzle about how the representational indispensability claim could possibly be true. Ruetsche ([2011] 335-339) proposes an interpretation based on the observation that finite spin-lattice models are also not exact, or truly fundamental, representations of their targets. If the world is actually described by a quantum theory with infinitely many degrees of freedom we could take the novel structures that appear in the thermodynamic limit to correspond to features of this more fundamental description which are missed by any finite model. The suggestion that the physics of the world's smallest, or largest, length scales needs to be invoked in order to account for the behaviour of tabletop ferromagnets clearly violates some notion of scale separation. Dismissing the proposal for that reason would beg the question, however, since whether claims about infinite scales are implicated in explanations of observed physical phenomena is precisely the point at issue. A fairer objection, perhaps, is that while postulating a more fundamental infinite theory may explain how degenerate equilibrium states are possible in general, we are left with the question of why SSB occurs in the particular systems, and at the particular values of the thermodynamic parameters, that it does.[18] What would be needed, it seems, is a derivation of all of the varied successful results obtained via the thermodynamic limit from a continuum field theoretic description, which we simply do not have. Furthermore, it is unclear how such a programme might proceed, rendering this reply problematically speculative.

The other line of response Ruetsche ([2011]) considers is to accept that the indispensability argument is blocked in quantum statistical mechanics but urges that in the QFT context there is no analogous reason to worry about the limit of infinite degrees of freedom, so that the argument goes through unhindered. We would then need to return to difficulties with identifying suitable cases of SSB in high energy physics mentioned above. But, even putting these issues aside, there is a tension in endorsing the representational indispensability of inequivalent representations selectively like this. The thermodynamic limit's status as an idealization does not simply act as a defeater on the explanatoriness of infinite systems in statistical mechanics. Rather, it leads us to reconsider what is underwriting the empirical and explanatory power of the limit of infinite degrees of freedom in standard treatments of SSB. Following this impulse, and the precedent set by deflationary treatments of phase transitions deployed by (Butterfield [2011]), (Callender and Menon [2013]), and others, a number of authors have developed ways of explaining the efficacy of infinite systems in the standard account of SSB in terms of properties they share with large finite systems.

James Fraser ([2016b]) distinguishes two general approaches to carrying out this programme. Firstly, we can point to the existence of very long-lived asymmetric states in large finite systems. Wallace ([2018]) uses the decoherent histories framework to spell out the sense in which the state space of a large quantum system,

[18] See (Mainwood [2006], 228-231) for a discussion of related worries.

which satisfies the standard definition of SSB in the limit, splits into approximately dynamically decoupled sectors when the number of degrees of freedom is large but finite. Transitions between these sectors take place on very long time periods—often longer than the age of the universe if the number of degrees of freedom is of the order of Avogadro's number. We can thus explain how the degenerate ground states found in the limit provide a good approximation of the behaviour of real systems without being forced to reify them.[19]

Secondly, we can point to the instability of the unique ground/equilibrium state of large finite systems to small asymmetric perturbations. Landsman ([2013]) shows that the energy gap between the ground and first excited state of many quantum systems that satisfy the standard definition of SSB in the limit vanishes exponentially with the number of degrees of freedom. This means that adding an otherwise undetectable symmetry-breaking perturbation to the dynamics will give rise to asymmetric behaviour that is well approximated by one of the degenerate stationary states found in the limit of infinite degrees of freedom. In the case of many real systems, we can expect symmetry-breaking perturbations to be physically instantiated: a real ferromagnet will be subject to small external magnetic fields and will contain symmetry-breaking lattice defects, for instance. Again, we can thus explain why the description afforded by the limit of infinite degrees of freedom is successful without taking the novel structures afforded by the limit of infinite degrees of freedom to be instantiated in the real world.

While work remains to be done in applying these approaches to empirically supported models, they at least demonstrate that the observed behaviour characteristic of SSB is possible when the number of degrees of freedom is finite. This mitigates against the claim that inequivalent representations afforded by the limit of infinite degrees of freedom must be taken to be a feature of the world in order to account for the occurrence of SSB. If it is admitted that finite models can accommodate SSB phenomena in quantum statistical mechanics why would one hold that they cannot in QFT? To justify this one would have to point to empirical or explanatory differences between the treatment of SSB in the two contexts which support an indispensability claim in the QFT case alone.[20] The well-known analogies between statistical physics and QFT end up being something of a false friend to proponents of the indispensability strategy then. What would, in a way, be a much more promising starting point for an indispensability argument is a signature of truly infinite systems which is observed in the high energy physics or cosmology context but not in condensed matter systems. SSB phenomena, at least as we presently understand them, surely do not provide this.

[19]Some explanation is needed for why we see symmetry breaking states in the first place when the true ground state is a symmetric superposition and consequently the viability of this approach appears to be conditional on a solution to the measurement problem (as Wallace ([2018]) acknowledges).

[20]Arguably, there are disanalogies in how the notion of SSB ought to be interpreted in QFT and statistical mechanics—see (Doreen Fraser [2012]). The question is whether these differences motivate endorsing the indispensability of the limit of infinite degrees of freedom in one case and not the other.

I have only considered one application of the inequivalent representations afforded by the limit of infinite degrees of freedom here, and nothing I have said shows that a stronger indispensability argument cannot be constructed from other cases. Superselection phenomena and the Unruh effect offer potential fodder in the QFT context. If one is willing to countenance the essential idealization reading of the thermodynamic limit suggested by Ruetsche then one might also look for applications of the thermodynamic limit which more emphatically resist attempts at de-idealization. In my view, however, the current state of this debate does not license high credence in the claim that our world is an infinite quantum system.

4 Extrapolation Arguments

A different approach to establishing epistemic warrant for the limit of infinite degrees of freedom is put forward by Baker ([2016]).[21] Baker thinks that a stronger case for taking inequivalent representations that appear in the limit seriously emerges when we combine the evidence for QFT with the evidence for the Λ-CDM model of cosmology. He concedes that we should not put weight on the inequivalent representations that arise from positing infinitely many ultraviolet degrees of freedom, chiefly, in his view, because quantum gravity research indicates that QFT will break down in that domain. He suggests that the infrared case is different, however, because: i) astronomical observations support a cosmological model with an infinite volume space-time, and ii) there is no analogous reason to think that the QFT framework, broadly construed, will break down on arbitrarily large length scales. While the empirical data collected at collider experiments might not support any claims about the world on arbitrarily large length scales directly, on this view we are licensed to extrapolate QFT into this regime, yielding the conclusion that the world does, indeed, exhibit infrared inequivalent representations. I will call this sort of argument an extrapolation argument.

4.1 Formulating the Argument

One obvious place to push back here is on the presumed strong evidence for the infinite volume of the universe, but let us grant that premise for the moment and focus on how the extrapolation argument might proceed on that basis. A natural reading of the argument is as an enumerative induction: we have empirically established that QFT gives a very good description of the behaviour of matter in our local region of space-time and conclude that it likely does so over much larger, and ultimately arbitrarily large, spatial regions. While this form of inference is surely legitimate in some scientific contexts, we have already discussed reasons for thinking that the large-length scale regime we are inductively extrapolating into has very different physics from the scales on which QFT models have been tested. As the standard model misses out gravity it fails already on human length

[21] I use Baker's ([2016]) relatively brief discussion as a starting point here, but as with Ruetsche in the previous section, I do not mean to attribute all of the positions considered in this section to Baker himself.

scales, never mind cosmological ones, and of course, it also fails to account for the dark matter and dark energy densities extracted from astronomical observations. Baker's idea, however, is that the QFT framework, broadly construed to include QFT on curved space-times, receives inductive support from the success of the standard model, and this framework can be safely extrapolated to arbitrarily large length scales, in the absence of reasons to think that QFT, and not just particular QFT models, break down in the infrared.

This version of the argument presupposes a strict asymmetry in how the problems of quantum gravity bear on the reliability of axiomatic QFT's extreme ultraviolet and extreme infrared posits. As I intimated in section 2, however, there are reasons to question, or at least sophisticate, this asymmetry thesis. While there might not be direct arguments for the breakdown of QFT in the infrared of the kind motivating novel Planck scale theories, there arguably are considerations which countenance reducing our confidence in QFT's claims about arbitrarily large length scales—perhaps motivating an agnostic, rather than pessimistic, attitude. The reason is the existence of major empirical and theoretical anomalies on cosmological scales, specifically the dark energy and dark matter problems, which currently find no resolution within the QFT framework. Crucially, there are also various live proposals which link these phenomena to the failure of QFT in the ultraviolet. Koberinski and Smeenk ([2022]) discuss multiple ways in which quantum gravity theories could turn out to be required to resolve the issues with dark energy and the cosmological constant. Dark matter candidates coming from quantum gravity theories—such as string theory axions—are another prima facie reasonable scenario of this kind.[22] My feeling is that this ought to motivate uncertainty about the infrared assumptions of QFT, which proportionally weakens the proposed enumerate induction. While it is certainly conceivable that a QFT successor of the standard model, set on a curved space-time background, could resolve all of the problems of contemporary cosmology, I find the suggestion that we know this is the case based on enumerative induction from the successes of LHC physics unconvincing.

Luckily, there may be a way of reformulating Baker's argument which does not rest on trusting QFT in the infrared, and thus gets around these objections. What I have in mind is a kind of no-alternatives argument, in the sense of (Dawid

[22]Since these scenarios connect ultraviolet and infrared phenomena a potential concern is that they violate scale separation in a problematic way. Could empirical evidence for scale separation be used to rule out these scenarios and repair the enumerate induction? As was mentioned in section 2.4, it is contentious which notions of scale separation are evidentially supported, and in what contexts. Some ways of connecting quantum gravity theories to infrared phenomena seem to be relatively tame. After all, we know that unknown ultraviolet physics can leave signatures on long-length scales: the standard model misses out gravitational interactions in the ultraviolet which pop up and dominate in the infrared, for instance. Some of the theories discussed by Koberinski and Smeenk (2022) violate scale separation in a more profound way, and they explicitly associate the cosmological constant problem with the failure of the effective field theory framework. They view this as a feature rather than a bug, however, and it is, at best, unclear whether any of these approaches can be legitimately ruled out on scale separation grounds.

[2013]; Dawid et al [2015]). While we do not know what the ultraviolet completion of the standard model will look like exactly, we might think it very likely that the basic degrees of freedom of that future theory will be quantum degrees of freedom. As long as this theory associates quantum degrees of freedom with finite regions of a 4-dimensional space-time manifold (whether space-time turns out to be fundamental or emergent), and the manifold is infinite in volume, it seems that there must be infinitely many infrared degrees of freedom and thus infrared inequivalent representations. This version of the extrapolation argument is not based on trusting existing axiomatizations of QFT in either the ultraviolet or infrared. Rather, it tries to establish warrant for one specific feature of those frameworks—the existence of infrared inequivalent representations. This seems to me to be the most promising way of developing the extrapolation strategy.

4.2 The Epistemology of Infinite Volume

We now need to return to the evidence for the infinite volume of the universe, however, since any version of the extrapolation argument is only as strong as the support for this premise. In fact, my own view is that that we are not in a position to claim to know that the universe is infinite in volume—with the implication being that an extrapolation argument does not establish knowledge of infinitely many quantum degrees of freedom.

Why might one have high credence in the infinite volume of the universe? One might reason as follows: the Λ-CDM model of cosmology, the equivalent of the standard model of particle physics for the universe's very long-distance physics, posits an infinite volume space-time, therefore, we have strong grounds to believe that this is the case. Note the similarity to the naive argument from the success of QFT to the existence of infinitely many quantum degrees of freedom criticised in section 2, however. As I said there, the empirical success, and community acceptance, of a theory does not imply that all of its semantic content is epistemically supported. We have to ask: do the empirical successes of the Λ-CDM model lend support to its claims about global space-time structure?

Let us look at how one gets to an infinite volume space-time in the usual textbook treatment. The Λ-CDM model is based on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, homogeneous isotropic solutions of the Einstein equations. This metric allows for three possibilities for the constant curvature of space-time's spatial sections depending on the overall energy density of the universe: if the energy density is equal to a critical value we get a flat geometry with zero spatial curvature; if it is above this value we get a spherical geometry with positive curvature; and if it is below it we get a hyperbolic geometry with negative curvature. Our current best estimates of the energy density of the observable universe are very close to the critical value. Assuming that it is legitimate to inductively extrapolate this observed value beyond the cosmic horizon, and to arbitrarily large space-time regions, we seem to reach the conclusion that the geometry of space is that of a Euclidean plane, so infinite in volume.

In fact, however, a hidden assumption is needed for the last step to go through—we need to assume that the topology is simply connected in order for flatness to imply infinite volume. The Einstein equations do not say anything about the global topology of space-time, relating, as they do, the local matter-energy density to the local metric. Apart from the connection between positive global curvature and closed space-times, global topology and curvature are independent features of a general relativity model and need to be tested separately.[23] Multiply connected topologies can give rise to finite volume zero curvature spaces that are fully compatible with current data: the most straightforward example being a hypertorus space-time. Furthermore, these are serious cosmological models with good scientific reasons to take them seriously, not contrived sceptical hypotheses. It has been suggested that models of the creation of the universe in quantum cosmology favour a flat finite volume universe (Linde [2004]) and that a finite spatial volume might help explain apparent empirical anomalies facing the Λ-CDM model (Aurich et al [2008]).

We can, again, think about the epistemic implications of this from the point of view of selective confirmation theories. One can start with the conventional Λ-CDM model and simultaneously vary its claims about global topology and volume in various ways (while keeping global curvature fixed) without making any difference to the relevant empirically verified predictions. This suggests that the posit of infinite volume is an idle wheel as far as its empirical success is concerned and therefore does not receive positive support from the model's agreement with astronomical observations. Furthermore, as I already pointed out, an enumerative induction from the energy density of the observable universe does not help here: at best, this gives us knowledge of the universe's global curvature, which leaves its volume underdetermined.

This calls the supposed support for the infinite volume of the universe into question. One might object at this point: why do cosmologists work with an infinite volume model then? Surely much stronger arguments are needed to overturn orthodox opinion in cosmology? Note, however, that I am not advocating overturning the consensus surrounding the Λ-CDM model. I take for granted that the acceptance of this model by the scientific community is rational, and indeed that it gets many things right about the long-distance structure of the world. Just as one can accept the standard model of particle physics while holding that it likely breaks down in the ultraviolet regime, one can selectively adopt a sceptical attitude towards the infinite volume hypothesis without rejecting the Λ-CDM tout court. After all, it is not difficult to find instances of cosmologists who accept the Λ-CDM model questioning whether we know that the universe is infinite in volume, and there are many reasons to work with an infinite volume model besides being confident that this feature is veridical. If there are multiple ways to fix global space-time topology which are compatible with present observations one is

[23]There is an extensive literature on the global topology of general relativity models, and possible empirical signatures of non-simply connected topologies, a standard reference being (Lachieze-Rey and Luminet [1995]).

free to do so in whatever way is simplest and most convenient.[24]

One might detect in this last comment a strategy for attempting to buttress the extrapolation argument, however. Could it be that a simply connected topology is more likely to obtain, in the absence of contravening evidence, because it is in some sense more parsimonious than a multiply-connected topology? This road quickly leads to vexed questions about both the general basis of parsimony arguments and their proper application in the present context. I am not optimistic that delving into these issues will yield decisive epistemic support for the infinite volume hypothesis. Even granting for the sake of argument that theoretical virtue considerations could justify increasing our credence in a simply connected model, I find it doubtful that this would be a sufficiently large boost to underwrite a knowledge claim. Note, also, that an advocate of this strategy cannot simply assume that parsimony considerations have an important bearing on the acceptance of the Λ-CDM model. As I said above, there are less commital reasons for settling on an infinite model, and determining how the cosmology community appraised the Λ-CDM in practice is a project for the history of science.

Philosophers with a more positive view of the justificatory power of parsimony arguments in the sciences might be left with more optimism about the promise of the extrapolation strategy. As with the indispensability strategy, I certainly do not want to be read as suggesting that this approach is intrinsically hopeless: careful philosophical work on the epistemology of global features of space-time, as well as new developments in cosmology, could lead to more powerful versions of the argument in the future. Still, my sense is that, while the versions of the extrapolation argument considered here may have more mileage than current indispensability arguments, and may license increasing our credence in the existence of infrared inequivalent representations to some degree, they do not support the very high level of confidence usually associated with scientific realist knowledge commitments. I remain significantly more confident in what the standard model of particle physics says about the 10^{-20}m regime, and what the Λ-CDM model says about 10^{26}m regime, than theoretical claims about arbitrarily large distances.

4.3 A Modalised Extrapolation Argument

In correspondence with me, Baker has suggested a way of weakening the conclusion of the extrapolation argument so that it is less vulnerable to the sorts of worries raised above. I have thus far focused on the question of whether or not the actual world has infinitely many quantum degrees of freedom. Baker suggests that a weaker claim, which is still philosophically interesting, is that it is physically possible for the world to have infinitely many quantum degrees of freedom, in the sense that it is compatible with its fundamental physical laws. Suppose we admit that we do not know that space-time is actually infinite in volume. We might still have reason to believe that the fundamental theory of our world admits

[24]This links to the reading of the limit of infinite degrees of freedom as a supporting auxiliary assumption in the QFT context, discussed in section 5.1.

models with infinite volume space-times (again whether space-time turns out to be fundamental or emergent). The fact that general relativity has infinite volume models might be taken to support this claim. If we again suppose that the fundamental degrees of freedom of our world are quantum degrees of freedom we are led to the conclusion that infrared inequivalent representations occur in some possible worlds in which the laws of our world hold.

I have two things to say about this modalized version of the extrapolation argument. Firstly, it is worth pointing out that there is still some uncertainty about the nomic possibility of infinite volume space-time—though I grant that our credence in it should be higher than our credence in the claim that the actual world is infinite in volume. Making the possibility claim amounts to betting that the global features of space-time which are contingent by the lights of general relativity will continue to be contingent in future physics. But, just as future theories may fix the values of the interaction couplings which the standard model treats as free parameters, it is conceivable that the global topological properties of space-time could turn out to be fixed with nomic necessity. These features of space-time may be derivable from the laws of the fundamental theory—perhaps the fundamental theory only has finite volume models. Alternatively, if claims about the volume and topology of space-time end up playing an important role in explaining otherwise unexplainable features of our universe they might be upgraded to laws on similar grounds to the low entropy past hypothesis, because they increase the strength of the best system of laws.[25] While the empirical evidence for general relativity does not rule out these scenarios, they might be thought far fetched for other reasons. They certainly run counter to the prevailing mood in quantum cosmology. The rise of multiverse cosmologies, and the string theory landscape, has cooled ambitions of explaining all of the features of the universe in nomic terms and eternal inflation models typically allow for the creation of finite and infinite volume universes. Evaluating the epistemic weight we should put behind these developments is another tricky problem, however.

Secondly, I can see reasons to question the significance of knowledge of the mere physical possibility of infrared inequivalent representations on its own. At least some of the extant philosophical discussion surrounding inequivalent representations seem to be prefaced on the Stone-von Neumann theorem failing in the actual world—the question of whether or not inequivalent representations threaten scientific realism mentioned earlier, for instance. Furthermore, it is difficult to see how the physical possibility of inequivalent representations can be put to work philosophically if we have not solved the problem of how we could ever come to believe that the actual world exhibits these structures. If one is sceptical about the possibility of answering this challenge one might doubt that there is anything to be learned from considering conditional statements about what would follow if space-time were infinite. Such conditionals cannot concern any effect which we can detect in finite volume space-time regions, it seems, otherwise, we would be

[25] Alexandre and Clough ([2019]) suggest that finite-size effects may play a part in the mechanism behind cosmic inflation, which would seem to push in this direction, for instance.

able to confirm that the universe is infinite in volume by observing them. Consequently, so the thought goes, these conditionals cannot be doing any substantive work in bona fide scientific explanations. While there is undoubtedly more to say on this issue, I suspect that attempts to draw interesting conclusions from the physical possibility of infrared inequivalent representations will lead us back to the justification of claims about the infinite scale structure of the actual world which have been examined in this paper.

5 Non-Implications

While I have not shown that these two strategies for establishing warrant for the novel properties of infinite quantum systems are doomed to failure, I suggest that they are not strong enough to underwrite knowledge claims in their current form. I am thus inclined to endorse Weak Infinite Scale Scepticism, the view that we do not presently have knowledge of the smallest and largest length scale structure of the world, nor do we know whether the world has infinitely many or finitely many degrees of freedom.

To avoid potential misunderstandings and to clarify its relevance to the inequivalent representations debate, this section discusses two non-implications of this position. 5.1 points out that this does not mean that infinite systems have no foundational import, and 5.2 discusses the relevance of topological inequivalent representations, which can occur when the number of degrees of freedom is finite, for the question of whether we have good reason to believe that the world is described by a theory which violates the assumptions of the Stone-von Neumann theorem.

5.1 Weaker Epistemic Roles for Infinite Systems

The philosophical interest of infinite quantum systems need not hang on the fate of the indispensability and extrapolation strategies. There are, in fact, at least three (mutually compatible) ways that one might try to motivate foundational work on infinite quantum systems which are fully compatible with Weak Infinite Scale Scepticism.

Firstly, one might argue that infinite quantum systems, and specifically, those characterized by standard axiomatizations of QFT, are intrinsically interesting owing to their bearing on questions about the internal relationships between important theoretical principles. This is at least one way of reading Doreen Fraser's defence of the philosophical significance of axiomatic field theory (Doreen Fraser [2009, 2011]). There she urges that the project of unifying quantum theory and special relativity is not complete until the existence of continuum QFTs set on Minkowski space-time have been verified and their physical content has been delineated, which will necessarily involve grappling with the consequences of ultraviolet and infrared inequivalent representations. Understood as an investigation of the internal relationships between relativistic and quantum principles this project does not hang on whether or not the claims these theories make about the extreme infrared and ultraviolet regimes accurately represent the actual world.[26]

Secondly, one might emphasize the potential heuristic payoffs of foundational work on infinite quantum systems for future theorizing. While we may not know that the world has infinitely many degrees of freedom we certainly do not know that it does not either, and theories that posit infinitely many ultraviolet and infrared degrees of freedom are live options on the frontiers of physical theorising. Interpretive debate about the physical content of infinite quantum systems might thus be hoped to play a role in the construction of successors to QFT. Exploring conceptual issues raised by their novel features might be recognized as speculative without being dismissed as idle speculation. An advocate of Strong Infinite Scale Scepticism might be unpersuaded by this defense, as they will suspect that epistemic problems with infinite scales will recur in future theories. As I said in section 2, however, it is far from clear that a generic pessimism about the possibility of knowledge of infinite scales is warranted. Furthermore, it is conceivable that engaging with foundational issues raised by the limit of infinite degrees of freedom could turn out to be pragmatically fruitful even if we never attain knowledge of the world's infinite scale structure.

Finally, whether they faithfully represent anything or not, novel properties of infinite quantum systems can play a supporting role in deriving, or precisely stating, facts that apply to both finite and infinite quantum systems. Vincent Rivasseau, a leading constructive field theorist, advances just this sort of reading of the significance of the limit of infinite degrees of freedom in QFT:

> In fact the most compelling reason for which we are interested in the continuous formulation of field theory is the same for which we are interested in the thermodynamic limit of statistical systems. In statistical mechanics this limit corresponds to systems of infinite volume. We know that in nature macroscopic systems are in fact finite, not infinite, but they are huge with respect to the atomic scale. The thermodynamic limit is an adequate simplification in this case, since it allows one to give a precise mathematical content to the physically relevant questions (like depedence of the limit on boundary conditions, existence of phase transitions etc...). Since a limit has been taken, the power of classical analysis may be applied to these questions. It would be much harder and less natural to try and define analogous notions for a large finite system, just as it is difficult and often inappropriate to make discrete approximations to some typically continuous mathematics like topology. (Rivasseau [1991], 4)

Studying infinite systems is undeniably a crucial (and perhaps indispensable) source of information about large finite systems. This is generally admitted by

[26]One might dismiss this line of investigation as a "mathematical game", as Wallace ([2011]) suggests. However, here we get into tricky questions about what is and is not valuable knowledge.

proponents of deflationary interpretations of the thermodynamic limit. The approaches to SSB in finite systems mentioned in section 3 do not eliminate reference to infinite systems entirely, rather the inferences drawn about how large finite systems are expected to behave proceed via an analysis of the limit system. Similarly, one might be interested in algebraic axiomatizations of QFT, not because we have good reasons to believe that the world satisfies all of the axioms, but because they allow us to derive results of foundational and philosophical importance which continue to apply when speculative assumptions about infinite scales are relaxed.[27]

5.2 Unitarily Inequivalent Representations in Finite Systems

Thus far in this paper, I have focused on the question of whether we have grounds to take the actual world to be an infinite quantum system. As I noted at the outset, however, this is not equivalent to the question of whether we have grounds for taking the world to be described by a theory which exhibits inequivalent representations.

While the extant philosophical literature has focused on the limit of infinite degrees of freedom, there are other situations in which inequivalent representations can arise. In addition to assuming that the number of degrees of freedom is finite, the Stone-Von Neumann theorem turns on assumptions about the topology of the configuration space of a quantum system; namely, it requires that it is structured like $\mathbb{R}^n$ where n is the (finite) number of degrees of freedom (Ruetsche [2011], section 3.2). Systems with different configuration space topologies thus fall outside of the purview of the theorem and can also exhibit unitarily inequivalent representations of their canonical commutation relations. In the case of the Aharanov-Bohm effect, for instance, in which an infinitely long, and perfectly insulated, the solenoid takes a chunk out of the electron's configuration space leading to a non-simply connected configuration space $\mathbb{R}^3/S^\infty$, inequivalent representations arise despite the number of degrees of freedom being finite (Earman [2019]). In addition to the ultraviolet and infrared inequivalent representations which we distinguished above then, we can also add topological inequivalent representations to our taxonomy.

In my discussion of Baker's extrapolation argument in section 4, I attached a great deal of importance to the possibility that space-time might turn out to be non-simply connected, allowing for a finite number of degrees of freedom to be consistent with global space-time flatness. However, it is possible that a non-trivial space-time topology may induce a non-trivial topology on the field theory's configuration space, and thus be an alternative source of inequivalent representations. While this question will likely be difficult to answer conclusively in the case of realistic field theories, Landsman identifies topological inequivalent rep-

[27] See Swanson ([2017]) for a discussion of how the claims about the extreme ultraviolet in axiomatic QFT can be understood as "auxiliary technical assumptions" employed in the derivation of important theorems, rather than epistemically justified claims about the world's ultraviolet structure.

resentations in a toy field theory on a circle, lending plausibility to this scenario (Landsman [1998], 420-424). This suggests the possibility of a strengthened version of Baker's extrapolation argument for taking inequivalent representations to be a feature of the fundamental theory of the actual world: either space-time is infinite in volume, in which case we get infrared inequivalent representations, or space-time is non-simply connected, in which case we get topological inequivalent representations. If such an argument could be made rigorous it would be a rather fabulous specimen of a no-alternatives argument. I leave it to more ambitious and technically savvy researchers to explore the promise of an argument of this kind.

Another situation in which non-uniqueness can occur in finite systems is if the configuration space is a quotient space of the form $Q = G/H$, where H is a non-trivial symmetry group (which can also be understood as a topological phenomenon—see Landman [1990]). A physically relevant situation in which this occurs is when a C*-algebra is constructed consisting only of gauge-invariant observables. Landsman ([1993]) carries out this procedure for a single particle moving in an external gauge field and finds inequivalent representations labelled by the global charge. Similarly, Kijowski and Rudolph ([2005]) construct algebras of gauge observables for quantum chromodynamics on a finite volume lattice and identify inequivalent representations labelled by global colour charge. Since the standard model is a gauge theory, this seems to present a much more direct route to establishing the relevance of inequivalent representations to real-world physics than the limit of infinite degrees of freedom. Indeed, this work by Landsman, Kijowski and Rudolf, hints that the inequivalent representations employed in the Doplicher-Haag-Roberts (DHR) theory of superselection, one of the crown jewels of algebraic QFT, spring from quotienting out a symmetry rather than speculative assumptions about the extreme ultraviolet and infrared.[28] In that case, philosophical work employing this formalism (for instance, Halvorson and Müger [2007]; Baker and Halvorson [2010]; Baker [2013]) would escape unscathed from the worries about the epistemic status of the limit of infinite degrees of freedom raised in this paper.

Whether these alternative sources of non-uniqueness raise the same interpretative and epistemological challenges mentioned at the outset is, of course, another crucial question. In any case, it is clear that the project of determining the physical significance of Hilbert space non-uniqueness requires a close study of other ways around the Stone-von-Neumann theorem than the limit of infinite degrees of freedom.

6 Conclusion

The goal of this paper has not been to invalidate philosophical work on infinite quantum systems and inequivalent representations but to bring relevant episte-

[28] Again, I must leave it to more ambitious and technically savvy researchers to investigate this connection.

mological issues into focus. I want to conclude by stressing that these are not idle questions: they have implications for how philosophical work in this area might proceed.

While the two strategies for establishing epistemic warrant for the inequivalent representations that appear in the limit of infinite degrees of freedom we examined in sections 3 and 4 were found to be insufficient to underwrite knowledge claims in their current form, I pointed out a number of ways that these approaches might be further developed. One interesting point to note here is that these two projects seem to pull in quite different methodological directions. Developing a strong indispensability argument turns on a detailed study of how particular structures of inequivalent representations are implicated in the predictive and explanatory success of our current best models. If it succeeds it will support the physical relevance of those particular structures—it might encourage us to take the non-trivial vacuum structure of a specific QFT model seriously, for instance. Baker's extrapolation argument, by contrast, moves the focus away from current QFTs, and even the QFT framework as we currently understand it. It, instead, leads in a more abstract direction, towards very general facts that follow from the existence (or mere physical possibility) of infinitely many quantum degrees of freedom alone.

Similarly, adopting one of the non-committal attitudes towards foundational work on infinite quantum systems discussed in section 5.1 will also have methodological consequences. If one hopes to contribute to future theory construction it behoves one to explore how these structures feature in active programmes in quantum gravity and quantum cosmology, for instance. And if we want to use the mathematics of the limit of infinite degrees of freedom as a derivational tool, as Rivasseau suggests, then we ultimately need to ensure that the conclusions drawn are suitably robust under variations of the relevant ultraviolet or infrared assumptions.

Finally, whichever of these routes one goes down, the brief reflections of section 5.2 suggest that the study of infinite quantum systems needs to be conducted alongside an investigation of other sources of non-uniqueness if the significance of the breakdown of the uniqueness of the Hilbert space representation of the canonical commutation relations for real-world physics is to be fully understood.

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