On Otávio Bueno on identity and quantification

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Abstract

Otávio Bueno has pointed an interesting and relevant topic about quantification. He stresses that the meaning of quantifiers can have sense if and only if the entities being quantified have well defined identity conditions. In this paper we discuss his view and show that this is not the case in all situations, being quite reasonable to quantify also over entities to which identity conditions fail to hold.

Keywords: quantification, identity, individuals, non-individuals, quasi-sets, Otávio Bueno.

"[A] Bose-Einstein condensate [is] a cloud of atoms *all* occupying the same quantum mechanical state."

W. Ketterle et al. [Ketterle et al., 1999], my emphasis

1 Introduction

The quotation of the epigraph came from a Nobel Prize winner (W. Ketterle) and collaborators. Ketterle received the prize working with bosonic condensates. In such condensates, an wide quantity of bosons are cooled to quite near the absolute zero and in such a situation, they became 'a big wave', behaving in unissono, all in the same quantum state [Leggett, 2009]. There are no differences among the bosons, not even in principle (see the above paper, and also [Krause, 2024, **?**]).

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In the quotation it is emphasised that in forming such quantum systems, the indistinguishability of the involved 'particles' is important, even essential. And, as we can see, it has a reference to *all* elements in a BEC (a Bose-Einstein condensate), that is, they are making use of an universal quantifier being applied to entities that cannot be discerned from one each other. In such situations, bosons don't have well defined identity conditions; more than that, we cannot attribute an identity to them in any way, and in particular they cannot be counted in the standard sense. Thus, this sample case shows that we really make use of quantifiers even for entities to which the notion of identity is in fault and which cannot be counted. We shall consider this situation with more care in that what follows. Another example involving the existential quantifier will be considered soon.

Otávio Bueno is a bright philosopher who has contributed a lot on logical and metaphysical foundations of quantum physics. In this discussion about quantifiers, he addresses that the meaning of quantification requires that the quantified entities do have identity conditions and need to be counted [Bueno, 2023]. His argumentation is clear and well done but in my opinion, despite he considers also 'non-standard' cases, is grounded on a *classical* way of thinking. In the next section I will revise some of the main points of his arguments. Then I show that quantification makes sense also for entities to which the standard notion of identity fails do hold; in particular, these entities cannot be counted in the usual way. The relevant cases are, of course, taken from quantum physics.

2 Standard quantification

In classical logic (and also in most logical systems), there is an analogy between $\forall$, $\exists$, and $\wedge$, $\vee$ [Kleene, 1952, p.177]. When interpreted in a finite non-empty domain $D = \{a_1, \ldots, a_n\}$, the formula $\forall x \alpha(x)$ is synonymous with $\alpha(x_1) \wedge \ldots \wedge \alpha(x_n)$, where $x_1, \ldots, x_n$ are formal names for $a_1, \ldots, a_n$ respectively. In the same vein, $\exists x \alpha(x)$ is synonymous with $\alpha(x_1) \vee \ldots \vee \alpha(x_n)$. Thus it is quite clear what is to mean that the elements of the domain need to have well defined identities. The infinite case would require infinitely long formulas, but the reasoning is the same: quantification seems to require identity; when we say that *some* entity has a certain property, we generally don't wish to say that all of them have the property, so some distinction among them need to be considered.

The semantics of quantification is also something that requires identity, as recalled by Bueno. Standard semantics, that is, one made in a standard set theory such as ZFC, says that given a structure $\mathfrak{A}$ with domain $D \neq \emptyset$, then a sequence s of elements of D satisfies the formula $\forall x \alpha(x)$ iff every sequence s' of elements of D that differ from s in at most the component that corresponds to x satisfy $\alpha(x)$. The condition for the existential quantifier is given accordingly [Mendelson, 1997, pp.59-60]. As Bueno observes, in order a sequence to *differ* from another in some place identity is required.

We observe that this is so in *standard* semantics, and similar things can be concluded in any kind of semantics we take into account. But Bueno himself defined in [Bueno, 2000] a semantics for a certain quantificational logic constructed in the *theory of quasi-sets* [de Barros et al., 2023, French and Krause, 2006] where there may exist entities devoid of identity conditions, and one can see there how to introduce corresponding notions of satisfaction and truth for domains with elements *without* identity. Apparently, in his new works, he didn't considered his own previous paper. More on this below.

3 Bueno's arguments and comments

Bueno is one of the few philosophers who, when speaks about identity, says what he understand by this notion. According to him, identity is an equivalence relation that obeys substitutivity. Let us accept this first-order definition. His arguments go also to claim that "quantification requires counting". We will start with this remark.

Bueno didn't say what is counting. But from his general ideas, we can presume that to count some things is to give an association from the things to an initial segment of the natural numbers, say from 1 to n. Since he not always work in a set-theoretical framework, we shall avoid to say that such an 'association' can be characterised by an injective function leaving it to an informal level. But it is clear that such a counting requires identity of the counted things.

Comment — Take the fingers of my left hand and let us count them. I can think of an informal and intuitive association such as 'thumb $\leftrightarrow$ 1', 'index $\leftrightarrow$ 2', 'middle $\leftrightarrow$ 3', 'ring $\leftrightarrow$ 4' and 'little $\leftrightarrow$ 5'. The association of course requires that the fingers have identity and that they are different from one each other. Furthermore, we can elaborate quantificational sentences such as 'there exists a finger in my left hand which is named "the ring finger" ' or then 'all fingers of my left hand are distinct from one each other'.

The most precise way of counting a number of elements is by means of the resources of set theory. Firstly we represent the elements as members of a set and them use the fact that to every set we can associate an ordinal (in the presence of the Axiom of Choice); the least ordinal equinumerous with the set is its cardinal. But this move involve several assumptions, such as that the elements to be counted are members of a set, which makes them distinct from one each other, that we can define a bijection between the set and the cardinal, etc. In my opinion, we can apply mathematics (and logic) to reality only in very restrictive cases; the general account requires *representation* by means of mathematical structures (usually, sets).

But we can also form quantificational sentences involving elements that are not *individuals*; to understand what I mean, let me define *individual* as something that obeys the theory of identity defined above (let us term it 'STI' for Standard Theory of Identity). One of the main characteristics of such a theory is that every object that satisfies it *preserves* its identity in different contexts or time;

so we can use H. Reichenbach's word borrowed from the Gestalt psychologist Kurt Lewin, *genidentity*: individuals have *genidentity*, what means that they are always *the same* in different contexts. If I have made reference to my ring finger once, when I refer to it again I shall be referring to *the same finger* and, more importantly, I can identify it again in different contexts, say when washing my left hand. I shall say that individuals do enable *re-identification*.

But now consider the two electrons in a neutral Helium atom. They have a quantum property called 'spin', which can be measured in a given direction and produce as outputs either UP or DOWN but never both at once (these are just names for the distinct outputs). Then, consider a direction, the z-direction; we can state that 'there exists one electron (of the He atom, etc.) with spin UP in the z-direction', which is a quantificational sentence. Quantum physics says that before measurement, the state of the join system of the two electrons is an *entangled* state; this means that the state cannot be 'factorized' (or simply 'separated') into two states, one for each electron. The system must be seen as a whole. Only after the measurement we can know the direction of the spin of the measured electron and if one of them is UP, the other will be DOWN.

Can we count these electrons before measurement? Not at all! There is no sense in defining an association from the electrons and the natural numbers 1 and 2. Of course we can numerate them or call them 'Peter' and 'Paul', but these numbers or names are not proper names according to standard semantics; they do not act as rigid designators, being nothing more than *mock names*, provisory labels that mean nothing at all. Even if we define an association 'Peter $\leftrightarrow$ 1' and 'Paul $\leftrightarrow$ 2', we cannot make sense to the idea that we can know which electron is Peter and which electron is Paul, contrary to what happens with my fingers and with individuals in general.

Furthermore, we can *ionise* the neutral atom and expunge an electron, getting a cation (a positive ion) He^+. The cation has now just one electron and you can ask: which electron was expunged? Peter or Paul? Quantum physics will say that such a question has no sense. Furthermore, the expunged electron does not preserve its supposed identification; once out of the atom, never more we shall be able to 'take *it* again': electrons, so as all quantum entities, do not have *genidentity*. Even if we make the cation to absorb an electron turning an He neutral atom again, we shall never be able to say that the captured electron is *the same* than the expunged one or that the 'new' neutral atom is the same as the old neutral atom we had before, despite they have all the same properties. It should be beyond doubt that Leibniz's Principle of the Identity of Indiscernibles (PII) is being questioned here.

In most situations, there is no counting in the quantum domain. Even in the case of fermions such as electrons, which must obey Pauli's Exclusion Principle which says that they cannot have the same quantum numbers, it would be wrong to say that they are 'different' by this reason. In fact, despite presenting a difference, which makes them *distinguishable*, this does not entail that they are individuals having identity. Let us exemplify in order to make things clear.

Suppose now that we have an atom with many electrons. Chemistry tells us how to know how many electrons there can be in each energy level, that is, how to attribute a quantity, a natural number, *without* counting or identifying the electrons. For instance, how can we know that in the second energy level there can be eight electrons? The answer is done by quantum numbers, which result from the solutions of the suitable Schrödinger equation.[1] In the case of the second energy level, we have $n = 2$ (the principal quantum number, indicating the level), $\ell = 0, 1$ (the angular momentum quantum number), $-\ell \leq m_\ell \leq \ell$ (the magnetic quantum number) and $m_s \in \{1/2, -1/2\}$ (the spin quantum number). So, for $\ell = 0$, $m_\ell = 0$ and $m_s \in \{1/2, -1/2\}$, which gives us two electrons. If $\ell = 1$, then $m_\ell \in \{-1, 0, 1\}$ and $m_s \in \{1/2, -1/2\}$ for each value of m_ℓ, which gives us six electrons. Then, 2+6=8 and we have eight places for electrons, *whatever they are*: the important thing is that the places are occupied with electrons; their 'identities' do not matter (by they way, they don't even exist!). Note: no counting, no identities: just *kinds* (of things) and *quantities*.

But we can form the quantificational sentence 'There exists an electron in the second energy level with quantum numbers $n = 2$, $\ell = 0$, $m_\ell = 0$ and $m_s = 1/2$'. Does this description provide an identity to the sole electron obeying it? Again, not at all. Electrons can jump from one energy level to another and even if some electron goes back to its original level, we shall never be able to say that it was *the same* electron that has jump and went back! We can have quantificational sentences with no identity.

Some could say that the referred quantum numbers give an identity to the electron *while it is there*, that is, while it is in the referred shell. But once we agree that in order to be qualified as an individual something must obey re-identification (if Julius Caesar was not *the same man* while in Rome and while in Egypt, we surely would have troubles with our historical explanations), but quantum entities don't have it. So, despite *temporarily* isolated, it cannot be qualified as something having identity, which would entail that it could be re-identified other times.

The view that quantum entities are viewed as *non-individuals*, meaning that they are not individuals obeying STI, a notion borrowed from some founding fathers of quantum mechanics such as Schrödinger, Weyl, Born and others, was termed the Received View (RV) [French and Krause, 2006, French, 2019].[2] But we need to take into account that the RV claims that it is *that* notion of identity formalised in the STI that would not hold for such entities. The reason is simple: if they obey STI, in being more than one they are necessarily *different* and this implies the existence of a predicate which is satisfied by just one of them; remember that some form of Leibniz's PII holds in STI: 'complete' indistinguishability, that is, agreement with respect to *all* properties, implies identity and reciprocally. But the given case of Bose-Einstein condensation clearly shows that this is not always the case in the quantum domain.

[1] A very clear explanation about 'how quantum numbers arise from the Schrödinger equation' can be found in the HyperPhysics homepage: `http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html`.

[2] These forerunners of quantum mechanics didn't speak of STI or of identity in logical terms. But we should acknowledge that the core idea of STI is to formalise the intuitive idea of identity as that thing an object has that makes it the object it is.

The RV is grounded on a mathematical theory called *quasi-set theory* (for details, see [French and Krause, 2006, de Barros et al., 2023]). This theory admits the existence of *quasi-sets* ('qsets') whose elements may be completely indiscernible and such that the standard notion of identity (STI) does not hold for them. Even so, the qset may have a cardinal, its *quasi-cardinal*. We shall not discuss here the way to attribute a cardinal to a collection of things that do not have identity, but indicate [Krause, 2024, Wajch, 2023]. One of the primitive notions of the theory is *indistinguishability*, treated as a binary relation '$\equiv$' which has the properties of an equivalence relation, but fail to obey substitutivity; hence, *it is not* the identity of STI.

Bueno claims that in postulating that $\forall x \forall y(x \equiv y \to y \equiv x)$, the theory is incurring into a mistake, since the variables would range over qsets requiring identity. Our answer is as follows. Take quantum physics once more, which is the paradigmatic case of the RV. When we say (by means of a quantified sentence) that for a neutral Sodium atom whose electronic decomposition reads $1s^2\ 2s^2\ 2p^6\ 3s^1$, we can say that all the electrons are either in the levels I or II or III, but we are not requiring their identities in the sense of STI. The sentence would be written as (with obvious meanings and the variable ranging over the qset of the electrons of the atom)

$$\forall x(x \in Na \to x \in I \vee x \in II \vee x \in III), \tag{1}$$

that is, a quantified sentence with no identified (as individuals) the quantified objects.

Bueno also mentions the reflexive law, namely, $\forall x(x \equiv x)$, saying that the variable x would refer to the same object in all occurrences. Well, syntactically, this sentence is equivalent to $\forall y(y \equiv y)$, and we could put in question whether x and y are referring to 'the same' things. But within the standard understanding, of course they are. Our remark is just that in saying that, Bueno is speaking semantically, so we need to interpret the sentence and of course it would be a categorical mistake to claim that the interpretation would assume some standard *set* whose elements have identity.

Comment — In a join paper [da Costa et al., 1995], Bueno says the following when discussing semantics for non-classical logical systems:

> "a set theoretical semantics for a non-classical logic (. . .) being constructed within classical set theory, (. . .) reveals itself, from a philosophical perspective, completely unsatisfactory. One reintroduces, so to speak, by the backdoors, exactly what was intended to be left on the entrance!

This is in fact true; a 'classical semantics' is usually grounded on classical logic, so departing from the very intention of the non-classical system. The semantics would be consonant with the logic. Their case is not one involving identity or the lack of it, but applies quite well to this case. If a semantics for a logic where the identity of STI is being left out for some entities is elaborated in a 'classical set theory' such as the ZFC system, we will enabling the identity to enter by the backdoor! We need to read differently, as the mentioned authors suggest.

In our sample case, when we say that the indistinguishability relation applies to *all* elements of a quasi-set, even if they lack identity conditions, we mean precisely this: *it holds for all elements*. As the philosopher Joseh Melia said, "if logician's '∀' doesn't mean 'all', what does it mean?" [Melia, 1995]. Even if the quasi-cardinal of the qset is finite, say n, in this case there is no the 'analogy' with ∧ put before; it has no sense to associate the universal quantifier with a sentence of the form $\alpha(x_1) \wedge \ldots \wedge \alpha(x_n)$ since the x_j do not act as names of the entities. The interpretation of the quantifiers must be different.

The interested reader can find in [French and Krause, 2006, Chap.8] a semantics for an intensional 'Schrödinger logic' built in the theory of quasi-sets; there, it is shown how quantified sentences behave semantically in the presence of entities devoid of identity conditions.

4 Conclusion

When leaving in a different world, we need to adapt ourselves to that world. It would be an error to try to bring to the new land our atavistic ideas and methods which were, as Einstein would said, "imbibed with their mother's milk"; we need to leave them. The XXth century brought us many non-classical logical systems which depart from such atavistic concepts grounded on classical ways of reasoning, such as the principles of non-contradiction, excluded middle, the Boolean behaviour of the logical connectives, and several others. But the taboo concerning that idea that everything must be endowed with identity still remains in most philosopher's minds. Perhaps it is time to leave this credo too.

References

[Bueno, 2000] Bueno, O. (2000). Quasi-truth in quasi-set theory. *Synthese*, 125:33–53.

[Bueno, 2023] Bueno, O. (2023). Identity and quantification. In Arenhart, J. R. B. and Arroyo, R. W., editors, *Non-Reflexive Logics, Non-Individuals, and the Philosophy of Quantum Mechanics: Essays in Honor of the Philosophy of Décio Krause*, Synthese Library, 476, pages 179–190. Springer, Cham, Switzerland.

[da Costa et al., 1995] da Costa, N. C. A., Bueno, O., and Béziau, J. Y. (1995). What is semantics? A brief note on a huge question. *Sorites*, 03:43–47.

[de Barros et al., 2023] de Barros, J. A., Holik, F., and Krause, D. (2023). *Distinguishing indistinguishabilities: Differences Between Classical and Quantum Regimes*. Springer, forthcoming.

[French, 2019] French, S. (2019). Identity and Individuality in Quantum Theory. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2019 edition.

[French and Krause, 2006] French, S. and Krause, D. (2006). *Identity in Physics: A Historical, Philosophical, and Formal Analysis*. Oxford Un. Press, Oxford.

[Ketterle et al., 1999] Ketterle, W., Durfee, D. S., and Stamper-Kurn, D. M. (1999). Making, probing and understanding bose-einstein condensates. *arXiv:cond-mat/9904034v2 5 Apr 1999*.

[Kleene, 1952] Kleene, S. C. (1952). *Introduction to Metamathematics*. Bibliotheca Mathematica, vol.1. North-Holland, Amsterdam and New York.

[Krause, 2024] Krause, D. (2024). On Dieks against the Received View. Preprint.

[Leggett, 2009] Leggett, A. J. (2009). Bose-Einstein condensation. In Greenberg, D., Hentschel, K., and Weinert, F., editors, *Compendium of Quantum Physics: Concepts, Experiments, History and Philosophy*, pages 71–74. Springer, Dordrecht, Heidelberg, New York, London.

[Melia, 1995] Melia, J. (1995). The significance of non-standard models. *Analysis*, 55(3):127–134.

[Mendelson, 1997] Mendelson, E. (1997). *Introduction to Mathematical Logic*. Chapman and Hall, London, 4th edition.

[Wajch, 2023] Wajch, E. (2023). Troublesome quasi-cardinals and the axiom of choice. In Arenhart, J. R. B. and Arroyo, R. W., editors, *Non-Reflexive Logics, Non-Individuals, and the Philosophy of Quantum Mechanics: Essays in Honor of the Philosophy of Décio Krause*, Synthese Library, 476, pages 203–222. Springer.