How bad is the postulation of a low entropy initial state of the universe?

Aldo Filomeno Farrerons*

Open Problems in Philosophy
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Abstract

I briefly summarize, through an informal interview, the main answers given to the 'Past Hypothesis', the postulation of a low-entropy initial state of the universe. I have chosen this as an open problem in contemporary philosophy, specifically in the foundations of physics. I hope this (too brief) overview helps the reader in gaining perspective and in appreciating the varied and fascinating landscape of arguments and proposals in this debate—a debate which is part of the quest to explain and interpret our scientific picture of the universe.

Keywords

Foundations of Thermodynamics; Entropy; Probabilistic Explanations

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*Instituto de Filosofía, Pontificia Universidad Católica de Valparaíso.
aldo.filomeno@pucv.cl.
https://philpeople.org/profiles/aldo-filomeno.

1 What do you think qualifies as an open problem in philosophy?

Something that only some philosophers of physics and physicists stress, which is a pressing open problem with dramatic consequences, is the need for a satisfactory explanation of the 'Past Hypothesis': the postulation of a very, very, very low value of the entropy of the universe at an early time, usually thought to be close to the Big Bang. Its postulation is necessary for the most accepted explanation of the second law of thermodynamics, as proposed by Boltzmann. The postulation becomes suspicious when we realize that it is not just a special, very unlikely value, but basically the opposite value that one would expect for the entropy of the early universe. Merely postulating the Past Hypothesis, without further explanation, constitutes an important open problem in the philosophical foundations of thermodynamics and cosmology. Without a proper explanation, we lack an explanation of the second law, thus we lack an account of some of the most ubiquitous phenomenological aspects of our reality, namely the rise of entropy towards equilibrium and the irreversibility of natural phenomena (and time's arrow, according to the majority who relate it with entropy's arrow).

I believe this is also an especially interesting open problem because many philosophers and physicists tend to neglect the issue, while others acknowledge the need to explain the Past Hypothesis but their explanations are clearly disputable. There's also an attempt by Callender (2004) to argue that, after all, the Past Hypothesis does not really need an explanation. Here, I would like to highlight that a central aspect of our scientific image is seriously flawed unless we provide at least one acceptable candidate explanation for the Past Hypothesis.

Also, an emphasis I would like to make in this brief overview is on a sometimes neglected matter of degree, which concerns the degree of severity of the mere postulation of the Past Hypothesis (PH). In other words, I will give a rough quantitative sense of the remarkably high level of demand for an explanation, which translates to an exceedingly low expectation that a rational agent should have regarding the universe beginning with such a small entropy value. In sum, the PH represents *the opposite* of what one would typically expect as the initial state of the universe, not merely "a very special or unexpected value," but *the most* special case.

2 Can you briefly reconstruct the history of the problem?

All begins with Boltzmann himself: he proposed a statistical explanation of the second law of thermodynamics, but as I explain below, its insufficiency was soon pointed out by Loschmidt. Then, Boltzmann proposed that the universe began in a very low entropy initial state.

The second law of thermodynamics has been formulated in various ways—a fascinating topic for philosophers, but that now we can set aside (for good introductions to the notion I recommend these textbooks: Atkins and De Paula, 2014; Stowe, 2007; Sethna, 2006; Reif, 1965; cf. Atkins, 2007). Now it suffices to refer to the second law of thermodynamics as follows:

(2nd Law): "The entropy of a closed system increases or remains constant."

The most widely accepted explanation of this law indicates that it is not a fundamental law of nature but rather a statistical macroscopic phenomenon, emergent from a fundamental microscopic dynamics. Hence, the second law is explained (or reduced) in mechanical terms.

Boltzmann's standard explanation resorts to combinatorial mathematics: Boltzmann (1877) noted that the vast majority of all possible micro-states correspond to one macro-state, the equilibrium macro-state M_{eq}, the macro-state in which the particles are approximately uniformly distributed throughout physical space, that is, the macro-state in which the entropy is maximum. In the measure considered natural, the uniform Lebesgue measure μ, $\mu(M_{eq})$ is extremely larger than any other region; in fact, M_{eq} occupies almost the entire phase space Γ.

An estimate of the ratio between M_{eq} and another generic region is usually close to 10^N, so for a $N = 10^{23}$, say, the ratio would be $10^{10^{23}}$.

Thus, Boltzmann defines the entropy S_B as the logarithm of the size of the region of the macro-state M:

$$S_B(M) = k_B \; ln|\Gamma_M|$$

where k_B is Boltzmann's constant, and Γ_M is the phase-space volume of the region corresponding to macro-state M. Then, due to the radical difference between the sizes of the equilibrium macro-region and the rest of the macro-regions, the non-decrease in entropy established by the 2nd law will be extremely more likely to happen. In fact, it turns out that $S_B(M_{Eq}) >> S_B(M_{\neg Eq})$ where $M_{\neg Eq}$ refers to any other macro-state.

Boltzmann's account would seem to explain the time asymmetric behavior stated by the 2nd law, preserving the classical time-symmetric picture of the world. The 2nd law, thus understood, is not a strict law but a statistical approximation.

However, even though many overlook the intricacies inherent in Boltzmann's approach, there exists a substantial body of literature comprising research by physicists, mathematicians, and philosophers who have been trying to unravel these intricacies (see, e.g., Price, 1996; Sklar, 2015; Frigg and Werndl, 2023; Hemmo and Shenker, 2012; Albert, 2000; Allori, 2020). The most prominent issue, originally identified by Loschmidt, points to a flaw in Boltzmann's explanation above, and is related to the need to account for the *irreversibility* that Nature seems to exhibit at a macroscopic scale, despite a fundamental dynamics that is time reversal invariant. If the account of the 2nd law predicts entropy increase towards the future, but also towards the past, then something in the explanation is failing. And this is what the initial account by Boltzmann, as himself noticed, does: the microscopic laws (classical or quantum), which predict entropy increase towards the future, are time-symmetric; hence they will also make 'retrodictions' of entropy increase towards the past (see figure 1).

Hence, it is thought that some asymmetric element must be added, in order to obtain the time asymmetry that our apparently irreversible universe displays. The most accepted proposal today, already proposed by Maxwell and Boltzmann, was postulating that the universe

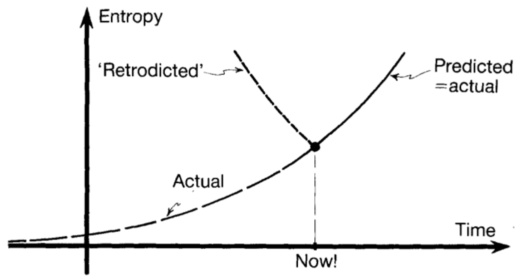

Figure 1: The initial statistical explanation by Boltzmann was insufficient, due to the reversibility problem, since the explanation also predicts an entropy increase towards the past (from (Penrose, 1989b)).

began in an initial state of very low entropy: the nowadays called 'Past Hypothesis'.

3 Why it constitutes an open problem?

At first glance, one might think along the lines of: "ignoring the behavior of the universe during such distant and energetically extreme regimes, in the absence of any physical theory or empirical evidence, it is conceivable and nothing rules out that the universe could have originated with any entropy value, including a low one." Such reasoning is, however, undermined by both probabilistic and scientific considerations (cf. Earman, 2006). The required low entropy value is the opposite of what we would expect, thus demanding an explanation.

The first basic issue, which I would like to emphasize, concerns the exceedingly small magnitude of the entropy value, so remarkably low that it is regarded as an exceptionally *special* value. It would be highly unexpected, strange, or (if probabilities are applicable in this context) extraordinarily unlikely.

Analogous problems in cosmology. And wait, it's not just the PH: this is just one of a number of several cases of *alleged* fine-tuning, that in the last decades we are finding in cosmology for a variety of parameters. It has been critically assessed in philosophy, where some have argued that it is relying on unwarranted probabilistic assumptions, so it is *not* really a problem. Hossenfelder (2018) has popularized the critical assessment of these probabilistic arguments, which was being carried out for many years in the philosophy of science (for instance, regarding the notion of naturalness, Richter, 2006; Wells, 2019; cf. Williams, 2015; Bain, 2019). Precursors of this debate can be seen in the criticisms of the anthropic cosmological principle (e.g. Mosterín, 2004), and more generally, of probabilistic reasoning in cosmology (e.g. McCoy, 2018; Norton, 2018), which in turn echo the same criticisms already found in the foundations of statistical mechanics (Sklar, 1993, 3.III). So, there is a wide number of fascinating open problems here! To settle whether they really are a problem, and if so, what to do about it, depends on each different case.

How bad it is. So, first of all: how acceptable the postulation of an hypothesis is can be a matter of degree, as here it is. The degree of dissatisfaction with the PH is, other considerations aside, inversely proportional to how small the required entropy value is. However, probabilistic explanations have been studied in philosophy and it is agreed that one can explain unlikely phenomena, which can just happen, even if they were very unlikely (see e.g. Strevens, 2008). This is especially acceptable if we know that there are no other alternative competing explanations: if your neighbour wins the lottery, and you know that the lottery is fair (for whatever reasons), then no matter how incredibly unlikely was that she won; it is rational to accept that she just did (the lottery paradox is related to this; see e.g. Wheeler, 2007).

However, that is not the scenario of the PH, for several reasons. There is still the aspect of how *special* a specific possibility (or subset of possibilities) is with respect to the rest of the space of possibilities. This distinction can make a difference, and lead us to never expect such a value; and if it does occur, to expect that there is some underlying explanation. In the case of a lottery, imagine that a single-event world-wide lottery takes place, and the winner turns out to be... the owner of the whole lottery machinery.

It seems reasonable to look for, or at least welcome, an explanation. It is perfectly possible, so it cannot be proved that the owner has cheated. On the implicit assumption of equiprobability, it is not more improbable than any other possibility, so we might be tempted to accept the winner—and not be jealous. However, the owner (and some few other individuals) constitute a *special* subset: the small group of people which had the ability to tweak the lottery. It seems, then, reasonable to investigate whether this is a clue of an alternative, now unconceived, explanation.

In our case, the special value amounts to S_B being so small. If the value is deemed as special, this means that an ideal rational agent should expect, and look for, an explanation. Now, we would be assuming too much if we would think that the value is the result of some process analogous to a lottery: in our context nothing warrants the randomization and blind selection that occurs in a fair lottery.

We can roughly quantify whether the needed value of S_B is special. First, since in Boltzmann's formula S_B is connected to the sizes of phase-space regions Γ, in the statistical explanation of a gas adiabatically expanding in a closed container, we already have quantified the volumes in phase space, showing how the size of the region of microstates corresponding to the equilibrium macrostate is many orders of magnitude larger than any other region. Previously, I have cited the incredibly high value $10^{10^{23}}$ referring to the ratio among the sizes of regions of phase space (the equilibrium macrostate with respect to the others), for the tractable case of an adiabatic gas.

In this very small context we can already realize the extreme disparity of sizes and, correspondingly, how extremely special is the low entropy initial state of all particles located approximately in the left corner of the container. When the setup is arranged by the scientist on purpose to have the particles initally arranged like that, we have an explanation for it. These numbers show how extremely surprising would be to spontaneously get such an initial arrangement.

With this in mind, let me now conclude citing the estimation that has been made at a cosmological scale. This assessment is tentative and relies on numerous highly speculative theoretical assumptions (including the thermodynamics of black holes, dark matter and dark energy, etc.). One popular rough estimation is due to (Penrose, 1989b,a) (adjusted in (Penrose, 2003, Ch.2.6) to include dark matter), who gives an estimate of the size of the state space volume corresponding to the smoothness acknowledged in the CMB, i.e. the low entropy, of the order of $10^{-10^{124}}$, in a standard $k = +1$ (i.e. spatially closed) Friedman-Lemaitre-Robertson-Walker cosmological model. Thus, stated in probabilistic terms (presupposing that one microstate is as likely to be the initial state as any other), it turns out that:

$$P(P.H.) = \frac{1}{10^{10^{124}}} \approx 0$$

In other words, on grounds of epistemic rationality, one should expect that the universe did *not* begin in such a low entropy initial state.

3.1 What are the contemporary motivations?

In the last decades the insights from contemporary cosmology entered the scene, and are thought to influence the debate. The first substantial novelty is that now we have a picture of the early universe: the celebrated cosmic microwave background radiation (CMB), shown in Figure 2. The standard interpretation is that, in spite of the appearances, this high degree of homogeneity and smoothness in the early universe corresponds indeed to a very low entropy state. The traditional story is that this is because in such energy/scale regimes *gravitational* effects must be taken into account (negligible in the classical gas in a box) which imply that the equilibrium state is exactly the opposite; that is, the least likely state to find would be such a uniform distribution of matter-energy. Hence, after this plot twist involving gravitation, the CMB seems to vindicate that the universe started in a very low entropy state.

This empirical acknowledgment, however, does not eliminate the need to explain why indeed the universe started in an extremely unexpected initial state. In fact, we can think that it is almost by the definition of Boltzmann's entropy S_B that, if in Boltzmann's theory it is proposed a very low entropy initial state, then an explanation will have to be sought. The reason is that Boltzmann's formula for entropy S_B is defined in terms of the sizes of phase-space macroregions, and this formula was so because it was aimed to quantify the 'likeliness' of the micro-states; if we agree with equating sizes with probabilities, then it is almost analytical that very low entropy amounts to very special ('atypical', 'not generic'), and 'very special' amounts to something that demands (or at least welcomes) an explanation.

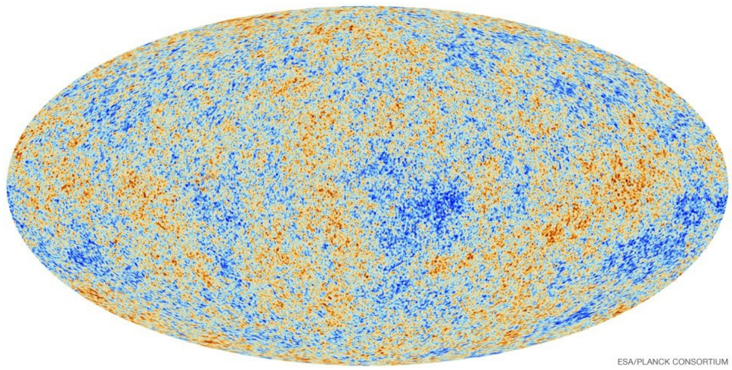

Figure 2: The cosmic microwave background radiation, which shows an early universe approximately homogeneous and isotropic. The changes in color show fluctuations departing from the average value, and only amount to about 1 part in 100.000.

4 Can you reconstruct approaches that may constitute significant avenues of research today?

Cosmological Inflation. In addition to the role of gravitation and empirical cosmological evidence, another contribution from cosmology is the alleged role of inflation. Inflation refers to the postulated period of exponential expansion in the early universe. This hypothesis has been a subject of extensive debate for decades, facing criticism on technical and philosophical grounds (Smeenk and Ellis 2017 and references therein). For now, let us set aside these debates and assume that some form of inflationary process did occur. Some argued that finally, inflation, after a century of debates in the foundations of thermodynamics, provides an explanation for the PH. The idea is that during the alleged stretching of matter-energy during the accelerated expansion, any irregularities were smoothed out, thus explaining the homogeneous low-entropy initial state post-inflation.

However, it has been pointed out by many (Price, 1996, Ch.4; Penrose, 2003, Ch.2.6; Penrose, 1989a; Earman, 2006, §7; Carroll, 2010, Ch.14) that this, unfortunately, might be too quick: inflation amplifies pre-existing disparities in the region that is going to suffer the expansion, and anyway it also requires very special initial conditions to be triggered. Hence, it is said that it already requires a prior degree of smoothness (in one way or another, but still amounts to a special requirement).

Probabilistic objections. A different sort of attempts tries to resist the urge to explain the PH, by appealing to worries in the above probabilistic exposition of the problem: most notably, the problem of selecting objectively, i.e. in a non-arbitrary way, a unique probability measure: in other words, how to justify the natural *uniform* probability measure ('Lebesgue measure') that is adopted in classical statistical mechanics? This is a point that, again, threatens all probabilistic reasoning in the philosophy of cosmology involved in the objections for fine-tuned parameters (cf. Earman, 2006; Filomeno, 2023). It is, in turn, related to the so-called 'reference class' problem, first noted by the mathematician Bertrand in his Bertrand paradoxes. See Van Fraassen (1989) for a nice illustration involving a cube factory.

Even ignoring the reference class problem, what grounds do we have to belief that any initial condition is equally likely? Since we are completely ignorant about such an era, in which our laws of physics are known to break down, it seems reasonable to avoid any unwarranted commitment to a probability measure. Although I understand that it seems intuitive to endorse the uniform distribution, in many contexts of ignorance this turns out to be unjustified—it could just not be that way that things are probabilistically distributed (see e.g. Albert, 2000). Even in the context of classical statistical mechanics, the identification of the probabilities with the measure of phase-space volumes is controversial: a condition of ergodicity[1] would justify it, because it's a sort of (weak) "randomization" condition that guarantees (an approximation) to a random wandering through phase-space, hence a uniform distribution in the long run. However, such ergodicity has been criticized, as neither necessary nor sufficient to explain thermodynamic behavior. In sum, this sort of criticism to the probabilities is one way in which some have tried to resist the criticisms and embrace the PH, because it undermines claims that the

[1]Ergodicity means, following one definition, that the system's point in phase-space spends an amount of time in the macro-regions proportional to the volume of those macro-regions. It is intended to convey the idea that the physical system will visit proportionately the more typical macro-states (by 'typical' I mean the macro.states that correspond to more micro-states). See Frigg and Werndl (2023).

PH is unlikely, for they are ill-defined.

The PH as a Humean law. The influential David Albert and Barry Loewer have been advocating for a naturalistic worldview (the 'mentaculus'), one of whose basic 3 ingredients is the PH. (Together with the two other ingredients, viz. the laws of nature and the Statistical Postulate, it is a sort of rough qualitative sketch of a "complete theory of the universe" (Allori, 2020, Ch.1), in that allegedly accounts for the statistical regularities of the history of the universe.) Other recent examples are Eddy Keming Chen's articles along the same line, delving into the initial state in the quantum domain (e.g. Chen, 2021). These authors do not exactly just assume the PH, but add it to the set of 'Humean laws'.

It seems ad-hoc to put in the list of the Best System whatever you need to postulate; and it becomes more problematic if it is a boundary condition. Then, it becomes even worse when it is not a generic (typical) boundary condition, but a very atypical one.

These criticisms aside, appealing to Humean laws is, curiously, worsening the situation even more, in that the Humean approach faces an analogous problem. The (neo-)Humean account of laws has undergone many sophistications, yet it continues to face a major issue that was recognized from the outset. It's an account of laws, of apparently non-accidental regularities, unable to explain the *ubiquity* of regularities of the Humean mosaic (the history of the universe). These regularities are *pervasive* throughout the entirety of the mosaic, the number of them (as summarized in the sum of textbooks of a physics library) is so large that basically no interaction is left to chance. In other words, nothing is treated as a fluke in the fundamental level, according to physics (even if we take fundamental laws to be indeterministic). Instead, the (neo-)Humean says no explanation is needed. Thus, the 'uniformity of Nature' (in David Hume's terms) is nothing but a cosmic coincidence ('cosmic' in the sense of being of cosmic dimensions). The Humean might refuse to explain cosmic coincidences, but this move seem to run against basic norms of rationality. This well known objection is spelled out in (Filomeno, 2021).

The interesting point now is that we can see that this dialectics is the same as with the

PH! Another extremely atypical value is just assumed as a brute fact. Then, by adding the PH to the Humean Best System only exacerbates the situation. Unless, of course, one rejects the probabilistic reasoning in the previous paragraphs, as some will do (wrongly, I'd say). Since the situation was already "quite" problematic prior to introducing the PH, it becomes challenging to quantify how much the situation worsens when the PH is incorporated as a law.

The PH as a dynamically explained asymmetric boundary condition. A more traditional move is due to Roger Penrose, who tried to explain the PH with a dynamical explanation which invokes a boundary condition at the initial spacetime singularity (Penrose, 1980, 1989b). He took into account the relevance of gravity in such scales, and referred to the Weyl curvature tensor, a geometric quantity associated with the curvature of spacetime. He then conjectured that the Weyl curvature tensor must have been extremely small (close to zero) at the initial Big Bang singularity, this being associated with a smooth spacetime geometry, which in turn would allegedly explain the low value of S_B. This condition is postulated as a sort of primitive principle—a fundamental law of nature. This, comprehensibly, has sounded to some as ad-hoc. Still, it shifts the focus to looking for explanations of this asymmetric boundary condition.

Another objection to Penrose's is an implicit way of reasoning that has plagued most of the treatments of the topic, including those previously cited. I refer to what Price (1996) has insisted and called it the "temporal double standard": the (alleged) illegitimacy in invoking asymmetric elements in an otherwise symmetric worldview given by modern physics (mostly, given time-reversal invariance and general relativity). In particular, Price criticizes any reasoning that involves presupposing (lacking any justification, as we lack in modern physics) any privilege of one time direction, and one "corner" of the universe, rather than the other—that is, presupposing that time flows from past to future, or that some initial condition held in the big bang but not in the big crunch. Eliminating this double standard (as in the symmetric approach by Thomas Gold) however, faces its own peculiar challenges. Of course, all this is very introductory, and does not make justice to any view. But I hope that this summary is able to make the reader appreciate the foundational questions at stake.

4.1 Do you have a preferred route to tackle this problem?

A variety of accounts of the 2nd law avoid needing to postulate the PH, although of course they face their own challenges. To begin with, again Boltzmann himself, together with his PhD student Schütz, considered that the universe is indeed in thermal equilibrium almost always, except for rare fluctuations; then, the era in which complex living systems like us have evolved is just in one of the slopes of the entropy gradient during these rare fluctuations. This impressive revisionary view, however, requires that the universe is extended in time for sufficiently long to give rise to such sufficiently strong—hence extremely unlikely—fluctuations away from equilibrium. Modern cosmology, in particular the alleged age of the universe (13.8 billion years since the Big Bang), constitutes an objection. There's also a weird thought experiment, that I think people have taken too seriously. It says that, in this picture, brains are more likely to spontaneously arise from those fluctuations than all the structure around us that has led to our human brains (for criticisms of this so-called 'Boltzmann-brains' thought experiment, see Dogramaci 2020; Chalmers 2018; Carroll 2017).

An alternative approach, which also eschews positing the PH, consists in considering that the fundamental dynamics is stochastic. This has been studied through various mathematical formalisms, see e.g. Van Kampen (2011); Davies (1974); Mackey (1992); Streater (1995). Mackey proceeds by denying the existence of an underlying deterministic or time-reversal invariant dynamics (similarly, Streater maintains an agnostic stance). For discussion, see Uffink (2009). I believe these stochastic explanations hold particular significance in investigating a related topic: the formation of lawful behavior (which scientists usually take for granted and don't seek to explain) from a non-lawful stochastic evolution (a subject I have devoted much of my research to; see Filomeno, 2019).

Other substantially different proposals, which also avoid postulating the PH, are those by Carroll and Chen (2004), and the project of 'shape dynamics' by Barbour, Koslowski, and Mercati (Barbour et al., 2014) (compared in Lazarovici and Reichert, 2020). I personally enjoy delving into original alternatives, but it's also important for scientific and philosophical progress (keep in mind the debates on scientific realism about unconceived alternatives, Stanford, 2006). We find little dogmatism in the philosophy of physics community today, compared to other areas of philosophy (which, remember, it had to be the temple against dogmatism!), but for some reason we also get mad at slight alternatives to our work. I say this because Carroll and Chen's proposal can be included in a kind of alternatives which, I must admit, some time ago I myself also found outrageous; now, not anymore, and I confess I even find good reasons to support them and the objections misguided. I refer to the heavily revisionary possibility of explaining very special (i.e. fine-tuned) features of our universe (such as S_B) by, first, noting that the cosmological model we have today is only a model of the *observable* universe; and that an extrapolation to the whole universe, of unknown size, is an inductive leap whose justification is unknown. It is then conceivable that our observable region is merely a tiny fraction of a significantly larger and different universe, where the peculiarities observed in our limited corner can be attributed to contingent or random features: these special features become less surprising when viewed as a small sample of a much larger universe with diverse parameter values. This approach, then, is complemented by the observation that complex structures, like human beings investigating the cosmos, are special, atypical, phenomena which can only arise and investigate their environment in certain subregions—one in which, for instance, the initial entropy began extremely low.

So, well, yes, this speculative proposal can be seen as a invoking a "multiverse", complemented by an anthropic argument. While some past anthropic arguments have been notably flawed, this does not mean that all such arguments are. On the other hand, scenarios involving a multiverse face the "slight" objection of not being scientifically testable. Nevertheless, this serious epistemological limitation does not determine whether a metaphysical thesis is or not the case. Sometimes we scientists and philosophers forget about it, but it's far from guaranteed that the true ultimate reality is empirically testable.

Last, but not least at all, my favorite route is a revision of the notion of entropy at a cosmological scale in light of the influence of gravitation, since I think that the standard picture of this relation is too simplistic. Earlier, I mentioned the role of gravitation to you, but I lied: that was the standard account, but I think the situation is more tricky. Wallace (2011) has already pointed out some confusion in the standard story (cf. Callender, 2011; Dougherty and

Callender). Ultimately, I think that depending on how we interpret the thermodynamic role of the gravitational degrees of freedom, the thermodynamic history of the universe could be very different. I don't have space to elaborate but, for instance, let me advance one thing: cosmology treats the history of the early universe as being in local thermal equilibrium, with constant entropy (in spite of the expansion). How to fit this with the standard story in the foundations of thermodynamics that I have just told you? Stay tuned for more, or feel free to contact me to discuss it!

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