Is the Deutsch-Wallace Theorem Redundant?

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Abstract

I defend the Deutsch-Wallace (DW) theorem against a dilemma presented by Dawid and Thébault (2014), and endorsed in part by Read (2018), and Brown and Porath (2020), according to which the theorem is either redundant or in conflict with general frequency-to-chance inferences. I argue that neither horn of the dilemma is well-posed. On the one hand, the DW theorem is not in conflict with general frequency-to-chance inferences on the most natural way of stating the theorem. On the other hand, the DW theorem is crucial for establishing the Born rule as a prediction of Everettian quantum mechanics (EQM), and so cannot be redundant within the theory.

Contents

1 Introduction

> [We] regard the complaint by Dawid and Thébault, and largely endorsed by Read, that the DW-theorem is, for all practical purposes, redundant, a serious challenge to those who endorse the arguments of the authors of the theorem. (195)

In the Everett interpretation of quantum mechanics, the Deutsch-Wallace theorem arises as an attempt to recover the usual probabilistic content of orthodox quantum theory within EQM. Deutsch and Wallace approach this problem by way of decision theory—showing that, given a suitable choice of rationality axioms, agents who believe that EQM is true and that the state of the system is $|\psi\rangle$ are rationally compelled to distribute their credences in accordance with the Born rule. As Wallace (2010, 259–260) stresses, the theorem is at its core a

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symmetry argument, albeit made rigorous through the decision-theoretic framework.[1]

Whilst much discussion has focussed on the tenability of the DW rationality axioms and the internal conceptual coherence of the Everettian decision-theoretic programme,[2] Dawid and Thébault (2014) have recently advanced another line of criticism of the DW theorem. These authors present a dilemma for the DW theorem, arguing that it is either redundant or in conflict with general frequency-to-chance inferences.[3] Their claims have found some qualified support, but not much opposition.

Here, I aim to respond to this argument. First, I review some essential background from the philosophy of probability, as well as the DW theorem, and Greaves and Myrvold's (2010) approach to Everettian statistical inference. I then, in §3, reconstruct Dawid and Thébault's criticisms of the DW approach, before addressing each horn of their dilemma in turn. This requires a detailed discussion of the issue of theory confirmation, which I undertake in §4. §5 concludes.

2 Two Decision-Theoretic Approaches

2.1 Background

Discussions of probability in the Everett interpretation are intimately tied to more general questions as to the nature of objective probability, and its relation to subjective probability. In brief:

Credences (subjective probabilities) quantify an agent's degrees of belief. As a first pass, this can be operationalised in terms of betting behaviour—to say that an agent has credence p in the proposition that some event E obtains is to say that the agent should be willing to pay a sum px in exchange for receiving a sum x in the event that E obtains. Credences, thus defined, can be argued to satisfy the probability calculus on the basis that an agent will otherwise be susceptible to a Dutch Book. As Wallace (2012, 134) notes, this definition extends naturally to the Everettian case—to say that an agent has credence p in the proposition that some event E obtains is to say that the agent should be willing to pay a sum px in exchange for her successors receiving a sum x in all branches where E obtains. Note, though, that on the DW approach to EQM, this is offered as a first approximation to a more subtle Savage-style decision-theoretic operationalisation of credence; see §2.2.

Chances (objective probabilities) meanwhile, are supposed to express agent-independent facts about the world—the half-life of some radioactive isotope, for example. Whilst the notion of chance is altogether more elusive, a minimal characterisation can be given in terms of the role it plays in our rational and inferential practices. One is the relation between chance and credence—Lewis's (1986) Principal Principle (PP):

PP: Let S be the statement that the chance of event E at time t is p, and let K be any admissible background knowledge (roughly, which excludes information regarding whether E happened). Then a rational agent's credence $Cr(E|S, K) = p$.

1. See Saunders (2022, 241) for a similar emphasis. Wallace (2012, 146–151) argues that such a symmetry-led derivation of the Born rule is *only* possible within EQM, though see Steeger (2022) for a dissenting view.

2. See e.g. Albert (2010), Kent (2010), and Price (2010); more recent examples include Dawid and Thébault (2015), Jansson (2016), and Mandolesi (2019).

3. I should note that Dawid and Thébault do not explicitly present their argument as a dilemma; my terminology is based on the fact that the central points of (Dawid and Thébault 2014) can be reconstructed after this fashion; see §3 for more on this.

Additionally, in Papineau's (1996) terminology, we have two operational links:

The inferential link: We use frequencies to estimate objective probabilities. If we observe a frequency of F for some type of result R in a finite sequence of trials of type T, then this is evidence that the objective probability of R in T is close to F.

The decision-theoretic link: We base rational choices on our knowledge of objective probabilities. In any chancy situation, a rational agent will consider the difference that alternative actions would make to the objective probabilities of desired results, and then opt for that action which maximises objective expected utility.

Both these links are captured by the PP. Most obviously, the decision theoretic link is just the PP applied to decision theory. And to recover the inferential link, consider an agent who updates their credence in various chance hypotheses via Bayesian conditionalization. If H_p is the statement that the chance of observing result R on each trial of type T is p, and $O_{M/N}$ the statement that R is observed in M out of N trials of type T, then the PP yields

$$Cr(H_p|O_{M/N}) = \frac{{}^{N}C_{M}p^{M}(1-p)^{(N-M)}Cr(H_p)}{Cr(O_{M/N})},$$

which, for large N, becomes strongly peaked about $p = M/N$.[4]

Anyone wishing to recover the usual probabilistic content of orthodox QM within the Everett interpretation therefore appears to face a twofold problem (Greaves 2007b). First, there is the thought that talk of non-trivial probabilities is simply incoherent in an Everettian universe. After all, for a knowledgeable Everettian agent, the result of a QM experiment is just to produce a decoherence-induced branching structure in the universal wavefunction, each branch associated with a distinct macroscopic state of affairs. And since this process is entirely deterministic, it is difficult to make sense of how there could be even an interesting *question* about the outcomes of experiments, let alone uncertainty as to which outcome occurs (answer: all of them do). Secondly, insofar as one can make sense of non-trivial probabilities in EQM, one might ask how it is that these should agree with the predictions of orthodox QM.

2.2 The DW theorem

Deutsch and Wallace seek to address these two problems by showing that, given a particular set of rationality axioms, one can prove a Savage-style representation theorem to the effect that rational agents who believe EQM to be true and that the QM state of the system is $|\psi\rangle$ will behave (in defining their preference ordering among acts) as if maximising expected utility, for some utility function, using the Born rule. It follows that, if credences are understood operationally (as whatever it is that appears in a Savage-style representation theorem expressing the betting preferences of rational agents), and the PP is adopted as a functional definition of chance, then the chances, in EQM, are given by the Born rule.[5] Moreover, the DW theorem appears to derive this result without modifying or appending anything to unitary QM—a point which is crucial if,

4. On assumption that the priors $Cr(H_p)$ are suitably non-dogmatic, that is.

5. For those who doubt the existence of chances, the import of the DW theorem will be slightly different; rather than providing a basis for reifying the branch weights as chances, the theorem will simply be taken to show that, within EQM, it is possible both to make sense of probabilistic claims and to derive the Born rule, providing the probabilities therein are understood as (rationally constrained) credences. See e.g. Deutsch (1999, 3136); Brown and Porath (2020) provide an extensive defence of this view.

as Wallace (2012, 36) urges, we are to understand EQM as a "straightforwardly realist" interpretation of the "bare quantum formalism."

2.3 The GM approach

There is, however, a second question relating to Everettian probability which remains open at this point—namely, how it is that EQM comes to be confirmed or disconfirmed on the basis of statistical evidence. For in the absence of some account of how statistical inference is supposed to work in a branching universe, the Everettian is faced with the (obviously unacceptable) suggestion that, since everything which can happen does, according to EQM, the theory must simply be confirmed come what may.

It is this problem which is the concern of Greaves (2007a) and Greaves and Myrvold (2010), who seek to develop a confirmation theory which applies, without prejudice, to both branching and non-branching theories. First, one defines a 'quasi-credence' function, which quantifies an agent's concerns subject to both the non-branching and branching versions of the PP: conditional on the proposition that the chance of E is p, it is to be set equal to p, conditional on the proposition that E occurs on branches with weight p, it is to be set equal to p. Greaves and Myrvold then show that the result of conditionalizing on the observed outcomes of experiments in an exchangeable sequence[6] is that an agent's quasi-credences in the chances or branch weights for E become increasingly peaked about the observed relative frequency. Call this approach to statistical inference GM.

Now suppose that agents assign credences to theories in accordance with such a quasi-credence function. It follows that, if the Born rule is among the predictions of EQM, then an agent who observes statistics which conform to the Born rule will be led to increase their credence in the proposition that EQM is true—and to decrease it where the statistics depart from the Born rule. We will see later that whether or not the Born rule is among the predictions of EQM is precisely the question at issue in what follows.

3 A Dilemma?

With these results in hand, we can now turn to Dawid and Thébault's (2014) criticisms of the DW theorem. As outlined in §1, I will reconstruct their arguments as a dilemma, though this differs somewhat from Dawid and Thébault's original presentation.

The dilemma is as follows. The Everettian agent is either on a deviant branch (those exhibiting anomolous statistics which differ from the Born rule), or a non-deviant branch. Suppose that they are on a deviant branch. If that agent believes that EQM is true, then they are rationally required by the DW theorem to distribute their credences in accordance with the Born rule. But any agent on a deviant branch is rationally required *not* to align their credences with the Born rule, via general frequency-to-chance inferences of the sort encoded in Bayesian conditionalization and the PP. The DW theorem therefore appears to be in conflict with general frequency-to-chance inferences, and hence, Dawid and Thébault (2014) claim, is not an "empirically viable" approach to QM.

Now consider an agent on a non-deviant branch. In this case, general frequency-to-chance inferences alone are sufficient to establish the rationality of betting in accordance with the Born rule. This appears to make the DW theorem redundant, since the betting strategy implied by the theorem "is being

6. Exchangeable, that is, with respect to the agent's quasi-credence function.

enforced anyway by the principle of inductive inference" (Dawid and Thébault 2014, 58).

I will begin by considering the first horn of this dilemma. Should we accept this? Read (2018) has recently argued that we should not. According to Read, agents who observe statistics which deviate from the Born rule will come to disbelieve that EQM is true, so that the DW theorem ceases to apply and they are no longer rationally required to align their credences with the Born rule. Read claims that this makes the DW theorem compatible with general frequency-to-chance inferences, since on deviant branches, the observed statistics will 'trump' the symmetry arguments of the DW theorem.

The claim seems plausible, so far as it goes. However, as Read himself notes, this argument runs into trouble once we are more specific about the belief-updating mechanism and the analysis of belief under consideration. In particular, if credences are updated exclusively via Bayesian conditionalization, and belief in a proposition P is analysed as an agent having credence 1 that P, it is *not* the case that observations of deviant statistics will lead agents who initially believe that EQM is true to come to disbelieve EQM. As a substitute, Read suggests, we might analyse an agent believing that P as their having credence $Cr(P) \geq x$ for some $0 \leq x \leq 1$ (Read 2018, 139).[7]

However, even this cannot be quite right—at least, if Read's proposed 'replacement' for belief is intended to reconcile the DW theorem with a Bayesian realisation of the 'observed statistics trump symmetry arguments' principle. On Read's analysis, agents who have credence greater than x in the proposition that EQM is true are rationally compelled to distribute their credences in accordance with the Born rule. But any agent who has non-zero credence in some chance hypothesis other than the Born rule is rationally compelled by the PP *not* to align their credences with the Born rule. The result is that, by Read's own lights, an agent who updates exclusively via Bayesian conditionalization can comply with both the DW theorem and PP in only two cases. The first is where their prior credence that the chances are given by the Born rule is 0 or 1—in which case, Dawid and Thébault's concern that the DW theorem is not empirically viable recurs, since given standard rules for Bayesian updating, an agent who initially has credence 0 or 1 that P will always, respectively, have credence 0 or 1 that P. The second is where their prior credence in EQM is less than x, and they happen to live on a deviant branch. Read's approach therefore fails, in all but this latter case, to reconcile the DW theorem with a commitment to Bayesian updating and the 'observed statistics trump symmetry arguments' principle—and only then because this is precisely the case in which neither the DW theorem nor the 'observed statistics trump symmetry arguments' principle ever apply.

Now, to be fair to Read, he does not explicitly state that an agent's 'belief' that P, so characterised, requires Bayesian updating of credences that P—and he might be happy to deny this.[8] Note, though, that whether belief that P is defined as having credence 1 that P, or we adopt Read's weaker characterisation, the end result is that the belief-updating mechanism in Read's analysis cannot be exclusively Bayesian—and once this has been conceded, we seem forced to accept Dawid and Thébault's conclusion that "an agent deliberating within the framework of Greaves and Myrvold cannot simultaneously operate within the framework of [DW] since their fundamental decision principles will conflict" (Dawid and Thébault 2014, 59).

7. Presumably, though, one would want to impose further constraints—perhaps that $1/2 < x$.

8. Though cf. (Read 2018, footnote 4) "Such behaviour [whereby agents who observe deviant statistics come to disbelieve EQM] could be codified in GM." which suggests that Read *does* have something like Bayesian updating in mind.

As such, I would like to suggest an alternative solution. On Read's formulation, the DW theorem is a conditional: if an agent believes that EQM is true and that the QM state of the system is $|\psi\rangle$, then they are rationally compelled to align their credences in accordance with the Born rule. And it is true that Wallace (2012, 163–164) introduces the quantum decision problem in terms of an agent who "knows" the QM state of the system and that unitary QM is correct. But there is nothing in the DW rationality axioms, or in the proof of the DW theorem, which requires that the agent in question actually *believe* (let alone know) that EQM is correct or that the QM state of the system is $|\psi\rangle$—merely that they behave, in defining their preference ordering among acts, *as if* these are true. On this reading, the DW theorem should never have been understood as an indicative conditional at all, but rather as a statement about conditional probabilities.[9] That is, for any rational agent, their credence $Cr(E|EQM,\psi)$ in some event E, conditional on the proposition that EQM is true and that the QM state of the system is $|\psi\rangle$, should satisfy

$$Cr(E|EQM,\psi) = \frac{\langle\psi|\,\Pi_E\,|\psi\rangle}{\langle\psi|\psi\rangle} \tag{1}$$

where Π_E is the projector onto E.

If this is correct, then Dawid and Thébault's concern that the DW theorem is in conflict with general frequency-to-chance inferences never arises. The DW theorem only compels an agent to bet in accordance with the Born rule, regardless of their statistical evidence, if they have 0 prior credence in all theories other than EQM or rival approaches to quantum mechanics. And the fact that agents with particularly dogmatic priors are essentially impervious to empirical evidence in this way is a well-known feature of Bayesian confirmation theory, and *not* a problem with the Everett interpretation *per se*.

Note that as well as dissolving the first horn of the dilemma, my suggestion that the DW theorem should be understood (exclusively) as a statement about conditional probabilities also has the advantage that it avoids getting into thorny issues concerning the relationship between credence and belief. For example, it is clear that if belief is analysed as credence 1 on a credence-first view then the indicative conditional version and conditional probabilities version of the DW theorem are consistent (in fact, the latter entails the former). However, the two are *not* in general consistent if belief that P is analysed as credence $Cr(P) \geq x$ for some $x < 1$ on a credence-first view; it is also not clear that the two are consistent (or rather: consistent under certain norms) on all dualist views which posit an additional normative relationship between belief and credence.[10] Whilst there might be room for both versions of the DW theorem on some belief-first or dualist views, adhering to just the conditional probabilities version avoids these worries, and captures what is needed for the DW approach to EQM.

So the first horn of Dawid and Thébault's dilemma does not hold up to scrutiny. What of the second horn of the dilemma—that the DW theorem is redundant? *Prima facie*, the 'observed statistics trump symmetry arguments' principle only makes this objection worse. On deviant branches, the DW theorem will be inapplicable. But if the 'observed statistics trump symmetry arguments' principle is true on deviant branches, then it must also hold on non-

9. The reader may be concerned that this is too quick, since *prima facie*, it seems plausible that the DW theorem could be understood *both* as an indicative conditional and as a statement about conditional probabilities. For more on this, see the subsequent discussion. I should also note that Wallace (2012) presents the theorem both ways—in (ch. 5) as an indicative conditional, and in (ch. 6) as a statement about conditional probabilities.

10. For example, if the norm in question is that an agent should believe that P if(f) they have credence $Cr(P) \geq x$ for some $x \leq 1$ then an agent who adheres to this norm cannot in general comply with both the indicative conditional and conditional probabilities version of the DW theorem apart from in the special case where $x = 1$.

deviant branches where the observed statistics *alone* suffice to establish the rationality of betting in accordance with the Born rule. As Read (2018, 140) puts it:

> the central case in which DW could have any relevance is the [...] scenario in which EQM is believed, but the agent in question has no statistical evidence for or against the theory. DW might deliver a tighter link between subjective probabilities and branch weights, and therefore put the justification of PP, and the status of quantum mechanical branch weights as objective probabilities, on firmer footing. However, DW is not necessary to establish the rationality of betting in accordance with Born rule probabilities *tout court*.

This is then taken to support the conclusion that the DW theorem is redundant, at least for all practical purposes (FAPP). For as Brown and Porath (2020, 192) note, "such an epistemologically-limited agent [one who has no statistical evidence for or against EQM] would be very hard to find in practice."

Let us focus on making clear what is established by this argument. The claim seems to be that the DW theorem, in all realistic cases (those where the agents in question do have statistical evidence for or against EQM), acts merely as an idle backup, either because it fails to apply to the case at hand or because it serves merely to establish a conclusion which has already been established inductively from the empirical evidence. We can reconstruct the argument as follows:

1. Either the observed statistics on a branch conform to the Born rule or not.

2. If not, the DW theorem simply fails to be applicable FAPP (by the 'observed statistics trump symmetry arguments' principle).

3. If so, then by the same principle, the DW theorem merely establishes a conclusion which the empirical evidence has already established FAPP.

C. So either way, the DW theorem is redundant FAPP.

As noted above, however, both Read, and Brown and Porath, admit that this argument does not cover all bases—the reason being that the DW theorem establishes the rationality of betting in accordance with the Born rule for *any* agent who believes EQM to be true, and therefore also applies to agents who have no empirical evidence for or against EQM. Instead, these authors seem to be working on the assumption that, given the "rather artificial" nature of this scenario, it could be of no relevance for realistic agents in an Everettian universe. Moreover, 2 as it stands does not go through if, as I have urged, the DW theorem is understood as a statement about conditional probabilities rather than an indicative conditional (the conditional credence applies on deviant branches as much as anywhere else). So perhaps a better reconstruction of their argument would be:

1′. Either the observed statistics on a branch conform to the Born rule or not.

2′. If not, then the empirical evidence establishes that it is not rational to bet in accordance with the Born rule FAPP, on that branch (by the 'observed statistics trump symmetry arguments' principle).

3′. If so, then the empirical evidence alone suffices to establish the rationality of betting in accordance with the Born rule FAPP, on that branch.

4′. The DW theorem is of practical relevance on a given branch only insofar as it acts to establish the rationality of betting in accordance with the Born rule for realistic agents on that branch.

5′. By 4′, if the empirical evidence establishes that is not rational to bet in accordance with the Born rule FAPP on a given branch, then the DW theorem is redundant FAPP on that branch.

6′. By 4′ and the 'observed statistics trump symmetry arguments' principle, if the empirical evidence alone suffices to establish the rationality of betting in accordance with the Born rule FAPP on a given branch, then the DW theorem is redundant FAPP on that branch.

C. So either way, the DW theorem is redundant FAPP.

2′ and 3′ follow immediately on the assumption of Bayesian conditionalization and the PP.[11] And *prima facie*, 4′ also seems like an eminently reasonable premise. But it is precisely 4′ which I wish to challenge. 4′ tacitly assumes that the only function of the Deutsch-Wallace theorem is to establish the rationality of betting in accordance with the Born rule. But the Deutsch-Wallace theorem aims to do more than this—it aims to show that the rationality of betting in accordance with the Born rule can be derived as a *prediction* of EQM. However, given the FAPP qualification in 4′, this argument does not suffice to block 4′. For this, it must also be shown that the fact that the DW theorem derives the Born rule as a prediction of EQM, rather than on any other basis, is relevant to agents in an Everettian universe, for at least some practical purposes.

4 Theory Confirmation, with and without the Deutsch-Wallace Theorem

Precisely such an example is provided by the question of theory confirmation. Suppose first that we are considering the DW 'Everettian package' which includes operationalism about credences, functionalism about chances, and the DW theorem (hence, the Born rule as a prediction). In this case, agents on non-deviant branches will regard the hypothesis that EQM is true to be confirmed upon receipt of statistical evidence about the outcomes of QM experiments; on deviant branches, they will regard EQM as disconfirmed.[12]

It is perhaps worth emphasising that the DW 'Everettian package' already contains all that is needed for this prescription for theory confirmation to go through. Formally, this is encoded in Wallace's (2012, 222-223) 'Everettian epistemic theorem', which shows (given some additional axioms from classical decision theory needed for the non-quantum part of the decision problem) that a rational agent unsure of the truth of EQM will have their preference ordering represented by a credence and utility function, and that this credence function is (a) updated via Bayesian conditionalization and (b) satisfies equation (1). Given the aforementioned operationalism about credences, this is sufficient to derive that EQM will be confirmed (disconfirmed) by observations of Born rule (non-Born rule) statistics in an analogous way to the justification of the inferential link in §2.1; note also that for the case of EQM in particular, we do not actually need to invoke the PP here (although it is still needed for the functional story about how the Everettian branch weights thereby qualify as chances).

11. Again, given suitable assumptions about non-dogmatic priors or appeal to convergence theorems.

12. Or, more carefully, the conjunction of EQM and the proposition that the state of the system is $|\psi\rangle$.

Now, theory confirmation is unquestionably an issue of practical relevance for agents in an Everettian universe—even agents whose statistical evidence is sufficient to establish the rationality of betting in accordance with the Born rule. (To take an extreme example, consider a community of agents who only regarded de Broglie-Bohm theory, rather than any other version of quantum mechanics, to be confirmed by observations of Born rule statistics. Such agents, one imagines, would be substantially more likely to divert resources towards such projects as developing a relativistic de Broglie-Bohm theory, or the search for evidence of non-equilibrium matter in the early universe, than their counterparts who also regarded orthodox QM, EQM, collapse theories etc. to be confirmed by the same evidence.) If then, the DW theorem is redundant, we would expect to be able to reach the same conclusion without it, at least FAPP. But it is far from obvious how this is supposed to work. Taken without the DW theorem, there is nothing in the formalism of unitary QM to tell us what value the chances are supposed to take. And if EQM simply falls silent on what the chances are, then it will neither be confirmed nor disconfirmed by statistical evidence about the outcomes of QM experiments.

Nor is the GM approach to statistical inference of any help here. For consider the GM 'Everettian package' consisting of operationalism about credences and the GM rationality axioms, but *not* the DW theorem.[13] Whilst it is indeed a prediction of this theory that rational agents in an Everettian universe will be led to infer that the chances are close to the observed frequencies on their branch, this tells us nothing about what values the chances take—since it is (if anything) a prediction of EQM that every possible relative frequency will be observed on at least some branch. (Of course, there is nothing to prevent one then extracting probabilistic claims about the chances from GM, providing there already exists some privileged measure over branches. But that there is such a measure is precisely what we are trying to establish!)

One might be tempted to protest, at this point, that GM also establishes that agents will have their quasi-credence function become increasingly peaked about the hypothesis that the branch weights are close to the observed frequencies. And surely it is at least a prediction of EQM that the *branch weights* are given by the Born rule, even without the DW theorem?

To address this objection, it is helpful to distinguish what two uses of the term 'branch weights' found in the literature:

- On Wallace's (2012) usage, branch weights are fixed and given by the Born rule, but may or may not be suited to play the role of chances.
- On Greaves's (2007), Greaves and Myrvold's (2010) usage, branch weights quantify how much a rational agent should 'care' about each branch, as encoded in the branching version of the PP (recall §2.3), but may or may not be given by the Born rule.

But then it immediately becomes clear that the objection rests on equivocating between these two uses. The GM 'Everettian package' predicts that rational agents will have their quasi-credence function become increasingly peaked about the hypothesis that the GM-style 'branch weights' are close to the observed frequencies; it also predicts that the Wallace-style 'branch weights' are given by the Born rule. Nowhere does it predict the equality 'GM-style branch weights = Wallace-style branch weights'. And if one did want to argue for this equality, on the basis that it is only the Born rule measure that could fulfil the role in constraining rational credence articulated by the branching version of the PP, then one needs either the DW theorem or something equivalent to it.

13. That is, without some of the DW rationality and richness axioms needed to derive the theorem.

Of course, agents on non-deviant branches would presumably *notice* that the (Wallace-style, henceforth my usage will exclusively follow Wallace's) branch weights match the chances, and perhaps inductively infer that, *as a brute fact about the world*, the chances are given by the branch weights. But unless the Born rule is then appended to EQM as an additional, primitive posit, this does nothing to connect the truth of EQM to the statistics observed by rational agents in an Everettian universe. And taking the Born rule as primitive hardly seems consonant with the stated aims of the Everett interpretation—namely, *not* to modify or append anything to the unitary quantum formalism.

There is also a further issue regarding how this inductive inference is supposed to work. For suppose that agents on a non-deviant branch conduct a variety of QM experiments, and conditionalize their credences about various chance hypotheses on the results. Whilst they might then notice that the Everettian branch weights match the chances, *for the outcomes of those types of QM experiments*, this says nothing about the chances for other types of QM experiments which are yet to be performed (or indeed, never will be performed), whereas the Born rule constrains the chances for all possible QM experiments.[14] This is precisely why science proceeds by articulating unified theories, rather than simple enumerative induction alone—something which those who claim that it is possible to recover the full content of the Born rule from GM seem to have forgotten.

We can now see in a little more detail where the second horn of the dilemma goes wrong. To return again to the claim as expressed by Read:

> the central case in which DW could have any relevance is the [...] scenario in which EQM is believed, but the agent in question has no statistical evidence for or against the theory.

which, in Brown and Porath's words, we are supposed to be justified in neglecting because

> such an epistemologically-limited agent would be very hard to find in practice.

But *pace* these authors, it is not only in the case where an agent has no statistical evidence for or against EQM that the DW theorem is relevant. The DW theorem establishes that the Born rule is a prediction of EQM—and this fact is *always* relevant to agents in an Everettian universe, regardless of the statistical evidence they possess. It is only with the Born rule as a prediction that EQM can be confirmed or disconfirmed by statistical evidence in the same way as orthodox QM.[15]

Now recall the discussion in §3. There, it was suggested that the problem with the redundancy argument is that it assumes that the DW theorem is of practical relevance only insofar as it acts to establish the rationality of betting in accordance with the Born rule, for realistic agents on a given branch. The foregoing makes it clear why this cannot be the case. Whilst it may well be

14. It would be possible to remedy this by appealing to induction on different types of QM experiments: since the branch weights match the chances for all previous types of QM experiments performed, they match the chances for all possible QM experiments. I merely wish to point out that the story about how the Born rule is recovered empirically from GM alone is more complicated than the presentations in e.g. Read (2018) and Brown and Porath (2020) might suggest.

15. Note that the same point applies even if, like Deutsch or Brown and Porath, one is a subjectivist about Everettian probabilities. In this case, one relies on showing that the QM branch weights play the role of rationally constrained credences in order to recover the Born rule as a prediction of EQM. But nothing in the foregoing depends on whether the Born rule is taken as a claim about chances or about rational credences; the same analysis goes through if 'chance' is replaced with 'rational credence' throughout.

rational to bet in accordance with the Born rule, this is not the only question which agents in an Everettian universe must address—they must also establish whether this follows from the truth of EQM. That it does follow is precisely what the DW theorem purports to show.

5 Conclusion

I have argued that the DW theorem is neither redundant nor in conflict with general frequency-to-chance inferences. On deviant branches, the DW theorem does not require agents to bet in accordance with the Born rule regardless of their statistical evidence, as a simple matter of formulating the theorem correctly. And whilst it may, on non-deviant branches, be possible to establish the rationality of betting in accordance with the Born rule by purely empirical means, this cannot supplant the DW theorem, which aims to derive the rationality of betting in accordance with the Born rule as a prediction of EQM. This distinction becomes relevant once we turn to the question of theory confirmation—without the DW theorem, EQM is simply devoid of non-trivial probabilistic content, and cannot, therefore, be confirmed or disconfirmed by statistical evidence regarding the outcomes of quantum experiments.

Of course, all this relies on it being the case that the Everettian cannot, or ought not, simply adopt the Born rule as a further postulate in addition to the unitary QM formalism. In support of this, note that this does seem to be the view of many Everettians.[16] But this is not to deny that such a theory would be coherent, nor that one could not make a case for understanding it as at least a *version* of EQM. It merely asserts that there is an interesting distinction between an Everettian quantum theory which does not supplement the bare quantum formalism with extra structure, and one which does. If it is the former in which we are interested, then the above arguments stand. And that the DW theorem should be redundant in the latter theory is unsurprising; the latter theory never stood in need of the DW theorem at all.

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16. See Wallace (2012) and Saunders (2010).

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