**“To Measure by a Known Measure”: Kepler’s Geometrical Epistemology in the Harmonices Mundi Libri V**

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**Abstract**:

In this paper, I address the epistemological role geometry plays in Kepler’s Harmonices Mundi Libri V and argue that the framework he develops there is meant to address concerns regarding the confirmation of astronomical hypotheses, which are evidenced by comments in earlier works regarding empirical underdetermination. The geometrical epistemology that he constructs to combat these concerns in the Harmonices Mundi is introduced in Book I and then is extended to his theory of harmonic proportion in Book III, finally providing the foundation for his derivation of the planetary motions in Book V. To argue for these claims, I begin by discussing Kepler’s concern with underdetermination in earlier works. Then I turn to the Harmonices Mundi and argue that Kepler seeks to provide a geometrical system that directly links the theorist with the world. Finally, I show how he applies this system to his astronomy via his harmonic theory. This account not only helps us to understand Kepler’s scientific methodology better but also sheds light on Kepler’s enthusiasm for the results of the Harmonices Mundi by showing how they provide an example of the successful application of his geometrical epistemology.

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**“To Measure by a Known Measure”: Kepler’s Geometrical Epistemology in the *Harmonices Mundi Libri V***

**1. Introduction**

In his *Harmonices Mundi Libri V*, Kepler takes up the task of uncovering the harmonic foundations of the universe and detailing the ways in which these harmonic principles manifest themselves. This project culminates in Book V, the penultimate chapter of which is dedicated to deriving the motions of the planets from these harmonic foundations. It is clear from the numerous enthusiastic comments peppered throughout the work that Kepler’s was tremendously pleased with the results of his project and regarded it as an overall success. Commentators largely confirm that Kepler viewed the work favorably – Caspar famously calls it Kepler’s “mind’s favorite child” in his biography (Caspar 1993, 288). However, it is unclear whether Kepler’s favorable assessment is warranted. Many readers of the *Harmonices Mundi* have regarded it as underwhelming, and while most commentators no longer regard it as a work of pure fantastical speculation, it has met with varying levels of ambivalence. Even among the growing body of literature that aims to take Kepler’s more “mystical” work as a serious and incorporated element of his scientific project, the *Harmonices Mundi* remains somewhat of an interpretive puzzle. This is especially true of the portion of the work of which Kepler seems most proud: the derivation of the planetary motions from harmonic principles. For instance, one author notes that “from anyone else, the carefully crafted excuses and scales would be considered the edifice of a madman” (Gingerich 1992, 60), and another writes, in a more measured tone, that “Kepler’s ecstatic judgment appears unwarranted” (Martens 2000, 115).[[1]](#endnote-2) This is due not only to the complexity of Kepler’s demonstrations, but also to the fact that he must make compromises in his quest to match the harmonic proportions to the motions of the planets.[[2]](#endnote-3)

In this paper, I aim to provide an account that, among other things, can help to clarify why Kepler viewed the results of the *Harmonices Mundi* in such a positive light. To understand his attitude, we must recognize some of Kepler’s primary epistemological aims and how he goes about achieving them in the *Harmonices Mundi*. I will argue that Kepler is motivated by serious epistemic concerns, manifested most saliently by the problem of underdetermination and the empirical equivalence of competing astronomical models, which Kepler discusses most directly in his *Apologia pro Tychone contra Ursum*. I will show how he attempts to address this concern in the *Harmonices Mundi* with an epistemology grounded in geometry, which is outlined in Book I. This foundation for knowledge is extended to his theory of harmonic proportion in Book III, and finally utilized to demonstrate the knowability of the planetary motions in Book V. The account I offer here is meant to build off of and complement existing scholarly treatments of Kepler’s *Harmonices Mundi* and his *Apologia*, but will supplement these by emphasizing the epistemological role of geometrical archetypes in connection with the problem of underdetermination. This, in turn, will help us to comprehend the way in which Kepler viewed his project in the *Harmonices Mundi* as a success. I will begin by discussing Kepler’s concern with underdetermination and the status of scientific hypotheses in his earlier works. I will then turn to the *Harmonices Mundi* and argue that Kepler seeks to provide a geometrical system that directly links the theorist with the world by highlighting conditions of knowledge for both the knower and the object to be known. Then I will show how he applies this system to his astronomy in Book V via his harmonic theory in Book III. Understanding this will help to explain some of Kepler’s enthusiasm for his project by showing how the results of the *Harmonices Mundi* provide an example of the successful application of Kepler’s geometrical epistemology.

**2. Underdetermination, Empirical Equivalence, and the Truth of Astronomical Hypotheses**

Kepler’s awareness of, and concern with, the fact that conflicting astronomical hypotheses can prove empirically adequate has been addressed by numerous commentators, so my aim in this section will mainly be to provide the relevant background for Section 3. [[3]](#endnote-4) His concern with this issue appears as early in his career as the *Mysterium Cosmographicum.* In this work, he addresses the competing Copernican and Ptolemaic accounts of planetary motion, both of which seemed able to capture the observed motions of the planets(Kepler 1937-, vol i, 15/1981, 75). One response that was offered to this problem, and which Kepler explicitly argued against, was to treat the problem of the truth value of the hypotheses as irresolvable. According to this view, in the face of this irresolution we should take the hypotheses as fictitious constructions whose assessment should be based solely on their ability to “save the phenomena.”[[4]](#endnote-5) A version of this view is endorsed by Nichol Baer (Ursus) in his *Tractatus*:

A hypothesis or fictitious supposition is a portrayal contrived out of certain imaginary circles of an imaginary form of the world-system, designed to keep track of the celestial motions, and thought up… for the purpose of keeping track of and saving the motions of the heavenly bodies and forming a method for calculating them. I say a contrived portrayal of an imaginary form of the world-system (not a true and genuine one, for that we cannot know), not the system itself… These contrived hypotheses are nothing but certain fabrications which we imagine and use to portray the world-system. So it is not in the least necessary… that those hypotheses correspond altogether… to the world-system itself (I believe it to be scarcely possible to establish such hypotheses…), provided only that they agree with and correspond to a method of calculation of the celestial motions… (Jardine 1984, 41).

In this passage, Ursus repeatedly uses phrases like “fictitious,” “contrived,” and “imaginary” to characterize hypotheses. The content of these hypotheses is contrasted with a “true and genuine” world system, or “the system itself.” This indicates that Ursus believed that astronomical hypotheses are best understood as mathematical constructions that solely aim to describe and predict celestial motions. While they may correspond in some way to the system they describe, they need not, and we can’t know if they do. Thus, according to Ursus, we should consider these hypotheses as fictitious models whose assessment should be based on their ability to capture observed motions.[[5]](#endnote-6)

Kepler’s response in the *Apologia*, which echoes his response to the problem in the *Mysterium*, is detailed and complex. As others have provided a thorough discussion, I will not discuss the entirety of it here, but rather will focus on the elements that are important for my examination of the *Harmonices Mundi* below. First, Kepler writes of the aims of science: “As in every discipline, so in astronomy also… we hold whatever there is in our conclusions to have been established as true. Besides, for the truth to be legitimately inferred the premises of a syllogism, that is, the hypotheses, must be true. For only when both hypotheses are true in all respects and have been made to yield the conclusion by the rule of the syllogism shall we achieve our end – to reveal the truth to the reader” (Kepler 1858-1872, vol i, 239/Jardine 1984, 139). Here Kepler states that the goal of scientific hypotheses, and astronomical hypotheses specifically, is not merely to serve as useful fictions, but to provide true premises that generate true conclusions.[[6]](#endnote-7) In the context of the debate, Kepler is asserting that astronomical hypotheses should ideally provide faithful representations of the system whose behavior they describe and predict. The key phrase in the passage above is that conclusions about observed motions must be “legitimately inferred.” So, while we might find cases where it seems like we can infer the observed motions of the planets from contradictory hypotheses, not all of these will be legitimate inferences, because a legitimate inference must derive a true conclusion from true premises. For Kepler, this can only occur when the premises accurately represent the relevant objects as they really are.

This, of course, raises the question of how we will be able to tell when a legitimate inference has been made. As we saw above, there was a high level of skepticism regarding the ability of any theorist to confirm that their astronomical hypotheses corresponded to the system in this way. And, as Kepler recognizes, if the only criterion for adjudicating between incompatible hypotheses is to appeal to the observed motions, it seems like we are in trouble. This is because, while Kepler believes that eventually the inadequacy of false hypotheses will become clear through consideration of additional data, this data might not be available or the false hypothesis in question might continue to be lucky.[[7]](#endnote-8) Kepler’s solution in the *Apologia* is to widen our criteria to include other considerations besides empirical adequacy or, as Kepler puts it, in cases where disparate hypotheses seem to capture the same motions, “there is often a difference between the conclusions because of some physical consideration” (Kepler 1858-1872, vol i, 240/Jardine 1984, 141). The ‘physical considerations’ that Kepler speaks of include topics that were traditionally considered to be in the domain of Aristotelian physics, rather than those that are restricted to the geometrical sphere of astronomy.[[8]](#endnote-9) This would mean that details about the nature of the objects in the system and their causal relations would be relevant for distinguishing between hypotheses. However, as other commentators also note, it seems like Kepler also uses the term ‘physical considerations’ here to cover other criteria, like simplicity, unification, explanatory fruitfulness, and archetypal foundation.[[9]](#endnote-10)

These interpretations are generally supported when we look at the examples Kepler cites in the passage. For instance, Kepler discusses a comparison between Brahe’s and Copernicus’s respective models, where they generate the same predictions in the motions of the planets, but Brahe constructs his with the intention to “avoid postulating the immensity of the fixed stars” (Kepler 1858-1872, vol i, 240/Jardine 1984, 141). Thus, Kepler includes commitments regarding physical features postulated of the system that aren’t directly implicated in the demonstration of the motions, but which constrain aspects of the hypotheses at play in that demonstration. And a bit further on in the *Apologia*, he writes: “Even if the conclusions of two hypotheses coincide in the geometrical realm, each hypothesis will have its own peculiar corollary in the physical realm. But practitioners are not always in the habit of taking account of that diversity in physical matters, and they themselves very often confine their own thinking within the bounds of geometry or astronomy and tackle the question of equipollence of hypotheses within one particular science, ignoring the diverse outcomes which dissolve and destroy the vaunted equipollence when one takes account of related sciences” (Kepler 1858-1872, vol i, 240/Jardine 1984, 141-142). In this passage, Kepler notes that if we take astronomical hypotheses as attempts at modeling the real physical system, different hypotheses will have different ramifications across different domains. So, while two sets of hypotheses might appear equivalent when we merely consider their results in one domain, they can be compared and assessed according to their relationship to other disciplines. Kepler explicitly mentions considerations in physics here and above, but we might think that results in other domains, especially those traditionally related to astronomy like harmonics, are also important to consider.[[10]](#endnote-11) I will return to this point about the connection between astronomy and music theory in my argument below. Finally, it is clear that he is also committed to simplicity as an indicator of preferability.[[11]](#endnote-12)

In addition to this, Kepler thinks that certain formal and metaphysical considerations will be relevant for adjudicating between systems of hypotheses. Kepler is not in a position in the *Apologia* to directly cite his own arguments in favor of the Copernican system since he is defending Brahe, but he does allude to his arguments “on Copernicus’ behalf” (Kepler 1858-1872, vol i, 243/Jardine 1984, 146). These arguments are found in his *Mysterium* where he claims that the number and spacing of the planets is determined by a privileged collection of polyhedra: the Platonic solids. He explicitly takes this to be an argument in favor of the Copernican system because it shows that an explanation for the number and spacing of the planets is available for the Copernican model, but not the Ptolemaic.[[12]](#endnote-13) Thus, in Kepler’s reference to his polyhedral hypothesis, we also see that he takes explanatory power as an indicator of a hypothesis’ preferability. Not only can the Copernican model predict the correct motions, but it is only according to this model that other questions, like “Why are there six planets?” and “Why are the planets spaced in the way that the theory claims?” can be answered.[[13]](#endnote-14)

Kepler’s reference to this “polyhedral hypothesis” from the *Mysterium* is an instance of a criterion that will be especially important in the discussion of the *Harmonices Mundi* below: support from geometrical archetypes. Throughout his career, Kepler held the view that the world was designed by God according to certain formal or structural principles. He believed that these principles, or *archetypes*, were generally related to quantity and, at least for the bulk of his career, were specifically geometrical.[[14]](#endnote-15) The archetypes are significant in several ways for Kepler, but the two of relevance here are their broad metaphysical and epistemological function. On the one hand, the archetypes function metaphysically as a principle of design for nature. They are coeternal with God and, as the blueprint for God’s creation, are present throughout the universe. On the other hand, they serve an important epistemic function due to the connection between the human mind and God’s mind. Kepler writes in numerous places of how the human mind is a reflection of God’s mind. For instance, he writes in Part IV of the *Harmonices Mundi*: “…This straight line is of course an element of a corporeal form. If this is spread out sideways, it now suggests a corporeal form, creating a plane; but a spherical shape cut by a plane gives the shape of a circle at its section, a true image of the created mind, which is in charge of ruling the body. It is in the same proportion to the spherical as the human mind is to the divine, that is to say as a line to a surface, though each is circular…” (Kepler 1937-, vol vi, 224/1997 305). Because the human mind has this geometrical relationship with God’s mind, it is in a unique position to recognize the archetypes in nature that God employed in the creation of the world. In particular, the human mind accesses the same geometrical archetypes in order to apprehend God’s creation.[[15]](#endnote-16) As I will discuss in more detail below in Section 3, this epistemological role of geometrical archetypes is key to understanding how the geometry in the *Harmonices Mundi* is meant to respond to the problem of the empirical equivalence of astronomical hypotheses and why Kepler believes the work to be a success.

The polyhedral hypothesis that Kepler offers in his *Mysterium* is a paradigmatic case of his use of archetypes. According to this view, the Platonic solids serve as a geometrical blueprint for God’s construction of the heavens, wherein the number of Platonic solids determines the number of the planets and their spacing is determined by the inscribed and circumscribed spheres for these polyhedra. There is, of course, much more to say about Kepler’s understanding of archetypes and I will go into this in more detail in my discussion of their importance in the *Harmonices Mundi*. For now, I would just like to note that Kepler includes geometrical archetypes in his criteria for adjudicating between hypotheses.

I will conclude this section by summarizing the main methodological points that Kepler brings forward in the *Apologia*. First, he asserts that hypotheses should not ultimately be considered as useful fictions but should be assessed as genuine attempts at representing objects and their relationships to one another in the real world, the truth of which consists in their correspondence with the objects they aim to represent. Altogether, this highlights that one of Kepler’s primary methodological goals was locating a way of matching observationally informed models to real physical systems. However, given that some models might be empirically equivalent, Kepler identifies several additional criteria that can make a difference in the assessment of competing systems of hypotheses. First, he believes that considerations in the domain of physics, like the nature of the objects in question and their causal powers, should be included in our criteria for assessing competing hypotheses. Moreover, the ability of a hypothesis to unify across different disciplines, including but not limited to physics, is an indicator of its preferability, as is its simplicity and explanatory power. Finally, he believes that metaphysical or archetypal considerations can contribute to the assessment of competing hypotheses, as evidenced by his citation of the polyhedral hypothesis from the *Mysterium*.

However, as I will discuss below, we might think that at least some of these criteria are not sufficient to determine the truth of a hypothesis. I will show how Kepler recognizes that, while these criteria are good indicators, many of them might still ultimately fail to guarantee the truth of a hypothesis. As a result, he relies specifically on the archetypes to provide the ultimate link between model and physical system, due to the archetypes’ foundational status. I will argue that this epistemological function comes across in high relief in the geometry of the *Harmonices Mundi*. In what follows, I will provide a detailed discussion of Kepler’s first book on geometry and its application in the *Harmonices Mundi* and argue that the geometrical foundation in Book I is offered as an antidote to Kepler’s worries about identifying true hypotheses. Specifically, I will show that Kepler’s offers his account of geometrical knowability as a method for confirming the match between model and world. This account of knowability is then extended to harmony and, finally, employed to support his explanation of observed celestial motions. In this discussion, we will see several of Kepler’s other criteria, like simplicity or explanatory power, make an appearance. However, as I will argue, it is the archetypal foundation that ultimately provides Kepler with the confirmation that he thinks he needs to establish that his model is the correct one.

**3. Geometry and Knowability in the *Harmonices Mundi***

***3.1 Geometrical Knowability and the Threat of Pessimistic Realism***

Before diving into the geometry of Book I of the *Harmonices Mundi*, I would like to highlight something about the picture of Kepler’s project we have constructed at this point. As we saw above, Kepler believed that the aim of astronomy is to capture accurately the system of celestial bodies and he provides criteria that might indicate when we have achieved this aim. However, one could agree with this aim and these criteria, but be a pessimist about the ultimate success about the project. This scenario could obtain even if we stipulate that our best model is both empirically adequate and fulfils other non-empirical criteria, like some of the considerations that Kepler lays out above. After all, it could be the case that we have a simple model that has a high degree of explanatory power, unifies across disciplines, and that describes and predicts all our observations, but that nevertheless just happens to fail to correspond to the actual world. We can call this view *pessimistic realism*.[[16]](#endnote-17) One of the reasons that pessimistic realism poses a threat to many of the criteria that Kepler outlines is that most of them hold only relative to a competing model. We can ask if a model is, say, simpler when compared to another one, but not whether it is simple *simpliciter*. A similar observation holds of many of the other criteria, like explanatory power or unification. One of the only criteria Kepler cites above that obtains absolutely, not relatively, is archetypal foundation. I will argue in what follows that Kepler identifies archetypal foundation as the criterion that can directly confront the threat of *pessimistic realism*, and that the geometry in the *Harmonices Mundi* highlights this in an especially salient way.

It is clear Kepler is aware that it is possible that some model might fulfil many of his criteria and nevertheless fail to be true. For instance, when he discusses the weight of physical considerations in the *Astronomia Nova*, he states, “in the manner of physics professors, I mingle the probable with the necessary and draw a plausible conclusion from the mixture” (Kepler 1937-, vol iii, 19/2020, 18). Here he admits that the physical considerations only contribute to making his explanation more plausible than other options, not that they confirm it absolutely. This awareness is also manifested earlier in his discussion of the Copernican account in the *Mysterium*:

Now if any age of the universe has discussed the arrangement of the universe on the basis of the assumption that there are six spheres moving round a motionless Sun, it has undoubtedly given a true account of astronomy. But Copernicus has six spheres of that sort, and each pair of spheres in such proportion to each other that all these five solids can very readily be fitted in between them, which is the essence of what follows. So we must concur with him, *until someone has either put forward hypotheses which give a better solution to these problems, or asserted a system which has been deduced by excellent reasoning from the very principles of Nature*… (Kepler 1937-, vol i, 26/1981, 98, emphasis mine).

In this passage, Kepler states that we should accept the Copernican model unless or until a better account comes along. This alternative would be favorable either in its ability to fit the hypothetical criteria better, or in its ability to proceed demonstrably from true first principles or causes. The latter is what Kepler believes would guarantee the truth of a hypothesis. This is because we would be assured that our demonstration is a legitimate one; that is, one that deduces true conclusions from true premises. This kind of deduction would enable the theorist to avoid the pessimistic realist conclusion by showing that the model can be derived from true first principles. As I will argue, Kepler believes his geometrical archetypes can provide this foundation.

Keeping this consideration in mind, we can now return to Kepler’s geometry in the *Harmonices Mundi*. Book I opens with two definitions dealing with geometrical knowledge. The first reads, “In geometrical matters, to know is to measure by a known measure, which known measure in our present concern, the inscription of Figures in a circle, is the diameter of the circle” (Definition VII). It is followed by the second: “A quantity is said to be knowable if it is either itself immediately measurable by the diameter… or by its [the diameter’s] square…or the quantity in question is at least formed from quantities such that by some definite geometrical connection, in some series [of operations] however long, they at last depend upon the diameter or its square” (Definition VIII, Kepler 1937-, vol vi, 21-22/1997 18-19). In the context of the worry presented by pessimistic realism, these definitions constitute the foundation of Kepler’s attempt at providing a method for ensuring the truth of his model in the *Harmonices Mundi*.

The first thing to note is that these definitions are formulated in a way that describes knowledge acquisition as a function of both the activity of the knower and the object to be known.[[17]](#endnote-18) In order to successfully acquire knowledge, the knower must apply the correct procedure – measuring by a known measure.[[18]](#endnote-19) However, it is also necessary that the object be measurable in the right way. For Kepler, what enables an object to be knowable is its limit or boundary. This is because infinite quantity cannot be conceptualized.[[19]](#endnote-20) The appropriate measure, on the other hand, comes from what is common between the knower and the object known, which for Kepler is the diameter of a circle. The choice of the diameter is a less obvious criterion and is connected to Kepler’s understanding of the human mind. According to Kepler, the circle is present in minds as an archetype, and Kepler often suggests that the circle captures the essence of the human mind and its relationship with God’s mind.[[20]](#endnote-21) I do not have space for a full discussion of Kepler’s philosophy of mind here, but what is important for us to recognize is that this measure for the world comes from the geometrical nature of the mind. Acquiring knowledge, then, becomes a matter of establishing the commensurability between the measure that is possessed by the theorist and the object in question. If the object is shown to be measurable by this proper measure, and our model is grounded in this measure, then Kepler believes that we can confirm that our hypothesis is true (i.e. confirm that our model is a match). Going forward, we will see Kepler explicitly applying these definitions of knowability to provide a foundation for his views on geometrical objects, harmonic proportions, and the motions of the planets.

Following his definitions of geometrical knowability *simpliciter*, Kepler defines figures with a “proper construction” and outlines a series of “degrees of knowability.” Figures with a proper construction are those where “the number either of the angles of the Figure itself, or of the figure related to it by having either double or half its number of sides, forms the middle term in finding the ratio of the side to the Diameter” (Kepler 1937-, vol vi, 22/1997, 19). In other words, a figure possesses a proper construction if its side can be determined by the diameter of a circle in which it is inscribed. Degrees of knowability are then determined by how close or distant the line or surface in question is to the diameter of the circle.[[21]](#endnote-22) The first, or closest, degree of knowledge occurs when the line in question is equal to the diameter, or the area of a surface is equal to the square of the diameter (Kepler 1937-, vol vi, 22-23/1997, 20). The degrees become higher as the procedure for determining the relationship between the diameter and the line or surface in question becomes more removed. Kepler outlines eight degrees of knowledge in total, with the second and third degrees dealing with expressible quantities, and the fourth through eighth dealing with various lines that are inexpressible but can be related to the diameter in other ways.[[22]](#endnote-23) I will refer to Kepler’s definitions of knowability and their degrees collectively in what follows as Kepler’s *geometrical epistemology*.

Once Kepler has outlined his geometrical epistemology, he applies it to his constructions of the regular polygons, beginning with the construction of the diameter itself and then moving on to the tetragon (or square), the trigon (or triangle), and the pentagon, as well as figures with double their numbers of sides.[[23]](#endnote-24) Each of these constructions states the degree of knowledge that characterizes the figure in question.[[24]](#endnote-25) After these propositions, Kepler attempts to show that any figure with a prime number of sides higher than five is impossible to construct. While it turns out that Kepler is wrong on this point, as proven by Gauss centuries later, the fact that he believed he was successful in proving this proposition is significant for his application of his geometrical epistemology in later parts of the *Harmonices Mundi*, which will be addressed below.[[25]](#endnote-26)

How does this relate to pessimistic realism? As outlined above, Kepler needs a method for confirming the truth of hypotheses, understood explicitly as a question of matching the content of a hypothesis to the system it describes. In this section, we have seen how Kepler’s geometrical epistemology in the *Harmonices Mundi* is formulated explicitly in terms of providing a reliable link between theorist and world. This system is first applied, unsurprisingly, to geometrical objects but, as I will discuss below, this is only the first step. In what follows, I will show how Kepler’s geometrical epistemology functions as a form of archetypal foundation that is employed to confirm the truth of his model. In outlining its employment, we can trace Kepler’s ongoing concern with establishing the truth of astronomical hypotheses and understand some of his enthusiasm for the success of his project in the *Harmonices Mundi*.

***3.2 Geometrical Knowability and Harmony***

The geometrical epistemology developed in Book I is first applied outside of geometry to musical harmony in Book III. In this portion of the work, the constructible polygons from Book I are employed to explain why certain musical proportions are harmonious, or *consonant*. For the purposes of the discussion in this section, I understand the term *musical consonance* to refer to a property possessed by certain musical intervals (i.e. two pitches sounding simultaneously) in virtue of which they are perceived as pleasant, smooth, or agreeable. Its contrast, *musical dissonance*, is a property possessed by musical intervals in virtue of which they are perceived as unpleasant, rough, or disagreeable.[[26]](#endnote-27) A primary question for music theory in this period was why only a certain small collection of musical intervals were consonant.[[27]](#endnote-28)

Almost all accounts of musical consonance in this period refer to the fact that any musical interval can be assigned a ratio that captures the relationship between the two pitches composing the interval. According to our contemporary understanding, these ratios refer to the proportion between the fundamental frequencies of the pitches, but in Kepler’s time they were understood to refer to the proportion between the lengths of string producing the pitches. The consonant intervals in this period, along with their ratios, were the octave (2:1), fifth (3:2), fourth (4:3), major and minor thirds (5:4 and 6:5), and major and minor sixths (5:3 and 8:5). Some theorists at the time appealed to the special character of the numbers making up the consonant intervals, while others stipulated a connection between the ratios and the physical characteristics of sound to explain consonance. Kepler, in contrast, explained consonance by reference to the constructible polygons and their relationship to a circle in which they are inscribed.

The heart of this view is expressed in Axiom I of Chapter 1 of Book III where Kepler writes, “the diameter of a circle, and the sides of the fundamental figures expounded in Book I, which have a proper construction, mark off a part of the circle which is consonant with the whole circle” (Kepler 1937-, vol vi, 102/1997, 144). When inscribed within a circle, these figures “mark off” sections of the circumference where their vertices intersect with it. The significance of the circle and its relationship to consonance is found in the elaboration on this axiom where Kepler tells us, “It is sufficient that a string stretched out straight can be divided in the same way as when it is bent round into a circle it is divided by the side of the inscribed figure” (ibid). Thus, the circle thus represents the string, and the vertices of the inscribed figures provide the points of its division. The ratios of the consonances are generated by comparing the whole circumference with a portion marked off by a side or sides of the constructible figures. For example, we can take the trigon, which is responsible for generating the perfect fifth. The whole circumference is divided into three parts by the vertices of this figure, which we compare with the portion cut off by two sides, giving us the fifth’s 3:2 proportion. Given the figures with a proper construction we are able to generate all of the consonances.[[28]](#endnote-29)

There are a few things that should be noted here. First, Kepler’s account of consonance is a case of unification between two related mathematical disciplines, geometry and harmonics, recalling one of the criteria from Section 2. His explanation of consonance also possesses a high degree of explanatory power. It not only explains the perceived pleasant character of the consonant intervals (i.e. captures the empirical evidence) but also answers further questions about why that specific collection of intervals, and only those, constitute the consonant class.[[29]](#endnote-30) This kind of strategy is reminiscent of Kepler’s view that his polyhedral hypothesis from the *Mysterium* provides support for the Copernican system. In both cases, Kepler’s account provides an explanation of why the system is set up the way that it is and not otherwise.[[30]](#endnote-31) In addition, the explanatory success of Kepler’s account of consonance when compared with its rivals is not to be overlooked. While his view might seem somewhat obscure to us, it is worth stressing that it was virtually the only account available at the time that provided a strict criterion for the division between the consonant and dissonant classes of intervals.[[31]](#endnote-32)

However, these considerations only show how Kepler’s account of consonance might be preferable, relatively speaking. What provides the foundation for the account, and thereby its confirmation for Kepler, is his geometrical epistemology. He explicitly employs the knowability of the constructible plane figures from Book I to explain the perceived pleasant character of the consonant intervals. Moreover, the degree of knowability corresponds to the relative degree of consonance between them, where the more knowable figure generates a more perfect or pleasing consonance.[[32]](#endnote-33) Thus, the explanation of consonance is secured by the circle and the constructible plane figures. And, as we saw in the previous section, the knowability of these geometrical entities is due to the status of the circle as an archetype, by which the plane figures can be measured and known. These plane figures then gain a kind of derivative archetypal status. Like how the regular polyhedra provided a geometrical explanation of the number and spacing of the planets in the *Mysterium*, the circle and the constructible polygons provide a geometrical explanation of musical consonance here. And since the circle and the constructible polygons have been established as paradigms of geometrical knowability, they provide the epistemological grounding for Kepler’s theory of harmony. In this section, we have seen Kepler take the first step in extending his definitions of geometrical knowability to objects outside of geometry. The way in which he does so fulfills the criteria he sets out in the *Apologia,* but ultimately relies on the definitions of knowability to provide the archetypal foundation. I will now turn to Kepler’s application of his geometrical epistemology to astronomy in Book V.

***3.3 Knowability, Harmony, and Astronomy***

The final book in the *Harmonices Mundi* is dedicated to explaining the motions of the heavenly bodies and, in particular, the eccentricities of their orbits. This involves applying the harmonic theory developed in Book III to his astronomy. In this section I will focus primarily on Chapter 9 of this book, where Kepler derives the motions of the planets from his harmonic theory.[[33]](#endnote-34) My aim in this section will be to show how Kepler’ grounds his astronomy in his geometrical epistemology via his harmonic theory. However, before getting into this task, I will briefly highlight some comments from earlier in Book V that capture Kepler’s thoughts on his work.

In the introduction to Book V, Kepler expresses a highly ebullient attitude toward the book’s contents:

The discovery which I foretold twenty-two years ago… at last, I say, I have brought [it] into the light, and have most truly grasped beyond what I could ever have hoped: that the whole nature of harmony, to its full extent, with all its parts, as expounded in Book III is to be discovered among the celestial motions. It is to be discovered indeed not in the way which I had mentally conceived (and this is not the least part of my joy), but in a totally different way, and also at the same time a quite outstanding and perfect way (Kepler 1937-, vol vi, 289/1997, 389-390).[[34]](#endnote-35)

As the passage above indicates, Kepler seems to regard the overall project as a success.[[35]](#endnote-36) However, as I mention above, Kepler’s enthusiasm has puzzled even those commentators who are sympathetic to his project, as there are certain discrepancies in fitting the harmonic ratios to his astronomy. I hope to provide an account that helps to bring out, at least to some extent, why Kepler was so pleased with his work, discrepancies and all.

After outlining the details of his astronomical theory and the presence of the harmonies in the heavens in the first eight chapters of Book V, Kepler sets out to derive the motions of the planets from a combination of harmonic and astronomical principles in Chapter 9. The chapter is extremely complex, presented in an axiomatic format composed of 49 numbered axioms, propositions, and corollaries, all divided into *a priori* arguments [*rationes*] (1-17) and *a posteriori* arguments (18-49). The *a priori* arguments deal with initial assignments of the harmonies to the planets and the *a posteriori* arguments deal with amendments and corrections that must be made to these to accommodate discrepancies in the system that are introduced by the motions of the Earth and Venus. In what follows, I will highlight the relevant elements of this chapter, beginning with the *a priori* arguments.

This portion of the chapter includes four axioms that serve as foundational rules determining the behavior of the system. The first axiom states that, whenever possible, harmonies are to be found between the extreme motions of the planets and that there should be as much variety as possible among these. [[36]](#endnote-37) The second axiom states that the spacing of the planetary spheres corresponds “to a certain extent” with the proportion between the circumscribed and inscribed spheres of the regular polygons (Kepler 1937-, vol vi, 331/1997, 452). The two other axioms are given a bit later, with the next asserting that there must be variety in the motions of the planets, the eccentricities of their orbits, and their distances from the sun (Kepler 1937-, vol vi, 332/1997, 453). The final axiom found in the *a priori* arguments states that, when there is a choice, the superior planet should take the more perfect or the larger harmonic proportion in its extreme motions (Kepler 1937-, vol vi, 334/1997, 456).

It is significant that the first two axioms capture the primary archetypal principles that govern Kepler’s system, both of which are rooted in geometry: the harmonic archetypes, which are based on Kepler’s geometrical epistemology, and his polyhedral hypothesis from the *Mysterium*. Thus, at the very beginning, Kepler combines his earlier archetypal foundation for the world with the geometrical epistemology of the *Harmonices*, signaling that these archetypes will provide the metaphysical and epistemological grounding for the system.The other thing to note is Kepler’s tentative language in these first two axioms: the harmonies obtain between the extreme motions “wherever it could be so” and the spacing between the planets corresponds to the spheres of the polyhedra “to a certain extent” (Kepler 1937-, vol vi, 331/1997, 452). This is because, as we will see, there are competing considerations that come up in the derivation of the planetary motions. To adjudicate between these, Kepler thinks we must take the first two axioms as guiding principles but recognizes that they also might have to be compromised in certain respects. The other two axioms build on this by providing further considerations that determine the outcome when such competing cases arise.

The next few propositions build directly on these axioms. In Proposition III, he applies the second axiom to assign the polyhedra that determine the distances between the planets.[[37]](#endnote-38)Then, in Proposition V, he states that each pair of neighboring planets will have two different harmonies, which is an application of Axioms I and IV (Kepler 1937-, vol vi, 331-332/1997, 453-454). The proof for this proposition also cites the astronomical theory in Chapter 3 where he established that the orbits of the planets are eccentric, and his application of harmonic theory to astronomy in Chapters 4 and 5, where he locates the harmonies that obtain between combinations of planetary motions. Following these, Kepler provides two propositions that deal with how the interaction between the first two axioms rules out the presence of certain celestial harmonies.[[38]](#endnote-39)

The primary thing to note at this stage is that Kepler is relying explicitly on his geometrical archetypes to set the stage in the demonstration. These archetypes include the Platonic solids from his original polyhedral hypothesis, but now also include the harmonic ratios that were derived from his geometrical epistemology. Together, they not only establish that the theory that follows accurately captures the behavior of the system, but also ensure that the details of Kepler’s astronomical model (e.g. elliptical orbits, central sun, etc.) correspond to the world it describes. This is evidenced at this early stage by how Kepler applies the archetypal axioms to the details of his astronomical theory that he summarizes for the reader in Chapter 3. For instance, the eccentricity of the orbits is not just stipulated to account for certain empirical observations but here is shown to follow directly from geometric archetypal considerations. These serve both to explain why the orbits are eccentric and to confirm that the eccentric orbits in the model correspond to the structure of the system.

The rest of the propositions in the *a priori* arguments provide initial assignments of harmonies to the motions of individual and pairs of planets. For instance, Kepler assigns the ratios of 1:2 and 1:3 (i.e. an octave and octave plus a fifth) to the converging and diverging motions of Saturn and Jupiter, respectively, and the ratios of 4:5 and 5:6 (i.e. a major and minor third) are assigned to the individual extreme motions of Saturn and Jupiter, respectively (Proposition XI).[[39]](#endnote-40) Most of the assignments in this first portion of the chapter proceed straightforwardly from the polyhedral hypothesis, so for reasons of space, I will not discuss all of them. However, there is one notable hiccup: Kepler’s assignment of the 1:8 and 5:24 (i.e. two octaves plus a minor third) ratios to the motions of Jupiter and Mars.

Kepler’s ‘demonstration’ of this proposition is somewhat strained.[[40]](#endnote-41) This is because, given that the proportion of the spheres of the tetrahedron (i.e. the polyhedron located between Jupiter and Mars) is twice that of the cube, Jupiter and Mars should be assigned ratios twice that of Saturn and Jupiter (i.e. 1:2 and 1:3). This would give him 1:4 and 1:9. However, 1:9 isn’t a harmonic proportion, and 1:4 is already taken by the diverging motions of Venus and Mercury. In cases like these, Kepler’s method is to appeal to the harmonies that would be closest. The closest harmonies to 1:9 would be either 1:8 or 1:10. He states that the tetrahedron has a greater affinity with 1:8, because the 1:10 proportion is grounded by the pentagon, which has no relation to the tetrahedron. Thus, he takes 1:8 as the proportion for the diverging motions of Jupiter and Mars. The closest harmonies to 1:4 are 5:24 and 3:10 and, since the 3:10 ratio is also grounded in the pentagon, 5:24 is taken as the assignment for their converging motions. If we accept this reasoning, the rest of the assignments in this section follow fairly straightforwardly.

Before moving onto the *a posteriori* arguments, I will pause to take stock. In some respects, it seems like Kepler has been successful so far. He has located harmonic proportions in the proper proportions of the three superior planets (i.e. Saturn, Jupiter, and Mars), in the converging motions of all pairs of planets, and in the diverging motions of all pairs of planets except Earth and Venus. However, some problems arise on closer inspection. First, there is the strained assignment of the diverging and converging motions of Jupiter and Mars, which has already been noted. Moreover, there is a more general issue with the assignment of the ratios based on the polyhedral hypothesis. Out of all the proportions between the spheres of the polyhedra, only the proportion of the tetrahedron’s spheres (i.e. 1:3) is harmonic. The proportion of the spheres of the cube and the octahedron are 1:√3 and the proportion of those of the icosahedron and dodecahedron are, as Kepler calls them, “inexpressible” [*ineffabilis*] (Kepler 1937-, vol vi, 295-296/1997,402).[[41]](#endnote-42) Thus, Kepler must make certain compromises to match harmonic proportions to the polyhedra, usually taking the closest available harmonic proportions.

This might make things seem bleak for Kepler but, surprisingly, when we compare these initial assignments of the harmonic ratios to observational data, they match up remarkably well (Kepler 1937-, vol vi, 312-305/1997, 424-430).[[42]](#endnote-43) So, if we accept Kepler’s reasoning, we are left at this stage with a theory that is empirically adequate and, more importantly for Kepler, is grounded in his geometrical epistemology. However, there is still a long way to go. Kepler is not satisfied with merely locating the harmonic proportions in the extreme motions of the individual planets and their converging and diverging motions with their neighbors. As he writes in Chapter 5, we must “see whether the individual harmonies stand separately, so that they have no affinity with the rest, or whether in fact they all agree with each other” (Kepler 1937-, vol vi, 317/1997, 431). Kepler believes that all the planets must be part of a larger universal harmony and the *a posteriori* arguments are dedicated to fleshing this out.

As with the prior series of arguments, this section begins with a collection of axioms. The first (Axiom XVIII) states that harmonies had to be established between the extreme motions of the six planets combined and, following this, Axiom XIX states that these harmonies occur over a range of motions at the planet’s extremes. Axioms XX and XXI establish that both kinds of musical scales (i.e. *durus*, or hard, and *mollis*, or soft) must be present in the extreme motions of the planets, with different harmonies from each being present in at least some of these.[[43]](#endnote-44) Kepler proceeds by introducing the pair of planets that serve as the foundation for the two types of harmony (i.e. *durus* and *mollis*). These are the Earth and Venus, whose proportions between their respective extreme motions must be the major and minor sixths (i.e. 3:5 and 5:8). This is because the distinction between the *durus* and *mollis* scales is made by these intervals.[[44]](#endnote-45) This is where another discrepancy creeps into Kepler’s system. The proportions between the Earth & Venus’ extreme motions do not obtain between their converging and diverging motions like with the other planetary proportions, but rather between the planets when they are both at aphelion or perihelion, which Kepler refers to as “of the same region” [*eiusdem plagae*]. This is because, if we were to locate the sixths between the converging and diverging motions, then either one planet’s individual motion would vary not at all (violating Axiom IV) or each planet’s individual motion would amount to an interval smaller than a diesis. Kepler finds this second possibility highly unsatisfactory because the diesis is the smallest of the melodic intervals that he discusses in Book III.[[45]](#endnote-46)

He then goes on to assign proportions to the proper motions of Earth and Venus, respectively. Kepler assigns the proportion of two dieses minus a comma (2,916:3,125, or close to 14:15) to the Earth and the proportion of a single diesis minus a comma (243:250, or close to 35:36) to Venus.[[46]](#endnote-47) As the reader will note, neither of these proportions is consonant. However, Kepler argues that if these planets’ proper proportions cannot be consonant due to the small size of their orbits, they should at least be melodic.[[47]](#endnote-48) As Kepler has shown that their respective proportions must be less than a tone, the only possible melodic intervals are the diesis and the semitone, which cannot be assigned respectively to each planet without violating their role of distinguishing between the durus and mollis scales. The assignment that Kepler settles on leaves the pair separated by the diesis necessary for their role in distinguishing the heavenly harmonies.

This seems to be far from an ideal instantiation of harmonic proportion in the heavens. Not only are the consonant proportions ruled out, but even the melodic intervals must be compromised somewhat. Kepler himself notes the apparent paucity of his reasoning, concluding the demonstration of this proposition with the question, “Do you ask whether the highest creative wisdom would have been taken up with searching out these thin little arguments?” (Kepler 1937- vol vi, 342/1997, 466). His answer to this question is telling, though. He writes, “If the nature of harmony has not supplied weightier arguments… it is not absurd for God to have followed even these, however thin they may appear, since he has ordered nothing without reason. For it would be far more absurd to declare that God has snatched these quantities… accidentally. Nor is it sufficient to say that He adopted that size because that size pleased Him. For in matters of geometry which are subject to freedom of choice it has not pleased God to do anything without some geometrical reason or other…” (Kepler 1937-, vol vi, 342-343/1997, 466). Echoing what we saw earlier, Kepler believes that God must have reasons for the way in which he creates the world. Moreover, there are constraints on what those reasons can be – they must be geometrical.

This brings us back to Kepler’s geometrical epistemology and the role it plays in the *Harmonices Mundi*. Geometry is what enables us to know the world – it is our link to God’s creation. Insofar as part of the job of the theorist is to locate God’s reasons for creating the world as it is, geometry provides us with the means of doing this. For Kepler, as we’ve seen, these reasons are traced back to the geometrical archetypes, whose knowability provides the connection between theorist and world. So, while he recognizes that the assignment of the proper proportions for Venus and Earth must seem somewhat tenuous, he thinks that it would be far more absurd to think that the assignment is arbitrary. The fact that Kepler can provide a reason grounded in geometry and geometrically based harmonic theory speaks to the success of his project in his eyes.

The propositions directly following these assignments finalize the proportions of Venus and Earth. The rest of the propositions in this section deal with implementing the *durus* and *mollis* scales among the planetary motions and the changes that must be made to the results of the *a priori* arguments to accommodate the Earth and Venus.[[48]](#endnote-49) Throughout these propositions, Kepler points out that, while these discrepancies are necessary for the sake of the greater harmonies between the planets, they are the smallest and least disruptive possible. In the final propositions of Chapter 9, Kepler returns to the intersection between the polyhedral hypothesis and the planetary harmonies, asserting that “there was not pure liberty for the inscription of the regular solid figures between the planetary spheres; for it was impeded over small details by the harmonies set up between the extreme motions” (Kepler 1937-, vol vi, 356/1997, 483). In the culminating proposition of the chapter, he derives the motions of the planets from the foregoing harmonic assignments. He begins by deriving the eccentricities of each planet’s orbit based on its proper proportion of extreme motions. This involves applying the astronomical principle that “the proportion of the extreme motions is the square of the inverse proportion of the corresponding distances from the Sun” (Kepler 1937-, vol vi, 356/1997, 484).[[49]](#endnote-50) Once the extreme distances are determined, Kepler is able to take their average to establish the orbital radius of each planet and compare this with the extremes to determine the eccentricities.

This completes Kepler’s project of determining the eccentricities of the planetary orbits from their harmonic proportions.[[50]](#endnote-51) In concluding, Kepler compares the results of this derivation to the original predictions of the polyhedral hypothesis. As expected, there are discrepancies (Kepler 1937-, vol vi, 359/1997, 487). However, the proportions are close, which allows Kepler to conclude in the final section of the chapter that, “in the genesis of the distances the solid figures should give way to the harmonic arguments, and the greater harmonies of the pairs to the universal harmonies of all, to the extent to which the latter was necessary” (Kepler 1937-, vol vi, 360/1997, 488). The polyhedra served as a rough guide to the Creator, while the harmonies provided the fine-tuning for the system. In the following final section, I will discuss Kepler’s own concluding remarks on the project, as well as providing my own. I hope to show that Kepler’s enthusiasm is comprehensible if we take seriously his concerns with providing true hypotheses and avoiding the threat of pessimistic realism outlined above.

**4. Concluding Remarks**

Before turning to Kepler’s remarks at the end of Chapter 9, let us take stock. First, in Section 2, we established Kepler’s concern with identifying astronomical hypotheses that faithfully capture their objects. Kepler’s recognition that empirical adequacy is not always sufficient for identifying the correct hypotheses led him to offer other additional criteria. These included considerations from domains that were not traditionally considered part of astronomy, as well as considerations regarding explanatory power and unification. However, in Section 3.1, we saw that it is still possible that our best-supported hypotheses might ultimately fail in their aim of capturing the objects they set out to represent. In response, I argued that Kepler relies on geometrical archetypes, in general, and his geometrical epistemology in the *Harmonices*, specifically, as an antidote to the worry presented by pessimistic realism. This is evidenced by how Kepler formulates his definitions for geometrical knowledge in terms of criteria for linking the knower with the object to be known. I then traced how Kepler applies his geometrical epistemology first to geometrical objects, then to harmonic proportions, and finally to the motions of the celestial bodies. In all these cases, Kepler’s geometrical epistemology provides a unifying principle across different disciplines and lends explanatory power to his account, fulfilling his criteria from his earlier works like the *Apologia*. In addition, though, the foundation provided by the geometrical epistemology in Book I provides more than just an indication that Kepler’s model is correct. Rather, it provides positive confirmation in that it links his hypotheses with the world they mean to capture. This is because each step of the process can be traced back to the criteria outlined by Kepler’s original principles of geometrical knowability.

So what about the discrepancies in Kepler’s ultimate derivation in Chapter 9? After all, it seems that by the end of Chapter 9 he has had to sacrifice much of his original hope for a perfect harmonic system. In the final section of Chapter 9 of the *Harmonices*, Kepler discusses his understanding not only of why the polyhedral hypothesis had to be complemented by the harmonic proportions, but also why some harmonies had to be changed. He begins by pointing out that, in both of these cases, a choice had to be made between competing perfections: “For where there is a choice between different things which do not allow each other to have sole possession, in that case the higher are to be preferred, and the lower must give way…” (Kepler 1937-, vol vi, 360/1997, 488). The idea is that the harmonies, and preferences among them, take a certain precedence due to their perfection. Most important for us, though, are his remarks regarding the adjustments that must be made to the spacing of the planets predicted by the polyhedral hypothesis.

He begins by telling the reader that the polyhedra relate to the spatial regions and the bodies of the planets while the harmonies relate to their motions. [[51]](#footnote-2)[[52]](#endnote-52) This means that both kinds of archetypes work together to determine the behavior of the celestial bodies. He then goes on to note that, as the polyhedra are related to the planetary bodies and the space they occupy, they are concerned with matter, which is infinitely divisible. The harmonies, on the other hand, are concerned with form, and comprise a fixed set:

As matter is diffuse and unlimited in itself, but form is limited, unified, and itself the boundary of matter; so also the number of the geometrical proportions is infinite, the harmonies are few. For although even among the geometrical proportions there are definite degrees of limitation and shape and restriction… yet even to these is attached an accidental property in common with all the rest, that is the presupposition of an infinite possible division of quantities… But the harmonic proportions are all expressible, and the terms of all of them are commensurable. Also, they have been taken from a definite and limited class of plane figures… Therefore, as matter strives for form… so the geometrical proportions in the figures strive for harmonies… (Kepler 1937-, vol vi, 360-361/1997, 488-489).

Some commentators have taken Kepler’s inclusion and emphasis on harmonic proportions as a radical reformulation of his theory of geometrical archetypes. For instance, while Martens asserts that Kepler remains committed to archetypal explanation in the *Harmonices*, she takes Kepler’s emphasis on harmony to be a “dramatic shift” from his earlier views on the importance of geometry (Martens 2000, 139-140). While I think it is clearly true that Kepler’s astronomical work caused him to revise and reformulate his metaphysical and epistemological commitments, I do not think that that his inclusion and emphasis on the harmonic proportions indicates that he rejected his earlier commitment to a geometrical foundation for his epistemology. After all, while the harmonic ratios supplement the polyhedral hypothesis of the *Mysterium Cosmographicum*, he does not abandon the latter. Moreover, as Martens herself notes, he goes to great lengths to provide a geometrical foundation for the former. So, we might think that it is not so much that his stance at the end of Chapter 9 of Book V commits him to geometrical archetypes and, *in addition*, harmonic ones. After all, the harmonic ratios are grounded in geometrical archetypes. Rather, we should see Kepler as remaining committed to the idea that geometrical archetypes provide the metaphysical foundation for God’s creation and our epistemic access to it, but realizing that this geometrical foundation could be more complex than initially supposed. The structure of the *Harmonices Mundi* makes it clear that geometrical knowability is the foundation for Kepler’s work in the later books. And, as I have argued, if we take this project in the context of the problem of underdetermination, we can see that providing a traceable link to a geometrical foundation is key for confirming his conclusions in Book V. This applies equally to the harmonic proportions and to the original polyhedral hypothesis it supplements.

To conclude, if we return to the passage from the *Harmonices Mundi* above, we see that Kepler emphasizes again the importance of limitation and boundary-setting. This should recall his comments from the very beginning of the *Harmonices* on knowability: that which can be known is that which possesses a boundary or limit. Here he reiterates this point and locates the harmonic proportions as limiting principles. Given that there are only a fixed set of these proportions, they provide significant constraints on how the universe could unfold. This not only supplies a theory with a higher degree of explanatory power in that it will be able to answer questions regarding why the universe is set up the way it is, but also helps us to restrict the domain of acceptable theories. Most importantly, though, it connects the results of the chapter back to the ultimate goal of knowability, understood in terms of the commensurability between the object to be known and the theorist’s measure. Thus, in these final remarks, we can identify a clear thread that runs all the way through the work; namely, Kepler’s concern with the knowability of the cosmos. In the Introduction to the *Harmonices Mundi*, he tells the reader that he will focus on the “features which distinguish geometrical objects to the mind” (Kepler 1937-, vol vi, 15/1997, 9). These features are furnished in Book I by his definitions of geometrical knowability, which state that to be known, a quantity must possess a boundary or measure and the knower must possess the appropriate measure for it. We then see him extending these principles throughout the work, beginning with geometrical objects, moving on to harmony, and then finally to the motions of the planets. Knowledge in each of these domains is secured by this geometrical epistemology.

We might think, then, that the reason that Kepler views his project with such pride is it secures our knowledge of these subjects. To understand the depth of his satisfaction, though, we must consider the backdrop provided by earlier works. Kepler’s concern with locating true hypotheses in science significantly informs his emphasis on knowability in the *Harmonices*. This is because he believes that the method for showing that one has arrived at true hypotheses involves showing that the theorist possesses the right measure; in other words, that the theorist is not only providing a consistent model that is backed up by empirical evidence, but that the model truly conforms to the world. Kepler’s geometrical epistemology in the *Harmonices* is meant to provide such a confirmation. So, while Kepler’s project in the *Harmonices* may seem inchoate at times, I hope to have shown that it is motivated by comprehensible concerns and that Kepler’s thorough dedication to his methodological principles is reflective of an overall commitment to the search for truth in the cosmos.

**References**

Barker, Peter & Bernard R. Goldstein. 1998. “Realism and Instrumentalism in Sixteenth Century Astronomy: A Reappraisal.” *Perspectives on Science* 6 (3): 232-258.

Bidelman, Gavin M. & Krishnan Ananthanarayan. 2009. “Neural Correlates of Consonance, Dissonance, and the Hierarchy of Musical Pitch in the Human Brainstem.” *Journal of Neuroscience* 21 (42): 13165-13171.

Boner, Patrick J. 2006. “Kepler’s Living Cosmology: Bridging the Celestial and Terrestrial Realms.” *Centaurus* 48: 32-39.

Bowling, Daniel L., Marisa Hoeschele, Kamraan Z. Gill, & W. Tecumseh Fitch. 2017. “The Nature and Nurture of Musical Consonance.” *Music Perception* 35 (1): 118-121.

Cantù, Paola. 2010. “Aristotle’s prohibition rule on kind-crossing and the definition of mathematics as a science of quantities.” *Synthese* 174: 225-235.

Caspar, Max. 1993. *Kepler*. New York: Dover.

Cifoletti, Giovanna. 1986. “Kepler’s *De quantitatibus.*” *Annals of Science* 43: 213-238.

Cohen, H. F. 1984. *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580-1650*. Dordrecht: D. Reidel Publishing Company.

Della Rocca, Michael. 2013. “Striving, Oomph, and Intelligibility in Spinoza.” In *Judgement and the Epistemic Foundation of Logic*, ed. Maria van der Schaar, pp. 49-65. Dordrecht: Springer.

Deutsch, Diana. 2013. “Grouping Mechanisms in Music.” In *Cognition and Perception, The Psychology of Music* (Second Edition), ed. by Diana Deutsch, 183-248. Academic Press.

Di Liscia, Daniel A. 2009. “Kepler’s *A Priori* Copernicanism in his *Mysterium Cosmographicum.*” In *Nouveau Ciel – Nouvelle Terre: La revolution copernicienne dans l’Allemagne de la Réforme (1530-1630)*, ed. Miguel Ángel Granada & Éduoard Mehl, 283-317. Paris: Belles lettres.

Duhem, Pierre. 1969. *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo*, translated by Edmund Dolan and Chaninah Maschler. Chicago, IL: University of Chicago Press.

Dupré, Sven. 2012. “Kepler’s Optics Without Hypotheses.” *Synthese* 185 (3, *Seeing the Causes: Optics and Epistemology in the Scientific Revolution*): 501-525.

Escobar, Jorge M. 2008. “Kepler’s theory of the soul: a study on epistemology.” *Studies in History and Philosophy of Science* 39: 15-41.

Field, J.V. 1988. *Kepler’s Geometrical Cosmology*. Chicago, IL: University of Chicago Press.

Gingerich, Owen. 1975. “The Origins of Kepler’s Third Law.” *Vistas in Astronomy* 18: 595-601.

Gingerich, Owen. 1992. “Kepler, Galilei, and the Harmony of the World.” In *Music and Science in the Age of Galileo*, ed. Victor Coelho, 45-63. Kluwer: Netherlands.

Itokazu, Anastasia Guidi. 2009. “On the Equivalence of Hypotheses in Part 1 of Johannes Kepler’s *New Astronomy.*” *Journal for the History of Astronomy* 40 (2): 173-190.

Jardine, Nicholas. 1979. “The Forging of Modern Realism: Clavius and Kepler Against the Sceptics.” *Studies in History and Philosophy of Science* 10 (2): 141-173.

Jardine, Nicholas. 1984. *The Birth of History and Philosophy of Science: Kepler’s* A Defence of Tycho against Ursus *with essays on its provenance and significance*. Cambridge: Cambridge University Press.

Kameoka, Akio & Mamoru Kuriyagawa. 1969. “Consonance Theory Part I: Consonance of Dyads.” *The Journal of the Acoustical Society of America* 45 (6): 1451-1459.

Kepler, Johannes. 1858-1872. *Joannis Kepleri astronomi opera omnia*, ed. Ch. Frisch; Frankfurt a.M. and Erlangen: Heyder and Zimmer.

---. 1937–. Johannes Kepler Gesammelte Werke, Walther von Dyck, Max Caspar and Franz Hammer, München: C.H. Beck.

---. 1981. *Mysterium Cosmographicum: The Secret of the Universe*, trans. A.M. Duncan, intro & comments E.J. Aiton, preface I. Cohen. Abaris Books: New York.

---. 1995. *Epitome of Copernican Astronomy & Harmonies of the World*, trans. Charles Glenn Wallis. Amhurst, NY: Prometheus Books.

---. 1997. *The Harmony of the World*, trans. E.J. Aiton, A.M. Duncan, & J.V. Field. Philadelphia: American Philosophical Society.

---. 2000. *Optics: Paralipomena to Witelo & Optical Part of Astronomy*. Trans. Donahue, Santa Fe, NM: Green Lion Press.

---. 2020. *Astronomia Nova: New Revised Edition*. Trans. Donahue, Santa Fe, NM: Green Lion Press.

Krumhansl, Carol L. 1990. *Cognitive Foundations of Musical Pitch*. New York: Oxford University Press.

Martens, Rhonda. 2000. *Kepler’s Philosophy and the New Astronomy*. Princeton, NJ: Princeton University Press.

Mittelstrass, Jürgen. 1972. “Methodological Elements of Keplerian Astronomy.” *Studies in History and Philosophy of Science* 3 (3): 203-232.

Miyake, Teru. 2015. “Underdetermination and Decomposition in Kepler’s *Astronomia Nova.*” *Studies in History and Philosophy of Science* 50: 20-27.

Nagel, Ernest. 1961. *The Structure of Science: Problems in the Logic of Explanation*, New York: Hargourt, Brace & Word, Inc.

Regier, Jonathan. 2013. “Method and the *a priori* in Keplerian metaphysics.” *Journal of Early Modern Studies* 2 (1): 147-162.

Regier, Jonathan. 2016. “An Unfolding Geometry: Appropriating Proclus in the ‘Harmonice mundi’ (1619).” In *Unifying Heaven and Earth: Essays in the History of Early Modern Cosmology*, eds. Miguel Á. Granada, Patrick J. Boner, & Dario Tessicini, 217-237. Barcelona: University of Barcelona.

Russell, John L. 1975. “Kepler and Scientific Method.” *Vistas in Astronomy* 18: 733-745.

Stephenson, Bruce. 1994. *The Music of the Heavens: Kepler’s Harmonic Astronomy*. Princeton, NJ: Princeton University Press.

Schoot, Albert van der. 2001. “Kepler’s Search for Form and Proportion.” *Renaissance Studies* 15 (1): 59-78.

Schöttler, Tobias. 2012. “From Causes to Relations: The Emergence of a Non-Aristotelian Concept of Geometrical Proof out of the *Quaestio de Certitudine Mathematicarum.*” *Society and Politics* 6 (2): 29-47.

Trainor, L. J., & B. M. Heinmiller. 1998. “The Development of Evaluative Responses to Music: Infants Prefer to Listen to Consonance Over Dissonance.” *Infant Behavior & Development* 21 (1): 77-88.

Walker, D.P. 1967. “Kepler’s Celestial Music.” *Journal of the Warburg and Courtauld Institutes,* 30: 228-250.

Westman, Robert S. 1972. “Kepler’s Theory of Hypothesis and the ‘Realist Dilemma.’” *Studies in History and Philosophy of Science* 3 (3): 233-264.

Zentner, Marcel R., & Jerome Kagan. 1998. “Infants’ Perception of Consonance and Dissonance in Music.” *Infant Behavior & Development* 21 (3): 483-492.

1. This is not to say that scholars as a whole have dismissed Kepler’s *Harmonices Mundi*, or that there have not been careful and considered discussions of the derivation of the planetary motions that occurs in Chapter 9 of Book V of the work. In addition to Martens’ careful assessment, which I reference above, there are also detailed examinations of this portion of Kepler’s work to be found in Field 1988, pp. 155 ff and Stephenson 1994, pp. 185 ff. This is just to say that numerous commentators have found the derivation complex and have not agreed altogether with Kepler’s enthusiastic assessment of it. [↑](#endnote-ref-2)
2. For thorough discussions of Kepler’s *Harmonices Mundi* and its importance for his overall project, see Mittelstrass 1972, Gingerich 1975, Field 1988, Stephenson 1994, Martens 2000, & van der Schoot 2001. Martens, specifically, provides a nuanced discussion of Kepler’s employment of archetypal metaphysics in solving epistemological and methodological problems, much of which agrees with my presentation here. For a discussion of Kepler’s *Harmonices Mundi* and its relationship with his work on optics, see Chen-Morris 2016. [↑](#endnote-ref-3)
3. For discussion of Kepler’s recognition of this problem and his response, see Westman 1972, Section III; Jardine 1984, 215-218; Martens 2000, Ch. 2; di Liscia 2009, Section IV; Itokazu 2009; and Miyake 2015. The work in this section is especially indebted to Jardine’s 1984 translation of the *Apologia* and attached essays. [↑](#endnote-ref-4)
4. This phrase is found in various historical texts, perhaps most famously in the title of Pierre Duhem’s essay (Duhem 1969). It is important to note here that Ursus’ position shouldn’t necessarily be understood as equivalent to contemporary instrumentalism, though (see Jardine 1979 & 1984). Moreover, the general debates in this period fail to correspond to the contemporary realist/instrumentalist distinction in significant ways (see Barker & Goldstein 1998, cf. Duhem 1969). As many of these commentators point out, knowledge of the real configurations of the heavens was not ruled out in principle but often in terms of practical accessibility, and there was no decided consensus on what specifically could or could not be known in this capacity. [↑](#endnote-ref-5)
5. In the paper, I will use the term ‘model’ to refer to a hypothesis, or set of hypotheses, that describes objects, their behaviors, and their relations to one another. These models can either be purely stipulative, as we see in the case of Ursus, or “real,” in the sense of aiming to accurately capture the behavior of the actual system in question. These can encompass both what Kepler calls “astronomical hypotheses,” or statements involving the actual or observed behavior of an object, and “geometrical hypotheses,” or mathematical means constructed to represent this behavior (Kepler 1858-1872, vol i, 246-247/Jardine 1984, 153-54). For a discussion of the status of hypotheses in this period, see Mittelstrass 1972, Westman 1972, Jardine 1979, Itokazu 2009, and Dupré 2012. [↑](#endnote-ref-6)
6. In the context of the passage, Kepler refers to “both the hypotheses,” or the premises of a syllogism on the model of an Aristotelian science. However, earlier in the *Apologia*, he makes it clear that a “totality of views of some… practitioner,” (Kepler 1858-1872, vol i, 239/ Jardine 1984, 139) or a collection of consistent propositions that are linked together for the purposes of demonstration, counts as an astronomical hypothesis. For a discussion of Kepler’s relation to the Aristotelian scientific tradition, see Jardine, 1979 & 1984, Ch.6; Martens 2000, Ch. 5; di Liscia 2009; and Regier 2013. [↑](#endnote-ref-7)
7. Kepler is confident that a false hypothesis will betray itself eventually: “… false hypotheses, which together yield the truth once by chance, do not in the course of a demonstration in which they have been combined with many others retain this habit of yielding the truth, but betray themselves” (Kepler 1858-1872, vol i, 239-240/Jardine 1984, 140). [↑](#endnote-ref-8)
8. For discussion of Kepler’s incorporation of the causal-explanatory domain of physics into astronomy, see Westman 1972, Section 3; Russell 1975; and Martens 2000, Ch. 5. For discussion of the scope of mathematics in the Aristotelian context, see Cantù 2010 and Schöttler 2012. [↑](#endnote-ref-9)
9. Westman suggests that Kepler’s physical considerations determine the truth of a mathematical hypothesis, “when it corresponds directly to real bodies in space and describes their motions in the simplest and most regular possible manner” (Westman 1972, 239). Jardine’s account is largely in agreement with this (Jardine 1984, 220), as is Martens’ (Martens 2000, Ch. 2-3). [↑](#endnote-ref-10)
10. Further support for this interpretation comes from considering his later *Epitome astronomiae Copernicanae* where he explicitly addresses the relationship between astronomy and other disciplines. There he tells the reader that, while astronomy has traditionally been understood apart from physics and metaphysics, astronomical hypotheses should be situated under metaphysical and physical principles. He also discusses there how astronomy is properly related to other mixed mathematical disciplines like optics, and how the philosopher should endeavor to formulate her hypotheses so they unify across these disciplines (Kepler, 1937-, vol vii, 24-25). Thanks to an anonymous reviewer for calling my attention to this passage. [↑](#endnote-ref-11)
11. For instance, Kepler writes in the *Mysterium Cosmographicum* that, “[nature] loves simplicity, she loves unity” (Kepler, 1937-, vol i, 16/1981, 77).For discussion of Kepler’s criteria, especially simplicity and unification, see Westman 1976. Martens also discusses this point and focuses on Kepler’s connection between astronomy, physics, and metaphysics explicitly (Martens 2000, Ch. 5). [↑](#endnote-ref-12)
12. Kepler provides a summary of his theory in Chapter II of the *Mysterium* (Kepler 1937-, vol i, 23-27/1981, 93-101), with the later chapters elaborating on this. [↑](#endnote-ref-13)
13. Kepler believed that there must be a reason for why the universe is the way that it is and not otherwise and, thus, a theory that has answers to these kinds of questions is a theory that is preferable. Here, and elsewhere, we see Kepler relying on something like what Leibniz would later canonize as the “Principle of Sufficient Reason”. For instance, as we will discuss in more detail below, he asserts in the *Harmonices Mundi* that “…it is not absurd for God to have followed even these, however thin they may appear, since he has done nothing without reason.” (Kepler 1937-, vol vi, 342-343/1997, 466). This connection between Kepler and Leibniz has been noted by other commentators, as well. For instance, see Martens 2000, p. 173. [↑](#endnote-ref-14)
14. In his *De quantatibus libelli*, Kepler speaks of quantity generally as a principle of creation and intelligibility but prioritizes geometrical quantity in most of his other works. For a thorough discussion of this point, as well as a broader treatment of the role of archetypes in Kepler’s theory of the soul, see Boner 2006 and Escobar 2008. For discussion of Kepler’s archetypes in his cosmology, see Field 1988 and for discussion of the role of archetypes in his metaphysics and epistemology, see Martens 2000 and di Liscia 2009. [↑](#endnote-ref-15)
15. Kepler discusses the importance of archetypes in apprehension of various aspects of the world. Most notably, he thinks they are crucial for human and non-human perception of harmony: “The ideas or formal causes of the harmonies… are completely innate in those who possess this power of recognition; but the are not after all taken within them by contemplation, but rather depend on a natural instinct, and are innate in them, as the number… of the leaves in the flower and of the segments in a fruit are innate in the forms of plants” (Kepler 1937-, vol vi, 226/1997 305). For a discussion of Kepler’s philosophy of mind and the epistemological role of the geometrical archetypes, see Barker 1997, Boner 2005, Claessens 2011, Escobar 2008, Jardine 1984, & Regier 2013. [↑](#endnote-ref-16)
16. It should be noted that my use of the term ‘pessimism’ is not meant to be evaluative, but rather is meant to bring out a particular kind of skepticism that a hypothetical theorist might harbor. In this hypothetical scenario, a theorist might agree with the realist about the aim of generating hypotheses (i.e. accurately describing target systems), while believing that the aim is impossible or unfulfillable (hence, being ‘pessimistic’ about the ultimate success of the project). I am not saying that Kepler held this kind of position – indeed, part of my aim here is to show that he was concerned with combatting it. The point of raising the specter of the ‘pessimistic realist’ is rather to bring out how certain theoretical criteria can still fail to provide a guarantee of confirmation. As I argue in this section, Kepler is concerned with providing such a guarantee and his geometrical epistemology is meant to fulfill this role. [↑](#endnote-ref-17)
17. Others have noted the importance of these definitions for Kepler’s project. Martens mentions the importance of knowability in the *Harmonices Mundi*, but only notes one side of the formulation, which doesn’t capture his emphasis on the match between knower and known object (Martens 2000, 118). Field also notes the importance of Kepler’s definition of ‘to know’ (Definition VII), but similarly only emphasizes one side of the ‘to know’/’known object’ pair of definitions (Field 1988, 102). Escobar captures both sides of the formulation in his discussion of Kepler’s theory of the soul (Escobar 2008, 29-30), but emphasizes Kepler’s focus on the possibility of knowledge. My discussion here diverges from Escobar’s in that I argue that Kepler is not only concerned with showing how knowledge is possible, but also with its justification and confirmation. Thus, while I am certainly not the first to note the importance of these definitions for Kepler, I take myself to be bringing out an aspect of their importance that has not been fully captured elsewhere. This pertains directly to the larger overall aim of the paper that is to emphasize the epistemological role of Kepler’s geometry in the *Harmonices Mundi* in the context of worries regarding underdetermination. [↑](#endnote-ref-18)
18. Kepler uses this kind of language to describe knowledge acquisition in several of his works. For instance, in his *De quantatibus* he tells his reader there that “measure is that by means of which it is known what a quantity is, and Aristotle shows that one is the measure of quantities” (Kepler 1858-1872, vol viii, 148/Cifoletti 1986, 223). While he makes this point in the context of a discussion of Aristotle’s *Metaphysics* X, it seems like much of the interpretation is Kepler’s own. [↑](#endnote-ref-19)
19. Kepler elaborates on this in several places in the *Harmonices Mundi*. For instance, in the Introduction to Book I, he writes “finite things which are circumscribed and shaped can also be grasped by the mind; infinite and unbounded things, insofar as they are such, can be held in by no bonds of knowledge, which is obtained from definitions, by no bonds of constructions” (Kepler 1937, vol vi, 15/1997, 9).It is worth pointing out that the Greek word for “definition” that is used in Euclid’s *Elements*, ὅροι, can also be translated as “circumspection” or “boundary,” and that, following this, we can understand definitions as being formed through imposing limits and distinctions. [↑](#endnote-ref-20)
20. “I shall adduce the affinity… of these souls, even the inferior ones, with the circle, in accordance with which, as with a rule or law, they have been arranged and shaped…” (Kepler 1937-, vol vi, 226/1997 308). This view emerges from Kepler’s understanding of how God’s divine light emanates in a sphere, which is then cut by a plane surface. This surface represents the body or corporeal form, while the circle that is created by the intersection between the surface and the sphere serves as “a true image of the created mind” (Kepler 1937-, vol vi, 224/1997 305). The view makes an appearance throughout Kepler’s work (for instance, he discusses it in the opening to his *Ad Vitellionem paralipomena* (Kepler 1937-, vol ii 6-7/Donahue 2000, 19-20)). The view is heavily influenced by Plato’s *Timaeus*, which Kepler references throughout the *Harmonices Mundi*, as well as by the neo-Platonic works of Proclus, which he also discusses favorably in numerous places (e.g. his lengthy citation of Proclus’ commentary on Book I of Euclid’s *Elements* in Book IV (Kepler 1937-, vol vi 218-221/ADF 298-301)). For a discussion of Proclus’ influence on Kepler, as well as Kepler’s departure from Proclus’ views, see Regier 2016. [↑](#endnote-ref-21)
21. It is worth noting the seeming peculiarity of the notion of differing “degrees” of knowability or intelligibility. *Prima facie*, it seems like something is either intelligible or not – intelligibility isn’t the kind of thing that comes in degrees. Furthermore, the whole notion of differing degrees of knowability is itself somewhat unintelligible. Does it mean that all objects that admit of a degree of knowability are only partially knowable, with some unknown remainder or property? Or are they all knowable *simpliciter*, but in different ways? I believe Kepler holds some version of the latter, where there are two categories, the knowable and the unknowable, and the degrees of knowability are species of the knowable. What makes something possess a higher or lower degree of knowability is its geometrical remoteness from the measure and the difficulty of the demonstration of its relation to that measure. As we will see below, Kepler employs these degrees to explain differences in the perceptual quality of the musical harmonies, providing an archetypal foundation to support and confirm Kepler’s theory of musical harmony. For a discussion of degrees of intelligibility in a different context, see Della Rocca 2013. [↑](#endnote-ref-22)
22. A quantity is expressible or expressible in square, respectively, “when if the diameter is divided into a certain definite number of equal parts, or its square is similarly divided, then the line or plane surface we are given is equal to one or more such parts” (Kepler 1937-, vol vi, 23/ADF 20-21). [↑](#endnote-ref-23)
23. Kepler provides constructions of the octagon, the “hekkaedecagon” (sixteen-sided figure), the hexagon, the dodecagon, and the decagon. These figures are related to the figures mentioned above by relations of successive doubling of sides, where if the original figure is constructible and has a prime number of sides *n,* any figure with 2*n* sides will also have a proper construction according to Kepler’s definition (Kepler 1937-, vol vi, 22/1997, 19). [↑](#endnote-ref-24)
24. For instance, he shows of the tetragon that “if it is inscribed in a circle this description is of the third degree of knowledge, the description of its square is of the second degree, and so is that of the area of the figure” (Kepler 1937-, vol vi, 36/1997 41). [↑](#endnote-ref-25)
25. Kepler’s definitions of the degrees of knowledge are largely based on Book X of Euclid’s *Elements*. What is original to Kepler, however, is his reorganization of Euclid’s discussion of commensurable and incommensurable magnitudes into an explicitly epistemological hierarchy, as well as the way he synthesizes it with the construction of the regular polygons. For a detailed discussion of Kepler’s mathematical contributions in the first two books of the *Harmonices Mundi,* see Aiton et al’s introduction to Kepler 1997, xxiv-xxvii and Field 1988, 103. [↑](#endnote-ref-26)
26. In Kepler’s words, “[Souls] take joy in the harmonic proportions in musical notes which they perceive, and grieve at those which are not harmonic. From these feelings of the soul the former (the harmonic) are entitled consonances, and the latter (those which are not harmonic) discords” (Kepler 1937-, vol vi, 105/1997, 147). [↑](#endnote-ref-27)
27. It is worth noting a couple of things here. First, almost all theorists at this time agreed on which intervals belonged to the consonant class. However, while we might attribute this agreement to cultural context and time-period, the consonance/dissonance distinction persists today, with the intervals in each class remaining relatively unchanged. There is also some evidence for agreement across cultures regarding which intervals are consonant, especially when considered in isolation (see Kameoka & Kuriyagawa 1969, Krumhansl 1990, Trainor & Heinmiller 1998, Zentner & Kagan 1998, Deutsch 2013, Bidelman & Krishnan 2009, and Bowling et al. 2017). [↑](#endnote-ref-28)
28. That is, the octave (2:1) from the diameter, the fifth (3:2) from the trigon, the fourth (4:3) from the tetragon, the major third (5:4) and the major sixth (5:3) from the pentagon, the minor third (6:5) from the hexagon, and the minor sixth (8:5) from the octagon. Kepler’s account of musical consonance is considerably more nuanced than is represented here, but I have refrained from a full discussion of it for considerations of space. For a detailed account, see Walker 1967. [↑](#endnote-ref-29)
29. The completeness of the set of consonant intervals is guaranteed (at least by Kepler’s lights) by his attempted proof of the impossibility of constructing figures with a prime number of sides higher than five. [↑](#endnote-ref-30)
30. Kepler’s long-standing concern with providing a geometrical account of musical harmony is noted by many Kepler scholars. For instance, Field makes note of this: “Kepler’s interest in giving a geometrical explanation of musical ratios dates back to the time when he was writing the *Mysterium Cosmographicum*” (Field 1988, 112). However, my point here is not so much to point out Kepler’s interest in this regard, but rather to show that certain explanatory features in his account of consonance mirror explanatory features of his polyhedral hypothesis in the *Mysterium*. [↑](#endnote-ref-31)
31. There is not space for a full comparison of Kepler’s theory of consonance with his contemporaries here. However, a detailed discussion of this point, and the relative success of Kepler’s theory, can be found in Cohen 1984, 13-34. [↑](#endnote-ref-32)
32. “To the same extent as the construction of a side is remote from the first degree, the consonance of a part of a circle, cut off by the side, with the whole circle, deviates from the most perfect consonance of unison; or, the allotted place of the figure of which it is the side among other figures is the same as the place of that consonance among the others” (Kepler 1937-, vol vi, 103/1997, 144). [↑](#endnote-ref-33)
33. For a comprehensive and detailed discussion of Book V of the *Harmonices Mundi*, see Stephenson 1994. [↑](#endnote-ref-34)
34. The foretelling that Kepler refers to is found in the *Mysterium*, where he discusses the potential causes of the eccentricities of the orbits. In a note on the second edition, he states that this is to be found in the harmonic ratios, which is captured in Book V of his *Harmonices* (Kepler 1937-, vol i, 103/1981, 187). [↑](#endnote-ref-35)
35. This is made even more apparent in the closing of the introduction, where he tells the reader it is his pleasure “to taunt mortal men with the candid acknowledgment that I am stealing the golden vessels of the Egyptians to build a tabernacle to my god from them” (Kepler 1937-, vol vi, 290/1997, 391). [↑](#endnote-ref-36)
36. Kepler uses the phrase “extreme motions” to refer to the motion of a planet at aphelion and perihelion. The extreme motions can be related between two planets in one of four ways: the “converging motions,” referring to the motion of the superior planet at perihelion and the inferior at aphelion, the “diverging motions,” referring to the motion of the superior planet at aphelion and the inferior at perihelion, and the motions “in the same region” [*eiusdem plagae*], referring to the motions of both planets at either aphelion or perihelion. The proportion of the planets’ converging motions will be the smallest and that between their diverging motions will be the largest. [↑](#endnote-ref-37)
37. This assignment corresponds to that given earlier in Chapter 2 of the *Harmonices Mundi* and in the *Mysterium.* [↑](#endnote-ref-38)
38. The major and minor thirds (4:5 & 5:6) are ruled out altogether by the polyhedral hypothesis: the smallest ratio between the spheres of the regular polyhedra obtains between those of the dodecahedron and icosahedron, which are larger than these proportions. If, according to Axiom II, the polyhedra determine the distance between the planets, these harmonies cannot be found between any planetary pairs. Similarly, the perfect fourth (3:4) can only be found in case the sum of the two planets’ proper proportions (i.e. their combined extreme motions) is greater than a fifth (Kepler 1937-, vol vi, 332-333/1997, 454-455). The proofs for these propositions also rely on details of Kepler’s astronomical theory in Chapter 3 and, specifically, his third (or harmonic) law of planetary motion. [↑](#endnote-ref-39)
39. This proposition follows directly from Axiom X, which states that when a choice must be made, the greater or more ‘perfect’ harmony should be assigned to the superior planet. Given Saturn and Jupiter’s combined motions of a fifth (2:3) and the fact that the only way to harmonically divide the fifth is into a major and minor third (4:5 & 5:6), the larger and more perfect is assigned to Saturn. [↑](#endnote-ref-40)
40. Stephenson calls it “an argument less convincing… than any other in book 5” (Stephenson 1994, 194). [↑](#endnote-ref-41)
41. As Stephenson notes, their proportions amount to 1: √15-6√5. [↑](#endnote-ref-42)
42. This is captured in Chapter 4 of Book V, where Kepler considers various parameters between observable relations of the planets as candidates for locating the harmonic proportions. He ultimately settles on the diurnal arcs (i.e. angular motion) of the planets apparent from the perspective of the sun, which correspond almost exactly to the assignments based on the polyhedral hypothesis. The only discrepancy is in Mars’ proper proportion: when assigning the closest harmonic proportions in Chapter 4 based on astronomical theory alone, Mars is assigned 2:3 (i.e. a fifth) but, as we saw above, this is changed in the derivation in Chapter 9 to 18:25 (i.e. slightly larger than a fourth). Martens includes a helpful table comparing the results of the arguments in Chapter 9 with the results from earlier chapters of Book 5 (Martens 2000, 129). [↑](#endnote-ref-43)
43. This refers to the two types of scales that were used in the musical tradition of Kepler’s time: the *durus* scale (corresponding roughly to our contemporary major scale) is one that includes a major third and sixth, while the *mollis* (corresponding roughly to our minor scale) includes a minor third and sixth. He provides a construction of these two kinds of scales in Chapter 6 of Book 3. Kepler justifies both Axioms XVIII and XXI with reference to the more general principle of variety in harmonic motions in Axiom I (Kepler 1937-, vol vi, 339/1997, 462). [↑](#endnote-ref-44)
44. Technically, this difference could be made either by the major and minor thirds or the major and minor sixths. The thirds are ruled out in Proposition VI due to constraints from the polyhedral hypothesis, so the sixths must do this work in the celestial harmonies. [↑](#endnote-ref-45)
45. The “melodic intervals” are those that, while not consonant when sounded simultaneously, are acceptable in succession for the formation of single-line melodies: “there is a great difference between the dissonant intervals, so that not only are the consonant intervals taught to us by Nature and approved by hearing at her prompting, but other smaller intervals are also established by the same sense which although they are dissonant are yet suitable for conveying melody” (Kepler 1937-, vol vi, 125/1997, 173). Kepler derives these from the consonances, writing “therefore the consonant intervals are by nature prior to the smaller intervals which we name melodic; and they are not composed of melodic intervals as if of elements… but on the contrary the melodic intervals arise from the consonances, as if from causes” (Kepler 1937-, vol vi, 126-127/1997, 174). These melodic intervals include the major tone (8:9), minor tone (9:10), semitone (15:16), and diesis (24:25). [↑](#endnote-ref-46)
46. The comma (80:81) arises from the difference between the major and minor tones (Kepler 1937-, vol vi, 129/1997, 178). [↑](#endnote-ref-47)
47. “And harmonic beauty urges that these planets’ own proportions, if owing to their small size they cannot be harmonic, should at least be among the melodic, if that is possible by Axiom 1” (Kepler 1937-, vol vi, 342/1997, 465). [↑](#endnote-ref-48)
48. For instance, Saturn and Jupiter’s respective proper proportions must be adjusted by small intervals (i.e. a comma is added to Saturn’s proportion and an interval of a little larger than a comma, about 62:63, is added to Jupiter’s proportion), as well as the proportion of the diverging motions of Jupiter and Mars (Proposition XL), Mars’ proper proportion (XLI), the diverging motions of Mars and Earth (XLII), and Mercury’s proper proportion (XLV). [↑](#endnote-ref-49)
49. For the reference in Ch. 3, see Kepler 1937-, vol vi, 300/1997, 409. [↑](#endnote-ref-50)
50. The reader might wonder how the derived eccentricities of the planetary orbits square with the empirical data available to Kepler. In the demonstration, Kepler merely states that “all of [the numbers expressing the converging distances of the pairs of planets] approach very closely the distances which I have found from the observations of Brahe” (Kepler 1937-, vol vi, 359/1997, 486). However, as is pointed out by several commentators, Kepler’s derivations are indeed quite close to the empirical measurements from Brahe’s data. According to calculations provided by Field, the largest discrepancy occurs for Jupiter’s aphelion distance (with 2.1% degree of error). The only other notable discrepancy occurs for Mercury’s aphelion distance (1.4%), but all others are under a single percentage point. See tables comparing the empirical measurements to Kepler’s derivations in Field 1988, 162 and Stephenson 1994, 226-227. Martens concurs (Martens 2000, 136). [↑](#endnote-ref-51)
51. [↑](#footnote-ref-2)
52. It is worth pointing out that Kepler retains this line of reasoning later in his *Epitome astronomiae Copernicanae*. In Book IV, he raises the question, “If the intervals [between the planets] approach so nearly the ratios of the figures [of the Platonic solids], why then does some discrepancy remain?” His answer is captured in the following:

    Because the archetype of the movable world is constituted not only by the five regular [solid] figures – by which the chariots of the planets and the number of courses were determined -- but also of the harmonic proportions with which the courses themselves were attuned, as it were, to the idea of celestial music or of a harmonic concord of six voices. Now since this musical ornamentation demanded a difference of movement in any given planet – a difference between the slowest and the fastest movement; and this difference is made by the variation of the interval between the planet and the sun; and since the magnitude or ratio of this variation was required to be different in different planets; hence it was necessary that some very small amount should be taken away from the intervals which are exhibited by the figures as uniform and without variation, and that it should be left to the freedom of the composer to represent the harmonies of movement (Kepler 1937-, vol vii, 275/1995, 31).

    Kepler also refers the reader to the *Harmonices Mundi*, indicating that he sees himself as showing this point successfully there. Thanks to an anonymous reviewer for bringing this point to my attention. [↑](#endnote-ref-52)