

Entropy cost of "Erasure" in Physically Irreversible Processes

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ABSTRACT. A restricted form of Landauer's Principle, independent of computational considerations, is shown to hold for thermal systems by reference to the joint entropy associated with conjugate observables. It is shown that the source of the compensating entropy for irreversible physical processes is due to the ontological uncertainty attending values of such mutually incompatible observables, rather than due to epistemic uncertainty as traditionally assumed in the information-theoretic approach. In particular, it is explicitly shown that erasure of logical (epistemic) information via reset operations is not equivalent to erasure of thermodynamic entropy, so that the traditional, information-theoretic form of Landauer's Principle is not supported by the physics. A further implication of the analysis is that there is no Maxwell's Demon in the real world.

1. Introduction.

Landauer's Principle (LP) was originally presented by Landauer in terms of computation. Specifically, Landauer (1961) proposed that "computing machines" engaging in logically irreversible steps incur a cost of the order of kT per each step. While LP has been largely accepted, there is a dissenting minority (e.g., Earman and Norton 1999; Norton 2005-2018; Hemmo and Shenker 2021). While the present authors join the dissenters in disputing the identification of physical irreversibility with logical/computational irreversibility inherent in Landauer's original proposal, we nevertheless adduce a physical basis for a restricted form of LP: one which is not identified with computation, but with a narrower class of genuinely irreversible physical processes. One might consider this a form of Szilard's Principle if measurement is a physically irreversible process; the present work suggests that it is.

In making this case, we wish to call attention to the crucial distinction between epistemic and ontological uncertainty (or 'information'), which tends to be glossed over in discussions of thermodynamics and the Second Law. We note that epistemic uncertainty, as assumed in classical statistical mechanics, arguably fails to non-circularly yield either the Second Law or Landauer's Principle (cf. Kastner 2017) and that ontological uncertainty is necessary for both to hold. This consideration implies a restricted form of LP that does not rely on epistemic uncertainty as traditionally assumed. Essentially, the LP is indeed a form of the Second Law, but only insofar as both are independent of epistemic considerations and instead supervene on specific physics involving the ontology of the system, independently of human knowledge or lack thereof. In contrast, if LP is *defined* as applying to epistemic uncertainty, we concur with Earman and Norton's refutations of the LP and provide an additional specific

refutation based on an analysis of the actual physics involved in epistemic 'erasure' or 'reset' operations.

2. Background

Landauer provided some heuristic arguments in support of his principle, but ultimately appealed to the Second Law of Thermodynamics (Landauer 1961, 187). Earman and Norton (1999) have correctly pointed out that attempts to use Landauer's Principle (LP) to save the Second Law from violation by such devices as "Maxwell's Demon" are circular whenever they invoke the Second Law in support of LP. EN (1999) also argue that extant attempts to save the Second Law and/or LP without resort to the Second Law are subject to counterexamples and are not universal or conclusive. In addition, Norton (2013) argues that the information-theoretic approach to thwarting the Demon is similarly circular and fails to identify a fundamental and universal principle.

Meanwhile, Bennett (1982) claimed that the relevant irreversible process is that of erasure of memory devices rather than measurement, which he portrayed as copying. Bennett's formulation has become the Received View in the literature on this topic, and a consensus has developed that Bennett refuted Brillouin's identification of measurement as the source of the entropy cost attending the entropy-reducing sorting of the Demon (Brillouin, 1951). However, the copying model of measurement applies only to the classical limit and is thus inadequate at a fundamental level. If a system is prepared in a state $\psi(x)$ with spread Δx , measuring its position is an irreversible process that, by the uncertainty principle, expands the uncertainty associated with its momentum (and vice versa). It appears that the Received View has neglected to take this point into account, and it is the focus of the present work.

3. Epistemic Uncertainty Fails to Uphold Either 2nd Law or Landauer's Principle

Before dealing with further specifics, we first consider the important concern of Norton (2013) that one cannot simply assume that "information entropy" I can be automatically identified with thermodynamic entropy S , a practice that has contributed to confusion and arguably to overstated claims about the findings of experiments (see, e.g., discussion of an experiment by Bérut *et al* in Norton (2013, 3.7). Here, we show that one cannot equate reset operations based only on epistemic uncertainty to phase space compressions (that are then putatively compensated by LP to save the Second Law). The tradition of doing so fails due to the neglect of crucial physics. For example, Ladyman *et al* (2014, 2281) argue that free energy is expended in a reset instantiating LP, since allegedly the same physical process takes place independently of the input state of the memory storage device. In fact, they regard this as the crucial meaning of a genuine "reset" operation that instantiates LP; they concede that LP fails if the reset is tailored to a specific memory state. However, the relevant physical content of the process is not exhausted by the reset operation but depends on the initial state of the system as well; and in any case, zero free energy is expended for either of the input states considered as determinate "facts of the matter" that are simply unknown to the experimenter. Thus, there

is no "reset" in the real world that upholds the LP if the uncertainty about the memory state is taken as epistemic. In what follows, we explicate this situation.

First, we need to make a clear distinction between:

- (a) the reset operation
- (b) the interactions taking place between the reset device and the system.

To our knowledge, this distinction has not previously been made in the literature, largely because of the tradition of defining entropy in terms of an ignorance, rather than ontological, interpretation of probability. It is indeed true that our ignorance requires that we cannot tailor the reset to the initial state of the system, which leaves (a) independent of the initial state. However, the actual physics requires that (b) depends on the initial state. When these details are taken into account, we find that the reset does not in fact correspond to phase space compression, despite (a). The assumption that the actual physics depends on our ignorance of the state is the essential fallacy that leads to the illicit overgeneralization that equates a decrease of logical/epistemic information to a decrease of thermodynamic entropy.

Let us now consider the physics involved concerning the expenditure of free energy (if any) in a reset process. Suppose the memory device is a box of volume V containing a gas molecule that could be either on the left or right sides of the box; the input states are thus L or R. The reset operation is the insertion of a piston from the right-hand side that imposes a constraint on the molecule such that only state L becomes available. This corresponds to (a), in that the reset operation must be independent of the initial state of the system for logical/computational reasons. First, we assume that there is no fact of the matter about the system's state; it involves a quantum system whose position is ontologically uncertain.

Recall that free energy F for an isothermal process is defined by $dF = dE - TdS$, where $dS \geq \frac{dQ}{T}$ (the equality holds in the reversible limit, which Norton (2017) correctly argues is an unphysical idealization)¹. Consider a system in a heat bath at temperature T while being compressed isothermally, so that an amount of heat ΔQ leaves the system and is dumped into the heat bath. Under these conditions, using the First Law $\Delta E = \Delta W + \Delta Q$ and for work $\Delta W = p\Delta V$ exerted on the system, the free energy of the system is:

$$\Delta F = p \Delta V + \Delta Q - T\Delta S \leq p \Delta V + \Delta Q - T \frac{\Delta Q}{T} \leq p \Delta V \quad (1)$$

The free energy expended in this process, ΔF_{exp} is thus defined by the work done on the system; specifically, $\Delta F_{exp} \geq p \Delta V$. As Norton notes, this is never an equality in the real world for any finite amount of heat transfer. But it is important to note that the above analysis

¹ To be totally precise, in the case of irreversibility the dQ would be replaced by a δQ . We neglect this detail here.

assumes that the system's ontological uncertainty takes up the entire volume V , so that there is no fact of the matter about the system--it is not functioning as a memory state.

Now, assume that the system *is* functioning as a memory such that it is actually in one of the two states L or R, and the uncertainty is merely epistemic. Firstly, the actual physical interaction between the reset device and the system differs for the two different memory states--this corresponds to observation (b). And in any case there is no real physical compression going on in this situation, i.e., when the uncertainty ('information') is epistemic. Both these points can be demonstrated as follows. If the molecule is L, then clearly $p=0$ and the motion of the piston is merely the placing of a constraint that does no work $\Delta W = p\Delta V$ on the molecule, since $p \Delta V = 0$. The latter point is crucial: *our ignorance has nothing to do with the quantity of work done on the molecule*; the physics is *not about us*. The physical process occurring when the molecule is on the L is no different from reducing the volume of an empty box, requiring zero work. Thus, it is *not at all true* that "only if we do not know which side of the compartment the atom is in do we expend free energy," despite Feynman's having fallen into this trap as well.² For this statement overlooks the fact that free energy F is defined *not* by any observer's state of knowledge, but by quantities of heat transferred and work done by, or on, the system. And zero work $p \Delta V$ is actually done in the "reset" when the atom is already in the "reset" state, despite the fact that the same mechanical process is implemented in an external sense (i.e., piston inserted).

Moreover, if the atom is in the state R, by definition it is not taking up the entire volume V but only the right-hand region of volume $V/2$. Therefore, the action of the piston is not a compression of the system's physical uncertainty from V to $V/2$; rather, $\Delta V = 0$ in this case. That is, the system is simply being pushed from one region of volume $V/2$ to a different region of volume $V/2$. Thus, while the specific physical interaction differs between the two memory states, still the relevant *thermodynamic* work $\Delta W = p\Delta V = 0$, even if there is a finite force F exerted against the atom in the case of state R. The latter serves merely to translate the position of the atom, while its associated phase space volume remains unchanged.

The conclusion is that one cannot equate "erasure" of epistemic information in a reset operation to reduction of thermodynamic entropy, even if the reset is constrained to be the same operation applied regardless of the memory's initial state. This is simply not physically valid in view of the analysis above. Nevertheless, there are still dissipative effects in the implementation of the reset, since it does require energy $\Delta E = F\Delta x$ to move the piston even if its motion does not result in nonzero thermodynamic work as defined above. But this energy expenditure is not related to "erasure" in the sense of phase space compression and therefore does not support LP.

² Feynman (1996), pp. 142-144) succumbs to this fallacy because he defines entropy in terms of epistemic probability, which leads to the endemic but erroneous conflation of "ignorance about a system's state" with physical phase space volume.

4. Erasure of ontological uncertainty involves an entropy cost

Nevertheless, the present work supports the relation of I to S under appropriate physical conditions. In teasing out the relation of I to S , we must clearly distinguish between epistemic uncertainty (in which a system is in a determinate but unknown outcome-state) and ontological uncertainty (in which a system cannot be said to possess a property corresponding to an outcome-state). Let us denote the former, epistemic form of I by I_E and the latter, ontological form of I by I_O . I_E , as Norton notes, needs to be distinguished from S : the fact that we do or don't happen to know which state a classical memory device is in does not affect its accessible phase space volume, and simply decreasing our ignorance cannot be equated to phase space compression on the part of the system itself.³ This point is emphasized in the analysis of the previous section.

In contrast, I_O is relevant to thermodynamic entropy due to the complementary relationship between incompatible observables in quantum systems and the fact that quantum states do not correspond to any determinate eigenvalue independently of measurement.⁴ The concept of joint information applicable to the quantum case was studied by Hirschman (1957) and Leipnik (1959), who built on the pioneering work of Weyl (1928). These studies yielded a quantitative expression for the joint information for Fourier transform pairs such as position and momentum. Specifically, we have the relation:⁵

$$-\int_{-\infty}^{\infty} |\psi(x)|^2 \ln |\psi(x)|^2 dx - \int_{-\infty}^{\infty} |\varphi(p)|^2 \ln |\varphi(p)|^2 dp \geq \ln \left(\frac{he}{2} \right) \quad (2)$$

where the left-hand side is the joint information denoted L in Leipnik's presentation. The associated dimensionless joint quantum information I_O is defined as $I_O = L - \ln h$:

$$I_O = -\int_{-\infty}^{\infty} |\psi(x)|^2 \ln |\psi(x)|^2 dx - \int_{-\infty}^{\infty} |\varphi(p)|^2 \ln |\varphi(p)|^2 dp - \ln h \geq \ln \left(\frac{e}{2} \right) \quad (3)$$

Note that this quantity reflects the accessible position and momentum state spaces corresponding to an objective uncertainty or spread in values, since for general $\psi(x)$ there is no fact of the matter about the system's possession of any particular value of either observable.

³ This is why erasure of a classically-modeled memory device does not correspond to a true phase space compression, as Norton notes (2013, 3.6): "Erasure reduces logical space but not physical phase space." See also Norton (2005).

⁴ Even under an assumption of hidden variables $\{\lambda\}$, I_O applies to a quantum system in state $|\psi\rangle$, since a putative property-possession λ does not define a state $|\psi\rangle$.

⁵ The differential form can in principle yield negative values, but may be viewed as an idealization, since the notion of a position and momentum continuum can also be viewed as idealizations. In addition, there is no true position observable at the fully relativistic level, so that localization is likely limited to a Planck volume (cf. Schlatter and Kastner, 2023). Thus (2) may be viewed as an excellent approximation for the macroscopic level.

For $\psi(x)$ a Gaussian, (2) and (3) become equalities. In this case, it is clear that any reduction in the information associated with either observable must be compensated by an increase by the same amount in the complementary observable. I.e., a reduction in position-information $I_{O,x}$ forces an increase in momentum-information, $I_{O,p}$, and vice-versa. Thus, since thermal states correspond to Gaussian distributions, the relation (3) implies at least an information-cost for reducing the information associated with either observable of a complementary pair, for any quantum system in thermal equilibrium.

Now, how do these considerations connect to thermodynamic entropy? Recall that classical thermodynamic entropy can be defined in terms of Boltzmann's quantity H,

$$S = k_B I_{O,p} = -k_B \int_{-\infty}^{\infty} |\varphi(p)|^2 \ln |\varphi(p)|^2 dp, \quad (4)$$

where we make explicit that this definition is physically valid for ontological (not epistemic) uncertainty. In general, "information erasure" is identified with the position observable; for example, true compression of a gas to a smaller volume (in contrast to the "reset" operation based on epistemic uncertainty) reduces its ontological position-information $I_{O,x}$ and that is paid for by a momentum-based entropy cost as defined in (4). Arguably, (3) together with (4) are the genuine physical basis for the Second law. Insofar as Landauer's Principle can be related to the Second Law, this is the only sense in which Landauer's Principle is physically justified.

This point--that it is quantum uncertainty that yields the Second Law (and a restricted, ontological form of LP)--has been obscured in the literature because the states of quantum systems in thermal equilibrium are usually given only in terms of the energy (momentum) basis, and the position basis is neglected; thus, the constraint represented by (2) is generally not evident and not taken into account. Moreover, the typical state description is that of an ostensibly proper mixed state such as

$$\rho = e^{-\beta \hat{H}} / Z, \quad (5)$$

which misrepresents the realistic situation in which a system is in a pure state of indeterminate energy (as in (2)), or in an improper mixed state (as in the case of entanglement with other quantum systems). In either case, it is not legitimate to view a subsystem as being in a determinate (pre-existing) but merely unknown energy state, so that measuring its energy is not a reversible copying procedure. Instead, measurement is, in general, irreversible.⁶

As a simple example of the relevance of (2), consider a single gas molecule in a large box of length l . This is a quantum system described to a good approximation by a pure state

⁶ Quantum measurement and its relation to entropy increase is discussed in detail in Kastner (2017). Of course, approaches assuming that quantum theory has only unitary evolution are subject to the same lack of true state space compression as the classical theory, which may be why the results offered herein have thus far been overlooked.

Gaussian $\psi_G(x, \sigma_x = l)$, such that its momentum space wave function is $\varphi_G(p, \sigma_p = \frac{h}{4\pi l})$. Using the molecule to do work via a piston would require detecting which side of the box it occupies, an irreversible position measurement that reduces the x-uncertainty such that the resulting state is $\psi'_G(x, \sigma = l/2)$. By the uncertainty principle, there is an accompanying increase in the spread of the momentum space wave function such that $\varphi'_G(p, \sigma_p = \frac{h}{2\pi l})$, and therefore the thermodynamic entropy (4) increases. This situation, rather than any considerations about classical/epistemic memory erasure (which, as noted earlier, does not actually affect physical entropy) or dissipation effects (to which there are arguably always exceptions, cf. Earman and Norton (1999)), is what enforces Landauer's Principle, but only insofar as it applies to reduction of *ontological* uncertainty (ontological erasure).

The increase in momentum-entropy corresponds to heating up the molecule, and indeed, in order to detect its position we must provide energy ΔQ . This is the familiar observation that there is no detection without the involvement of photons, and that the more precisely we wish to measure the position, the higher the energy of the photon(s) required. Thus reduction in $I_{O,x}$ inevitably leads via (1) to an associated increase in entropy $\frac{\Delta Q}{T}$ that thwarts usage of the system to extract useful work.

For the isothermal case, in which the system's internal energy E remains constant in a heat bath at temperature T, we can derive the Landauer result for *ontological* erasure of one bit of position-information (halving of the volume) via the First Law of Thermodynamics as follows (using the ideal gas approximation):

$$dE = 0 = dW + dQ \Rightarrow dW = -dQ \quad (6)$$

$$\int dW = \int p dV = \int_V^{V/2} \frac{kT}{V} dV = kT[\ln(V/2) - \ln V] = -kT \ln 2 \quad (7)$$

$$\Rightarrow \Delta Q = kT \ln 2, \quad \Delta S \geq k \ln 2 \quad (8)$$

Again, however, this derivation is only valid for ontological uncertainty of the gas with respect to the volume, since no actual compression of the volume takes place in the case of epistemic uncertainty, as shown in Section 3.

The case of momentum measurement corresponds to that of Maxwell's Demon, who is conceived as sorting faster and slower molecules into each half of the box in order to violate the Second Law. It is traditionally assumed that the molecules have a well-defined but unknown momentum, but of course this is not the case at the quantum level; thus, there is never a "Maxwell's Demon" as Maxwell originally envisioned. Treatments that presuppose this classical picture must inevitably fall short of the relevant physics. Any real "Demon" must deal with the quantum nature of the gas molecules. In practice this means that any success the Demon has in measuring the momentum of a molecule must leave its position so indeterminate as to prevent

him from getting it through the trap door. In other words, he now knows where to put it, but it has been so delocalized as to prevent physical sorting.

In more quantitative terms, the door width d must of the order of the position spread of the molecules to prevent leakage at temperature T , $d \sim \frac{h}{4\pi\sigma_p}$, where $\sigma_p \sim p_{rms} = \sqrt{3mkT}$. However, an accurate measurement of momentum involves a low-energy photon, $h\nu \ll kT$. This results in a position spread of the measured molecule of $\sigma_x \sim \frac{h}{4\pi\sqrt{m h\nu}} \gg d$. We thus note that exorcism of the Demon does not even require recourse to specific entropic considerations: the quantum uncertainty principle precludes the sorting required, so the question of "How is the entropy reduction from sorting compensated?" never arises in the first place. The Demon cannot even begin to sort a quantum gas, and a real gas is a quantum gas.

Finally, it should be noted that the Second Law is defended from these sorts of challenges not by any argument about the specific pragmatics of thwarting the various mechanical manipulations involved in any particular scenario, but by the joint ontological information constraint (2), which is an inviolable property of any Fourier transform pair of which the quantum uncertainty principle is an example. This prevents, in principle, violation of the Second Law, and in particular provides an "exorcism of the Demon" that does not depend on illicit circular reference to the Second Law. There is no Maxwell's Demon in the real world.

5. Conclusion.

It has been shown that the information-theoretic form of Landauer's Principle, insofar as it equates epistemic uncertainty to thermodynamic entropy, fails due to neglect of relevant physics. Nevertheless, a restricted form of Landauer's Principle corresponding to ontological (as opposed to epistemic) information is implied for a thermal system by the joint entropy corresponding to the quantum uncertainty principle, independently of the Second Law. This remedies the current situation, in which Landauer's Principle has often been invoked as a surrogate for the Second Law when it is the Second Law itself that is being challenged. It also arguably provides the true physical basis for the Second Law, and reminds us why there can be no Maxwell's Demon.

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