Locality Implies Reality of the Wave Function: Hardy's Theorem Revisited

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Abstract

Based on an analysis of Hardy's theorem, I argue that the reality of the wave function can be established based on a locality assumption for product states. Moreover, this locality assumption is weaker than the preparation independence assumption of the Pusey-Barrett-Rudolph (PBR) theorem. One implication of this new result is that it excludes the local ψ-epistemic quantum theories that evade the PBR theorem.

Key words: quantum mechanics; wave function; psi-ontic view; Hardy's theorem; PBR theorem; locality assumption

1 Introduction

The reality of the wave function has been a hot topic of debate since the early days of quantum mechanics. Is the wave function real, directly representing the ontic state of a physical system, or epistemic, merely representing a state of incomplete knowledge about the underlying ontic state? In recent years, a general and rigorous approach called ontological models framework has been proposed to distinguish the ψ-ontic and ψ-epistemic views (Harrigan and Spekkens, 2010). Moreover, several important ψ-ontology theorems that establish the reality of the wave function have been proved in the framework, two of which are the Pusey-Barrett-Rudolph (PBR) theorem (Pusey, Barrett and Rudolph, 2012) and Hardy's theorem (Hardy, 2013). These theorems are based on auxiliary assumptions, such as the preparation independence assumption for the PBR theorem, and the restricted ontic indifference assumption for Hardy's theorem. It is widely thought that the PBR theorem makes the strongest case for ψ-ontology. As the first ψ-ontology theorem, the PBR theorem has also been widely discussed in the literature. By comparison, Hardy's theorem has not received much attention. A possible reason is that the restricted ontic indifference assumption of Hardy's theorem is much stronger than the preparation independence assumption of the PBR theorem (Hardy, 2013; Leifer, 2014). In this paper, I will present a new analysis of Hardy's theorem. In particular, I will argue that when replacing the restricted ontic indifference assumption with a much weaker locality assumption for product states, the proof of Hardy's theorem can still go through.

The rest of this paper is organized as follows. In Section 2, I briefly introduce the ontological models framework in which Hardy's theorem is proved. In Section 3, I introduce Hardy's theorem and the result that the restricted ontic indifference assumption of the theorem can be derived from two sub-assumptions: an ontic state assumption and a locality assumption in the usual first-quantized description of quantum states. In Section 4, I analyze the ontic state assumption and argue that it is valid in both the first-quantized and the second-quantized descriptions of quantum states. In Section 5, I argue that although the restricted ontic indifference assumption cannot be derived from the locality assumption when considering the existence of the vacuum state in the second-quantized description of quantum states (as rightly pointed out by Leifer and Hardy), the reality of the wave function can still be proved based on the locality assumption in this case. In Section 6, I further analyze the locality assumption and argue that it is a locality assumption for product states and it is weaker than the preparation independence assumption of the PBR theorem. Conclusions are given in the last section.

2 The ontological models framework

Before presenting my analysis of Hardy's theorem, I will briefly introduce the ontological models framework in which the theorem is proved.

The ontological models framework provides a rigorous approach to address the question of the nature of the wave function (Harrigan and Spekkens, 2010).[1] It has two fundamental assumptions. The first assumption is about the existence of the underlying state of reality. It says that if a quantum system is prepared such that quantum mechanics assigns a pure state to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. This assumption is necessary for the analysis of the ontological status of the wave function, since if there are no any underlying ontic states,

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it will be meaningless to ask whether or not the wave functions describe them.

Here a strict ψ-ontic/epistemic distinction can be made. In a ψ-ontic ontological model, the ontic state of a physical system determines its wave function uniquely, and the wave function represents the ontic state of the system. While in a ψ-epistemic ontological model, the ontic state of a physical system can be compatible with different wave functions, and the wave function represents a state of incomplete knowledge – an epistemic state – about the actual ontic state of the system. Concretely speaking, the wave function corresponds to a probability distribution $p(\lambda|P)$ over all possible ontic states when the preparation is known to be P, and the probability distributions corresponding to two different wave functions may overlap.

In order to investigate whether an ontological model is consistent with the empirical predictions of quantum mechanics, we also need a rule of connecting the underlying ontic states with the results of measurements. This is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is only determined by the ontic state of the system, along with the physical properties of the measuring device. More specifically, the framework assumes that for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the predictions of quantum mechanics then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|P) = p(k|M, P)$, where $p(k|M, P)$ is the Born probability of k given M and P. A direct inference of this relation is that different orthogonal states correspond to different ontic states.

In recent years, there have appeared several ψ-ontology theorems in the ontological models framework which attempt to refute the ψ-epistemic view. They include the PBR theorem (Pusey, Barrett and Rudolph, 2012), the Colbeck-Renner theorem (Colbeck and Renner, 2012), and Hardy's theorem (Hardy, 2013). Leifer (2014) gives a comprehensive review of these ψ-ontology theorems and related work. The key assumption of the ψ-epistemic view is that there exist two nonorthogonal states which are compatible with the same ontic state (i.e. the probability distributions corresponding to these two nonorthogonal states overlap). A general strategy of these ψ-ontology theorems is to prove the consequences of this assumption are inconsistent with the predictions of quantum mechanics (under certain auxiliary assumptions). In the following, I will introduce Hardy's theorem and present my new analysis of the theorem.

3 Hardy's theorem

Hardy's theorem states that under the assumption of restricted ontic indifference, which says that the unitary transformation that leaves a wave function invariant also leaves the underlying ontic state invariant exists at least for one wave function, the reality of the wave function can be established in the ontological models framework (Hardy, 2013).[2]

The basic idea to prove Hardy's theorem can be illustrated with a simple example (Leifer, 2014). Consider two non-orthogonal states $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ and $|\psi_L\rangle$, where $|\psi_L\rangle$ and $|\psi_R\rangle$ are two spatially separated wave packets of a particle localized in regions L and R, respectively, at a given instant.[3] The reality of the wave function requires that these two non-orthogonal states should be not compatible with the same ontic state. In order to prove this result, we apply a local unitary transformation in the region R. For the superposed state $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$, this unitary transformation adds a phase π to the branch $|\psi_R\rangle$. Then the superposed state changes to its orthogonal state $\frac{1}{\sqrt{2}}(|\psi_L\rangle - |\psi_R\rangle)$. Since two orthogonal states correspond to different ontic states, the ontic state of the particle must be changed by the unitary transformation. Now assume that this unitary transformation, which leaves the wave function $|\psi_L\rangle$ invariant, also leaves its underlying ontic state invariant, namely restricted ontic indifference holds true for the wave function $|\psi_L\rangle$. Then, the ontic state of the particle will not be changed. This leads to a contradiction. Therefore, under the assumption of restricted ontic indifference, the two non-orthogonal states $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ and $|\psi_L\rangle$ cannot be compatible with the same ontic state, and thus they are ontologically distinct.

The proof of Hardy's theorem is uncontroversial. However, it is still unclear how to understand the assumption of restricted ontic indifference on which the theorem is based. As pointed out by Hardy (2013), restricted ontic indifference would follow (in the usual first-quantized description of quantum states) if (1) the ontic state of a localized particle exists in the region where the particle is localized, and (2) the ontic state of a localized particle in a region is not affected by unitary transformations implemented

[2]It is worth pointing out that Hardy's theorem can be proved based on a weaker version of the ontological models framework, in which the first reality assumption is kept but the second measurement response assumption, which says that the ontic state determines the probabilities of measurement results, can be replaced by a weaker possibilistic completeness assumption, which says that the ontic state determines whether any result of any measurement has probability equal to zero of occurring or not (Hardy, 2013). In addition, it is also worth noting that since Hardy's theorem concerns only a single copy of the system in question, it has no issues of the PBR theorem related to multiple copies, e.g. it can be extended to apply to relational quantum mechanics (cf. Oldofredi and Calosi, 2021).

[3]If one requires that the two wave packets are completely separated, one may consider two ground states in two identical boxes which are well separated in space.

outside this region. This result can be proved as follows. According to the first ontic state assumption, the ontic state of the particle whose wave function is $|\psi_L\rangle$ is localized in the region L. Moreover, according to the second locality assumption, the unitary transformation applied in the region R, which leaves the wave function $|\psi_L\rangle$ invariant, does not change the ontic state of the particle, which is localized in the region L. Thus, the assumption of restricted ontic indifference is true. In the following, I will present a more detailed analysis of the two assumptions from which restricted ontic indifference can be derived.

4 The ontic state assumption

The ontic state assumption says that the ontic state of a localized particle exists in the region where the particle is localized. In this section, I will analyze this assumption.

First of all, I will argue that the ontic state assumption is valid in the usual first-quantized description of quantum states. In this description, one assigns a wave function to a particle, and the wave function corresponds to a probability distribution over all possible ontic states. Then, if the ontic state of the particle exists in a region, the support of the particle's wave function must also include that region.[4] Conversely, if the support of the wave function of a particle is not defined in a region, the ontic state of the particle will not exist in the region either. In other words, the ontic state of a particle exists only in the support of its wave function in space. For a localized particle, this means that its ontic state exists in the region where (the wave function of) the particle is localized.

Next, I will argue that the ontic state assumption is also valid in the second-quantized description of quantum states. In this description, a localized wave function such as $|\psi_L\rangle$ in the previous example will be replaced by $|1\rangle_L |0\rangle_R$, where $|1\rangle_L$ represents the one-particle state in the regions L, and $|0\rangle_R$ represents the vacuum state in the region R. It has been claimed that the ontic state assumption is violated in this second-quantized description of quantum states (Leifer, 2014).[5] It seems that Hardy (2013) also agreed with this claim.[6] The reason is supposed to be that the vacuum ontic state existing in the region R should be also regarded as one part of the ontic state

[4] For the ψ-epistemic view, the support will include not only that region.

[5] Leifer (2014) identified this ontic state assumption as the assumption that local ontic state spaces compose according to the direct sum.

[6] Hardy (2013) wrote, "In the model of Martin and Spekkens there are ontic states associated with a path of an interferometer even when the particle goes along the other path (i.e. it violates (i) from the preceding paragraph)." In this quote, "ontic states" denotes the vacuum ontic states, and "(i)" denotes the ontic state assumption, which, according to Hardy, says that "all the ontic variables associated with a localized particle are "situated" in the region where that particle is localized" (Hardy, 2013).

of the particle localized in the regions L (If not, the ontic state assumption will be true). However, it can be argued that this reason is not convincing.

First, the total quantum state of the particle in the region L and the vacuum in the region R is a product state $|1\rangle_L |0\rangle_R$. If the vacuum ontic state in the region R is one part of the ontic state of the particle in space, then this quantum state should be a sum state such as $|1\rangle_L + |0\rangle_R$, like the previous superposed state $|\psi_L\rangle + |\psi_R\rangle$. Second, the properties of the vacuum state are not the same as those of any particle such as a photon or an electron. For example, an electron has mass and charge, while the vacuum state has no mass and charge. Third, the total probability of detecting the particle in the region L is already one. This indicates that the ontic state of the particle should exist only in the region L. If it also existed outside the region L such as in the region R, then either it has no efficacy of the particle so that the particle cannot be detected there (in this case, the ontic state would not belong to the particle) or it has the efficacy of the particle so that the particle can also be detected there (in this case, the total probability of detecting the particle in the whole space would be larger than one). Finally, the vacuum state in a region such as region R is the same for all particles outside the region. When assuming two independent particles being in a product state have independent ontic states, the vacuum ontic state in a region cannot be a part of the ontic state of any particular particle outside the region.

5 Is the vacuum state relevant?

Although the vacuum ontic state existing in one region is not one part of the ontic state of the particle localized in the other region, it is still one part of the ontic state of the whole system in both regions. This fact will indeed invalidate the proof of Hardy's theorem, as Leifer (2014) rightly pointed out. Let us see why this is the case.

In the second-quantized description of quantum states, the two non-orthogonal states in the previous example $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ and $|\psi_L\rangle$ will be $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$, where $|1\rangle_L$ and $|1\rangle_R$ represent the one-particle states in the regions L and R, respectively, and $|0\rangle_L$ and $|0\rangle_R$ represent the vacuum states in the regions L and R, respectively. In order to determine whether the these two non-orthogonal states are compatible with the same ontic state, we apply a local unitary transformation in the region R. As before, this unitary transformation changes the superposed state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ to its orthogonal state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$ and thus changes the underlying ontic state. However, for the state $|1\rangle_L |0\rangle_R$, although the ontic state of the particle in the region L (corresponding to $|1\rangle_L$) does not change according to the locality assumption, the vacuum ontic state in the region R (corresponding to $|0\rangle_R$) may be changed by the unitary transformation applied in the region R when assuming that the vacuum quantum state corresponds to more than one vacuum ontic state as the ψ-epistemic view assumes. Then, we cannot derive a contradiction as before, since the underlying ontic states may both change for the two non-orthogonal states. This also means that restricted ontic indifference cannot be derived from the ontic state assumption and the locality assumption and it is not valid either in the second-quantized description of quantum states.

Based on this argument, Leifer (2014) concluded and Hardy (2013) also agreed that when considering the existence of the vacuum state, one cannot prove the reality of the wave function such as that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are not compatible with the same ontic state under the ontic state assumption and the locality assumption. In the following, I will argue that this conclusion is problematic. The key is to realize that the vacuum ontic state may not affect the ontic state of the particle and the interference result about the detection of the particle.

Consider a typical Mach-Zehnder interferometer. Let $|1\rangle_L |0\rangle_R$ be the state that the particle is in one path L, and the other path R has no particle and it is in the vacuum state. Similarly, $|0\rangle_L |1\rangle_R$ is the state that the particle is in path R, and the other path L has no particle and it is in the vacuum state. Now assume that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are compatible with the same ontic state. This means that for the superposed state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$, there is an underlying ontic state in which the ontic state of the particle exists in path L, and the vacuum ontic state exists in the other path R. Then, in order to explain the different interference results for the two cases of no phase shifter and a π phase shifter placed in path R (i.e. for the two orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$), it is required that the information of a π phase change encoded by the vacuum ontic state in path R (where there is no particle but only the vacuum ontic state) should reach the second beamsplitter of the interferometer exactly at the time when the particle in path L reaches the beamsplitter.[7] If this requirement is not satisfied, for instance, if the arrival of the information is later than the arrival of the particle, then the phase shifter will not affect the interference result, or in other words, the interference result will be the same no matter whether a π phase shifter is placed in path R. This contradicts

[7] It is worth noting that interference is not the only way of distinguishing two orthogonal states such as $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$. We may also change each state to a two-particle entangled state and then use a joint measurement on these two particles to determine the relative phase of the two branches of each entangled superposition (see Aharonov and Vaidman, 2000). In this way, we need not to wait until the two branches of each entangled superposition meet together to measure the interference as in the Mach-Zehnder interferometer; rather, we can directly make a joint measurement on the two branches to determine their relative phase immediately after one branch passes through the phase shifter or undergoes another local unitary transformation.

quantum mechanics and experiments.

However, it can be argued that this requirement cannot be satisfied.[8] The essential reason is that no matter what the vaccum ontic state is and how it evolves in time, it is different from the ontic state of a particle, and thus they cannot always propagate with the same velocity under various conditions.[9] For example, the vacuum state and the vacuum ontic states are the same for all types of particles including photons, electrons and neutrons, as well as for the same type of particles with different momenta. Then, even though the information encoded by the vacuum ontic state arrives at the beamsplitter at the same time as a particle with a certain momentum arrives, they cannot arrive at the beamsplitter at the same time when the particle has a different momentum. Besides, even though the information and a photon arrives at the beamsplitter at the same time, the information and an electron cannot arrive at the beamsplitter at the same time, since different types of particles such as a photon and an electron usually moves with different speeds.

Now if the information encoded by the vacuum ontic state in one path and the particle in the other path cannot arrive at the second beamsplitter of the interferometer at the same time, e.g. the information arrives later than the particle, then the vacuum ontic state will not affect the ontic state of the particle and the interference result about the detection of the particle. In other words, the interference result will be determined only by the ontic state of the particle. In this case, one can still prove that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are not compatible with the same ontic state.[10] On the one hand, for the state $|1\rangle_L |0\rangle_R$, the ontic state of the particle in the region L is not changed by the π phase shifter placed in path R according to the locality assumption. On the other hand, for the state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$, the ontic state of the particle must be changed by the π phase shifter placed in path R, since the interference results for

[8]Note that due to this reason, the toy field theory proposed by Catani et al (2021), which is a ψ-epistemic model based on the second-quantized description, cannot reproduce the Mach-Zehnder phenomenology in terms of local causal influences (i.e. without violating the locality assumption).

[9]In fact, the vacuum state has zero momentum (i.e. $\hat{P}|0\rangle = 0$). Thus, the vacuum state, as well as the underlying vacuum ontic state, does not propagate in a definite direction with a nonzero speed. By comparison, a particle can have a definite nonzero momentum, which means that the particle state can propagate in a definite direction with a nonzero speed.

[10]There is another possible way to understand this result in a more general context. If one consider only processes that conserve the number of particles, as we do throughout this paper, then the second-quantized formulation is empirically equivalent to the first-quantized formulation, although it facilitates the application of the theory in many cases. Then, when assuming that the ψ-epistemic ontological models which can explain the relevant phenomenology of quantum interference are not underdetermined by experience, the proof of the reality of the wave function based on an ontological analysis in the first-quantized formulation should be also valid in the second-quantized formulation.

the two cases of no phase shifter and a π phase shifter placed in path R (i.e. for the two orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$) are different and the interference result is determined only by the ontic state of the particle.

The above analysis can also be extended to a general case. Consider two non-orthogonal states $\alpha |\psi_L\rangle + \beta |\psi_R\rangle$ and $|\psi_L\rangle$ in an N-dimensional Hilbert space, where α and β satisfy the normalization relation. According to Hardy (2013), when a certain condition is satisfied, there are N unitary transformations U_i (performed outside region L) that leave $|\psi_L\rangle$ invariant and N bases $|d_i\rangle$ ($i = 1$ to N) for which when performing the unitary U_i on the state $\alpha |\psi_L\rangle + \beta |\psi_R\rangle$ and then measuring in the $|d_i\rangle$ basis we obtain the null result. In other words, we have $p(d_i|\lambda_i, M) = 0$ for every i, where λ_i is an ontic state in the ontic support of the transformed state $U_i[\alpha |\psi_L\rangle + \beta |\psi_R\rangle]$. Now assume that the two non-orthogonal states $\alpha |\psi_L\rangle + \beta |\psi_R\rangle$ and $|\psi_L\rangle$ are compatible with the same ontic state λ. Then by restricted ontic indifference for the state $|\psi_L\rangle$, λ will keep unchanged after every unitary transformation U_i. Thus we have $p(d_i|\lambda, M) = 0$ for every i. Since there is the normalization relation $\sum_{i=0}^{N} p(d_i|\lambda, M) = 1$, this leads to a contradiction.

This is the proof of Hardy's theorem for a general case in the first-quantized description of quantum states. As noted before, this proof cannot go through in the second-quantized description of quantum states. In this description, the above two non-orthogonal states will be $\alpha |\psi_L\rangle |0\rangle_R + \beta |0\rangle_L |\psi_R\rangle$ and $|\psi_L\rangle |0\rangle_R$, and restricted ontic indifference is no longer valid for the state $|\psi_L\rangle |0\rangle_R$. Then, the ontic state λ, which includes both the ontic state of the particle and the vacuum ontic state, will not keep unchanged after every unitary transformation U_i, and thus there will be no contradiction. However, according to the above analysis, since the measurement result is determined only by the ontic state of the particle and it is not affected by the vacuum ontic state, the ontic state λ in $p(d_i|\lambda, M)$ may include only the ontic state of the particle. Then, since the ontic state of the particle keeps unchanged after every unitary transformation U_i (performed outside the region of the particle) by the locality assumption, we still have the relation $p(d_i|\lambda, M) = 0$ for every i, where λ denotes the ontic state of the particle. Thus, we can also derive a contradiction as before.

6 The locality assumption

I have argued that the reality of the wave function can be proved based on the locality assumption in both the first-quantized and the second-quantized descriptions of quantum states. In this section, I will present a more detailed analysis of the locality assumption. In particular, I will argue that the locality assumption is weaker than the preparation independence assumption of the PBR theorem.

First of all, it can be argued that the locality assumption is a locality assumption for product states, not a locality assumption for entangled states. The locality assumption says that the ontic state of a localized particle in a region is not affected by unitary transformations implemented outside this region. This means that there are two spatially separated systems, a particle in one region and a system in another region which implements a unitary transformation such as a phase shifter, and they are in a product state, and the ontic state of the particle is not affected by the system via action at a distance. In this way, the locality assumption can be stated as follows: for two spatially separated systems being in a product state, the ontic state of one system (e.g. a particle) in one region is not affected by the other system (e.g. a phase shifter) in the other region via action at a distance. Then, this locality assumption is not refuted by Bell's theorem which applies to entangled states (Bell, 1964). Moreover, for two independent, non-interacting spatially separated systems being in a product state, it seems that assuming one system has action at a distance on the other system can hardly be justified. This is quite different from the case of two systems being in an entangled state, for which the two systems can be regarded as a whole and one system may affect the other system by action at a distance.

Next, it is arguable that the locality assumption for product states is weaker than the preparation independence assumption of the PBR theorem. On the one hand, the violation of the latter does not entail the violation of the former. If the preparation independence assumption is violated, the ontic states of two independently prepared systems will be correlated. But the correlation may result from a common cause in the past, and it does not require that one system must has action at a distance on the other system, i.e. the locality assumption for product states must be violated. On the other hand, if the locality assumption for product states is violated, then a system will be able to change the ontic state of another independently prepared system via action at a distance. Then, the ontic states of the two systems (which are in a product state) will be correlated in general. This means that the preparation independence assumption will be also violated. Therefore, the locality assumption for product states is arguably weaker than the preparation independence assumption.

Last but not least, there is also another reason why the locality assumption for product states is a weak assumption for the ψ-epistemic view. It is that almost all ψ-epistemic quantum theories are local, and they aim to remove "spooky action at a distance" from quantum mechanics. Then, the above proof of the reality of the wave function based on the locality assumption will have more strength than the PBR theorem in setting restrictions on or even excluding these theories. This point is rather relevant to recent studies on the limitations of the PBR theorem in restricting some local ψ-epistemic quantum theories (see, e.g. Oldofredi and Calosi, 2021; Hance and Hossenfelder, 2022). I will analyze this issue in more detail in another paper.

7 Conclusion

Hardy's theorem is an important ψ-ontology theorem besides the PBR theorem, and yet it receives little attention in the literature. The main reason is that the restricted ontic indifference assumption of the theorem is much stronger than the preparation independence assumption of the PBR theorem from the ψ-epistemic view, and in particular, the assumption is not true when considering the existence of the vacuum state in the second-quantized description of quantum states. In this paper, I present a new analysis of Hardy's theorem and argue that the theorem can be revised to obtain a stronger result. The key is to realize that although the existence of the vacuum ontic state may invalidate the restricted ontic indifference assumption and block the proof of Hardy's theorem, it does not affect the ontic state of the particle and the result about the detection of the particle. Then, the reality of the wave function can still be proved based on the locality assumption suggested by Hardy. Moreover, the locality assumption is arguably a locality assumption for product states and weaker than the preparation independence assumption of the PBR theorem. This result will set new restrictions on ψ-epistemic quantum theories, e.g. it will exclude the local ψ-epistemic theories that evade the PBR theorem.

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