Transfer of quantum information in teleportation

Abstract

The controversial issue of information transfer in the quantum teleportation procedure is analyzed in the framework of the many-worlds interpretation of quantum mechanics. It is argued that quantum information, considered as a measurable property for an observer in a particular world, is transferred in a nonlocal way in teleportation process. This, however, does not lead to an action at a distance on the level of the universe which includes all parallel worlds. The alternative approach of Deutsch and Hayden is discussed.

1 Introduction

Recently, we have witnessed a rapid development of quantum information science fueled by a quantum technology revolution, which allowed the experimental implementation of many theoretical ideas. Philosophical analyses of quantum concepts, which were introduced at the birth of quantum theory but never reached a consensus, become more relevant than ever. Here, Ianalyze arguably the most bizarre quantum information protocol: quantum teleportation, a transfer of a quantum state with surprisingly small resources.

When Asher Peres, a coauthor of the teleportation paper (Bennett et al. 1993) was asked by a reporter if quantum teleportation could teleport the soul as well as the body, he answered: "No, not the body, just the soul." What is transferred in the teleportation protocol, and how, is still the matter of controversy. The indistinguishability of quantum particle made Saunders (2006) to ask the question: "Are quantum particles objects?". But this indistinguishability is what made teleportation possible: the particle (the "body") is not moved in the teleportation protocol. It is the quantum state of a particle (the "soul") in one site that is transferred to a particle in another site.

People are not teleported today from one city to another and it is safe to say that it will never happen, but the teleportation protocol has become one of the cornerstones of quantum information. The mathematics of teleportation is uncontroversial, but we still need to gain understanding of the paradoxical features of this process (see Vaidman 1994a): how one can send a quantum state, specification of which requires a large amount of information, by sending only a tiny amount of information through a classical channel:

two bits instead of two angles for sending a qubit (Bennett et al. 1993), or two real numbers instead of two real valued functions for sending a quantum state of a continuous variable (Vaidman 1994b).

The approach of Deutsch and Hayden (2000) to the question of information flow in the teleportation protocol created a large controversy regarding the concept of "quantum information", see Duwell (2001, 2003), Timpson (2005, 2013), Wallace and Timpson (2007), Deutsch (2011), Lombardi et al. (2016), Lopez and Lombardi (2018), Bedard (2021a, 2021b). Timpson (2006, 2013) argued that the way out of this conundrum is to realize that "information" is an abstract concept and it is a mistake to take the view that "something travels from Alice to Bob in teleportation... in a spatio-temporally continuous fashion".

Another line of research that attempts to explore the meaning of quantum teleportation might be mentioned. It culminated in a paper "Classical Teleportation of a Quantum Bit" (Cerf et al. 2000). A natural interpretation of such a title is a transfer of a quantum state from Alice to Bob, both of whom have quantum capabilities but do not have a quantum channel. The authors write instead: "Classical teleportation is defined as a scenario where the sender is given the classical description of an arbitrary quantum state while the receiver simulates any measurement on it." It is surprising that this task can be achieved with shared randomness instead of an entanglement channel and a very small amount of transferred classical information, but this scenario does not provide what is promised in the title of the paper: a quantum bit was not teleported in this procedure.

I do not view quantum mechanics as a particular probabilistic theory that predicts statistics and correlations of the results of measurement. Of course, quantum theory can describe ensembles, but it also describes single systems (see Vaidman 2014). In the teleportation scenario, the sender should get a qubit, not its classical description. More importantly, the output should be a qubit. I give Alice a qubit and come to Bob with *my* measuring device to test it. For proper teleportation, I expect that a verification measurement of the state of the qubit will succeed with probability 1. This cannot be done without entanglement.

I believe that it is possible to have a coherent picture of quantum information transfer in teleportation. The core of the controversy is that relevant concepts are often understood in different ways. Although I, as Deutsch and Hayden, perform the analysis in the framework of the many-worlds interpretation (MWI) (Everett 1957), my conclusions are very different: the nonlocality of Everett's worlds is the basis of the teleportation of quantum information. The difference in conclusions is not necessarily a contradiction. Our disagreement follows from the difference in the ontologies of our approaches. I assume that the wavefunction of the universe is the only ontology. Apparently, the nonlocality of the worlds which follows from my assumption led Deutsch and Hayden and their followers to search for an ontology which avoids this nonlocality.

Here is the plan for the rest of the paper: In Section 2 explains how I use the term "information". In Section 3 I introduce the concepts of a classical bit, a rabit (something between bit and qubit), and a qubit. In Section 4 I analyze the transfer of information in the encoding procedure of a qubit to clarify the concepts of a qubit and "information about the qubit". Section 5 analyzes teleportaion of a qubit. An alternative approach by Deutsch and Hayden is presented in Section 6. Section 7 describes the teleportation of a rabit, which is arguably the simplest protocol demonstrating the paradoxical property of teleportation, transferring an object that requires much more than a bit for its description by sending just a single bit. Section 8 summarizes the results of the paper.

2 Information

I want to discuss information transfer, but the word "information" can have many different meanings. Google provides the following definition of the word information:
1. Facts provided or learned about something or someone.
2. What is conveyed or represented by a particular arrangement or sequence of things.
In computing: data as processed, stored, or transmitted by a computer.
In information theory: a mathematical quantity expressing the probability of occurrence of a particular sequence of symbols.
"2" and "data" describe best what I adopt here as the concept of information. I analyze physical processes that implement protocols for transfering qubits which also involve the transfer of bits. Qubits and bits are not facts. A common protocol is transferring a secret key which does not represent any facts. I also discuss transferring an object which I call rabit (random bit) the definition of which involves probability, but I consider the transfer of physical objects, not mathematical quantities.

I do not consider "information$_\mathrm{t}$" a technical term in Shannon theory for analysis of channel capacity etc., see Timpson (2013). Classical physics does not have sources creating bits with some probabilities, the basis of Shannon theory. Quantum information that I consider is also not a quantum "information$_\mathrm{t}$", a technical term in the framework of Schumacher (1995) in his generalization of information theory to the quantum domain. But contrary to the classical case, quantum information cannot be understood easily in layman terms. It is relevant for quantum physicists who analyze quantum computation or other tasks in the new field of quantum communication which started from "quantum money" of Wiesner (1983).

The classical and quantum data require physical objects as the carriers information, but the particular physical properties of these carriers are not part of the definition of the information. The same information can be carried by very different systems, but the information does not exist without physical systems. This allows one to define location and flow of information through location and motion of these physical systems.

I will discuss information transfer in the processes of transferring bit, rabit, and qubit. These objects are often considered elementary units of information of different types. In the next section, I will provide their definition by specifying descriptions of physical systems corresponding to their existence. Then, e.g. a qubit, will be identified with the system carrying the qubit, but without physical identity of this system. A qubit is an example of quantum information, but the definition of quantum information is wider. In the process of transferring a qubit, often there will be a stage in which there will be no system carrying a qubit according to the definition given below. Still, if the transfer protocol has, in principle, a certainty of success, I will say that information about the qubit is present.

Let me summarize the definition of information as follows. Physical systems carry information about the qubit if it is possible to create the qubit by interaction with these systems. The quantum state of these systems without physical identity of the systems is defined as quantum information. The location of these systems defines the location of the information. Similarly, the information about bit is classical information: physical systems encoding a bit without their physical identity (e.g. a paper with marks of ink, or a pixels on a laptop screen). Physical systems which allow the creation of a rabit (again, without their physical properties) represent information about a random bit. Classical physics does not have randomness, so this is also a type of quantum information.

Qubit (as rabit) can also be defined by using a description of the procedure with macroscopic devices that creates the qubit. Essentially, it is given by the two numbers (p, ϕ). I will name it as classical information about qubit. Contrary to quantum information about the qubit, it allows one to create multiple copies of the qubit. The teleportation of a qubit I consider is when we have a qubit in one location and it is moved to another location. The input is quantum information of the qubit. It was first implemented by Bowmeester et al. (1998) while the experiment by Boschi et al. (1998) performed at the same time had as input the classical information of the qubit. See discussion in Vaidman (1998).

3 Qubits, bits and rabits

I consider three types of elementary units of information: classical, quantum, and random. They are named bit, qubit, and rabit (the latter is introduced in this work). A physical system with two distinguishable states (which we will name 0 and 1) can carry a qubit, bit, or rabit according to the conditions which are defined below. The particular physical nature of the system (photon with polarization states, atom with two energy eigenstates, or anything else) is not part of the characterization of the information.

Qubit is a superposition of the corresponding states $|0\rangle$ and $|1\rangle$. It can be written in a form

$$|\psi\rangle_{\rm qubit} = \sqrt{p}\ |0\rangle + e^{i\varphi}\sqrt{1-p}\ |1\rangle, \tag{1}$$

so it is characterized by two numbers $p \in [0,1], \quad \varphi \in [0, 2\pi)$. It can be measured in the $|0\rangle, |1\rangle$ basis and then the probability of finding the state $|0\rangle$ is p, but it can also be measured in any other basis, including a basis in which a particular outcome is obtained with probability 1. This basis and the corresponding state are an alternative characterization of the qubit.

Bit is the choice of one of the states $|0\rangle$ or $|1\rangle$. It is characterized by a member of a set of two numbers $x \in \{0,1\}$. Measurements are defined only in the $|0\rangle, |1\rangle$ basis, and the result x is obtained with certainty.

Rabit is a "mixture" of the two states $|0\rangle$ or $|1\rangle$. In some sense, both are present. Measurements are defined only in the basis $|0\rangle, |1\rangle$ and the outcome 0 is obtained with probability p. The parameter that characterizes the rabit is $p \in (0,1)$.

When our system is entangled with other (microscopic) systems and its Schmidt decomposition can be written in the $|0\rangle$, $|1\rangle$ basis, it carries a rabit. In general, a system might not carry any of the information concepts defined here: qubit, bit, or rabit, so the treatment of teleportation is not the most general, but it provides a wide stage for discussion of quantum (non)locality issues.

The framework of the current analysis is quantum theory without the collapse postulate. The complete ontology of the universe is the wavefunction of the universe (Vaidman 2016). To connect the ontology to agents which can discuss information transfer in quantum protocols, the wavefunction of the universe is decomposed into a superposition of the wavefunctions corresponding to worlds specified by the requirement that within a world all macroscopic objects are well localized (see definition in Vaidman 2021):

$$|\Psi\rangle_{\rm UNIVERSE} = \sum \alpha_i |\Psi\rangle_{{\rm WORLD}\ i}, \tag{2}$$

The decomposition (2) is not defined precisely by physical parameters. "Macroscopic" and "well localized" are concepts specified by agents to help them explain their experience. Essentially, each world is one of the possible worlds of the standard approach to quantum mechanics in which every quantum measurement ends up with a single outcome.

In a world there is a qubit (p, φ) if the wavefunction of the world is a product state of a system S and the rest of the world:

$$|\Psi\rangle_{\rm WORLD} = \left(\sqrt{p}\ |0\rangle_S + e^{i\varphi}\sqrt{1-p}\ |1\rangle_S\right) |\Psi\rangle_{\rm REST}. \tag{3}$$

The system S must be microscopic, otherwise (3) will not correspond to a world.

To have a bit in a world, the world wavefunction must have one of the following forms:

$$|\Psi\rangle_{\rm WORLD} = |0\rangle_S|\Psi\rangle_{\rm REST}, \quad \text{or} \quad |\Psi\rangle_{\rm WORLD} = |1\rangle_S|\Psi\rangle_{\rm REST}. \tag{4}$$

A quantum measurement process measuring states $|0\rangle_S, |1\rangle_S$ (with macroscopic device) leads to such situations.

One may wonder how the definition of a bit can include concepts which are only vaguely defined: "measurement", "macroscopic". This is because bit is not a concept of an exact physical theory, but a concept of conscious beings in a world which helps them to explain their experience. A bit is a precisely defined concept in information (Shannon) theory, but this is a mathematical theory which does not have a corresponding precise counterpart in physical (quantum) reality.

The concept of a rabit has a similar difficulty. Mathematically, rabit is well defined: a dichotomic random variable. It is a basic element of the probability theory, but classical physics does not have anything which represents it. The quantum physics describing all worlds together, also cannot be used to represent a rabit. However, we can have a rabit in a world of the MWI and this is what is needed, since it is a concept of an agent living in a world. We need that in the wavefunction of the world our two-state system is entangled with one or many microscopic systems (ancillas) which are not in reach of the agent. A rabit p is present in a world if the wavefunction of the world is

$$|\Psi\rangle_{\rm WORLD} = \left(\sqrt{p}\ |0\rangle_S|0\rangle_{\rm anc} + \sqrt{1-p}\ |1\rangle_S|1\rangle_{\rm anc}\right)|\Psi\rangle_{\rm REST}. \tag{5}$$

The ancilla must not be a macroscopic system, since then we will get two worlds with a bit in every world and no world with a rabit. (Note that for an agent living in Everett's world (Everett 1957) defined relative to a definite state of the agent, entanglement with a macroscopic ancilla isolated from the agent can be considered as a rabit, but I use here semantics of the MWI defined in (Vaidman 2021).)

To summarize, bit, qubit, and rabit are defined on a system with two orthogonal states, and the system is not part of the definition. These are concepts of an observer who lives in a world and, therefore, defined within a world. The state (1), appearing as a product term in a wavefunction of a world, represents a qubit. A qubit entangled with a microscopic (remote) ancilla in a world, described by (5), represents a rabit. A bit appears as a product term $|0\rangle$ or $|1\rangle$ in the wavefunction of a world and can be viewed as a qubit with $p = 1$, or $p = 0$.

In various communication protocols, bits, qubits or rabits are transferred from one location to another. While there is no difference between transferring a bit and its mathematical description, transferring a qubit does not mean transferring the pair of numbers (p, φ), and transferring a rabit does not mean transferring the number p. Although (p, φ)

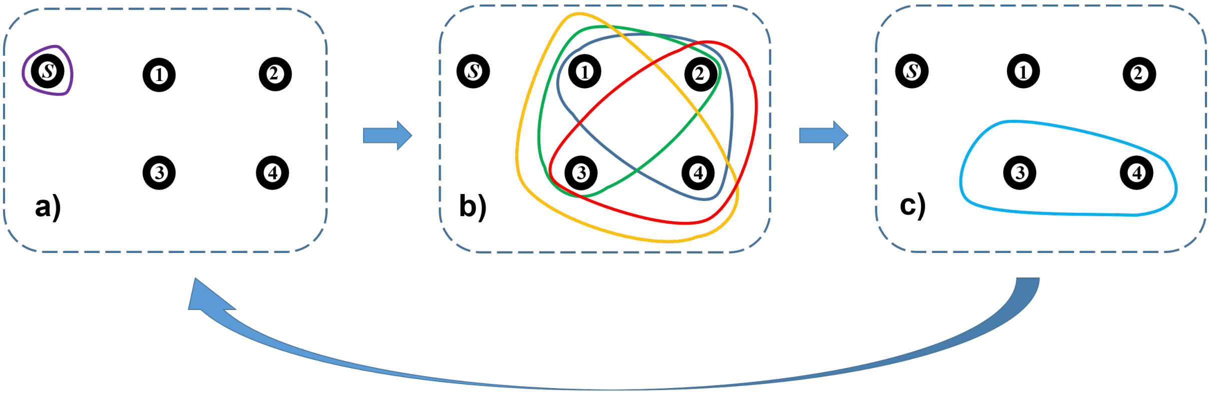

Figure 1: **Information transfer of the qubit in encoding and decoding procedure.** Coloured closed curves show where the full information about the qubit is present. a) At the beginning the qubit (1) is fully and solely in system S. b) After encoding by transformation (6), the information about qubit is present in any set of three out the four particles described by state (7). c) After the interaction between the pair of particles 3 and 4 and particle 1, the information about the qubit is in particles 3 and 4 described by state (8). A final swap can move the qubit back to S.

cannot be found from a single qubit, these numbers characterize the qubit. They define the measurement on the qubit, the result of which has probability 1. Similarly, p cannot be found from the rabit, but it characterizes the rabit. If an agent gets a dollar when $|0\rangle$ is found, it is rational for him to pay $100\,p$ cents for this game.

4 Copying quantum information

There is no constraint on copying a bit, so we can spread the information about a bit to many systems, making many clones of a given bit. In contrast, we cannot clone a qubit. Otherwise, by making many copies we could perform tomography and specify (p, φ) which would identify the qubit with the pair of numbers. However, as we have learned from Shor's method of error correction (Shor 1995), we can create some redundancy by performing a unitary encoding of the qubit in a system of several two-state systems. Consider here an encoding of a qubit in four particles in an error prevention code (Vaidman et al. 1996). The encoding and decoding procedure is described in Fig. 1.

We start by preparing two pairs of maximally entangled particles 1,2 and 3,4. Then we apply the following unitary transformation (swap) between our system in state (1) and the four particles:

$$\begin{aligned}
&\frac{1}{2}|0\rangle_S(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)(|0\rangle_3|0\rangle_4+|1\rangle_3|1\rangle_4) \to \frac{1}{2}|0\rangle_S(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)(|0\rangle_3|0\rangle_4+|1\rangle_3|1\rangle_4),\\
&\frac{1}{2}|1\rangle_S(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)(|0\rangle_3|0\rangle_4+|1\rangle_3|1\rangle_4) \to \frac{1}{2}|0\rangle_S(|0\rangle_1|0\rangle_2-|1\rangle_1|1\rangle_2)(|0\rangle_3|0\rangle_4-|1\rangle_3|1\rangle_4).
\end{aligned} \tag{6}$$

As a result, our original system will have no information about the qubit (1), but the four other systems will be in a state

$$\sqrt{p}\frac{(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)(|0\rangle_3|0\rangle_4+|1\rangle_3|1\rangle_4)}{2}+e^{i\varphi}\sqrt{1-p}\frac{(|0\rangle_1|0\rangle_2-|1\rangle_1|1\rangle_2)(|0\rangle_3|0\rangle_4-|1\rangle_3|1\rangle_4)}{2}, \tag{7}$$

which encodes the information about the qubit. Considering our four particles as a single composite system, (7) corresponds to the definition of qubit. Any three of four particles are not described by a state corresponding to a qubit, but, somewhat surprisingly, they allow for full reconstruction of the qubit, so, according to my definition, they carry (quantum) information about the qubit. Indeed, assume that, say, particle 2 is lost. Conditioned on the state of the pair of particles 3 and 4, we can change the relative phase from $\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2-|1\rangle_1|1\rangle_2)$ to $\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)$ by interaction only with particle 1. This will lead to "decoupling" of the pair of particles 1 and 2, they will be in a product state with the pair of particles 3 and 4 that now will encode the qubit:

$$\sqrt{p}\;\frac{|0\rangle_3|0\rangle_4+|1\rangle_3|1\rangle_4}{\sqrt{2}}+e^{i\varphi}\sqrt{1-p}\;\frac{|0\rangle_3|0\rangle_4-|1\rangle_3|1\rangle_4}{\sqrt{2}}. \tag{8}$$

Another unitary swap can put the qubit back on our system. Note that although any three out of the four particles in state (7) contain full information about the qubit, we cannot get any information about the qubit from any single particle. The pairs 1,2 and 3,4 contain (separately) some information: they represent rabits with value p.

We cannot clone a qubit and, similarly, we cannot clone a rabit, i.e. create another system which independently has probability p to be found in state $|0\rangle$. (If we could clone a rabit, we would be able to create a large ensemble of rabits described by p and thus find this value contradicting the Holevo bound.) However, it is much easier to spread the quantum information of a rabit among many systems in an efficient way. A simple unitary transformation involving the system carrying the rabit and additional system "1"

$$\begin{aligned}|0\rangle_S|0\rangle_1 &\to |0\rangle_S|0\rangle_1,\\ |1\rangle_S|0\rangle_1 &\to |1\rangle_S|1\rangle_1,\end{aligned} \tag{9}$$

will make system 1 to carry the rabit p identical to the original rabit. Indeed, the quantum description of rabit p in a world is

$$\sqrt{p}|0\rangle_S|0\rangle_{\rm anc}+\sqrt{1-p}|1\rangle_S|1\rangle_{\rm anc}. \tag{10}$$

The ancilla is not within the reach of the agent, it is not a macroscopic object (but may contain many microscopic systems), and ${}_{\rm anc}\langle 0|1\rangle_{\rm anc}=0$. In the world with the rabit and a particle 1 starting in a pure state $|0\rangle_1$, the transformation (9) leads to the quantum state

$$\sqrt{p}|0\rangle_S|0\rangle_1|0\rangle_{\rm anc} + \sqrt{1-p}|1\rangle_S|1\rangle_1|1\rangle_{\rm anc}, \tag{11}$$

which describes our system representing the rabit p as before, and particle 1 representing the *same* rabit p. This procedure can be repeated any number of times creating many identical systems representing the same rabit. This redundancy helps for storing the rabit: even if all but one system are lost, we still have the original rabit. Rabits described by (11) however, are not independent. Observing one rabit collapses the other to a bit, because it creates entanglement with a macroscopic system - the measuring device. Thus, this operation cannot help for estimation of the rabit value p.

I suggest to name the system and particle 1 described in (11) an entangled random pair. My reason is that such a pair can be used as a channel in a protocol (discussed below) that is natural to name as teleportation of a rabit. Note that according to the standard convention (Horodecki et al. 2009), the particles described by (11) are not entangled, since they can be written as a convex combination of product states.

The procedures for encoding information described in this section do not require any kind of action at a distance: local coupling between the particles results in transferring information between them. There is some kind of nonlocality feature in the fact that when particles 1-4 are moved to spatially separated sites, the information is encoded in a nonseparable way: we cannot get full information in a single local site without quantum channels between the sites.

5 Teleportation of a qubit

In the teleportation of a qubit (4), spatially separated Alice and Bob share a maximally entangled pair of particles 1 and 2:

$$|\psi\rangle_{\rm EPR} = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2). \tag{12}$$

Alice has a qubit (1) on a system S and she performs a Bell measurement on her qubit S and particle 1, see Fig. 2. One way of performing the Bell measurement is performing two consecutive measurements. The first is the measurement of the modular sum of variables s for which states $|0\rangle$ and $|1\rangle$ are eigenvalues (see Aharonov et al. (1986) for description of such a measurement): $s|i\rangle = i|i\rangle$. The result of the measurement will be written in bit 3:

$$(s + s_1) \bmod 2 \equiv b_3. \tag{13}$$

The second measurement is of the same kind, but in a different basis. The result will be written in bit 4:

$$(\tilde{s} + \tilde{s}_1) \bmod 2 \equiv b_4, \tag{14}$$

where

$$|\tilde{0}\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \qquad |\tilde{1}\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \qquad \tilde{s}|\tilde{i}\rangle = i|\tilde{i}\rangle. \tag{15}$$

The world will split into four worlds according to the outcomes of the macroscopic measuring devices of b_3 and b_4. In these worlds, the quantum states of all microscopic systems involved, including the carriers of the results of the measurements to be sent to Bob, are:

$$|0\rangle_3|\tilde{0}\rangle_4 \ \frac{1}{\sqrt{2}}(|0\rangle_S|0\rangle_1 + |1\rangle_S|1\rangle_1)(\sqrt{p}\ |0\rangle_2 + e^{i\varphi}\sqrt{1-p}\ |1\rangle_2), \tag{16}$$

$$|0\rangle_3|\tilde{1}\rangle_4 \ \frac{1}{\sqrt{2}}(|0\rangle_S|0\rangle_1 - |1\rangle_S|1\rangle_1)(\sqrt{p}\ |0\rangle_2 - e^{i\varphi}\sqrt{1-p}\ |1\rangle_2), \tag{17}$$

$$|1\rangle_3|\tilde{0}\rangle_4 \ \frac{1}{\sqrt{2}}(|0\rangle_S|1\rangle_1 + |1\rangle_S|0\rangle_1)(\sqrt{p}\ |1\rangle_2 + e^{i\varphi}\sqrt{1-p}\ |0\rangle_2), \tag{18}$$

$$|1\rangle_3|\tilde{1}\rangle_4 \ \frac{1}{\sqrt{2}}(|0\rangle_S|1\rangle_1 - |1\rangle_S|0\rangle_1)(\sqrt{p}\ |1\rangle_2 - e^{i\varphi}\sqrt{1-p}\ |0\rangle_2). \tag{19}$$

We can see explicitly that at this stage, for every result of these measurements (which can be seen in states of the information carriers $|i\rangle_3$ and $|\tilde{i}\rangle_4$), the information about the qubit (p, φ) is encoded in particle 2. Quantum states of all other systems are independent of p and φ.

In each world, the particle 2 together with the identity of the world are enough to reconstruct the qubit. The identity of the world can be learned from bits 3 and 4. In world $(0_3, \tilde{0}_4)$ particle 2 as is represents the qubit, see (16). In world $(0_3, \tilde{1}_4)$ we should introduce phase π to state $|1\rangle_2$, see (17). In world $(1_3, \tilde{0}_4)$ we should flip the qubit $|0\rangle_2 \to |1\rangle_2$, $|1\rangle_2 \to |0\rangle_2$, see (18). Finally in world $(1_3, \tilde{1}_4)$ we should introduce phase π to state $|1\rangle_2$ and flip the qubit, see (19).

The identity of the world can be read from the (macroscopic) measuring devices, or from systems 3 and 4 which are sent to Bob, or from the Bell states of the composite system which include the system S and particle 1. Note that all systems which carry information about the identity of the world are located, at this stage, at Alice's site, see Fig. 2b. Without this information, Bob cannot learn anything about the qubit. Of

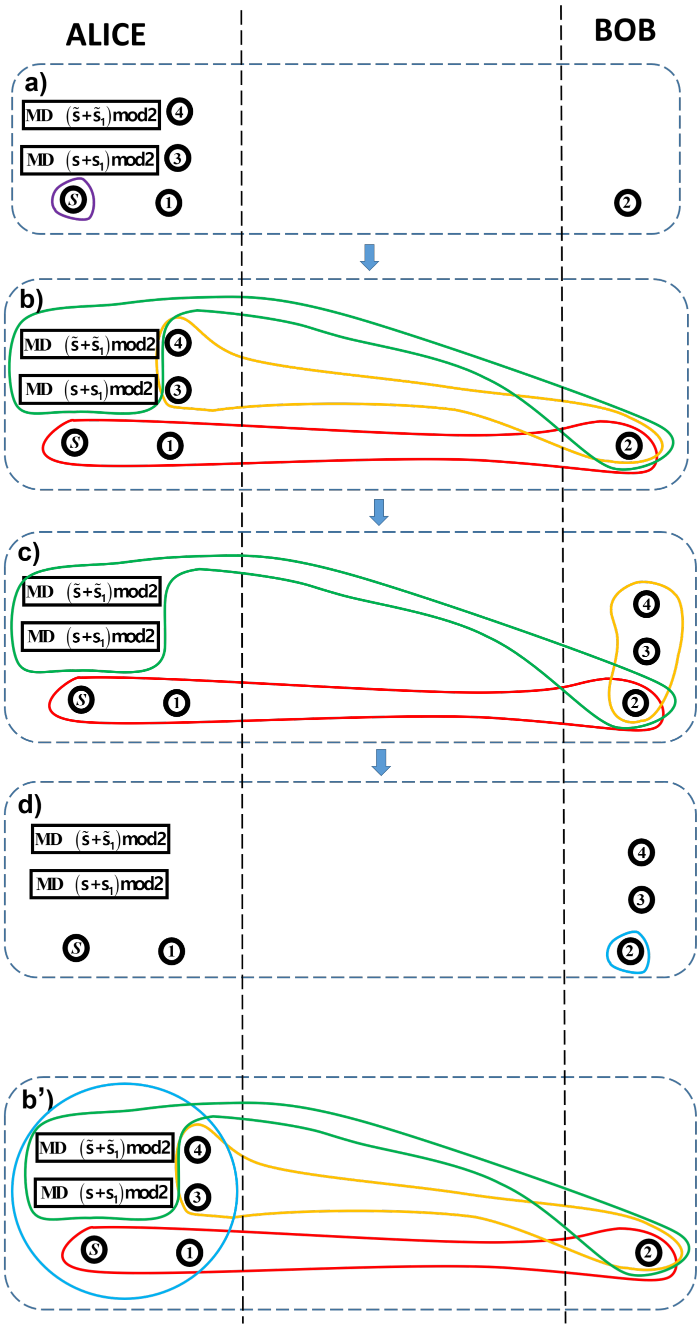

Figure 2: **Information transfer in the teleportation of a qubit.** Coloured closed curves show where the full information about the qubit is present. Rectangular boxes represent macroscopic measuring devices. a) At the beginning, the qubit (1) is fully and solely in system S. b) Location of the information about the qubit after performing the Bell measurement on the system and particle 1 (which was entangled with remote particle 2) within every world corresponding to all possible outcomes (16-19). In all worlds we need particle 2 to reconstruct the qubit. (Not all sets are shown, e.g., particle 2, system 3, and the measuring device of $\tilde{s}_4$.) c) The carriers of the results of the Bell measurement, particles 3 and 4, are moved to Bob. d) After unitary transformation of the state of particle 2 conditioned on the information brought by systems 3 and 4, particle 2 at Bob's site, and only particle 2, carries the qubit. b') If we consider all worlds together, (and not separately as was done in (b)), then the full information about the qubit is present also in all systems coupled to the system S or to the systems coupled to the systems which coupled to S, etc. within the light cone of the Bell measurement event (blue circle).

course, it cannot be otherwise, since superluminal signalling is impossible. Only after the carriers of the information about the world identity, systems 3 and 4, are moved to Bob, Fig. 2c, he reconstructs the qubit on particle 2, Fig. 2d.

After the Bell measurement, within every world corresponding to a particular outcome of the measurement, the information about the qubit is present in sets of systems all of which include particle 2 located far away from system S, the original carrier of the qubit, see Fig. 2b. So, although superluminal signaling is not present here, we do have some superluminal feature in the teleportation procedure. It is a counterpart of the superluminal property of the collapse of the wavefunction (spooky action at a distance). Just before the Bell measurement, the information about the qubit is present solely at Alice's site. Immediately after, in every world created by the Bell measurement, full information about the qubit is not present at Alice's site but spread out among spatially separated particles. (It is also true just after completion of the measurement (13).) At every world the information about the qubit is distributed across several systems including one (particle 2) in a spacelike (relative to Bell measurement event) location. Particle 2 has to be supplemented with some information to reconstruct the qubit. This additional information is brought by particles 3 and 4, see Fig. 2c-d.

My claim that the Bell measurement creates some kind of superluminal action needs clarification. The world-splitting view prevents the concept of diachronic identity between a world now and a world at a later time. The world in which the Bell measurement was performed evolves into a multitude of worlds. So, it is not obvious how to discuss time evolution within a world when splitting occurs. One way to do this is a four-dimentional "worm view" of agents (Wilhelm 2023). However, I cannot understand how it explains an experience of an agent at a particular time. My future is not part of my experience now. My past is, it is recorded in my brain. What can be done at one moment of time is to consider the history as recorded in the brain of an agent and in other objects of the agent's world. In these records we can see a "four-dimensional" agent with a well defined evolution backwards in time from Bob's manipulation of particle 2, to backward motion of the carriers of the result of Bell measurement to a particular outcome of Bell measurement etc. We can analyze this history forward in time. It becomes a usual time evolution which we experience: preparation of Bell measurement, obtaining one of the results, etc. It is not a unitary evolution. It can be described by the usual Von Neumann story with two processes, the evolution according Schrödinger equation and collapses. We can ask questions about presence of superluminal phenomena in this evolution. The nonlocality feature of the teleportation procedure is the following temporal sequence: presence of a qubit in one location (Alice), immediate disappearance of the information about the qubit from this location and its appearance in a state of several spatially separated particles. Contrary to the action at a distance of the collapse process, this superluminal feature is not an actual process in space-time, but a property of records at a particular time describing the history of a particular world.

In the teleportation procedure, the nonlocality feature is stronger than in the encoding procedure. In addition to the nonseparability feature of the information distributed in two locations at some stages, we have an action at a distance when we consider histories within separate worlds: creation of distributed information occurs nonlocally. A measurement on Alice's site transfers (superluminally) some part of the information to particle 2 at Bob's site. Note that we do not get any amount of locally accessible information in particle 2 in a superluminal way. But we know that in every world the part of the information is there, because without particle 2 the qubit cannot be recovered.

6 Deutsch-Hayden approach and the view of an agent equipped with super-technology

Let us now put ourselves in the position of Wigner (1961), equipped with super-technology which allows the analysis of his friend performing the Bell measurement. Wigner, who considers four worlds with all possible outcomes of the Bell measurement together, is not forced to say that there is a superluminal feature in the Bell measurement which puts some information in space-like separated particle 2. Even with this global consideration, the local coupling of the measuring device to the system S and particle 1, which is entangled with particle 2, leads to a distribution of the information about the qubit into sets of particles which include particle 2. But, particle 2 is needed only if we do not consider all the remaining systems. The local systems involved in the coupling to the qubit (without particle 2) are enough to reconstruct the qubit, see Fig. 2b'. The reconstruction can be done by "reverse evolution" which erases the information from all systems and places the qubit back on the original system. (Braunstein (1996) discussed this fact considering teleportation procedure without macroscopic detection, where the erasure does not require the super-technology of Wigner.) The "reverse evolution" does not involve remote systems, so there is no superluminal process when we consider all worlds together.

How can we see superluminal phenomena inside a world with a particular outcome of the Bell measurement? Worlds are nonlocal entities. The local Bell measurement splits the world into worlds that have different properties in remote particle 2 due to the initial entanglement between particles 1 and 2. This is an effective action at a distance within each world. However, the mixture of four states of particle 2 corresponding to the four worlds with different outcomes of the Bell measurement is identical to the original mixed state of particle 2 in the Einstein-Podolsky-Rosen (EPR) state (Einstein et al. 1937). Thus, we see again that there is no superluminal effect on the physical level of all worlds together.

The Deutsch-Hayden (2000) analysis of information flow was also performed in the framework of the MWI and considered all worlds together, so it is not surprising that in the abstract they made a similar claim:

> Measurement or interaction with a quantum system S in another way has no effect on distant systems from which S is dynamically isolated, even if they are entangled with S.

However, the other message of their work is very different from my approach:

> All information in quantum systems is, notwithstanding Bell's theorem, localized. ... Using the Heisenberg picture to analyse quantum information processing makes this locality explicit, and reveals that under some circumstances (in particular, in Einstein-Podolsky-Rosen experiments and in quantum teleportation), quantum information is transmitted through 'classical' (i.e. decoherent) information channels.

So, Deutsch and Hayden claim that the full information about the qubit is stored in particles 3 and 4. I, however, have shown that in every world (corresponding to a particular outcome of a Bell measurement) we cannot reconstruct the qubit without particle 2. Even if we consider all worlds together, systems 3 and 4 alone are not enough to reconstruct the qubit.

How can we reconcile the differences in the analyses? The following quotation from Deutsch-Hayden makes the differences clear.

> When analysing information flow in the Schrodinger picture, it is essential to realize that it is impossible to characterize quantum information at a given instant using the state vector alone.

In contrast, the main postulate of quantum theory, as I understand it, is that *everything* is described by the quantum state. It is a complete description at a particular time. Experiences of all conscious beings in all worlds at that time supervene on the wavefunction of the universe at that time. Deutsch and Hayden apparently want to describe more. They write:

> The latter [Schrödinger picture] is optimized for predicting the outcomes of processes given how they were prepared, but (notoriously) not for explaining how the outcomes come about...

They expect the quantum picture to explain the situation not just now, at time t but also at other times. Indeed, the roots of their approach stem from Gottesman (1998) analysis which dealt with practical aspects of quantum computation. A particular intermediate state of the computer makes sense as a part of the computation when a computer program is given. We need some information about the history of the system to give the meaning of the state as a computational step. The Heisenberg picture includes this history. This can be seen from the continuation of Deutsch and Hayden's writing.

> To investigate where information is located, one must also take into account how that state came about. In the Heisenberg picture, this is taken care of automatically, precisely because the Heisenberg picture gives a description that is both complete and local.

This "complete and local" description is an example of a local realistic model of quantum mechanics which must exist, as argued by Brassard and Raymond-Robichaud (2019), since quantum mechanics is a nonsignalling theory. The description, based on "descriptors" introduced by Deutsch-Hayden, further developed by several authors (Hewitt-Horsman and Vedral 2007; Waegell 2018; Raymond-Robichaud 2021), comes for a high complexity price, (see Bedard 2021b). The description is local because it is based on local descriptors affected by local interactions. By adding the assumption of a known initial state, we obtain a picture in which all information is about local facts (interactions), and it is stored locally in local descriptors. However, the interaction of a particle with every quantum system increases the Hilbert space of the descriptors (Bedard 2021b), so the description becomes very complex. Conditioning on a particular outcome when the interaction is a measurement with a macroscopic measuring device, i.e., considering descriptors in a particular world, (see Kuypers and Deutsch 2021) reduces the complexity, but only a little.

In my view, the main disadvantage of this approach is that it loses *gedanken measurability*. In the language of Raymond-Robichaud (2021), the local description is based on "noumenal" states. Given an ensemble of universes in a particular quantum Schrödinger state at a particular moment and external omnipotent quantum devices (like Wigner's measuring device capable of measuring the quantum state of his friend), we can perform tomography and find the quantum state of the universe. In contrast, access to the ensemble of universes at a particular time is not enough to specify the description with descriptors, we need access during the history of the universe.

7 Teleportation of a rabit

"Half" of the teleportation process described above is enough for teleporting of a rabit. This is apparently the simplest demonstration of the paradoxical situation in which transferring one bit is enough for transferring (in some sense) a real number $p \in (0, 1)$ which characterises the rabit.

The rabit p to be teleported is given in a system S entangled with an ancilla 1

$$\sqrt{p}|0\rangle_S|0\rangle_{\rm anc1} + \sqrt{1-p}|1\rangle_S|1\rangle_{\rm anc1}. \tag{20}$$

We need to perform only one measurement of the modular sum of the variable s of the system and the variable s_1 of Alice's particle of the teleportation channel. We also do not need a pure EPR channel (12), a "decohered" EPR state (which corresponds to "entangled" random pair with $p = \frac{1}{2}$) is enough:

$$|\psi\rangle_{\text{EPRde}} = \frac{1}{\sqrt{2}} \left(|0\rangle_1 |0\rangle_2 |0\rangle_{\text{anc2}} + |1\rangle_1 |1\rangle_2 |1\rangle_{\text{anc2}}\right). \tag{21}$$

The two possible outcomes $b = 0, 1$ of the measurement of the modular sum $b \equiv (s + s_1) \bmod 2$ correspond to the two worlds. The quantum state of all systems involved, including ancilla 1 of the transmitted rabit (20) and ancilla 2 of the decohered entanglement channel (21), in the world $b = 0$ is

$$\sqrt{p}\ |0\rangle_S |0\rangle_{\text{anc1}} |0\rangle_1 |0\rangle_{\text{anc2}} |0\rangle_2 + \sqrt{1-p}\ |1\rangle_S |1\rangle_{\text{anc1}} |1\rangle_1 |1\rangle_{\text{anc2}} |1\rangle_2. \tag{22}$$

In this world particle 2 is rabit p. The two ancilla particles and Alice's system S play the role of the ancilla of the rabit. In the world $b = 1$ the quantum state is

$$\sqrt{p}\ |0\rangle_S |0\rangle_{\text{anc1}} |1\rangle_1 |1\rangle_{\text{anc2}} |1\rangle_2 + \sqrt{1-p}\ |1\rangle_S |1\rangle_{\text{anc1}} |0\rangle_1 |0\rangle_{\text{anc2}} |0\rangle_2. \tag{23}$$

By observing bit b, Bob splits the world into two worlds with different Bobs. Bob in the world $b = 0$ does nothing, but Bob in the world $b = 1$ flips the state of particle 2. Thus, the particle 2 becomes the rabit p in both worlds.

If instead of measurement of $(s + s_1) \bmod 2$ using a macroscopic measuring device, we just perform coupling between systems s, s_1 and the two-state device b , i.e. if we perform only the coherent part of the von Neumann measurement procedure, the resulting quantum state will be

$$\begin{aligned}
&\frac{1}{\sqrt{2}} |0\rangle_b (\sqrt{p}\ |0\rangle_S |0\rangle_{\text{an1}} |0\rangle_1 |0\rangle_{\text{anc2}} |0\rangle_2 + \sqrt{1-p}\ |1\rangle_S |1\rangle_{\text{anc1}} |1\rangle_1 |1\rangle_{\text{anc2}} |1\rangle_2) + \\
&\frac{1}{\sqrt{2}} |1\rangle_b (\sqrt{p}\ |0\rangle_S |0\rangle_{\text{anc1}} |1\rangle_1 |1\rangle_{\text{anc2}} |1\rangle_2 + \sqrt{1-p}\ |1\rangle_S |1\rangle_{\text{anc1}} |0\rangle_1 |0\rangle_{\text{anc2}} |0\rangle_2).
\end{aligned} \tag{24}$$

Sending the two-state device b to Bob, who coherently performs the conditional flip of state of particle 2, leads to the quantum state

$$\frac{1}{\sqrt{2}} \left(|0\rangle_b |0\rangle_1 |0\rangle_{\text{anc2}} + |1\rangle_b |1\rangle_1 |1\rangle_{\text{anc2}}\right) \left(\sqrt{p}\ |0\rangle_S |0\rangle_{\text{anc1}} |0\rangle_2 + \sqrt{1-p}\ |1\rangle_S |1\rangle_{\text{anc1}} |1\rangle_2\right). \tag{25}$$

We see that this procedure also transfers the rabit from Alice to Bob.

Interestingly, before the conditional flip by Bob, he had two rabits of value $p = \frac{1}{2}$, one in particle 2, and one in device b. The rabits are not independent. A conditional flip of one rabit depending on the value of the other creates rabit p. The situation is symmetric: Bob can flip the state of device b conditioned on 2 instead of flipping 2 conditioned on b. This questions the natural assumption that "most" information is transferred in a nonlocal way in a Bell measurement and the particle moving from Alice to Bob provides only 1 bit of information about the identity of the world. However, teleportation without macroscopic measurement (Braunstein 1996) when transferring an isolated quantum system is not really a teleportation. The two-state device b we move is a qubit, so there is nothing surprising in the ability to transfer a rabit. The channel can also allow entanglement with (microscopic) systems of the environment: it does not spoil the procedure, but also not make it more interesting. A decohered qubit is a rabit, so it is not surprising that we can move a rabit in such a channel. The difficult task is to move the rabit when we transfer only one bit, a two-state system in a definite state within our world.

One may wonder: is there a conceptual difference between the teleportation of a rabit and "Classical teleportation of a qubit" (Cerf et al. 2000) based on "shared randomness"? Shared randomness is defined as "identical (possibly infinite) list of random numbers" shared by Alice and Bob. In this case Alice, who is given a *known* qubit can, by sending only a few bits, allow Bob to perfectly simulate all possible measurements performed on this qubit. The reason for suspicion of similarity of the methods is that in both cases (rabit and "classical" teleportation) we need an ensemble for verification. The difference I see here is that at the end of a rabit teleportation process Bob has a two-state system with genuinely uncertain values that encode value p. In contrast, at the end of the "classical teleportation" procedure Bob has a single bit value without uncertainty. This value does not encode p. Only if we repeat the procedure many times, the ensemble of Bob's records will correspond to p. The result of the classical teleportation procedure is a bit that is operationally "random" for Bob, who cannot deduce its value before observing it. However, this is not a rabit. The value of this bit is not genuinely uncertain. It can be deduced from the shared list, Alice's qubit, and the choice of Bob's measurement. In contrast, the result of measurement of the system representing a rabit cannot be deduced before observation.

8 Conclusions

I reviewed various approaches to the question of information transfer in the process of quantum teleportation, a controversial topic which has not reached a consensus. I presented arguments in support of Vaidman's proposal made after the discovery of teleportation according to which nonlocality of worlds in the MWI is the basis of the explanation (Vaidman 1994a): Alice's local Bell measurement splits the world in different ways depending on the quantum state she receives to teleport. The operation creates worlds with information about Alice's qubit in Bob's (far away) location, which, however, cannot be transferred into a qubit itself without information about the identity of the world.

To provide an explanation, I analyzed the spread of quantum information in error prevention encoding, information transfer in qubit teleportation in which the Bell measurement was done through two consecutive measurements of a modular sum, and information transfer in a simplified procedure which teleports the rabit, the concept I define here, which represents a random bit.

The controversy about information transfer arises from the lack of a precise definition of the concept of information and the difficulties related to the quantum measurement problem. If we accept the reality of collapse in quantum measurements, then the teleportation procedure demonstrates a very problematic action at a distance. A local property in one location is changed immediately by action at remote location. The MWI, which does not contain action at a distance, does provide a coherent framework for discussing this problem; however, to achieve a clear picture, it is necessary to carefully consider the many-world structure of the physical universe. A useful concept of information has to be considered within a world, and not to be confused with the propagation of a locally created pattern in the space-time description of the wavefunction of the universe which incorporates all parallel worlds.

The apparent contradiction with the information transfer picture of Deutsch and Hayden follows from different questions which were asked: local description with descriptors describes not just the situation at a particular moment but also information about the past. And the locality of this description relies on a (strong) assumption about the past which cannot be verified at a particular time, even if we are given super-technology and an ensemble of universes.

The coherent and elegant picture of information transfer in the teleportation procedure in the framework of the MWI presented in the Schrödinger representation provides, in my view, strong support for the MWI, especially relative to collapse theories: it avoids randomness (see Vaidman 2014) and action at distance (see Vaidman 2015). The local picture of Deutsch and Hayden avoids, in addition, the nonlocality (nonseparability) of worlds of the MWI, but for the very high price in complexity of new ontology.

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