Is decoherence necessary for the emergence of many worlds?

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

November 19, 2023

Abstract

It is usually thought that decoherence is necessary for the emergence of many worlds. In this paper, I argue that this may be not the case. First, I argue that the original synchronic decoherence condition leads to a contradiction in a thought experiment. Next, I argue that although the diachronic environmental decoherence condition may avoid the contradiction, it is not a necessary condition for the emergence of many worlds on a sufficiently short timescale. Finally, I argue that a more plausible necessary condition is the synchronic no-interference condition, and it can also avoid the contradiction.

The many-worlds interpretation of quantum mechanics (MWI) assumes that the wave function of a physical system is a complete description of the system, and it always evolves in accord with the linear Schrödinger equation. In order to solve the measurement problem, MWI further assumes that after a measurement with many possible results there appear many equally real worlds, in each of which a definite result occurs (Everett, 1957; Barrett, 2018; Vaidman, 2021). This many-worlds assumption is supported by an extensive analysis of decoherence and emergence in the modern formulation of MWI (Wallace, 2012). In this paper, I will present a new analysis of the decoherence condition for the emergence of worlds in MWI.

Suppose there is a closed system containing two experimenters Alice and Bob.[1] They are initially in an entangled state:

[1]This thought experiment may also use two measuring devices or two common macroscopic systems. In order to omit the effect of rapid environmental decoherence, we only discuss the behavior of the system during a very short time intertal. In this sense, the system can be regarded as closed.

$$|0\rangle_{Alice}|1\rangle_{Bob} + |1\rangle_{Alice}|0\rangle_{Bob}, \tag{1}$$

where $|0\rangle_{Alice}$ and $|1\rangle_{Alice}$ are two result states of Alice in which she obtains the results 0 and 1, respectively, and $|0\rangle_{Bob}$ and $|1\rangle_{Bob}$ are two result states of Bob in which he obtains the results 0 and 1, respectively.

Consider a unitary time evolution operator U_N which changes $|0\rangle_{Alice}$ to $|1\rangle_{Alice}$ and $|1\rangle_{Alice}$ to $|0\rangle_{Alice}$ after a very short time interval T.[2] Then by the linearity of the dynamics, the time evolution of the initial state under U_N is

$$|0\rangle_{Alice}|1\rangle_{Bob} + |1\rangle_{Alice}|0\rangle_{Bob} \to |1\rangle_{Alice}|1\rangle_{Bob} + |0\rangle_{Alice}|0\rangle_{Bob} \tag{2}$$

At each instant $t \in [0, T]$ during the evolution, $U_N(t)$ can be defined as follows:

$$U_N(t)|0\rangle_{Alice} = \alpha(t)|0\rangle_{Alice} + \beta(t)|1\rangle_{Alice}, \tag{3}$$

$$U_N(t)|1\rangle_{Alice} = \alpha'(t)|0\rangle_{Alice} + \beta'(t)|1\rangle_{Alice}, \tag{4}$$

where $\alpha(0) = 1$, $\alpha(T) = 0$, $\alpha'(0) = 0$, $\alpha'(T) = 1$, $\beta(0) = 0$, $\beta(T) = 1$, $\beta'(0) = 1$, and $\beta'(T) = 0$. Note that the unitarity of $U_N(t)$ will keep the orthogonality of the two states of Alice during the evolution.

Now the state of Alice and Bob at each instant t during the evolution is

$$[\alpha(t)|0\rangle_{Alice} + \beta(t)|1\rangle_{Alice}]|0\rangle_{Bob} + [\alpha'(t)|0\rangle_{Alice} + \beta'(t)|1\rangle_{Alice}]|1\rangle_{Bob} \tag{5}$$

This state can also be written as follows:

$$[\alpha(t)|0\rangle_{Bob} + \alpha'(t)|1\rangle_{Bob}]|0\rangle_{Alice} + [\beta(t)|0\rangle_{Bob} + \beta'(t)|1\rangle_{Bob}]|1\rangle_{Alice} \tag{6}$$

Then, $U_N(t)$ will equivalently evolve the states of Bob as follows:

$$U_N(t)|0\rangle_{Bob} = \alpha(t)|0\rangle_{Bob} + \alpha'(t)|1\rangle_{Bob} \tag{7}$$

$$U_N(t)|1\rangle_{Bob} = \beta(t)|0\rangle_{Bob} + \beta'(t)|1\rangle_{Bob} \tag{8}$$

[2]For a Hilbert space with dimension greater than two, the swap operator U_N can be accomplished in many ways, such as with a 180 degree rotation about the ray halfway between the two state vectors. Admittedly U_N involves anti-thermodynamic manipulation of macrocopically many degrees of freedom. But for a unitary theory like MWI, U_N can be accomplished in principle, although the accomplishment is extremely difficult. Note that a similar thought experiment involving the swap operator U_N was first proposed and discussed by Gao (2019).

This means that the unitarity of $U_N(t)$ will ensure that the two states of Bob are also orthogonal during the evolution.[3]

Now an interesting question arises: what worlds does the state (5) or (6) correspond to in MWI? Let's first use the original definition of decoherence to determine the emergence of worlds. That is to say, when the reduced density matrix of a measuring device or an observer is (almost) diagonalized with respect to definite result states at a given time, a superposition of these result states will correspond to many worlds, in each of which there is a definite result. According to this decoherence condition, for the superposed state (5), since the two states of Alice, $\alpha(t)\left|0\right\rangle_{Alice} + \beta(t)\left|1\right\rangle_{Alice}$ and $\alpha'(t)\left|0\right\rangle_{Alice} + \beta'(t)\left|1\right\rangle_{Alice}$, are orthogonal and thus Bob's two result states are decohered, there are two worlds, in one of which Bob obtains a definite result 0, and in the other Bob obtains a definite result 1. Moreover, in each of these worlds, Alice does not obtain a definite result, since the decoherence condition is not satisfied for her. On the other hand, for the superposed state (6), due to the similar reason, there are also two worlds, in one of which Alice obtains a definite result 0, and in the other Alice obtains a definite result 1. Moreover, in each of these worlds, Bob does not obtain a definite result. Since (5) and (6) are only two different decompositions of the same state, the above two answers are incompatible with each other, and there is a contradiction here. Note that the state (5) or (6) only corresponds to two worlds and it does not correspond to four worlds according to the above decoherence condition.

There is another formulation of the contradiction concerning the changes of worlds. According to the above decoherence condition, the initial state of Alice and Bob corresponds to two worlds. In one world, Alice obtains a definite result 0 and Bob obtains a definite result 0, and in the other world, Alice obtains a definite result 1 and Bob obtains a definite result 1. Similarly, the final state of Alice and Bob also corresponds to two worlds. In one world, Alice obtains a definite result 0 and Bob obtains a definite result 1, and in the other world, Alice obtains a definite result 1 and Bob obtains a definite result 0. Then, how do the states of Alice and Bob change in each world during the above time evolution?

Since the states of Alice and Bob in the branches of both the initial state and the final state are symmetrical, the answers to this question for Alice and Bob must be the same. But, as we will see, this is impossible. By Eqs. (3) and (4), since the states of Alice, $\alpha(t)\left|0\right\rangle_{Alice} + \beta(t)\left|1\right\rangle_{Alice}$ and $\alpha'(t)\left|0\right\rangle_{Alice} + \beta'(t)\left|1\right\rangle_{Alice}$, are always orthogonal during the time evolution and thus the decoherence condition is satisfied for Bob, Bob's state should not change in each world during the evolution. On the other hand, by

[3] In a two-dimensional Hilbert sub-space, $U_N(t)$ can be represented as $\begin{pmatrix} \alpha(t) & \alpha'(t) \\ \beta(t) & \beta'(t) \end{pmatrix}$. Then its unitarity implies the relation $\alpha(t)\beta^*(t)+\alpha'(t)\beta'^*(t) = 0$, which means that the two states of Bob, $\alpha(t)\left|0\right\rangle_{Bob} + \alpha'(t)\left|1\right\rangle_{Bob}$ and $\beta(t)\left|0\right\rangle_{Bob} + \beta'(t)\left|1\right\rangle_{Bob}$, are also orthogonal.

Eqs. (7) and (8), since the states of Bob, $\alpha(t)\left|0\right\rangle_{Bob} + \alpha'(t)\left|1\right\rangle_{Bob}$ and $\beta(t)\left|0\right\rangle_{Bob} + \beta'(t)\left|1\right\rangle_{Bob}$, are always orthogonal during the time evolution and thus the decoherence condition is also satisfied for Alice, Alice's state should not change in each world during the evolution either. However, the time evolution (2) shows that Alice's and Bob's states cannot both keep unchanged in each world; when Alice's state keep unchanged, such as from $\left|0\right\rangle_{Alice}$ to $\left|0\right\rangle_{Alice}$, Bob's state must change, such as from $\left|0\right\rangle_{Bob}$ to $\left|1\right\rangle_{Bob}$, and vice versa. This is a contradiction.[4]

The above analysis shows that decoherence by its original definition cannot be used as the condition for the emergence of many worlds. This result may be not beyond expectations for the proponents of modern MWI. According to the decoherence-based approach to MWI (Wallace, 2012), world branching results from environmental decoherence, which is an effectively irreversible process that can form temporally extended and stable branches. By this environmental decoherence condition, there is no branching at all during the above unitary evolution of the initial entangled state (1), and both Alice and Bob exist in a single world, and they have been in an indefinite state and they obtained no definite results.

However, this answer seems counter-intuitive. Intuition may tell us that the initial entangled state (1) and the final entangled state (i.e. the right hand side of (2)) should correspond to two worlds, in each of which both Alice and Bob obtain a definite result. Moreover, the environmental decoherence condition is arguably too strong for the emergence of worlds on a sufficiently short timescale such as in the above example.

According to Wallace's (2012, p.62) analysis of the emergence of worlds, if only structures instantiated by the macroscopic degrees of freedom of a quantum system do not overlap and cancel out when the system is in a superposition of macroscopically definite states, these different structures will correspond to different worlds, and the superposition will correspond to many worlds. In other words, if only there is no interference between the macroscopically definite states in a superposition, the superposition will contain non-overlaping structures and correspond to many worlds.

Certainly, environmental decoherence is an effective way to prevent interference and erasure of structures. In this sense, it is a sufficient condition for the emergence of worlds. But it is not a necessary condition, since certain no-decoherence dynamics such as free Schrödinger evolution may also do its job on a sufficiently short timescale, if only the dynamics keeps the macroscopically definite states in a superposition separated without interference, namely it satisfies the no-interference condition. Only after the macroscopically definite states have interference with each other at later times, does

[4]The contradiction can also be formulated in a higher-dimensional Hilbert space. For example, in a 3D Hilbert space, the time evolution will be $\left|0\right\rangle_{Alice}\left|1\right\rangle_{Bob} + \left|1\right\rangle_{Alice}\left|2\right\rangle_{Bob} + \left|2\right\rangle_{Alice}\left|0\right\rangle_{Bob} \rightarrow \left|1\right\rangle_{Alice}\left|1\right\rangle_{Bob} + \left|2\right\rangle_{Alice}\left|2\right\rangle_{Bob} + \left|0\right\rangle_{Alice}\left|0\right\rangle_{Bob}$.

the superposition not correspond to many worlds.

This analysis of the emergence of worlds is also supported by a general argument. First, whether there are many worlds at each instant is a definite fact. For example, whether Alice and Bob obtain a definite result at a given instant in the above example is a fact that Alice and Bob can find and verify. Next, whether a superposed state corrresponds to many worlds at an instant is supposed to be determined only by the superposed state, such as whether the superposed state contains non-overlaping structures according to Wallace's analysis. Then, since a superposed state may evolve from an earlier state by or not by a process of environmental decoherence, environmental decoherence is not the necessary condition for the existence of many worlds; even if a superposed state is not generated by environmental decoherence, it may also corrrespond to many worlds.

Therefore, it is arguable that the diachronic environmental decoherence condition should be replaced by the synchronic no-interference condition as the condition for the emergence of worlds in MWI on a sufficiently short timescale. Although the emergent worlds resulting from the no-interference condition are in general short-lived and also different from our familar classical worlds resulting from the environmental decoherence condition (since they have different dynamics), they are qualified to be worlds according to Wallace's analysis.

It can be seen that the synchronic no-interference condition can also avoid the contradiction resulting from the synchronic decoherence condition in the above exmaple, and at the same time, it is consistent with our intuition. According to the no-interference condition, the initial entangled state (1) and the final entangled state (i.e. the right hand side of (2)) correspond to two worlds, in each of which both Alice and Bob obtain a definite result. The original contradiction results from the fact that the inbetween superposed state (5) or (6) only corresponds to two worlds and it does not correspond to four worlds according to the synchronic decoherence condition. Now according to the synchronic no-interference condition, the superposed state (5) or (6) corresponds to four worlds, in each of which both Alice and Bob obtain a definite result, and thus there is no contradiction.

To sum up, I proposed a thought experiment and argued that the decoherence condition for the emergence of worlds in MWI should be replaced by the no-interference condition on a sufficiently short timescale. However, environmental decoherence is still an effective way to stitch together these very-short-timescale worlds into a temporally extended world.[5]

[5]I thank David Wallace for helpful discussion about this point.

References

[1] Barrett, J. (2018). Everett's Relative-State Formulation of Quantum Mechanics. The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/archives/win2018/entries/qm-everett/.

[2] Everett, H. (1957). 'Relative state' formulation of quantum mechanics. Rev. Mod. Phys. 29, 454-462.

[3] Gao, S. (2019). The measurement problem revisited. Synthese, 196, 299-311.

[4] Vaidman, L. (2021). Many-Worlds Interpretation of Quantum Mechanics. The Stanford Encyclopedia of Philosophy (Fall 2021 Edition), Edward N. Zalta (ed.). https://plato.stanford.edu/archives/fall2021/entries/qm-manyworlds/.

[5] Wallace, D. (2012). The Emergent Multiverse: Quantum Theory according to the Everett Interpretation. Oxford: Oxford University Press.