

How Haag-tied is QFT, really?

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Abstract

Haag’s theorem cries out for explanation and critical assessment: it sounds the alarm that something is (perhaps) not right in one of the standard way of constructing interacting fields to be used in generating predictions for scattering experiments. Viewpoints as to the precise nature of the problem, the appropriate solution, and subsequently-called-for developments in areas of physics, mathematics, and philosophy differ widely. In this paper, we develop and deploy a conceptual framework for critically assessing these disparate responses to Haag’s theorem. Doing so reveals the driving force of more general questions as to the nature and purpose of foundational work in physics.

1 Introduction

The edifice of science is not raised like a dwelling, in which the foundations are first firmly laid and only then one proceeds to construct and to enlarge the rooms. Science prefers to secure as soon as possible comfortable spaces to wander around and only subsequently, when signs appear here and there that the loose foundations are not able to sustain the expansion of the rooms, it sets about supporting and fortifying them. This is not a weakness, but rather the right and healthy path of development.

[Hilbert 1905, 102; translation by Corry 2004, 127]

Proven over six decades ago, Haag’s theorem appears to present a problem for particle physics. The theorem seems to block a key technique—namely, the interaction picture

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and its attendant calculational methods—that has been widely used to generate successful predictions. It is clear that the theorem points to some sort of problem, driven by the empirical success of the calculations employing the interaction picture on the one hand and the logical force of the theorem on the other. Thus, while particle physics has secured for itself a “comfortable space” around which to wander, Haag’s theorem appears as a sign that the foundation are too loose “to sustain the expansion of the rooms.”

This paper aims to provide a framework for possible answers to a single, if double-faced, question: *What does Haag’s theorem tell us about quantum field theory, present and future?* Several divergent answers have been given already. Indeed, these will shape the paper’s framework substantially. Nevertheless, the shape the framework should take is less straightforward than it may at first seem. Even before getting to the nitty-gritty analysis of Haag’s theorem, a framework must grapple with the problem of viewpoint, as the following exercise makes clear:

Regardless of your actual field, take whatever career stage you are at—early graduate student, doctoral candidate, early-, middle- or late-career researcher—and imagine yourself instead as a particle physicist. You may imagine yourself as an experimentalist or a theoretician, expert in QED or QCD—whatever comes to mind. Regardless, several things are true of you. First, you are committed to the development of particle physics (no matter what this means in practice). Second, you are steeped in, and reliant on, the interaction picture for your research in and teaching. And third, you have just learned of Haag’s theorem and the trouble that it spells for the interaction picture. How might you respond?

Set the exercise up again, except now you are a mathematician committed to contributing to the development of QFT (no matter what this means in practice). Second, you are steeped in the implications of Haag’s theorem, and you are intimately familiar with the axiomatic or algebraic approach to quantum field theory. And third, you believe that a full, conceptually coherent physically realistic replacement for the interaction picture is (presently) unavailable.

Set it up one last time, except now you are a philosopher of science. You may consider yourself a realist or an instrumentalist, interested in metaphysics or methodology—whatever comes to mind. Regardless, several things are true of you. First, you are committed to understanding the foundations of QFT (no matter what this means in practice). Second, you are familiar both with the major advances in conventional (Lagrangian) QFT that use the interaction picture as well as those based on algebraic QFT (AQFT).¹ But third, it is unclear to you if, or how, these advances can form a consistent whole.

It is not a given that your physicist, mathematician, and philosopher selves will share a single outlook on RQFT, even before considering Haag’s theorem. Nor is it clear that they should. Inevitably, this will affect your reaction to our guiding question. Thus, a major contribution of our framework will be to highlight the influence these extra-Haagian outlooks have on understanding Haag’s theorem and on assessing its implications for philosophy and for theoretical physics. As we show in section 4, many (but not all) of the disagreements about Haag’s theorem derive ultimately from different extra-Haagian outlooks, such that the disagreement is far less about Haag’s theorem itself than it is about how to do (foundations of) physics.

This work also aims to contribute to a larger discussion in philosophy of physics. As

¹See [Fraser, 2011] and [Wallace, 2011] for the classic debate over these two formulations of QFT.

a quintessential and live example of work at the foundations of physics, the discussions of Haag’s theorem draw our attention to important methodological questions: What role does (should) foundational work play in progress in physics? How is foundational work coordinated with non-foundational work, or how should it be? And, what does (should) foundational work even look like? These are undoubtedly heady questions, and we feign no complete answers. Nevertheless, our framework will reveal some of the answers that are being given by the authors under survey, and in so doing these answers open themselves to investigation. As we conclude the paper, we suggest several investigations we expect will sober up future discussions of these heady methodological questions.

The remainder of the paper is structured as follows. In section 2 we provide a synopsis of a standard proof of Haag’s theorem and discuss the sense in which it raises an alarm that something is not right with the interaction picture. In section 3 we motivate the need for a framework for understanding the literature on Haag’s theorem. The framework itself is given in section 4. The framework employs and extends Hilbert’s construction analogy: the framework understands each author as something like a contractor giving their **assessment** of the problem in the foundations of physics heralded by Haag’s theorem, their recommendation for **repair** work on the foundations, and their expectations of the needed long-term **maintenance or future renovations** to QFT’s area of the edifice of science. This section applies the framework to seven leading contemporary viewpoints on Haag’s theorem; the key results of this application are given in table 1. Section 5 demonstrates the framework’s judicious balance of conceptual structure and flexibility in order to bring clarity to the space of responses to Haag’s theorem; it further argues that, at the end of the day, the most important lesson for philosophers to take from Haag’s theorem is that we need to put our own energies into clearly answering meta-level questions regarding the nature and purpose of foundational work in physics. Concluding remarks are given in section 6.

We will use the following terms to disambiguate the different approaches to and versions of QFT. By *canonical* QFT we mean what [Wallace, 2001] does by ‘Lagrangian QFT’: an approach that, for the most part, proceeds from specification of a classical field theory, to its quantization, and finally to its (Wilsonian) renormalization. By *formal variants* of QFT we mean the most familiar collection of approaches focused on mathematical rigor and precision, including algebraic QFT, constructive QFT (CQFT), and axiomatic QFT; these center around the Wightman, Osterwalder-Schrader, or Haag-Kastler axioms.

2 Haag’s Theorem

2.1 History of Haag’s Theorem

Haag’s theorem is the culmination of two approaches in early quantum field theory. On the one hand, Dyson had combined Feynman’s rules and the Tomonaga-Schwinger formalism into a reliable and practical approach for calculating the results of scattering experiments in relativistic quantum field theory. This approach brought with it a shift toward considering scattering amplitudes between *free, asymptotic states* (see [Blum, 2017]), there accomplished by the so-called interaction picture to model interactions [Schwinger, 1948b] (see 2.2). The approach was wildly successful, in particular in its use to calculate the anomalous magnetic

moment of the electron [Schwinger, 1948a]. This is the genesis of canonical QFT.

On the other hand, spurred by Wigner’s precise characterization of special relativity’s implications for quantum theory [Wigner, 1939], a more mathematical approach to understanding the structure of relativistic quantum field theories maintained an interest in determining the properties of *instantaneous states* in addition to free, asymptotic states. This is the genesis of formal variants QFT. Naturally, a transparent representation of relativistic QFT’s mathematical structure was prized, leading to an enumeration of several assumptions. However, the assumptions underlying the former approach’s use of the interaction picture were unclear. Haag, in his lecture series given at CERN in 1952–3,² began by stating assumptions for models of relativistic QFT for a single, fundamental field. These included, following Wigner’s suggestion, requiring an irreducible Hilbert space representation of the Poincaré group. Then, Haag used the mathematical tools at his disposal to characterize the interaction picture (e.g., the particle representation of field states, unitary intertwiner). The problem (Haag’s theorem [Haag, 1955]) was that the interaction picture, thus characterized, could not be used to represent non-trivial interactions. This result was clarified and generalized by [Hall and Wightman, 1957], to which most discussions of Haag’s theorem refer. Essentially, Haag, Hall, and Wightman showed that “to give different physical predictions, two theories of an interacting field which satisfies the canonical commutation relations must use inequivalent representations of the commutation relations” [Hall and Wightman, 1957, 2].

We should make two historical remarks. First, Haag’s theorem was not unique in observing the existence and importance of unitarily inequivalent representations of the CCRs. [Reed and Simon, 1975, 329] suggest—and [Haag, 1955, 21] asserts—that von Neumann may have known of the existence of inequivalent representations by the time he wrote [von Neumann, 1938], and, von Neumann notwithstanding, the particle representation originating in [Dirac, 1927] was the only one used until the late 1940s. But by 1951, Friedrichs had characterized a new representation and demonstrated its general inequivalence with the particle representation. He moreover argued that “in a certain sense [such fields] *do occur in nature* [Friedrichs, 1953, 142;153], and, in fact, the representation allows one to treat problems previously-insoluble because of the so-called infrared catastrophe. Similarly, [Wightman and Schweber, 1955] produced inequivalent representations (using von Neumann’s techniques) and suggest their physical relevance. Garding and Wightman then published their classification of all representations of the CARs in [Garding and Wightman, 1954], noting along the way that unitary equivalence also depends on the “terminal behavior” of the involved unitary operators when the measure (partially) characterizing the representation is nondiscrete [Garding and Wightman, 1954, 621]. This line of thinking led ultimately to Theorem X.46 of [Reed and Simon, 1975, 233], which says that free, neutral scalar fields of inequivalent mass carry unitarily inequivalent irreducible representations. This theorem will be relevant in §4, where e.g., [Duncan, 2012, Klaczynski, 2016, Earman and Fraser, 2006], construe this theorem as the conceptual core of Haag’s theorem.

Second, the early reception of Haag’s theorem emphasized its implications for a particle

²See [Haag, 2010] for his recollections of this aspect of the history. Note that Haag says (incorrectly) that the lectures were given in 1953–4; see [Lupher, 2005].

interpretation but neither *reduced* its significance to its interpretive implications nor viewed it as ultimately damning for particles. It will suffice to discuss Haag himself. While he emphasized its consequences for the “possibility of defining a theory[...] which describes the interaction processes of particles,” he nevertheless (a) notes in summary that relaxing the equal-time vanishing of the CCRs suffices to evade the theorem and (b) claims the assumptions he makes are not only compatible but even “have physical significance” in the lowest orders of a perturbation expansion [Haag, 1955, 36-7]. Haag actually emphasizes two escape paths taken by later commentators, namely that his use of strong convergence diverges from the typical convergence factor and that satisfactory physical descriptions do not require use of infinite-dimensional Hilbert spaces. The point for Haag, it seems, was to probe the limits of the conventional field theory scheme [Haag, 1955, 32].

2.2 The Interaction Picture

Haag’s theorem is often construed as a no-go theorem for the interaction picture. As noted, this picture has been widely used for the calculation of many physical quantities that have matched experimental results to a high degree of accuracy, for example the celebrated computation of the anomalous magnetic dipole moment of the electron by [Schwinger, 1948a], and is a mainstay of undergraduate and graduate textbooks and has facilitated the calculation of many physical quantities that have matched experimental results to a high degree of accuracy.³ As its name suggests, the interaction picture is one way to model interacting fields in conventional quantum field theory. Let us suppose we have a field, ϕ , with conjugate momentum π , generally taken (either explicitly or implicitly) to obey the equal time canonical commutation relations and other axioms of QFT.

The interaction picture is intermediate between the Schrödinger and Heisenberg pictures. In the Schrödinger picture of quantum mechanics, states evolve in time under the full Hamiltonian, whilst operators are stationary. In the Heisenberg picture of quantum mechanics, the operators evolve under the full Hamiltonian, whilst states are stationary. To form the interaction picture, we split the full Hamiltonian into a free and a (time-dependent) interaction part, $H = H_F + H_I$. The evolution of operators is governed by the free Hamiltonian, H_F , so the fields are free. The evolution of states is governed by the interaction part, H_I .

³ As one crude measure of how salient the interaction picture is for actual physics, a search for the exact phrase “interaction picture” in Physics Reviews conducted on Nov 4, 2021 returned 4,377 hits. The interaction picture forms the basis of calculations using time-dependent perturbation theory. Crucially, the Gell-Mann and Low theorem makes use of the interaction picture. The original paper [Gell-Mann and Low, 1951] presenting this formula has (as of Nov 4, 2021) 1544 citations recorded on google scholar. This is likely a rather low estimate of how widely the formula is in fact used since the formula is now at the level of common knowledge among physicists, and thus in many cases it might be used without citation. For instance, the Gell-Mann and Low theorem was used to rigorously derive the Bethe–Salpeter equation, which describes the bound states of a system of two fermions in a relativistic formalism [Salpeter and Bethe, 1951]. This equation was used in subsequent quantum electrodynamics calculations of the fine structure of Hydrogen-like atoms [Salpeter, 1952], Helium [Douglas and Kroll, 1974] and heavy atoms [Mohr et al., 1998] and contemporary precision calculations of the electron magnetic moment [Aoyama et al., 2015]. Likewise, the equations have been utilized in the calculation of nucleon-nucleon potentials [Partovi and Lomon, 1969, Partovi and Lomon, 1972, Lomon, 1976, Partovi, 1978] and the derivation of scaling laws for interactions with large momentum transfer, confirmed by scattering experiments [Brodsky and Farrar, 1975], among many other examples.

We stipulate that the operator fields coincide with those of the Heisenberg picture at some time, t_0 . Let $V(t_2, t_1)$ represent the unitary evolution of the interaction picture states from time t_1 to t_2 , generated by the interacting part of the Hamiltonian, H_I ,

$$(1) \quad V(t_2, t_1) = e^{-iH_I(t_2-t_1)} = e^{+iH_F(t_2-t_1)}e^{-iH(t_2-t_1)}.$$

We call this operator the intertwiner, or Dyson operator. Then, at all times, t , the Heisenberg (subscript F) and interaction (subscript I) operators are related by the intertwiner as follows,

$$(2) \quad \phi_I(t, \mathbf{x}) = V^{-1}(t, t_0)\phi_H(t, \mathbf{x})V(t, t_0),$$

$$(3) \quad \pi_I(t, \mathbf{x}) = V^{-1}(t, t_0)\pi_H(t, \mathbf{x})V(t, t_0),$$

where at time t_0 the operators interaction picture and Heisenberg operators coincide, $\phi_I(t_0, \mathbf{x}) = \phi_H(t_0, \mathbf{x})$ and $\pi_I(t_0, \mathbf{x}) = \pi_H(t_0, \mathbf{x})$. As we will see, it is central to Haag's theorem that the relation of the interaction field, ϕ_I , to the free field, ϕ_F is characterized by a unitary map.

Generally, we seek to calculate physical amplitudes, taken in the limit in which the fields are free at times $t \rightarrow \pm\infty$. This is meant to capture the intuition that in an interaction, particles begin infinitely far apart (and hence not interacting) and then separate again infinitely far apart after an interaction. The interaction picture may be useful if we can treat the effects of H_I as a small, time-dependent perturbation on the evolution under H_F . In perturbation theory we perform approximate calculations by expanding the desired physical quantities in powers of the small interaction, H_I .

2.3 Proof of Haag's Theorem for Spin-free, Neutral, Scalar Fields

The core strategy is due to [Hall and Wightman, 1957] rather than [Haag, 1955]. The main technical advance of the former over the latter was a series of theorems concerning the analytic continuation of functions invariant under the orthochronous inhomogeneous Lorentz group, which allowed them to extend the equality of two-, three-, and four-point vacuum expectation values at equal times to any times. For simplicity, we eschew reference to this complex, noting its intuitive gloss as “a quantitative formulation of the intuitive feeling that in a Lorentz invariant theory the equivalence of descriptions in different Lorentz frames should somehow restrict the possible correlations between the values of physical quantities at different points in space-time” [Hall and Wightman, 1957, 35]. We likewise appeal to the simpler Jost-Schroer theorem, which at any rate affords extension to all vacuum expectation values (when one of the fields is free). For convenience, we also adopt some of the notation and conventions of [Earman and Fraser, 2006] and [Seidewitz, 2017], including the numbering due to the latter.

2.3.1 Wightman Axioms

Axiom 0. States. *We have a physical Hilbert Space, \mathcal{H} , for which the states, $|\phi\rangle$, are rays, such that,*

1. The states transform according to a continuous unitary representation of the Poincaré group, $U(\Delta\mathbf{x}, \Lambda)$, under Poincaré transformations, $\{\Delta\mathbf{x}, \Lambda\}$.
2. There is a unique, invariant vacuum state, $|0\rangle$, in \mathcal{H} , invariant under U : $U(\Delta\mathbf{x}, \Lambda)|0\rangle = |0\rangle$.
3. Let $U(\Delta\mathbf{x}, \Lambda) = e^{iP^\mu\Delta x_\mu}$. Then, $P^\mu P_\mu = -m^2$. We interpret P^μ as an energy-momentum operator and m as a mass. The eigenvalues of P^μ lie in the future lightcone.

Axiom 1. Domain and continuity of fields. The field $\phi(x)$ and its adjoint $\phi^\dagger(x)$ are defined on a domain D of states dense in \mathcal{H} containing the vacuum state $|0\rangle$. The $U(\Delta\mathbf{x}, \Lambda)$, $\phi(x)$ and $\phi^\dagger(x)$ all transform vectors in D to vectors in D .

Axiom 2. Field transformation law. The fields transform under Poincaré transformations as,

$$(4) \quad U(\Delta\mathbf{x}, \Lambda)\phi(x)U^{-1}(\Delta\mathbf{x}, \Lambda) = \phi(\Lambda x + \Delta\mathbf{x}).$$

Axiom 3. Local commutativity. If x and x' are two spacetime positions,

$$(5) \quad [\phi(x), \phi(x')] = [\phi^\dagger(x), \phi^\dagger(x')] = 0$$

Furthermore, if x and x' are space-like separated,

$$(6) \quad [\phi(x), \phi^\dagger(x')] = 0.$$

Axiom 4. Cyclicity of the vacuum. The vacuum state $|0\rangle$ is cyclic for the fields, $\phi(x)$. That is, polynomials in the fields and their adjoints, when applied to the vacuum state, yield a set D_0 dense in \mathcal{H} .

2.3.2 Proof of Haag's theorem

Haag's theorem follows from the results of two other theorems. For clarity of exposition, here we only prove the theorem for neutral, scalar fields; the generalization for other types of fields follows in a straightforward manner. From here on, it is convenient to decompose four-vectors as $x = (t, \mathbf{x})$, where the right hand side are the temporal and spatial components, respectively.

Theorem 1. Equality of equal-time vacuum expectation values. Let ϕ_1 and ϕ_2 be two field operators, with associated conjugate momentum operators, π_1 and π_2 , defined in respective Hilbert spaces \mathcal{H}_1 and \mathcal{H}_2 , satisfying the Wightman axioms listed above, and satisfying the equal time commutation relations,

$$(7) \quad [\phi_i(t, \mathbf{x}), \pi_i(t, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'),$$

$$(8) \quad [\phi_i(t, \mathbf{x}), \phi_i(t, \mathbf{x}')] = [\pi_i(t, \mathbf{x}), \pi_i(t, \mathbf{x}')] = 0.$$

Suppose that there exists a unitary operator G such that, at some specific time t_0 ,

$$(9) \quad \phi_2(t_0, \mathbf{x}) = G\phi_1(t_0, \mathbf{x})G^{-1},$$

$$(10) \quad \pi_2(t_0, \mathbf{x}) = G\pi_1(t_0, \mathbf{x})G^{-1}.$$

We call G an intertwiner for the fields ϕ_1 and ϕ_2 . Then the equal-time vacuum expectation values of the fields coincide. (Note that so far, this holds only at the particular time, t_0 .)

Proof. Let $U_i(\Delta\mathbf{x}, \mathbf{R})$ be a continuous, unitary representation of the inhomogeneous Euclidean group of translations, $\Delta\mathbf{x}$, and three-dimensional rotations \mathbf{R} , defined on each \mathcal{H}_i , $i = 1, 2$. Let us further suppose that transformations $U_i(\Delta\mathbf{x}, \mathbf{R})$ induce Euclidean transformations of the field

$$(11) \quad U_i(\Delta\mathbf{x}, \mathbf{R})\phi_i(t_0, \mathbf{x})U_i^{-1}(\Delta\mathbf{x}, \mathbf{R}) = \phi(t_0, \mathbf{R}\mathbf{x} + \Delta\mathbf{x}),$$

$$(12) \quad U_i(\Delta\mathbf{x}, \mathbf{R})\pi_i(t_0, \mathbf{x})U_i^{-1}(\Delta\mathbf{x}, \mathbf{R}) = \pi(t_0, \mathbf{R}\mathbf{x} + \Delta\mathbf{x}),$$

as in axiom 0.1. From our supposition (equations 9 and 10), it follows that

$$(13) \quad U_2(\Delta\mathbf{x}, \mathbf{R}) = GU_1(\Delta\mathbf{x}, \mathbf{R})G^{-1}.$$

But since the representations possess unique invariant vacuum states $|0\rangle_i$ such that $U_i(\Delta\mathbf{x}, \Lambda)|0\rangle_i = |0\rangle_i$, as in axiom 0.2.,

$$(14) \quad c|0\rangle_2 = G|0\rangle_1,$$

where c is a complex number of absolute value 1, $|c| = 1$. In other words, up to a phase factor, $G|0\rangle_1$ is the vacuum state for field ϕ_2 at time t_0 .

It further follows that the equal-time vacuum expectation values (Wightman functions, also known as correlation functions) of the two fields are the same,

$$(15) \quad {}_1\langle 0|\phi_1(t_0, \mathbf{x}_1), \dots, \phi_1(t_0, \mathbf{x}_n)|0\rangle_1 = {}_2\langle 0|\phi_2(t_0, \mathbf{x}_1), \dots, \phi_2(t_0, \mathbf{x}_n)|0\rangle_2,$$

for $\mathbf{x}_1, \mathbf{x}_2$ up to at most \mathbf{x}_4 , all at the same fixed time, t_0 .⁴ □

Theorem 2. Jost-Schroer theorem. *For any free scalar field ϕ , the two-point vacuum expectation values are given by*

$$(16) \quad \langle 0|\phi(x_1)\phi^\dagger(x_2)|0\rangle = \Delta^+(x_1 - x_2),$$

⁴If at least one of ϕ_1 or ϕ_2 is a free field, then this can be proven to hold for all Wightman functions, i.e. for $\mathbf{x}_1, \mathbf{x}_2$ up to any \mathbf{x}_n .

where $x_1 = (t_1, \mathbf{x}_1)$ and $x_2 = (t_2, \mathbf{x}_2)$ are arbitrary spacetime four-vectors. Δ^+ is the advanced Feynman propagator⁵, given by,

$$(17) \quad \Delta^+(x_1 - x_2) = (2\pi)^{-3} \int d^3p \frac{e^{i[-\omega_{\mathbf{p}}(t_1-t_2) + \mathbf{p} \cdot (\mathbf{x}_1 - \mathbf{x}_2)]}}{2\omega_{\mathbf{p}}},$$

and $\omega_p = \sqrt{\mathbf{p}^2 + m^2}$.

If, for any arbitrary field, the vacuum expectation values are given by equation 16, then that field is a free field.

This result was first proved by Jost [Jost, 1961] for fields of positive mass, and extended to fields of zero mass by Pohlmeier [Pohlmeier, 1969].

Theorem 3. Haag's theorem for scalar fields. Let ϕ_1 be a free, scalar field, which therefore satisfies equation 16. Let ϕ_2 be a second, locally Lorentz-covariant scalar field. Let us assume that ϕ_2 is unitarily related to ϕ_1 at time t_0 , as in equations 9 and 10. Further let us assume that the conjugate momenta fields (written as adjoints ϕ_1^\dagger and ϕ_2^\dagger) satisfy the hypotheses of Theorem 1. Then $\phi_2(x)$ is also a free field.

Proof. Theorem 1 tells us that the equal time vacuum expectation values of the two fields, ϕ_1 and ϕ_2 must coincide at t_0 , i.e.,

$$(18) \quad {}_1\langle 0 | \phi_1(t_0, \mathbf{x}_1) \phi_1(t_0, \mathbf{x}_2) | 0 \rangle_1 = {}_2\langle 0 | \phi_2(t_0, \mathbf{x}_1) \phi_2(t_0, \mathbf{x}_2) | 0 \rangle_2.$$

Since the field ϕ_1 is free, it follows from Theorem 2 (equation 16) and equation 20 that the two-point vacuum expectation values for the two fields coincide at t_0 , i.e.,

$$(19) \quad {}_2\langle 0 | \phi_2(t_0, \mathbf{x}_1) \phi_2^\dagger(t_0, \mathbf{x}_2) | 0 \rangle_2 = {}_1\langle 0 | \phi_1(t_0, \mathbf{x}_1) \phi_1^\dagger(t_0, \mathbf{x}_2) | 0 \rangle_1.$$

So far, this holds *only* at t_0 . However, any two spacelike separated position vectors, (t_1, x_1) and (t_2, x_2) , can be brought into the equal time plane $t_1 = t_2$ by a Lorentz transformation. Thus, the Lorentz-covariance of ϕ_2 allows us to extend the satisfaction of equation 19 to any two spacelike positions, and then, because time translation is equivalent to a Lorentz boost plus spatial translation plus Lorentz boost [Earman and Fraser, 2006, 317], to any two positions:

$$(20) \quad {}_2\langle 0 | \phi_2(x_1) \phi_2^\dagger(x_2) | 0 \rangle_2 = \Delta^+(x_1 - x_2).$$

Therefore, by Theorem 2, $\phi_2(x)$ must be a free field. □

⁵For example, see [Duncan, 2012, pages 209-210 and 256]

2.4 Implications of Haag’s Theorem

By the criteria used by particle physicists, the interaction picture has undoubtedly produced numerous notable successes (see note 3). However, the interaction picture is generally taken to depend upon all of the assumptions needed for Haag’s theorem, including Poincaré invariance and the existence of a unitary operator relating the two fields, that is, that the fields are unitarily equivalent. The apparent clash between the interaction picture and Haag’s theorem arises as follows. If the fields, ϕ_F and ϕ_I , obey the usual axioms of QFT, and we have a unitary operator intertwining the two fields at even a single time (as in equations 9 and 10), and ϕ_F is free, then according to Haag’s theorem ϕ_I must also be free. So the interaction must be trivial.

So Haag’s Theorem appears to be a no-go theorem for calculations that use the interaction picture. At a glance, it would be mathematically inconsistent to use the interaction picture for calculations involving any of the non-trivial interactions that we care about in particle physics. However, a closer look reveals a whole labyrinth of philosophical, mathematical, and physical issues at stake in understanding the full significance of Haag’s theorem. We take that closer look in the next section.

3 What the Haag Is Going on?

There are a few points on which all parties generally agree. First, judged by its own particular standards of success, particle physics—including the interaction picture—is highly successful.

Second, there is a consensus that Haag’s theorem poses a *bona fide* problem for the standard presentation of the interaction picture.⁶ The theorem itself is mathematically correct: as Kłaczynski puts it, “it is a mathematical theorem in the truest sense of the word; it brings with it the ‘hardness of the logical must’.” [Kłaczynski, 2016]. Further, it is not disputed that the assumptions of Haag’s theorem hold in the case of the standard textbook presentations of the interaction picture.⁷ No one, to our knowledge, argues that the problem posed by Haag’s theorem is simply illusory.⁸ It is, rather, the severity and appropriate

⁶That is, the pre-renormalization interaction picture.

⁷For example, standard presentations assume, either implicitly or explicitly, a Poincaré invariant theory, and that there are distinct free and interacting fields, related by a unitary operator.

⁸It may very well be that many practicing physicists would say that the problem, if not illusory, is trivial. Perhaps many physicists think that the problem posed by Haag’s theorem is at most rooted in an unrealistic idealization of non-interacting states at temporal infinity. And perhaps they therefore choose not to address Haag’s theorem in their textbooks, lecture notes, or research articles. We can do little more than speculate that such an unspoken consensus fully explains the dearth of references to Haag’s theorem in standard accounts of QFT—simple ignorance of the theorem may be just as strong of a causal factor. David Tong’s lecture notes on QFT, for instance, do not explicitly mention Haag’s theorem; but they do say that the assumption of non-interacting states in the interaction picture is wrong and should be replaced by the interactions-are-always-on interpretation of the LSZ reduction formula [Tong, 2012] p. 54-55, 79-80. We might tentatively read this as an attitude towards Haag’s theorem as minimally useful: it points out that an initial assumption was wrong, and in wise pedagogical fashion, Tong will correct that assumption in due course when more complexity and sophistication can be digested. But no need to belabor the point by naming and deeply discussing the theorem. Thus, while it is possible that this sort of largely dismissive response to Haag’s theorem is widespread, too little of it exists in print to be extensively covered in the remainder of this paper.

remedy of the problem that is subject to debate.

However, the severity of the problem raised by Haag’s theorem clearly stops short of spelling the demise of that research program predicated on the application of quantum theories of fields to scattering experiments, commonly called particle physics.⁹ Even the practitioners of AQFT, while operating far afield from the details of the Standard Model’s phenomenology, still conceive of their work as contributing to the scientific enterprise whose primary goal is to develop the quantum theory of fields as the appropriate theoretical apparatus for understanding scattering experiments. So, then, what *is* going on with Haag’s theorem such that these two stances—the interaction picture has been used successfully, and Haag’s theorem poses a substantive problem for the interaction picture—can be held together? This central tension is widely recognized as a point of agreement. As Teller succinctly puts it,

Everyone must agree that as a piece of mathematics Haag’s theorem is a valid result that at least appears to call into question the mathematical foundation of interacting quantum field theory, and agree that at the same time the theory has proved astonishingly successful in application to experimental results. [Teller, 1995, 115]

And yet, despite this initial agreement, extant responses to Haag’s theorem form a confusing lot. The literature on Haag’s theorem reflects a number of different assessments of the import of the theorem for both (mathematical) physics and philosophy. Moreover, the extent to which these different assessments make meaningful contact with each other is often unclear. In this section, we briefly illustrate the nature of the confusion in this literature. First, the confusion is *not* about the status of Haag’s theorem as a mathematical result. All parties agree that the original theorem, and its several generalizations, have valid proofs. The confusion enters when trying to trace out the ramifications of Haag’s theorem for the foundations of QFT. [Earman and Fraser, 2006] put it well, saying that “the theorem provides an entry point into a labyrinth of issues that must be confronted in any satisfactory account of the foundations of QFT” (p. 334). Once we have entered into this labyrinth via Haag’s theorem, we encounter a host of conceptual and interpretive issues, enmeshed in technical issues of mathematics and physics, making even the range of options for a way forward through the labyrinth unclear, much less which one may be the best.

A reader interested in Haag’s theorem and its implications for the use of the interaction picture in physics may first look for insight from Earman’s and D. Fraser’s seminal paper, “Haag’s theorem and its implications for the foundations of quantum field theory.” They conclude, “On any reading Haag’s theorem undermines the interaction picture and the attendant approach to scattering theory” [Earman and Fraser, 2006, 333]. So, the reader naturally thinks, *the interaction picture is no good*. And yet for Duncan, “the proper response to Haag’s theorem is simply a frank admission that the same regularizations needed to make proper mathematical sense of the dynamics of an interacting field theory at each stage of a perturbative calculation will do double duty in restoring the applicability of the

⁹Though see the discussion of Kastner (section 4.6) and Seidewitz (section 4.7) below for genuine proposals of new physical theories, each drawing some motivation from Haag’s theorem.

interaction picture at intermediate stages of the calculation” [Duncan, 2012, 370]—the interaction picture survives! Miller concurs, adding that the success of calculations delivered from regularized and renormalized theories is explained by the conjecture that “perturbative expansions are asymptotic to exact solutions of a theory that generates them” [Miller, 2018, 818]. *So*, the reader concludes triumphantly, *the interaction picture works!*

Still more paths begin to emerge, however. According to Klaczynski, these renormalized theories evade Haag’s theorem precisely by *denying* that the interaction picture exists [Klaczynski, 2016]. Maiezza and Vasquez agree, arguing that “due to *Haag’s theorem*, it is *impossible to define QFT* starting from the interaction picture with free fields” [Maiezza and Vasquez, 2020, 10] (italics in original); indeed, they seem to argue that the interaction picture fails precisely because of the failure of the conjecture Miller relies on to save it. Yet confusingly, Maiezza and Vasquez *also* disagree with Klaczynski on what saves perturbative calculations from Haag’s theorem.

The paths so far, while many, nevertheless seem to turn on what to say about the mathematical coherence of the interaction picture. Thus, *a labyrinth though it may be*, the reader thinks, *I can at least see its basic structure*. The reader has judged too soon, however, for it is not only the interaction picture *per se* that is at stake, but the metaphysics: “*either the assumptions of Haag’s theorem do not hold, in which case there is no particle notion applicable to a scattering experiment at intermediate times, or they do, in which case the particle notion applicable at intermediate times is incommensurable with the ingoing/outgoing particle notions, if the interaction is non-trivial*” [Ruetsche, 2011, 252] (italics in original). *So*, the reader thinks, *the path out of the Haagian labyrinth requires the banishment of particles and the embracing of fields* (as [Halvorson and Clifton, 2002] suggest)! Not so, says Kastner: *particles* exist, and *fields* must be banished [Kastner, 2015].

The reader is thus confronted with paths out of the Haagian labyrinth diverging on both metaphysical and mathematical grounds and, worse still, she can’t tell from her place in the labyrinth whether or where these paths coincide. As if this weren’t bad enough, a fog sets in. *Are we even trapped at all?*, the reader asks, [Seidewitz, 2017, 356] in hand, for Haag’s theorem arises in traditional QFT only because time is not “treated comparably to the three space coordinates, rather than as an evolution parameter.” Thus, the Haagian labyrinth could have been entirely avoided had time been treated in a relativistically sensible manner at the outset.

How should the reader react to this? Is there a Haagian labyrinth? And if so, which path will lead us out? The framework given in the next section gives a fixed structure for organizing and assessing this confusing network of responses to Haag’s theorem, thereby creating several distinct mappings of the Haagian labyrinth. More general lessons from studying these maps are given in section 5.

4 The Framework: Assessment, Repair, and Renovation and Maintenance

Before presenting the framework, we should carefully consider its goal. The purpose of this framework, as we said in the introduction, is to fruitfully structure and organize answers

to the question “What does Haag’s theorem tell us about quantum field theory, present and future?” Two desiderata for such a framework are fairly obvious. First, it should sensibly organize the answers that have already been given to this question. As such, we will apply the framework to prominent answers in the literature. Second, the framework should leave significant room for future developments. Given the interdisciplinary nature of the study of Haag’s theorem, and given the recent increase of interest from physicists ([Klaczynski, 2016], [Maiezza and Vasquez, 2020], and [Seidewitz, 2017]), we expect there is much more to be said on this topic. A good framework for organization, therefore, must have space to accommodate these expected future developments.

Expanding on Hilbert’s construction analogy, we will construe QFT as a building within the “edifice of science” that is continually under construction. Responses to Haag’s theorem, then, will be organized and analyzed according to their assessment or diagnosis of the problem posed by the theorem, the appropriate immediate repairs to that problem, and any longer-term renovations or maintenance measures called for in light of the assessment:

- **Assessment:** What precisely is the problem posed by Haag’s theorem, if any? For what objective(s) is this a problem? Or, are there multiple problems?
- **Repair:** How should this problem be remediated?
- **Maintenance or Renovation:** Where should resources (time, attention, grant funding, conference and journal platforms, etc.) for the next (relevant) phase of research be allocated?

On one hand, the authors surveyed below may be thought of as subcontractors brought in to assess and diagnose the building’s ability to support further growth in light of Haag’s theorem. On the other hand, we the authors of this present paper, may be thought of as general contractors. As such we aim to arrange and make comparable these subcontractors’ assessments, which we do in 4.

However, our ultimate goal as ‘general contractor’ is to provide guidance on the building’s construction—specifically, guidance as to how stakeholders in the future of QFT can coordinate their efforts on their shared interdisciplinary goals (see section 5 below). This requires us to explain any disagreements amongst the subcontractors. The points of general agreement provide a convenient starting point. Recall from the outset of section 3 that all authors agree:

1. particle physics is successful *by its own standards*; and
2. Haag’s theorem presents a *bona fide* problem for using the interaction picture to relate distinct fields *prior to renormalization*.

Differences of Assessment clarify where interlocutors in fact part ways beyond these points. Differences of Repair and Renovation reveal differences of motivation, expectation, and aim, rather than matters of fact. We have called these *extra-Haagian outlooks* in order to stress that they are more abstract, give rise to less adversarial disagreements, and stem from commitments that extend far beyond the technical reach of Haag’s theorem. Each of our two initial points of agreement gives rise to a distinctive locus of extra-Haagian outlooks.

Agreement (1) belies the different standards one might hold predictions to. It is agreed that the calculational methods produce reliable predictions. That is, the methods have generated successful predictions, and their application has been systematic enough to warrant belief in their continued success. Yet, insofar as they are generated using the interaction picture, these predictions rely on low-order perturbative approximations made at asymptotic times and relative to detector resolution. Moreover, since the full perturbative series diverges, it is difficult to specify what we are successfully approximating in a physical sense. *Can we, or should we, expect more of a theory’s predictions? of a relativistic QFT’s predictions?*

Agreement (2) belies the different expectations one might have for the form of QFT. Beyond furnishing a method of generating predictions, one pre-Haagian appeal of the interaction picture was its intuitive representation of interaction dynamics. The asymptotic nature of calculations aside, interactions looked familiar: the full state space was time-evolved, from beginning to end, by a specified dynamical operator. As the agreement on (2) reflects, Haag’s theorem undermines the interaction picture’s ability to represent interactions in this way. And while renormalization delivers the agreed-reliable predictions of (1), it does so at the cost of further complicating attempts to represent interactions as originally hoped. But whether any of this is problematic, and if so to what degree, depends on one’s hopes and expectations. *What should count as a fully satisfactory theory? as a fully satisfactory theory of relativistic QFT?*

In thus organizing the various responses to Haag’s theorem, and in bringing the background commitments of the extra-Haagian outlooks into broad daylight, the framework itself does not adjudicate among the responses to Haag’s theorem. Rather, it clarifies the various lines of genuine debate, both at the level of disagreement of technical matters of fact, and at the level of deeper methodological commitments. As such, these outlooks are directly implicated in 5. Since there is no single “owner” of QFT, let alone the edifice of science, the information that any inquirer of QFT will want—be they physicist, philosopher, or mathematician—will vary according to their own outlook. In our role as ‘general contractors’, we indulge these owners by providing sample advice that either arranges the responses according to their outlook and even provisionally adopts an outlook.

A final note before we proceed. While the responses to Haag’s theorem we review below proceed as if it presents a single problem demanding a singular response, this is not universal. In particular, [Ruetsche, 2011, Chs. 9–11] notes several potential problems that unitary inequivalence phenomena, Haag’s theorem among them, pose to a fundamental particle interpretation. Her response to these is pluralist insofar as she recognizes that “*different interpretations will be indexed to—and adulterated by—different aspirations,*” where these aspirations “adulterate because they arise *along with* particular applications of the theory” [Ruetsche, 2011, 246]. She argues that to demand a univocal interpretation, which would require a singular response to Haag’s theorem, would “stym[ie] attempts to harness the theory [...] in fruitful projects of explanation and expansion” [Ruetsche, 2011, 260]. While this pluralist response is rich and worth discussion, we do not review it here because of its complexity and because it is not centered on Haag’s theorem *per se*.

Name	Assessment	Repair	Renovation or Maintenance
Earman & D. Fraser	Interaction picture (IP) rests on a set of inconsistent assumptions	Abandon the interaction picture; adopt Haag-Ruelle scattering	Interpret and assess formal variants of QFT
Pristine Theory Interpreters	Standard particle notion inconsistent with axioms of QFT	Abandon standard particle interpretation	Develop univocal interpretation of axioms of QFT
Duncan & Miller	Pre-renormalization IP is inconsistent	Renormalization (breaks Poincaré invariance)	Continue studying renormalization and EFTs, don't worry
Klaczynski	IP is inconsistent	Renormalization leads us to abandon the IP	Replace the IP's unitary intertwiner
Maiezza & Vasquez	The full perturbative series has renormalon divergences	Resummation methods (but these are ambiguous)	New resummation methods and physical insights are needed
Kastner	Physics is non-local	Abandon notion of an independent field	Develop direct-action theories
Seidewitz	Use of time as the evolution parameter is at fault	Revise axioms to introduce new evolution parameter	Extend parameterized QFT to gauge theories and non-Abelian interactions

Table 1: Framework application summary

4.1 Earman and D. Fraser

The philosophical literature on Haag’s theorem is significantly influenced by [Fraser, 2006] and [Earman and Fraser, 2006]. [Fraser, 2006] addresses the significance of Haag’s theorem for interacting QFT, and [Earman and Fraser, 2006] argue that there are multiple, significant implications of Haag’s theorem which are worthy of philosophical attention. However, the multiplicity of these implications all stem from the same root: Haag’s theorem says that the interaction picture is predicated on an inconsistent set of assumptions. From this root, one stem concerns the implications for scattering theory, another holds implications for the use of non-Fock representations in describing interacting fields, and still another concerns the practitioner’s choice among many unitarily inequivalent representations.

Extra-Haagian outlooks. D. Fraser’s dissertation, *Haag’s Theorem and the Interpretation of Quantum Field Theories with Interactions*, set the stage for much of the philosophical literature on Haag’s theorem over the last sixteen years ([Fraser, 2006]). We will therefore give a more detailed assessment of this source than subsequent works. She writes, “Haag’s theorem frames answers to the question ‘What is QFT?’ ” (p. 171). There are several contenders for the prize of being QFT. Fraser divides what we have called “canonical QFT” into three subcategories: unrenormalized canonical QFT, renormalized canonical QFT, and canonical QFT with cutoffs. Contending against these for the prize of being QFT are several different formal variants of QFT: bottom up axiomatic work based on either the Wightman or Haag-Kastler axioms; constructive QFT; and algebraic QFT. Haag’s theorem shows us that unrenormalized canonical QFT is predicated on an inconsistent set of initial assumptions, and is therefore disqualified. Neither renormalization nor the introduction of cut-offs are satisfactory in the face of this inconsistency:

We understand both renormalization and the introduction of cutoffs as expedients that permit the derivation of predictions for interacting systems, but do not address the root cause of the problem (namely, the inconsistency of the initial assumptions of the canonical framework). Renormalized canonical QFT and canonical QFT with cutoffs approximate (in some unspecified way!) a correct, completely finite theory; however, this theory is not string theory or some successor to QFT, but the correct formulation of QFT itself. [Fraser, 2006] (p. 174)

Thus, as we seek to answer the question “What is QFT?” D. Fraser argues that we eliminate contenders as follows: unrenormalized canonical QFT is disqualified for its inconsistency, demonstrated by Haag’s theorem; renormalized canonical QFT is disqualified for being “mathematically ill-defined” (p. 173); whilst canonical QFT with cut-offs changes the goalposts. The goal was to integrate special relativity with NRQM, and that requires one to produce a theory on infinite, continuous space, “not a lattice of finite spacial extent” (p. 173). This leaves the formal variants of QFT as the only programs aimed at the “correct formulation of QFT” to which the other approaches are approximations.

In subsequent work, D. Fraser is explicit about the role that correcting the inconsistency flagged by Haag’s theorem plays in guiding the wise choice of variant of QFT to give philosophical attention:

[C]onsistency is also a relevant criterion because quantum field theory is, by definition, the theory that integrates quantum theory and the special theory of relativity. Consistency is relevant to QFT for theoretical reasons—not for practical reasons (e.g., the derivation of predictions). As a result, it is necessary to either formulate a consistent theory or else show that this criterion cannot be satisfied (i.e., that there is no consistent theory with both quantum and special relativistic principles). . . . The formal variant is the only variant that satisfies the criterion; its set of theoretical principles are both consistent and well motivated. [Fraser, 2009a] (p. 563)

Thus we have the key dialectical currents of the Fraser-Wallace debate fueling this line of response to Haag’s theorem. Bedrock issues about what QFT even is, and what it takes to formulate it in a way that is apt for deep philosophical work on its conceptual-nomological machinery place Haag’s theorem in the spotlight: *this no-go theorem regiments answers to the central question at hand, namely, What is QFT?* In contrast, the average theoretical particle physicist contents themselves with combining SR and QM to the best of their current ability, rather than demanding perfectly coherent and mathematically rigorous unification up front. At least for now, our current best practice gives up, among other things, the strict Poincaré invariance demanded by SR. Arguably, the ‘best of their ability’ has improved over the decades such that, the average theoretical particle physicist could claim, we will inevitably *find* such a complete and honest marriage *if one is forthcoming*. In the meantime, we have successful, if incomplete, marriages of SR with QM with which we can keep experimental and theoretical particle physics going. While it is likewise a live option for D. Fraser (and presumably Earman) that an honest and complete marriage of QFT is not possible, this is a question of *logic*, not practical or empirical feasibility.¹⁰

We note briefly here that the physicists’ attitude of settling for good-enough-to-be-getting-on-with variants of QFT clearly extends far beyond Haag’s theorem. As just one other example, theoretical particle physicists are not bothered when they learn that their QFT path integrals cannot actually be integrals, given that there is no measure on the infinite-dimensional space of paths. Sorting out where path integrals properly live in the mathematical universe is a mathematician’s job—not a physicist’s. Meanwhile, that these path integrals are currently mathematically homeless in no way diminishes their calculational and conceptual fruitfulness for theoretical physics. Haag’s theorem is one result among many indicating that the formal and canonical variants of QFT are not communicating well with each other: either, as Earman and D. Fraser see it, the foundations of QFT are in excellent shape, and they are waiting for more mathematical success in erecting interacting models of

¹⁰ Of course, D. Fraser’s full corpus on the philosophy of QFT extends beyond the scope of issues in this present paper. In her dissertation she seemed optimistic that, with more time, the as-of-yet unfinished project of constructing a fully rigorous interacting QFT in four spacetime dimensions could be completed. Crucially, she allowed for the possibility that this will require revision to the axioms ([Fraser, 2006] section 5). Subsequently, she put it to the proponents of renormalized and cutoff variants to wrestle with the question of whether such a completion is impossible: “Neither the infinitely renormalized nor cutoff variant furnishes an argument that a consistent formulation of QFT is impossible; such an argument would require making the case that the axiomatic program cannot be completed.” [Fraser, 2009a] (p. 563). See [Koberinski and Fraser, 2023] for her most recent views on the epistemological status of effective field theory approaches in particle physics.

realistic matter in a realistic number of spacetime dimensions; or, as Wallace sees it, QFT needs no foundational work since it doesn't really even need (mathematically) rigorous foundations. We, the authors, think that this communication breakdown between formal and informal variants of QFT is problematic. We will return to these concerns in section 5.

Assessment. [Earman and Fraser, 2006] give us the cold, hard logical truth: Haag's theorem "demonstrates that the interaction picture is predicated on an inconsistent set of assumptions. In response to this *reductio* of the assumptions, at least one must be abandoned" (p. 322). As discussed above, the goal for Earman and D. Fraser is to determine what QFT is, i.e., to completely and honestly unify SR and QM. A successful unification, if it is possible, will at least be able to model interacting systems. Thus, according to Earman and D. Fraser, Haag's theorem is a demonstration of the logical impossibility of modeling interactions using the interaction picture; all that the interaction picture is capable of modeling is, provably, free fields—a spectacular failure. "On any reading, Haag's theorem undermines the interaction picture and the attendant approach to scattering theory" (p. 333).

Repair. The repair work needed is in the task of modeling interactions in QFT. This might proceed either by abandoning the interaction picture altogether (e.g. with Haag-Ruelle scattering), or else by substantially revising it through giving up one or more of its assumptions. While assessing this second possible response, Earman and D. Fraser find the robustness of Haag's theorem, and its generalizations, to be significant: "Subsequent no-go results do not show that field theorists do not have to worry about Haag's theorem because some of its assumptions do not hold in all cases of interest; rather, what the subsequent results show [is] that even more assumptions have to be abandoned in order to obtain well-defined Hilbert space descriptions of interacting fields" (318). Thus, Earman's and D. Fraser's preferred remedy for Haag's theorem is to abandon the interaction picture altogether in favor of alternative ways of modeling interactions in QFT. They discuss Haag-Ruelle scattering at length, presenting it as a readily available "mathematically consistent alternative framework for scattering theory" (p. 333). As a similar type of response, [Bain, 2000] argues that the innovations of the LSZ formalism provide the right solution to the heart of the conceptual problem posed by Haag's theorem—namely, "the apparent incoherence of using the interaction picture in a situation in which its use dictates its non-existence" (p. 383).¹¹

Note that this accords with Earman and D. Fraser's stated goal of unifying SR and QM. Regularization and renormalization techniques are not options available to them as they change the game or execute the unification dishonestly, respectively. Thus, the only option is to give up the interaction picture: Haag's theorem does not pose a problem for QFT per se, but it "does pose problems for some of the techniques used in textbook physics for extracting physical prediction from the theory" [Earman and Fraser, 2006](p. 306). QFT, therefore, is not to be identified with textbook physics. The textbook physics is one attempt

¹¹[Bain, 2000] asserts that the LSZ formalism achieves this by replacing the strong convergence condition with weak convergence. [Earman and Fraser, 2006, note 23] disagree, arguing that the information contained in the in and out states of the theory is already contained in the behavior of the interpolating Heisenberg field of Haag-Ruelle. Rather, the weak convergence condition of LSZ takes the further step of explicitly linking the S-matrix and vacuum expectation values of the free field. See [Duncan, 2012, 275-289] for more discussion of the relationship between LSZ and Haag-Ruelle.

at doing QFT, and Haag’s theorem exposes serious problems in that particular attempt. The immediate technical moral of Haag’s theorem is that (better attempts at) QFT must embrace the use of unitarily inequivalent representations of the CCR.¹² The philosophical insight from this technical result is that it designates the role of unitarily inequivalent representations as a distinctive feature of QFT in contrast to QM.¹³

In addition, Earman and D. Fraser recognize that an explanation is needed for the success of the interaction picture and perturbation theory in the face of Haag’s theorem. They “suspect that the full explanation will have a number of different pieces” and offer their analysis of Haag-Ruelle scattering as one such piece (p. 322). The Duncan & Miller response discussed below (section 4.3) should be thought of as adding one additional such piece, not refuting Earman and D. Fraser.

Renovation. As a consequence of this logical diagnosis and logical remedy, Earman and D. Fraser advocate for future philosophical work and resources to be deployed in philosophical projects about formal variants of QFT and their attendant non-interaction-picture approaches to interactions. There are at least two such broad philosophical projects that benefit from the ways in which practicing mathematicians have “digested the lesson of Haag’s theorem” [Earman and Fraser, 2006] (p. 334). First there is the project of interpreting the mathematical structures used in algebraic QFT and in constructive QFT. Second, and relatedly, there is the subtle matter of assessing the implications of theorems of axiomatic approaches to QFT for the philosophy of those areas of fundamental physics that make use of QFT.

4.2 Theory Interpreters—The Particle Problem

There is a segment of the philosophical literature addressing Haag’s theorem that is primarily motivated by the question, as put by Ruetsche, ‘Is particle physics particle physics?’ [Ruetsche, 2011, 190]. The issue is whether or not the quantum field theories employed in particle physics admit of particle interpretations. The majority view is that the answer is ‘no’, with Haag’s theorem playing a starring role some lines of reasoning within a larger body of evidence in support of this position. We include an application of our framework to the lines of reasoning within this literature that appeal to Haag’s theorem. However, since many of the arguments at play in the works surveyed below extend beyond the reach of Haag’s theorem, we will not attempt a comprehensive analysis of the arguments for and against a particle interpretation of QFT.¹⁴

Most notably [Halvorson and Clifton, 2002] and [Fraser, 2008] treat Haag’s theorem as a no-go result for the particle interpretation of QFT. The heart of the argument runs, roughly, as follows. At a minimum, a particle interpretation requires that the theory admits of a formulation in which a number operator exists—you can’t very well say that the theory

¹² “[A] single, universal Hilbert space representation does not suffice for describing both free and interacting fields; instead, unitarily inequivalent representation of the CCR must be employed” (p. 333).

¹³ “Haag’s theorem was instrumental in convincing physicists that inequivalent representation of the CCR are not mere mathematical playthings but are essential in the description of quantum fields” (p. 319).

¹⁴ Readers interested in the many lines of argument here beyond those involving Haag’s theorem are encouraged to see chapter 11 of [Ruetsche, 2011] and references therein.

says it is possible that matter is fundamentally particles if the theory also says that it is not possible (in theoretical principle) for those particles to be counted. The Fock space representation of a free field yields a well-defined number operator suitable for a particle interpretation. Might we then, by means of the IP, use the Fock space of a free field for representing and counting particle states in an interacting system as well? Haag’s theorem blocks this path, since it entails that representations of the ETCCRs for free and interacting systems are unitarily inequivalent. (see Assessment below for more details). Others disagree with this assessment (e.g. [Wallace, 2011] and [Bain, 2011]).

Extra-Haagian outlooks. What unifies this sub-literature on Haag’s theorem is that it aims at a decidedly *interpretive* task: authors of such work aim to answer questions about what ontological commitments are supported by QFT. Thus, this literature is not ultimately motivated by considerations of theory consistency, application, development and innovation, or conceptual-nomological machinery (‘how does this theory work?’) that guide the work of groups surveyed in other subsections. Although such considerations sometime arise in this literature (as, indeed, matters of theory interpretation sometimes arises as a secondary consideration in other segments of the Haag’s theorem literature), they do so as steps within larger arguments ultimately aimed at interpretation. (And of course, many authors take up distinctive projects across this interpretation/not-interpretation divide.)

Within this interpretation-focused literature, the further extra-Haagian outlooks driving the discussion concern the nature of theory interpretation: what is the goal of interpretation; what does it take for a theory to be formulated in a way that is appropriate to the task of interpretation; and to what extent do the contingent details of how the theory is applied in various scientific contexts matter for the project of interpretation. Much of this literature tacitly adheres to what [Ruetsche, 2011] calls the ideal of pristine interpretation. In pristine interpretation, a theory is formulated cleanly in terms of a few powerful laws and principles. The interpreter then seeks to determine where, among all possible worlds, the theory draws a sharp line in “one fell swoop” between what is physically possible and what is physically impossible (p. 3). Contingent details of how the theory gets applied in various contexts are of no consequence for the project of pristine interpretation.

As Ruetsche noted when coining the phrase, this ideal is rarely explicitly stated (p. 3). Nevertheless, sympathy for the ideal of pristine interpretation appears widespread, particularly in the philosophy of QFT.¹⁵ [Halvorson and Clifton, 2002] and [Fraser, 2008] approach the particle problem from a methodological stance of pristine interpretation. The pristine interpreters within this group are engaged with a project that has certain commitments from the outset regarding what it takes for a physical theory to be ripe for interpretation. On this approach, the theory to be interpreted *is* the formal, mathematically well-posed version of the scientific achievement under study ([Fraser, 2009a], [Fraser, 2011]). No-go theorems proved within these formal theoretical structures, then, powerfully and persuasively constrain interpretive options¹⁶ [Fraser, 2008] is explicit on this point. In describing the role of Haag’s

¹⁵See [Araçgeorgis, 1995, 119–125] for an early interpretation-focused discussion of Haag’s theorem. We do not discuss this work here because its status as pristine or not is unclear.

¹⁶See [Freeborn et al., 2023] for further discussion on alternative stances toward no-go theorems in general, and Haag’s theorem in particular.

theorem for blocking one attempt at developing a particle interpretation of interaction QFT, she writes:

Haag’s theorem pinpoints the source of the problem with the strategy of obtaining a quanta interpretation for an interacting system from the Fock representation for some free system. The consensus among axiomatic quantum field theorists is that Haag’s theorem entails that a Fock representation for a free field cannot be used to represent an interacting field. (p. 847)

Moreover, according to Wightman, it is a “*necessary* condition” that any physically sensible interacting field theory use a representation of the ETCCRs that is unitarily equivalent to a Fock representation. Having found a necessary condition for a particle interpretation, pristine interpreters use this as the clean dividing line between theories that do admit of a particle interpretation (i.e. free fields) and those that do not (i.e. interacting fields). Thus, a presupposition of favoring mathematical rigor and axiomatic approaches is at play in the background of these pristine interpretations of Haag’s theorem.¹⁷

Non-pristine interpreters, in contrast, take a more flexible and opportunistic approach to no-go theorems. To them, Wightman’s “necessary condition” looks more like an interesting suggestion or a useful starting point. From such a starting point, they turn their attention toward opportunities within the messy details of how interacting QFT gets applied to develop serviceable notion of particles without meeting the stringent requirements of finding a representation unitarily equivalent to a Fock space representation. The usual response along these lines appeals to a scale-variant or emergent particle interpretation ([Wallace, 2011, 124], [Wallace, 2001]), or else to an asymptotic notion of particle states within the interacting theory ([Bain, 2000]). More generally, [Ruetsche, 2011] gives an extended argument against pristine interpretation. While this difference in outlook is often implicit, D. Fraser (in response to [Wallace, 2001]) is transparently skeptical of even the cogency of an ontology with non-fundamental entities or which is not built on an exact similarity relation with the mathematical model at hand [Fraser, 2008, 857–8].

This difference in outlook clearly informs reactions to the particle problem. In addition to attitudes towards the ideal of pristine interpretation, there is a further extra-Haagian outlook operative under the surface of this literature. This outlook concerns the basic nature of the question at stake. For some, the question to be answered is circumscribed as a *highly* constrained puzzle within a formalization of our intuitions of particle-like things. Others, in contrast, aim to assess the viability of a particle interpretation understood much more loosely and flexibly from the outset. [Fraser, 2009a, 857] explains this difference (situating herself in the former camp) as coming down to solving a problem within a research program versus setting out on a different program altogether: those who, like Wallace and Bain (and in a different way [Halvorson and Clifton, 2002] and [Feintzeig et al., 2021]), aim at rehabilitating (our talk of) particles under a revised, emergent, or approximate notion of particle offer “a program, not a solution” to the original puzzle.

¹⁷See [Hans Halvorson, 2007] p. 731-732 for a concise statement of this attitude towards interpretation and mathematical precision in theory formulation.

Assessment. For pristine interpreters, Haag’s theorem is a diagnostic no-go result concerning interpretation: QFT cannot be interpreted as a theory of particles. For instance [Halvorson and Clifton, 2002, 23] appeal to Haag’s theorem, among other no-go results, when they conclude that the theory of quantum fields “does not permit an ontology of localizable particles; and so, strictly speaking, our talk about localizable particles is a fiction”, useful though this fiction may be. As [Halvorson and Clifton, 2002, 20] briefly describe, Haag’s theorem precludes a particle ontology as follows. One first requires that there is a representation of the Weyl relations having a global occupation number operator. These are usually furnished with the interpretation of counting the number of particles in a state (i.e., it is the particle representation discussed in 2.1).¹⁸ Since number operators only exist for representations unitarily equivalent to a free-field Fock representation (theorem 3.3 of [Chaiken, 1968]), and since Haag’s theorem implies that the latter cannot represent an interacting field, interacting field theory does not admit of a particle interpretation.

[Fraser, 2008, 847-849] demonstrates the force of Haag’s theorem against particle interpretation in more detail. She describes three distinct attempts to generalize a quanta interpretation of free fields to interacting fields, arguing that each of these fail. The first of these attempts is foiled by Haag’s theorem; the others fail for different reasons.¹⁹ The first attempt tries to rather straightforwardly extend the notion of particle available to free field theories to interacting ones, just as [Halvorson and Clifton, 2002] discuss. In the free case, we obtain a well defined number operator, whose eigenvectors may be interpreted as counting definite numbers of particles so long as we are in a Fock representation. But Haag’s theorem shows us that we cannot use a Fock representation of free fields to represent interacting ones.²⁰ And of course, when we are looking for a particle interpretation of QFT, we care significantly more about the interacting case than we do the free case.

Thus, for these authors, Haag’s theorem sits alongside results like the Reeh–Schlieder theorem [Reeh and Schlieder, 1961], the Unruh effect ([Fulling, 1973], [Davies, 1975], [Unruh, 1976] and discussed more recently in [Crispino et al., 2008] [Earman, 2011]), or the Hegerfeldt [Hegerfeldt, 1998a] [Hegerfeldt, 1998b] and Malament [Malament, 1996] theorems (and extensions thereof [Halvorson and Clifton, 2002]) as evidence against the existence of a fundamental particle interpretation for RQFT. Indeed, canon seems to dictate that these results be discussed together when addressing the particle problem (e.g., [Bain, 2000, 380] [Halvorson and Clifton, 2002, 20] [Fraser, 2008, 842] [Ruetsche, 2011, Chs. 9–11] [Wallace, 2001, §2.4].)

Crucially, the existence of a global number operator is necessary for the standard fundamental particle interpretation since these operators enforce the presumed right notion of localizability and countability inherent in the particle concept. For pristine interpreters, there is a further desideratum: the selfsame particle concept must be available throughout each timeslice of a model of particle interactions. Put so crudely, it is this latter aspect of

¹⁸See [Fraser, 2008, 845-846] for an exposition as to why these operators are apt for counting particles, rather than counting energy levels or other discrete quantum quantities.

¹⁹But they, like the approach in [Bain, 2013] to the particle problem, count the evasion of Haag’s theorem among their merits.

²⁰See also [Heathcote, 1989] pp. 91-97 for a discussion of the consequences of Haag’s theorem for Fock space representations and axiomatic approaches to modeling interactions, and see [Earman and Fraser, 2006] section 7 for their critical assessment thereof.

the criterion that seems to rule out emergence- and coalescence-type particle notions like that found in [Wallace, 2001].²¹

Repair. Interpreters addressing the highly constrained puzzle of developing a particle interpretation of interacting QFT bite the bullet and accept that there are no fundamental particles. Doing so entails providing an alternative fundamental ontology and, consequently, alternative explanations of erstwhile “particle” physics phenomena. Halvorson and Clifton conclude that “relativistic quantum field theory does permit *talk* about particles—albeit, if we understand this talk as really being about the properties of, and interactions among, quantized fields” [Halvorson and Clifton, 2002, 24]. A large part of the repair offered in [Halvorson and Clifton, 2002] is an account of how our talk about ‘particles’ is still useful, despite its fictitious status. Similarly, albeit with less patience for the talk of particles, D. Fraser concludes that special relativity and the non-linearity of the field equation for an interacting system—key ingredients for Haag’s theorem—conspire against a quanta interpretation and lending some weak support to a field interpretation instead (“At least on the surface QFT is a theory of fields” and “there is no quanta interpretation and there are no quanta” [Fraser, 2008, 857-8].²² We may think of the repair work advocated here as follows. It is as though the ‘owner’ of our QFT building had intended to furnish the interacting QFT ‘room’ with a fundamental particle ontology to account for particle-like phenomena; this group of subcontracts is saying that the deep structure of the building simply will not allow for that kind of ‘furniture’ in that particular ‘room.’ The best we can do is paint some faux particles on the walls to save the usefulness of particle-talk.

If, however, we set aside the highly constrained puzzle within the program of rigorous formulations of QFT, we can pursue alternative programs of particle interpretations rooted in revised formal notions of ‘particle.’ These alternative programs can be pursued according to either pristine or non-pristine interpretive standards, but in either case one must revise the relevant definitions at play in the no-go on particles. One such option revises the basic approach to scattering theory (Haag–Ruelle) [Reed and Simon, 1975]. Another option revises the condition for localizability and state equivalence (LSZ formalism) [Bain, 2000]. Or we might instead consider an emergent notion of particles in QFT. Instead of proposing repairs to either our ontology or QFT, it suffices for [Wallace, 2001] to show that a concept of particle can be recovered from a field-theoretic description. Given Wallace’s background presumption that approximate concepts are legitimate in physics, it does not matter that the particle concept he deploys is vaguely defined and applies only to bosonic, massive QFTs; likewise, the satisfaction with approximate concepts removes the sting of Haag’s theorem for a fundamental particle notion. For Ruetsche, Haag’s theorem is only one chapter (namely, Ch. 9) in the story of particles [Ruetsche, 2011, Chs. 9–11], and that it rules out the standard particle interpretation *in general* is unremarkable in the larger story. Indeed, she

²¹See also [Feintzeig et al., 2021] for recent work on accounting for emergent particle phenomena using the classical limit of QFT.

²²However, it should be noted that the most obvious way of constructing a field ontology appears to run into the same problems as the particle ontology [Baker, 2009]. Baker’s argument relies on the restriction that the field wave functionals are square-integrable. If we relax this restriction, then the space of wave functionals will be larger than the space of particle wave functions (see [Jackiw, 1990] and [Sebens, 2022], but see [Wallace, 2006] for a defense of the square-integrability restriction).

concludes, “[s]ometimes particle physics is, adulteratedly, particle physics, and that’s a good thing” [Ruetsche, 2011, 260].

Renovation. Non-pristine interpreters, we can safely assume, set their long term sights on increasing the thoroughness of their multiple, context-dependent interpretations of QFT. Haag’s theorem itself is not an especially valuable guide to such a project. The long-term desire of the pristine interpreter group—including each of the above repairs—is to further develop and refine a pristine interpretation of QFT. Given this group’s expectations for how one should be developed, a mathematically precise, general, and consistent formulation of QFT, both in its foundations and in specific models, is a goal. And under the auspices of pristine interpretation, Haag’s theorem is a key guidepost. Whereas Duncan and Miller satisfy themselves with identifying violations of the assumptions of Haag’s theorem in particular settings, thus potentially making available *in those settings* a particle interpretation amenable to the non-pristine interpreter, the pristine interpreter requires a satisfactory response applicable across *all* settings. This feeds directly into the resource allocations proposed by pristine interpreters: since a response must work across all settings, axiomatic approaches to inquiry are naturally preferred. For this reason, satisfying responses are typically sought from the formal variants of QFT; the effective field theory approach, in particular, is not the right place to look for long term stability. Thus, expected renovations for this group are largely mathematical. We note here that current mathematically rigorous models of QFT are deficient in various ways, such as the decades long failed search for an interacting model in four spacetime dimensions.²³ One noteworthy recent development on what kinds of field theories can be rigorously constructed in four space-time dimensions is [Aizenman and Duminil-Copin, 2021]. Likewise, revisions of the relevant definitions (e.g., Bain’s re-definition of ‘particle’) have led to difficulties [Fraser, 2008]. Long term successful renovations in this area will likely require deep conceptual work in tandem with mathematical precise explications of those concepts.

Another line of renovation work is advocated in [Bain, 2011]. He argues that our intuitions about what formal requirements capture the appropriate senses of localizability and countability constitutive of the particle notion at stake in these debates are themselves non-relativistic. Holding a putative particle interpretation of *relativistic* QFT to non-relativistic standards of localizability and countability is misguided. Instead, Bain urges, renovation efforts ought to center on developing an appropriately relativistic representation of localizability and countability. Similarly, projects aimed at explicating an emergent notion of particles ([Wallace, 2001], [Feintzeig et al., 2021]) advocate long term renovation work in terms of conceptual criteria for particle interpretations.

4.3 Duncan and Miller

Tony Duncan [Duncan, 2012] and Michael Miller [Miller, 2018] argue that at least one of the assumptions needed to prove Haag’s theorem is violated at some point in the actual process of calculating scattering theory results in perturbative QFT. Thus, in Duncan’s words, we can

²³Recall D. Fraser’s assessment of this search in note 10.

“stop worrying” about Haag’s theorem. These calculational processes include the methods of regularization and renormalization. ²⁴

A similar view on Haag’s theorem is given in [Fraser, 2017]. Here, James Fraser is primarily concerned with the much broader question of the status of perturbative QFT and diagnosing the ‘real problem’ with this area of physics. He argues that perturbative QFT is “a method for producing approximations without addressing the project of constructing interacting QFT models” (4). ²⁵ As a consequence of this view, the threat of inconsistency posed by Haag’s theorem is defused for much the same reasons as given by Duncan and Miller. J. Fraser concludes section 4, “The perturbative method simply does not assert the set of claims shown to be inconsistent by Haag’s theorem” (18). We set aside [Fraser, 2017] for the remainder of this section since Haag’s theorem is not the primary target of that article; but readers interested in the broader question of how to assess perturbative QFT are encouraged to look there for an important contribution on that topic.

Extra-Haagian outlooks. For Duncan and Miller, it is a given that the interaction picture has been consistently applied in calculating various specific theoretical predictions. For Duncan, QFT as it is used in particle physics—including its use of the interaction picture—is “the most powerful, beautiful, and effective theoretical edifice ever constructed in the physical sciences” [Duncan, 2012, iv]. His goal, therefore, cannot coincide exactly with Earman and D. Fraser’s of strictly unifying SR with QM. Rather, Duncan’s broader goal for the book is to provide a “deep and satisfying comprehension” of QFT by addressing the important conceptual issues for which the traditional, “utilitarian” texts fail to provide careful explanations (pp. iii–iv).

Likewise, Miller is focused on understanding the QFT noted for its empirical successes [Miller, 2018, 802], hence not the QFT of Earman and D. Fraser. Ultimately, Miller’s aim is to address “a general tension that exists in much of the literature engaged in the philosophical appraisal of the foundations of quantum field theory” (p. 803). This tension is essentially that between Earman and D. Fraser and the theory interpreters’ approach to QFT on the one hand and the empirically-tractable QFT on the other: while the former is mathematically rigorous and hence (relatively) straightforward to interpret using standard philosophical tools, it has yet to produce a realistic model, so it is unclear how it could inform claims about the actual world; conversely, while the latter has generated wildly successful empirical predictions, it has done so through changes to the mathematical formalism whose interpretive significance is far from obvious (p. 803).

For both authors, then, the preferred approach seems to be to bring the philosopher’s penchant for logical and conceptual clarity to bear on QFT *as it is actually used*. Both are sure (up to fallibility) that particle physicists succeed at applying QFT for predictions for

²⁴Regularization is a mathematical procedure used to temporarily control the (UV or IR) divergences that arise in QFT calculations. After regularization, it becomes possible to extract finite results from the perturbative series, order-by-order. However any regularization technique will introduce new, arbitrary parameters (“regularization scales”). Renormalization is a subsequent procedure in which the arbitrary dependence on these regularization scales is absorbed by redefining the bare parameters of the theory. For a fuller explanation, see [Collins, 1984].

²⁵Miller also addresses the question of what happens after we remove UV and IR cutoffs and thus restore exact Poincare covariance, which J. Fraser does not directly discuss.

scattering processes. From this standpoint, Duncan and Miller seek to account for *how* the use of the interaction picture in QFT in practice overcomes the challenge of Haag’s theorem, rather than questioning *whether* it can.

Assessment. Both Duncan and Miller recognize the logical nature of the problem posed by Haag’s theorem—i.e., as the conflict between the interaction picture and free-field representation assumptions—and they take consistency to be a requirement for a viable theory. Indeed, they recognize that a response to Haag’s theorem must come from negating at least one of its assumptions. However, unlike Earman and D. Fraser, their goal is to “explain why theoretical predictions for realistic experimental observables give empirically adequate results” [Miller, 2018, 803]. These predictions *in fact* apply (a version of) the interaction picture, so they cannot simply jettison the framework of the interaction picture altogether**. Rather, they aim to identify where within the practical calculational techniques such violations of the assumptions must already take place for reasons entirely independent of Haag’s theorem.

[Duncan, 2012, pages 359-370] and [Miller, 2018] each argue that we circumvent Haag’s theorem in the messy calculational details of how the interaction picture is used in practice. In fact, they identify several areas which might demand explanation in the light of Haag’s theorem.

First, is the question of why perturbative calculations, built upon the interaction picture, are able to yield empirically adequate results at each finite order. Duncan and Miller argue this takes place in a two-step process.

In the first step, the regularization procedure evades Haag’s theorem. Both Duncan and Miller focus on the case of cutoff regularization, in which short and long distance cutoffs (or, equivalently, high and low momentum cutoffs) are introduced, reducing the theory to a finite number of degrees of freedom. Such a procedure clearly breaks the Poincaré invariance (and generally the unitarity) of the S-matrix, and so the regularized theory evades the assumptions of Haag’s theorem.

In the second step, renormalization, the theory’s parameters are redefined to remove the dependence on the cutoff scales that were introduced during regularization. As such the Poincaré invariance and unitarity of the theory are restored. Each individual term in the renormalized perturbative series is well-defined, and the series can be used up to finite order to make calculations that are compared to experiment. Thus according to [Duncan, 2012, 370], “the proper response to Haag’s theorem is simply a frank admission that the same regularizations needed to make proper mathematical sense of the dynamics of an interacting field theory at each stage of a perturbative calculation will do double duty in restoring the applicability of the interaction picture at intermediate stages of the calculation.”

Note that the violation of Poincaré invariance cannot be the explanation in all cases. Many, widely used methods of regularization (such as dimensional regularization) do not break Poincaré invariance. This raises the question as to how this story should apply in such cases. In response, [Miller, 2018, pages 814-815] suggests that the basic structure of the argument still stands. Any such regularization technique must provide *some* means for controlling infrared divergences. In so doing, one expects that one or more of the assumptions of Haag’s theorem *must* have been violated through the regularization process.

However this raises a second question: after renormalization, the full symmetries of the theory are restored, so why do we continue to get empirically adequate results? After all, restoring such symmetries should once again render the entire formalism ill-defined. Here, [Miller, 2018, p. 815] argues, “The best available explanation of this fact is that the observables that get compared to experiment are insensitive to the removal of the infrared cut-off.”²⁶ As [Miller, 2021, 1134] explains, this involves expressing physical quantities in terms of detector resolution, which conditions “on which kinds of soft radiation can be undetected in the final state.” This conditioning is appropriate insofar as it merely articulates “the precise nature of the question we are asking about the outgoing state by executing the particular measuring process that we chose to execute.” While exactly which assumption is given up varies, the standard toolkits used by physicists to generate practical calculations nevertheless allow them to avoid any possible traps laid by Haag’s theorem.

Even with this in place, Duncan and Miller point to a further theoretical problem with the perturbative series. Although the individual terms in the properly renormalized perturbative series are finite, the series as a whole still diverges [Miller, 2018, 187] [Duncan, 2012, 374-411]²⁷. However, Duncan does not attribute this problem to the consequences of Haag’s theorem and Miller goes further arguing the problem is “not related to Haag’s theorem” [Miller, 2018, 817]. Thus for both Duncan and Miller, whatever theoretical problems may remain with the perturbative series, the issue of how it can evade Haag’s theorem in particular has been solved.

Repair. The interaction picture in practice makes use of renormalization and regularization techniques that, in pulling “double duty”, both produce finite perturbative results and defuse the assumptions of Haag’s theorem. The renormalization and regularization ‘repairs’ also evade Haag’s theorem as a positive side effect. No additional repair is needed.

Maintenance. Maintenance is straightforward: maintain current practices of proper use of renormalization and regularization techniques. For the conceptually and logically curious, following in Duncan and Miller’s footsteps, this maintenance will likely also involve identifying precisely which assumptions have been violated, and where. While it remains to be shown that the perturbative power-series converge and thus correspond to exact non-perturbative objects post-regularization and -renormalization, [Miller, 2018, page 815] contends that this problem is both unrelated to Haag’s theorem and more serious (compare [Miller, 2021, 1135]).

4.4 Kłaczynski

Extra-Haagian outlooks. Like the authors above, Kłaczynski recognizes a bifurcation in QFT research. On the one hand there is canonical QFT. Canonical QFT has been spec-

²⁶Miller points to the KLN theorem of [Lee and Nauenberg, 1964, Kinoshita, 1962]. The KLN theorem ensures that IR divergences, which appear in individual terms of the perturbative series, cancel out when summing over all terms. The cancellation happens when considering the entire series of terms, not in each individual term. So, while each term might have an IR divergence, the full result, corresponding to a physically measurable quantity, does not.

²⁷Arguably, there is still significant explanatory work to do regarding those cases in which the perturbative series is not Borel-summable.

tacularly successful not only for making precise predictions but, as Kłaczynski emphasizes, “*predict[ing] the existence of hitherto unknown particles* [Kłaczynski, 2016, 2]. Nevertheless, canonical QFT is mathematically ill-defined: “canonical QFT presents itself as a stupendous and intricate jigsaw puzzle. While some massive chunks are for themselves coherent, we shall see that some connecting pieces are still only tenuously locked, though simply taken for granted by many practising physicists, both of phenomenological and of theoretical creed” [Kłaczynski, 2016, 2]. One major contributor to this ill-definedness is owed to the use of renormalization.

On the other hand is constructive QFT, which use operator theory and stochastic analysis to attempt to construct models of quantum fields in a mathematically well-defined manner. A number of important results have been obtained by this research program, including many triviality results that can be seen as calling into question basic features expected of any rigorous QFT. Nevertheless, the construction of a renormalizable theory in 4 dimensions—i.e., realistic—has neither been achieved nor seems achievable using current methods [Kłaczynski, 2016, 3]. That is, no rigorous and realistic model exists. Kłaczynski’s aim is to reconcile canonical and constructive QFT by elucidating the coherence brought about by renormalization.

Assessment. Given the above, Kłaczynski approaches Haag’s theorem intent on understanding what it says about canonical QFT. At first blush, the result appears to be negative. For instance, the Gell-Mann-Low formula, relating the ground states of the interacting and non-interacting fields, is built on the assumption that the time-evolution operator in the interaction picture, which relates the two fields, is unitary; this is exactly what Haag’s theorem rules out (recall note 3 on the significance and wide-spread use of the Gell-Mann-Low formula.). However, on closer inspection, the contradiction is resolved, if nevertheless unfavorably: like J. Fraser and Duncan, Kłaczynski blames Haag’s theorem for the divergence of the perturbative expansion of the Gell-Mann-Low formula. This leads Kłaczynski to conclude that the interaction picture is ill-defined and trivial [Kłaczynski, 2016, 59].

While he points to similar symptoms as Duncan and Miller, Kłaczynski comes to a different conclusion. In his final assessment, the interaction picture itself, relying as it does on a unitary intertwiner, is flawed—even renormalization does not save the interaction picture. While standard regularization methods may break Poincaré invariance, this is physically unacceptable; moreover, Poincaré invariance broken by regularization is restored when we take the adiabatic limit. This means that Haag’s theorem again applies, albeit now to the renormalized theory [Kłaczynski, 2016, 62–3].

Repair. Like Duncan and Miller, Kłaczynski thinks that renormalization procedure still repairs the problem, but gives a different reason for why it works. When we renormalize our theories, we replace the bare quantities with their renormalised counterparts. This process of renormalization does not merely cancel the divergences, but also fundamentally changes the theory, by bringing about a coupling-dependent mass shift. As a result, the renormalized free and interacting fields now have different masses. Two quantum fields of different mass

are overwhelmingly likely to be unitarily inequivalent at any given time.²⁸, and so the conditions for Haag’s theorem do not apply in our renormalized theories.

Nevertheless, the un-renormalized and renormalized theories *are* still related—just not by a unitary transformation, as physicists still believe. Thus, we are working with something *like* the interaction picture insofar as the two theories are still related by an intertwiner of sorts, but *unlike* the interaction picture insofar as this intertwiner is manifestly non-unitary. So whilst renormalization is the correct treatment, physicists have not fully grasped how it allows us to evade Haag’s theorem. By using renormalization to fix the divergences, physicists have “muddled our way through to successfully applying perturbation theory” [Klaczynski, personal communication, Sept. 8, 2021].

Renovation. Given that the interaction-picture presentation of canonical QFT still dominates, renovation is necessary. This has two parts. First, physicists have misunderstood, or at least mis-described, the tools they are using when performing scattering calculations with renormalized fields. While Haag’s theorem says this tool cannot be the interaction picture, that does not mean renormalization is unconstrained or incoherent. Rather, renormalization “follows rules which have a neat underlying algebraic structure [the Hopf algebra] and are not those of a random whack-a-mole game” [Klaczynski, 2016, page 4].

Second, this program of identifying the structure of renormalization must continue. First and foremost, there are some mathematical lacunae in this process, as well as procedures that are not defined wholly rigorously; these should be addressed. Klaczynski believes that renormalized quantum field theory “provides us with peepholes through which we are allowed to glimpse at least some parts of that ‘true’ theory” [Klaczynski, 2016, page 4]. As such, the ultimate goal here would be to glimpse what we can of the ‘true’ theory through the peepholes this work affords.²⁹

4.5 Maiezza and Vasquez

Extra-Haagian outlooks. Like Klaczynski, Maiezza and Vasquez are interested in determining the precise mathematical structure of canonical QFT. However, they seek a “consistent and generic (non-perturbative) formulation of QFT” [Maiezza and Vasquez, 2020, 2], in contrast to the perturbative formulation of canonical QFT. Given the centrality of the interaction picture to canonical QFT, the natural starting point for finding such a consis-

²⁸ The mathematical ill-definedness of the renormalized terms makes it hard to prove that renormalized free and interacting fields will be unitarily inequivalent in general. However, Klaczynski uses a theorem X.46 of [Reed and Simon, 1975] to argue that it is “plausible beyond doubt” [Klaczynski, 2016, page 68]. Theorem X.46 says that if there are two free, scalar, neutral fields with a unitary intertwiner at some time t_0 , with *different* masses, then these fields must be unitarily inequivalent. Klaczynski uses the theorem to argue we should not expect a unitary intertwiner at any time between the renormalized interacting and free fields. Recall that Haag’s theorem requires precisely such a unitary equivalence between the free and interacting fields at some time (10).

²⁹This view can be fruitfully compared to that of [Fraser, 2017]. Both agree that the perturbative renormalization offers a consistent calculational procedure that can both generate empirical predictions and offers some insights about underlying “true” physics. However, J. Fraser argues for a selectively realist attitude towards some parts of the effective theory. Klaczynski seems to argue only that this theory might provide some insights about a “true” physical theory, one which gives a true representation of the physical system.

tent and generic formulation of QFT would be to use the interaction picture, starting with free fields. But, Maiezza and Vasquez ask, is it possible to build such a non-perturbative formulation from the interaction picture and perturbative renormalization? Or, rather, is something new needed?

Assessment. In Maiezza and Vasquez’s assessment, Haag’s theorem reveals a central problem when we try to improve upon the standard methods of perturbatively renormalized canonical QFT. We see this problem clearly when we consider the entire perturbative expansion series. The problem stems from vacuum polarization: because of vacuum polarization, the full power-series expansion diverges³⁰. Maiezza and Vasquez prove that renormalon singularities arise in the total perturbative series, and trace these back to Haag’s theorem. These renormalon singularities “are the concrete manifestation” of the impossibility of generating an unambiguous finite result in such cases [Maiezza and Vasquez, 2020, page 10].
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Attempts to generate a finite result for the whole series through analytic continuation methods (such as using a Borel-Laplace transformation) necessarily rely on a choice of arbitrary constants. Maiezza and Vasquez describe these dependencies on an arbitrary choice as “renormalization ambiguities.” Thus, because of these ambiguities, ultimately arising from Haag’s theorem, the interaction picture with perturbative renormalization cannot lead to their desired non-perturbative QFT.

Repair. According to Maiezza and Vasquez, there is no repair we can make at this time to address this problem. There remains a concerning flaw in perturbative renormalization, namely, the renormalized series’ dependence on an arbitrary choice of constant. As Maiezza and Vasquez put it, perturbative renormalization “cannot be a self-consistent cure, because perturbative renormalization needs to be completed, or in practice resummed” [Maiezza and Vasquez, 2020, page 10].

In particular, they argue that the repair suggested by Klaczynski—essentially, to replace the interaction picture’s unitary intertwiner with a non-unitary one—cannot work. The Dyson operator relating the free and interacting fields is unitary by construction, in such a way that simply undoing the time-ordered product present in its definition (as suggested by [Klaczynski, 2016]) does not therefore make it non-unitary [Maiezza and Vasquez, 2020, 5].
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Something more is needed to repair the interaction picture, in particular to make it suitable for calculations using non-perturbative QFT.

³⁰This was first noted in [Dyson, 1952] concerning QED.

³¹Renormalons are singularities that arise as a function of the complex transform parameter when a formally divergent series is summed using Borel summation, see [Beneke, 1999] for a review. Whilst Haag’s theorem is generally associated with infrared divergences of individual terms in the perturbative series (see [Earman and Fraser, 2006, p. 319], [Fraser, 2006, pp 63-66], [Sbisà, 2020, p. 34], [Van Hove, 1952]), Maiezza and Vasquez associate the *overall divergence* of the perturbative series back to Haag’s theorem. As such, Maiezza and Vasquez associate both IR (low momentum) and UV (high momentum) renormalon divergences with Haag’s theorem.

³²Maiezza and Vasquez point out that it can be written in a manifestly unitary form (see [Maiezza and Vasquez, 2020, p. 80]). More generally, as a quantum mechanical evolution operator, the Dyson operator must be unitary, otherwise it would not preserve the normalization of quantum states.

That said, Maiezza and Vasquez are quick to note that the practical uses made of the interaction picture with perturbative renormalization need no repair: they evade Haag’s theorem. Haag’s theorem is, after all, a non-perturbative result, in that it applies to the *entire* power-series expansion, not just the perturbative expansion at some finite order. Because the power-series expansion is asymptotic, truncating at finite orders can give meaningful approximations, while also missing out on the non-perturbative effects that lead to the complete series’ divergence. Thus, while regularization and renormalization do not ‘fix’ the problem posed by the divergences tracked by Haag’s theorem, this is essentially because Haag’s theorem does not apply in cases where the series is truncated. Perturbative renormalization fixes the divergences of individual terms within the series, but it does not address the non-perturbative divergences of Haag’s theorem.

Renovation. They suggest two forms of renovation. First, in the short run, research efforts should aim at improving our understanding of the non-perturbative singularities that arise, such as renormalons. Further research should be directed into mathematical techniques to better understand neglected non-perturbative effects (for example [Maiezza and Vasquez, 2021b, Maiezza and Vasquez, 2021a]). These include resurgence, a method for reading off certain non-perturbative effects from the perturbative expansions (for a review, see [Dunne and Ünsal, 2016]).

Second, further work is needed *within* canonical QFT to account for non-perturbative effects. Ultimately, new physical insights are needed to complete quantum field theory. These must move beyond simply reformulating the perturbative quantum field theory framework [Maiezza and Vasquez, personal communication, Sept. 8, 2021].

4.6 Kastner

Extra-Haagian outlooks. [Kastner, 2015]’s outlook differs from the previous authors. Insofar as she seeks the correct formulation of our theory of quantum phenomena, she sits with the physicists above. However, insofar as she thinks this correct formulation requires a fundamental reinterpretation of quantum phenomena, she sits with the theory interpreters, who are overwhelmingly philosophers. Kastner advocates replacing QFT with a direct action theory (DAT). As such, she has *already* given up on mediating interactions locally in favor of nonlocal, direct interactions between field sources. This is a profound departure from QFT.

Assessment. Kastner locates the problem ‘further back’ conceptually. Rather, Kastner begins with the commitment that QFT is wrong from the outset. As such, her assessment of QFT in the light of Haag’s theorem is grim: “QFT is not the correct model; a different, yet empirically equivalent, model is needed” (57). The problem lies with the foundational assumption of QFT—namely, that we use fields as mediators between particles (crudely, as “a ‘bucket brigade’ that is invoked in order to restrict causal influence to a local, continuous conveyance from spacetime point to spacetime point” (59)). Thus, Haag’s theorem “simply tells us what we already know: the interaction picture of quantized fields does not really exist” [Kastner, 2015, 58]. Haag’s theorem is therefore an additional motivation for abandoning QFT in favor of DATs.

Repair. The treatment for such a deep problem is a major change in the modeling procedure for this area of physics. If we accept that Haag’s theorem shows us that it simply does not work to model interactions by field operators creating and destroying Fock space states, then, Kastner urges, we are to instead revive direct action theories (DAT), in which we model interactions through a direct, nonlocal interaction between sources of the field [Kastner, 2015, 57].

Rather than modeling the evolution from free to interacting states (as in the interaction picture of QFT), on the direct action approach, we do away with fields as mediators of interactions. Instead, we posit a direct, nonlocal between particles or sources of the field, without needing any intermediary field, so that the issue of constructing a unitary operator to map between free and interacting theories does not arise. More fundamentally, in a DAT, quantum mechanics is only incorporated at the level of the sources and receivers of a field, and is not enforced through the imposition of canonical commutation relations on the fields (see [Davies, 1970]). As such, there is no requirement of local commutativity between the field operators at spacelike separations, unlike in ordinary QFT (recall axiom 3).

Kastner believes that this is a viable repair because she is convinced that DATs—which differ radically from QFTs—are empirically equivalent to QFTs. This places significant weight on (purported) demonstrations of the equivalence of direct-action theories and QFT (e.g., [Narlikar, 1968]). If the demonstrations are sound, *and* they extend such that all currently-used QFTs are approximations to some DAT, then they would go some way towards explaining how calculations in renormalized quantum field theories are able to generate successful predictions: QFT’s “mathematical inconsistencies can be rendered inconsequential since they can be understood as arising from its ‘makeshift,’ nonfundamental character” [63].³³

Kastner further invokes the abandonment of the idea of the ether due to the Michelson-Morley experiment to justify abandoning QFT, saying that abandoning interactions through local quantum fields in response to Haag’s theorem would have an analogous “interpretive elegance” (58). She likens Haag’s theorem to the EPR experiment insofar as it presents a serious challenge to the assumption that the laws of nature are entirely local.³⁴ We repair all these problems, including those raised by Haag’s theorem, by abandoning locality in favor of DATs.

Renovation. The prescribed renovation, on Kastner’s view, is to develop a research program for DATs. This research program includes the rejection of alternative responses to Haag’s theorem such as Haag-Ruelle scattering and constructive QFT; Kastner views these workarounds as “*ad hoc*, approximate, or partial measures” (63). Future work in the research program would need to assess the question of the calculational, pragmatic viability of DATs as well as the crucial need to check thoroughly for empirical equivalence with all of the accepted theoretical results of ordinary QFT. Kastner cites work from Narlikar and

³³However, this is a non-trivial ‘if’, especially given the recent results suggesting that ‘empirical equivalence’ is a less straightforward concept than philosophers often presume. See [Weatherall, 2019] and references therein for more detail.

³⁴Recall that Haag’s theorem depends on an axiom of local commutativity (axiom 3). Kastner’s approach can evade Haag precisely because of this non-local character, avoiding such a local commutativity requirement.

Davies developing a DAT empirically equivalent to QED; more work is needed to determine if other sectors of the Standard Model of particle physics can be recovered in the DAT approach. Further renovation efforts should also explore the possibility that a DAT approach would lead to novel, empirically testable predictions, as much of theoretical particle physics is currently disappointed with the lack of direction from particle accelerator experiments for new lines of research.

Given the wide extent of Kastner’s proposed work—not least, replacing the entire Standard Model with DATs—it would perhaps be more appropriate to describe the proposal as an entirely new foundation than as a renovation of the one that exists.

4.7 Seidewitz

Extra-Haagian outlook. Seidewitz’s outlook is interesting in that it substantially coincides with several of the others already discussed, while at the same time departing from them. First, he seems to think of theory interpretation in a way similar to Earman and D. Fraser and the theory interpreters. Like the latter, he appears to take mathematical rigor as a precondition for providing a satisfactory interpretation [Seidewitz, 2017, 369]. However, Seidewitz sees axiomatic QFT built on the Wightman axioms as an *incorrect* marriage of special relativity with quantum theory. In the light of this, Seidewitz thinks that these axioms will need to face significant revision.³⁵ Seidewitz departs from the (pristine) theory interpreters insofar as a univocal characterization of particle-ness is not forthcoming. This is because particles now come in two varieties—virtual and real—and a new definition of ‘particle’ will have to account for the significant mathematical distinction between the two.³⁶

Second, Seidewitz’s outlook coincides with Kastner’s. On the one hand, processes can evolve in a space-like way. That is, physics is non-local. Specifically, virtual processes need not evolve in a local fashion. On the other hand, particles come in two varieties—virtual and real. Indeed, the two agree on how these varieties are constituted with respect to locality. However, Seidewitz disagrees with Kastner that the non-locality of physics is a reason to abandon field theories. Rather, he suggests an alternative formulation of quantum field theory.

Assessment. Thus, [Seidewitz, 2017]’s assessment of Haag’s theorem is that it is the symptom of a larger problem, namely the inconsistent treatment of time in traditional QFT. Seidewitz reads Haag’s theorem as a corollary of two previous results, where these previous results are the more significant. First, let $\hat{\psi}_1(t, \mathbf{x})$ and $\hat{\psi}_2(t, \mathbf{x})$ be field operators defined in Hilbert spaces \mathcal{H}_1 and \mathcal{H}_2 , respectively, and suppose there exists a unitary operator \hat{G} such that, at a specific time t , $\hat{\psi}_2(t, \mathbf{x}) = \hat{G}\hat{\psi}_1(t, \mathbf{x})\hat{G}^{-1}$. Then (Theorem 1) the equal-time vacuum expectation values of the fields at time t are the same. Second (Theorem 2), a given

³⁵See [Fraser, 2006, Section 5] for an extended discussion of the sense in which Fraser takes the axioms of QFT to be provisional.

³⁶In particular, we should expect (an analog of) the (global) number operator characterization to hold only for real particles, given that the space-like evolution possible for virtual particles prevents their characterization as Fock space states to be created or destroyed (where this is done by operators used to define number operators).

field’s two-point expectation values satisfy a certain equation (Eq. (3) in Seidewitz) if, and only if, that field is free. Letting $\hat{\psi}_1$ be a free field and $\hat{\psi}_2$ a field related to the former as in Theorem 1, Haag’s theorem merely observes that $\hat{\psi}_2$ also satisfies Theorem 2’s equation for spacetime points at time t (by Theorem 1) and, by the Lorentz-covariance of $\hat{\psi}_2$ and analytic continuation, extends $\hat{\psi}_2$ ’s satisfaction of the equation to any two positions.

As Seidewitz sees it, Haag’s theorem “essentially relies on a conflict between the presumption that the fields are Lorentz-covariant and the special identification of time in the assumptions of Theorem 1” [Seidewitz, 2017, 360]. This special identification is apparent in that the time-evolution is given by the (frame-dependent) Hamiltonian. This requires particles to evolve on time-like trajectories. Essentially, the (frame-dependent) Hamiltonian operator is playing double-duty as the generator of time translation, in addition to the generator of state evolution, with t as the evolution parameter for each. However, because of this, the translation group of t —the unitary operators $\hat{G}(t)$ —coincides with the group of Lorentz transformations, which guarantees that one can extend the coincidence $\hat{\psi}_2(t, \mathbf{x}) = \hat{G}\hat{\psi}_1(t, \mathbf{x})\hat{G}^{-1}$ at a specific time t to *all* times t .

Repair. The immediate repair is to break the identification of the Hamiltonian with the generator of time translation and the energy observable. Seidewitz proposes we do so by dropping (i) the requirement that there is a unique Poincaré-invariant vacuum state and (ii) the spectral condition, i.e., the requirement that states are *on shell*. Essentially, this involves dropping the assumption that states transform according to an *irreducible* representation of the Poincaré group. This frees us up to define vacuum states as *relative* to the choice of a Hamiltonian operator, where the Hamiltonian is no longer equated with the time-translation generator. Instead, a Hamiltonian is required only to be Hermitian, commute with all spacetime translations, and have a unique null eigenstate.³⁷

Then, one can consider the one-dimensional group generated by each Hamiltonian, where λ (instead of t) is the (now frame-independent!) evolution parameter. (Physically, λ can be thought of as the parameterization of a particle’s path in spacetime—now unburdened from the constraint that the path be timelike.) Haag’s theorem is evaded by limiting the equality of vacuum expectation values (Theorem 1) to equal values of λ , which is neither surprising—each Hamiltonian has a unique Poincaré invariant vacuum state—nor problematic—one can no longer use Lorentz transformations to extend the equality any farther. In particular, one cannot demonstrate that the fields coincide (Haag’s theorem). Thus, interacting fields need not coincide with free fields, and yet a version of the interaction picture is reinstated since interacting fields can be related to free fields by a unitary intertwiner [Seidewitz, 2017, 369].

Renovation. Like Kastner, the renovation Seidewitz proposes is far-reaching. Most important, it will involve expanding the reach of parameterized QFT.³⁸ As it stands, the

³⁷Note that this model is more general than standard AQFT. The Hamiltonian operators do not need to be positive definite or represent the energy observable. Likewise, each Hamiltonian will have a corresponding “vacuum state”, but these need not correspond to the conventional sense of the “ground state”.

³⁸In one sense, Seidewitz has formulated a more general framework than standard AQFT, one in which the vacuum state and Hamiltonian are both representation-dependent. In standard AQFT models, the energy operator is always the Hamiltonian and the evolution parameter is always time. Seidewitz’ framework allows for additional models in which these alignments are broken.

theory does not cover gauge theories or non-Abelian interactions, nor does it resolve all of the problems with standard QFT. As such, significant work is required before this approach is of practical use. Nevertheless, insofar as it is conceptually closer to constructive QFT, one might reasonably suspect that the renovations required will be less thorough-going than those proposed by Kastner.

4.8 Framework Results Summary

Our framework accomplishes two main objectives. First, it maps out an otherwise wild jungle of scholarship on Haag’s theorem. This organization is especially helpful as groundwork for making the interdisciplinary exchanges of ideas on Haag’s theorem more efficient: it is not always easy for physicists, philosophers, and mathematicians to communicate. This is in part due to appropriately different aims, as well as differences in how these separate communities have developed their own forms of discourse, vocabulary, and standards of rigour. But at some point, the disparate aims of these communities reconnect, and facilitating interdisciplinary communication becomes essential to progress.

Thus, second, our framework goes beyond the organization of viewpoints on Haag’s theorem by pulling each viewpoint’s underlying disciplinary and methodological values to the foreground (see again table 2). How one diagnoses and treats Haag’s theorem is profoundly influenced by what one expects of QFT, and of the interaction picture in particular, as well as one’s expectations of theoretical physics in general. One’s expectations for the mathematician’s and the philosopher’s appropriate relationship to theoretical physics also has an influence.

The viewpoints above are characterized by several different purposes. Duncan and Miller are both concerned with a specific explanatory task: how, in fact, do physical predictions evade Haag’s theorem? This takes for granted the acceptability of current predictions and appropriateness of QFT’s scale-relative form. While Klaczynski makes similar observations, he moreover desires a QFT form making the algebraic structure of interaction dynamics mathematically manifest. Similarly, Maiezza and Vasquez hope to precisely characterize the non-perturbative structure of QFT. Klaczynski and Maiezza and Vasquez therefore coincide with Duncan and Miller in their attention to canonical QFT, but diverge in setting their sights on an unambiguous and non-perturbative structure underlying it. In contrast, Kastner and Seidewitz divert their attention from canonical QFT altogether in search of a more satisfactory marriage of special relativity and quantum theory. This search beyond canonical QFT is shared by many theory interpreters, who seek a QFT form that straightforwardly exposes its ontology. Among these, the pristine interpreters’ search is allied with Earman and D. Fraser, insofar as they expect a straightforward exposure of QFT’s ontology will proceed from a theory whose consequences are logically-straightforward derivations from conceptually clear assumptions.

5 Where and how to make future progress

Our ultimate goal has been to provide guidance on QFT’s construction in light of Haag’s theorem. Section 3 suggested that this would not be easy, given the literature’s apparent

Name	Extra-Haagian Outlooks
Earman & D. Fraser	Which QM and SR assumptions characterize QFT? We desire well-defined representations of interactions, which requires carefully tracking our assumptions as is done in the formal variants of QFT. The process by which predictions are actually generated is irrelevant.
Pristine Interpreters	Should we interpret QFT interactions as the interaction of (free-field) particles? Only the formal variants of QFT are ripe for interpretation. The process by which predictions are actually generated is irrelevant.
Non-Pristine Interpreters	What is the particle content of particle physics? No restrictions are placed on representations, and the actual predictive process may be, and often is, relevant.
Duncan & Miller	Why are predictions in QFT reliable? It is to be shown that the actual process of generating predictions in canonical QFT, via renormalization and perturbation theory, is consistent. The details of the actual predictive process are not only relevant but central to this project.
Klaczynski	What is the coherent structure underlying renormalized canonical QFT? We desire well-defined representations of interactions that can deliver unambiguous non-perturbative predictions.
Maiezza & Vasquez	To what extent can we recover an unambiguous non-perturbative formulation of QFT from the perturbative formulation of canonical QFT? We desire a well-defined representation of QFT that can address non-perturbative singularities.
Kastner	How should QM be extended to the phenomena described by QFT? We desire a well-defined theory eschewing the CCRs whose predictions coincide with those of canonical QFT.
Seidewitz	How should SR be incorporated into QFT? We desire a well-defined theory that treats time analogously to the spatial coordinates.

Table 2: Extra-Haagian outlooks summary.

labyrinthine set of disagreements. Deploying our framework in 4 regimented and catalogued the various responses to Haag’s theorem in such a way that those actively working on the philosophy, mathematics, or physics of QFT can more readily appreciate where the real differences between these proposals lie. We now turn to giving our ‘general contractor’s’ viewpoint on the insight Haag’s theorem gives us into QFT, present and future. Section 5.1 uses the results of the framework application to disentangle three separate lines of debate on Haag’s theorem. In each of these debates, there is a clear guiding question to the discussion, such that debates are not entirely talking past each other. Nevertheless, we highlight for each debate the role that extra-Haagian outlooks continue to play in shaping the discourse. Our contribution is to sort out and elucidate the driving forces in the various debates around Haag’s theorem, without ourselves defending a position in any of those debates. However, in section 5.2, we take a clear stand on the general methodological lessons to be learned from this study of Haag’s theorem. We discuss prospects for future progress on the challenges for QFT raised by Haag’s theorem. We argue that stakeholders in physics, mathematics, and philosophy ought to give more specific characterizations of their proposed methods for making progress, together with more detailed arguments as to why those preferred methods are likely to success.

5.1 Three distinct lines of debate

Does the interaction picture exist? The most salient of disagreements concerned the status of the interaction picture. On the one hand, Earman and D. Fraser, Klaczynski, and Maiezza and Vasquez each suggest its existence is undermined by Haag’s theorem. On the other hand, Duncan and Miller both suggest it exists and its use is justified. The difference here is, of course, that Duncan and Miller speak of the interaction picture at intermediate stages in a perturbative calculation, where regularization ensures finitely-many degrees of freedom. In those contexts, the interaction picture is perfectly well defined. Our framework was not necessary for this observation, nor was an appeal to extra-Haagian outlooks.

Yet there is more to say here, as revealed by the framework. First, the two groups differ substantially on what QFT should look like and, therefore, what counts as a sure foundation. Duncan and Miller are focused first and foremost on the reliability of the predictions generated by “the algorithm of quantum field theory” [Miller, 2021, 1135]. While Miller clearly believes a non-perturbative structure is desirable [Miller, 2018, 818], he nevertheless argues that the perturbative nature of our data does not undermine a realist interpretation of the theory [Miller, 2021, 1135]. Thus, if the ultimate goal is to characterize a theory about which we can be realists, it would seem canonical QFT has a sufficiently sure foundation. Earman and D. Fraser, Klaczynski, and Maiezza and Vasquez do not seem similarly satisfied with canonical QFT’s present perturbative structure.

Second, members of the former group are not entirely alike in their accounts of the consequences of Haag’s theorem. At issue is how deeply the authors trace Haag’s theorem’s consequence for the existence of an intertwiner. For Earman and D. Fraser, the focus is on the IP as an intertwiner between the free field and the *unrenormalized* interacting field. For Klaczynski and Maiezza and Vasquez, it is on the IP as an intertwiner between the free field and the *renormalized* interacting field. The differing focuses lead to different proposed strategies for renovating QFT.

Is particle physics particle physics or not? Section 4.2 also revealed disagreement about the theorem’s consequences for a particle interpretation. While many theory interpreters, including Halvorson and Clifton, take it as evidence against a particle interpretation, Bain and Kastner, among others, do not. As we discussed above, the primary difference between (e.g.) Halvorson and Clifton and Bain is how wedded we are to the free-field conception of particles. Halvorson and Clifton are so wedded. Bain, instead, is not, for he aims to capture the sense in which particle physics *is* about particles—if this is not the free-field one, so much the worse for the free-field conception.

The reader might think Kastner fits in with Bain. This isn’t correct, however. Kastner not only recommends a different interpretation of particle physics from Halvorson and Clifton but, as her proposed renovations reveal, suggests QFT must be replaced altogether. That is, rather than derive an interpretation from a presupposed formalism, she instead derives a formalism from a presupposed interpretation. Among other things, this interpretation is committed to a particle conception of particle physics. This commitment ultimately derives from a belief that relativistic quantum mechanics should behave more non-locally than QFT does, as, she thinks, non-relativistic quantum mechanics counsels. Moreover, there is at least one respect in which Kastner fits more easily with Halvorson and Clifton: despite its (supposed) empirical equivalence to DATs, QFT is to be replaced. This suggests two disanalogies with Bain. First, the comparatively unmoored approximations of canonical QFT are not enough—we should demand exact calculations, or at least approximations to exact solutions. Second, a fully satisfactory theory should wear its metaphysics on its face more plainly than does canonical QFT.

What is relativistic QFT? Spelling out the renovations proposed also reveals commitments we might have missed. Focusing just on broad-strokes aims as a way to separate various responses to Haag’s theorem, Kastner and Seidewitz would seem far removed from the pristine interpreters—the former propose new theoretical forms for QFT, while the pristine interpreters restrict themselves to the formal variants. However, it is worth noting that these authors seem to share the same outlook on axiomatization. In particular, both seem at least weakly committed to claiming that QFT phenomena has a structure that can be captured in a closed-form as something like a finite set of axioms *and that we can actually specify these axioms*. Likewise, the pristine interpreters’ project is predicated on a belief that such a form exists and that we have found it. In contrast, Earman and D. Fraser appear committed only to the belief that axiomatization could be *helpful* for comprehending current QFT, or even for finding a more scrutable form. D. Fraser, at least, is quite clear that CQFT does not need to build a realistic model in order to have been useful—its utility lies rather in the fleshing-out of the relationship among the axioms. While Kastner and Seidewitz certainly could take such a view, the emphasis they place on the correctness of their chosen axioms suggests this is not their current conception.

This is useful, too, for understanding the varied responses to the question of which assumption of Haag’s theorem ‘should’ be dropped. Kastner, Seidewitz, and the pristine interpreters each have a clear answer to this question. Things get murkier as we consider the others, however. Not only do none of them explicitly affirm some set of precise assumptions, they are not entirely clear about which assumptions are problematic and how. This

comes in degrees, of course. Earman and D. Fraser are perhaps clearest, suggesting that the IP’s assumption of a global unitary intertwiner is problematic but that the local equivalence delivered by Haag-Ruelle appears to suffice. Klaczynski and Maiezza and Vasquez are a bit less clear. Insofar as they each hope for an explicit formalism, unlike canonical QFT, they appear sympathetic to axiomatic approaches. Yet Klaczynski only suggests that renormalization *somehow* replaces the unitarity of the intertwiner, and Maiezza and Vasquez are even less precise about what is problematic. In a sense, Duncan is even less clear, albeit for a principled reason. Obviously respectful of axiomatization’s utility, he nevertheless emphasizes again and again that the truly remarkable feature of QFT is the “scale separation” property of local field theory [Duncan, 2012, 570-1]. Building one’s theory around this fact, as the effective Lagrangian approach of canonical QFT has done, would seem to undermine specification of any single set of assumptions since, e.g., symmetry structure can vary. Duncan, it would seem, cannot be clearer about what is denied.

5.2 Lessons for making progress

The framework’s breakdown of responses into assessments, repairs, and renovations and consequent revealing of extra-Haagian outlooks has thus distinguished the foregoing three separate essential questions raised by Haag’s theorem. Are the machinations of the framework strictly necessary for pulling apart these distinctive debates? For the respondents we review, perhaps not: they are each quite explicit about their goals and, therefore, about how fundamental an evasion of Haag’s theorem is to their goal. But not all responses, especially those that engage only glancingly with Haag’s theorem, will be or can be so explicit. To be sure, alternative frameworks could be used for sorting out the various viewpoints on Haag’s theorem. We could, for instance, simply generate a list of different questions various researches ask, with Haag’s theorem playing some role in their answers. This strategy, however, is likely to miss out on the connections between what we have called assessments and the longer term work of maintenance and renovation. While we give no argument as to the superiority of our framework above all others, nevertheless, we hope that the reader has found our framework to be of positive epistemic utility in the breadth of issues it captures and in the dialectical organization it delivers.

We began with the question, how Haag-tied is QFT, really? That is, how big of a problem for QFT is the problem (or problems) identified by Haag’s theorem? Clearly, there is no straightforward answer to that question. How one thinks about whatever problems Haag’s theorem raises, how those problems ought to be solved, and what future major work is called for in this area, depends heavily on one’s extra-Haagian outlooks. The reason why the literature on Haag’s theorem initially presents as a dizzying conceptual labyrinth (recall section 3), prior to some framework or other regimenting the various responses to the theorem, is that the extra-Haagian outlooks are the driving forces in this dialectic. Thus, the discourse on Haag’s theorem isn’t so much about Haag’s theorem, as it is about what hopes and expectations each member of the debates has for QFT.

Bringing these extra-Haagian outlooks out of the background and into the foreground sheds light on deeper commitments about what goals are well-suited for making progress in physics, as well as convictions about which methods are most apt to succeed at achieving those goals. Interestingly, despite the many disagreements amongst authors discussed in

this paper, one central agreement about hoped-for progress stands at the end of the day. Specifically for QFT, all agree that finding the unambiguous, non-perturbative structure underlying canonical QFT would be enthusiastically welcomed progress in this area of physics. In other words, it would be progress if we could identify an exact, non-perturbative solution.

In the meantime, however, accepting that such an unambiguous, non-perturbative underlying structure has not yet been developed, how significant of an impediment to progress on smaller-scale research objectives is Haag’s theorem? That is, how much of an impediment is Haag’s theorem to “expanding the rooms” of QFT? By now it will come as no surprise that the answer varies, and that it varies according to the “expansions” respondents have in mind (see Figure 1 for a summary). Since it presents no impediment to building a tower of effective field theories or producing asymptotic calculations, Haag’s theorem is not a fundamental impediment to Duncan and Miller, nor to Wallace or Bain. In one sense, it is also not problematic for Kastner and Seidewitz—after all, QFT incorrectly represents quantum (resp. relativistic) principles anyways. Of course, Haag’s theorem *would* be an impediment if we were wedded to the scaffolding around which we have built the room, i.e., the assumptions of Haag’s theorem. For Earman and D. Fraser it no longer presents a significant barrier to representing interactions, since Haag-Ruelle and CQFT offer conceptually coherent alternative approaches to scattering theory. Indeed, “[a]xiomatic and constructive field theorists have digested the moral [of Haag’s theorem] and moved on to other problems” [Earman and Fraser, 2006, 334].

Haag’s theorem seems to be a more significant impediment for other proposed expansions. For Klaczynski and Maiezza and Vasquez, the desired expansion is a non-perturbative, unambiguous mathematical structure supporting canonical QFT. Circumventing Haag’s theorem is fundamental to this expansion, and appeal to CQFT and Haag-Ruelle is not an option since neither can support canonical QFT’s calculations. It also presents a substantial problem for pristine interpreters and those who desire a particle interpretation of interactions rooted in the free-field Fock space notion of particle. It is less of a problem for those open to weaker or non-fundamental, emergent conceptions of particle, like Bain, Wallace, or Feintzeig.

One is tempted to ask: how fundamental Haag’s theorem is *really*? As we construe it here, fundamentality is relative to an imagined expansion. But since which expansion one imagines depends on what one expects of a theory, Haag’s theorem’s fundamentality depends ultimately on one’s extra-Haagian outlook. Thus, to ask how fundamental Haag’s theorem is *really* is to ask which extra-Haagian outlooks is most reasonable. And as we saw in response to ‘What is relativistic QFT?’, agreement on a goal doesn’t necessarily settle all methodological questions. In particular, answers to the following can still differ:

1. **What role does (should) foundational work play in progress in physics?**
2. **How does (should) foundational work coordinate with non-foundational work?**
3. **And, what does (should) foundational work even look like?**

These are not new questions: the so-called Wallace-D. Fraser debate concerns exactly these kinds of questions [Wallace, 2006, Wallace, 2011, Fraser, 2011, Fraser, 2009b] (see also [Koberinski, 2016, Koberinski, 2022] for reception of the debate). Nevertheless, their implication here again

highlights the utility of more careful assessments of answers than have been provided. For example, what does the relevant history suggest we can reasonably expect from axiomatic-adjacent approaches, and how does that compare to what is, in fact, being expected of them? Some work has been done here. For example, [Koberinski, 2023, 14] has argued that the axiomatic-adjacent approach of framework generalizations is reasonable insofar as it “expand[s] the domain of indirect tests able to find evidence of new physics.” In another vein, [Mitsch, 2022] argues that no-go theorems can anchor mathematical research programs in serious interpretive issues. [Freeborn et al., 2023] argue, moreover, that no-go theorems can serve as helpful guides to building adequate models. Further work in this area is needed.

The importance of Haag’s theorem is, in fact, an excellent example of the importance of these foundational questions. Recall that many folks agree that finding an unambiguous structure underlying canonical QFT’s perturbative calculations would be significant progress. Insofar as finding this structure would necessitate identifying the analytic structure to which the perturbative calculations are approximations, the phenomena of Haag’s theorem are directly implicated. Yet even with Haag’s theorem being so ‘fundamental’ in this sense, it is unclear how central these phenomena should be in our efforts to find this unambiguous structure. Indeed, perhaps its identification will come by completely ignoring Haag’s theorem and other foundational issues in favor of performing more high-energy experiments. The questions above, then, suggest we should concern ourselves with what method(s) we can reasonably expect to get us closer to this structure.

“Expanding the rooms” of QFT may not, in the end, require answers to these questions. That is, we may identify a generally acceptable expansion of QFT without any further reflection or clarity on the role of foundational work. Indeed, such an expansion could even determine our path out of the Haagian labyrinth. But this does not mean answering these questions would be useless. At worst, a more robust understanding of the methodological tradeoffs could provide heuristic guidance to individual researchers. Better still, it could focus effort on methods more likely to bear fruit, and it could help foster the clarity and humility needed to change course when one’s preferred method is no longer making progress. At best, it could identify the single best method for a given goal. In any case, it is worth gathering more evidence on the past success of foundational views and their prospects for success today.

Different extra-Haagian outlooks shape different long term goals for the development and interpretation of QFT, which in turn lead to different types of primary disciplinary long term renovation work. And yet, if work on QFT in physics, mathematics, and philosophy continues to aspire to be ultimately about the *same* part of the edifice of science, then scholarly discourse on results such as Haag’s theorem needs to have clear and efficient cross-disciplinary communication. How might we all—mathematicians, physicists, and philosophers—best direct our efforts in a well-coordinated pursuit of our various inter-connected goals? As summarized in figure 2, we can readily (in schematic, simplified form) sort proposals for potential methodologies to employ in service of these goals according to primary disciplinary homes. It is not, however, enough to advocate for one’s preferred methodology in the absence of a substantive and detailed argument in favor of that methodology’s likelihood for success. We might fruitfully draw here upon Ruetsche’s advocacy for non-pristine theory interpretation: just as context matters for explanatorily adequate theory interpretation, context matters for the reliability of inter-disciplinary methods. We therefore urge all those with a stake in the

future of QFT to give serious thought to our three foundations-questions. These foundations-questions are policy generating questions, and good policy requires precisely-characterized methods with clear and persuasive evidence as to their sufficiency for solving the problem at hand. A high degree of specificity is necessary for making wise judgments about how to optimize the interdisciplinary community's distribution of resources amongst these methodologies. QFT is not an epistemic island: there is a rich history of physics—and indeed of other sciences!—on which we can draw for evidence concerning the specific conditions in which a given methodology is likely (or not) to succeed in pursuit of goals that implicate the foundations of a given theory.

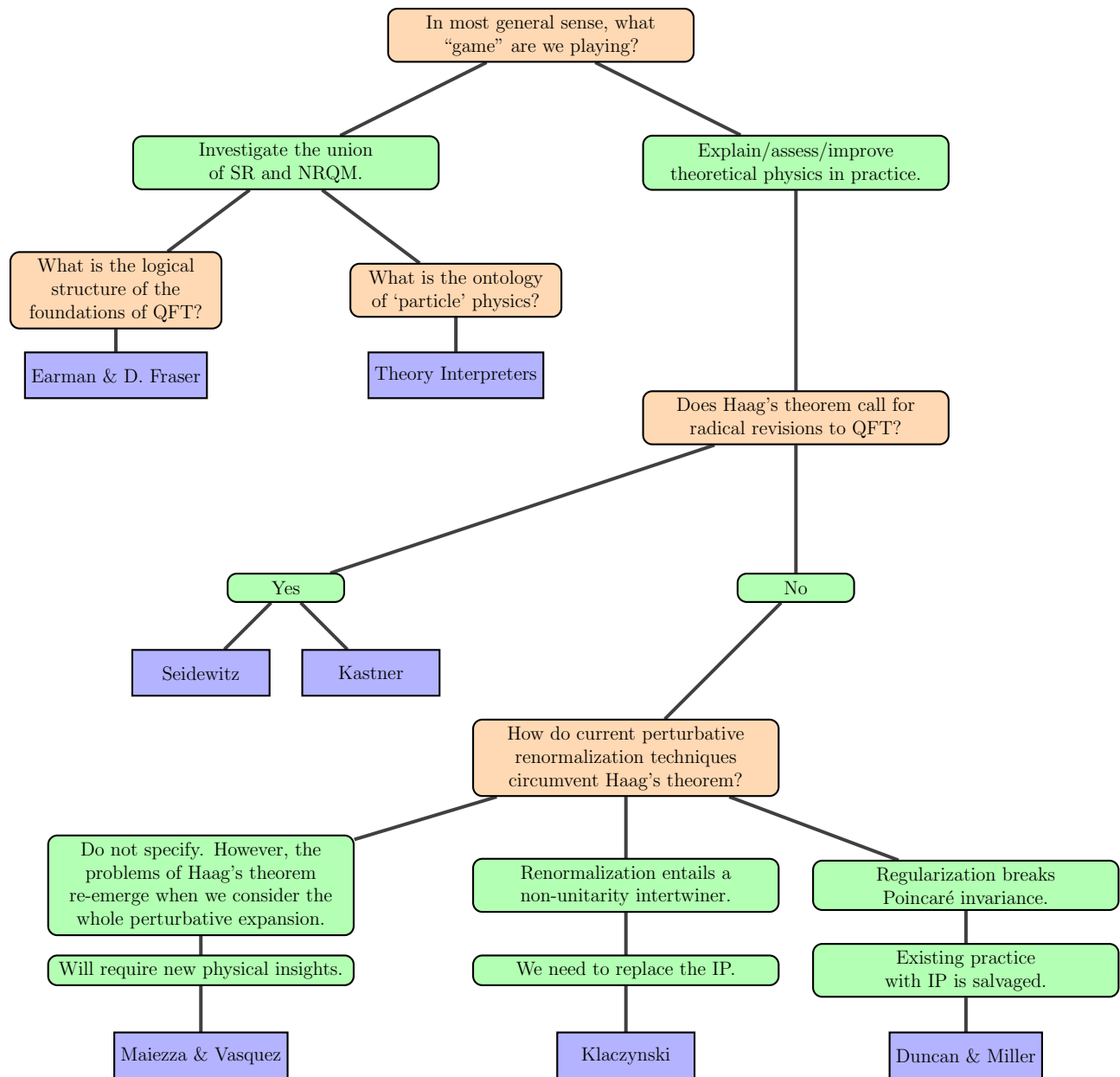


Figure 1: A mapping of the responses to Haag's theorem according to the driving motivations.

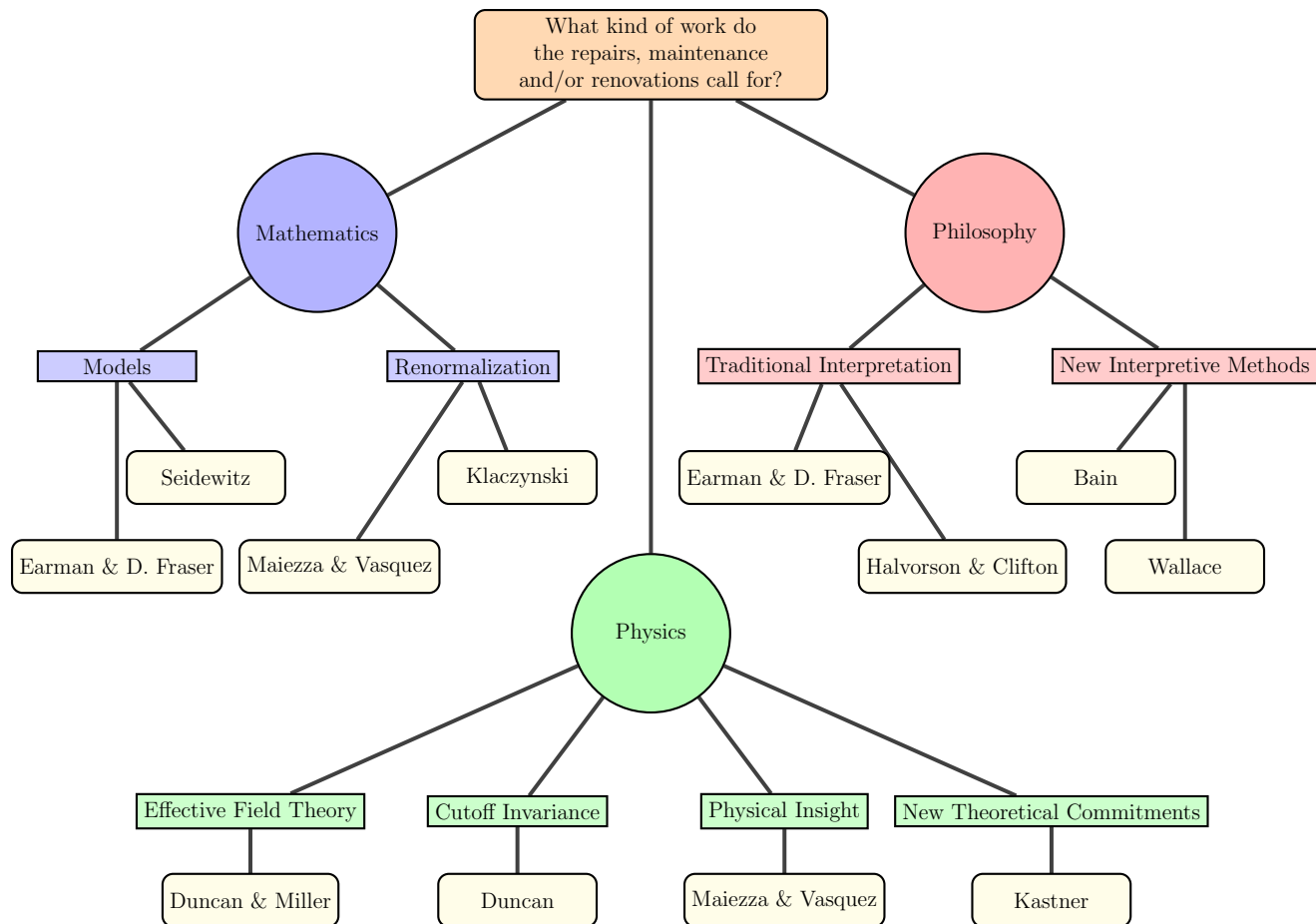


Figure 2: Disciplinary homes of proposed renovations projects.

6 Conclusion

Haag’s theorem cries out for explanation and critical assessment: it sounds the alarm that something is (perhaps) not right in how QFT has been built. Viewpoints as to the precise nature of the problem (assessment), the appropriate solution (repair), and subsequently called-for developments in areas of physics, mathematics, and philosophy (maintenance or renovation) differ widely. Moreover, the extant literature presenting these differing views constitutes a complex mix of arguments at cross-purposes, generating substantive confusion as to the precise issues to be addressed. In this paper, we have worked to address this confusion by cataloging and comparing a number of these viewpoints. We have developed and then deployed a framework for accomplishing that task. The application of our framework reveals each authors’ background disciplinary and methodological commitments and expectations of QFT—what we have termed extra-Haagian outlooks.

We have argued that these extra-Haagian outlooks are the primary driving forces in the various debates on Haag’s theorem, and that these furthermore shape the different viewpoints as to what are the most important future projects regarding QFT, as well as on the appropriate methods for accomplishing those projects. We have urged stakeholders in

the future of QFT to reconsider their own expectations for QFT, reflecting seriously on meta-level questions regarding the nature of foundational work in physics. What role does (should) foundational work play in the progress in physics? How does (should) foundational work coordinate with non-foundational work? What does (should) foundational work even look like? In calling attention to these questions, we have argued that stakeholders in physics, mathematics, and philosophy ought to give more specific characterizations of their proposed methods for making progress, together with more detailed arguments as to why those preferred methods are likely to succeed.

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