# Virtuality in modern physics in the 1920s and 1930s: meaning(s) of an emerging notion

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### Abstract

This article discusses the meaning of the notion of virtuality in modern physics. To this end, it develops considerations on the introduction and establishment in nuclear physics of two independent concepts at the turn of the 1920s and 1930s: that of the virtual state, used in the context of neutron scattering studies, and that of the virtual transition, useful for the theoretical understanding of strong nuclear forces, which forms the basis of what are now called virtual particles. Their comparative analysis highlights the theoretical nature of virtual entities and processes in modern physics. It also shows how the virtual has been associated with various purely physical attributes, leading to a form of polysemy of the term, from the beginning of the application of these concepts.

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## Introduction

In the years 1936 and 1937, Hans Bethe—assisted in part by Robert Bacher and Milton Livingston—published in *Reviews of Modern Physics* a series of three long articles (nearly 500 pages in total) which soon became known as "Bethe's Bible" of nuclear physics (Bethe and Bacher 1936; Bethe 1937; Bethe and Livingston 1937). This impressive work, which gave a comprehensive overview of everything known theoretically and experimentally about the atomic nucleus, repeatedly used the terminology of the virtual. While this terminology had a long tradition in physics—let us mention virtual images in geometrical optics<sup>2</sup> or virtual work in classical mechanics—Bethe's Bible is a sign of a new breath for the notion of virtuality, which had found its place in modern physics. Among Bethe's uses is the description of strong nuclear forces that hold nucleons together using a system of two neutrons, one of which "emits 'virtually' [...] an electron and a neutrino, and then absorbs the particles emitted by the other neutron" (Bethe and Bacher 1936, p. 204). Although incorrect,<sup>3</sup> the process described here refers to the emission and absorption of what we currently call a "virtual particle." This fundamental concept of

<sup>&</sup>lt;sup>2</sup> More on virtual images in Arianna Borrelli's contribution to this volume.

<sup>&</sup>lt;sup>3</sup> To be discussed in section 2.

quantum field theory and particle physics has now become for many a standard illustration of what a virtual entity can be in science.

Short-lived, off shell—i.e., not satisfying the energy-momentum relation and consequently unobservable, virtual particles are the object of a dispute over their ontological status.<sup>4</sup> Depending on the position defended by the various actors, these considerations have sometimes emphasized an understanding of the term *virtual* as a genuine antonym of *real* or *observable*. However, it should be stressed that such debates, part of centuries-old philosophical disagreements, only very rarely address the notion of virtuality in itself. Notably, they do not raise the question of the nature and significance of its introduction into modern physics. This article aims to discuss this question by going back to the origins of the use of the word *virtual* and its cognates in the context of quantum physics in the first half of the last century.

In this sense, Bethe's Bible and the field of nuclear physics provide us with an interesting case study.<sup>5</sup> Indeed, the physicist's uses of the terminology of

<sup>&</sup>lt;sup>4</sup> Mary Hesse's discussion of virtual particles in *Forces and Fields* in 1961, as well as Mario Bunge's charge against their reality in the early 1970s, were important moments in the opening of these debates (Hesse 1961; Bunge 1970). For more recent contributions, see among others: Fox (2008); Valente (2011); Jaeger (2019).

<sup>&</sup>lt;sup>5</sup> We could, of course, have relied on other works from the second half of the 1930s which have the same characteristics. See, for example: Feather (1938).

the virtual were not limited to physical phenomena related to the theoretical understanding of nuclear forces and what would later be identified as virtual particles. Most of them actually dealt with the more experimental area of neutron scattering, which aims at understanding how free neutrons interact with matter. In this domain, the notion of virtuality is also still relevant to this day. A comparative historical treatment reveals that the use of the virtual for neutron scattering and nuclear forces has its roots in the developments of quantum mechanics at the turn of the 1920s and 1930s. In fact, it highlights that the notion of virtuality has imposed itself in modern physics along two axes, still distinct today. On the one hand, as far as neutron scattering is concerned, the notion of virtuality was initially linked to the concept of a state of a system. A "virtual state" was considered to be responsible for a resonance phenomenon in particle scattering. On the other hand, when it comes to nuclear forces, virtuality was originally related to physical processes in quantum electrodynamics. In particular, a quantum jump to an intermediate state was regarded as a "virtual transition."<sup>6</sup> Nevertheless, despite clear differences in the specific contexts of the respective emergence of these axes, such a comparative approach also points to the possibility of a common

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<sup>&</sup>lt;sup>6</sup> The terminology *virtual state* has been (and still is) used on some occasions by actors discussing physical processes in quantum field theory. However, for the sake of clarity, we will refer to their work as simply dealing with virtual transitions.

influence with the fleeting 1924 Bohr-Kramers-Slater (BKS) theory of radiation. Its consideration allows ultimately to emphasize the idea of theoretical constructs for the complete understanding in modern physics of the notion of virtuality. Lastly, this perspective raises the question of the rapid alteration of the latter's initial meaning in the 1930s towards a form of applied polysemy.

Sections 1 and 2 of this paper aim to reveal the origins of the terminology of the virtual in neutron scattering and nuclear forces studies, respectively. Section 3 puts forward the BKS theory as a potential common influence on both cases. Finally, Section 4 directly addresses the meaning(s) of the notion of virtuality in modern physics.

# 1. Neutron scattering: "virtual states"

In Bethe's Bible, the term *virtual* was first introduced in a subsection dedicated to the scattering of neutrons by protons: "Now if the singlet state is a real bound state, the phases of the waves scattered by the two protons will be the same, whereas they will be opposite if the singlet state is a virtual state" (Bethe and Bacher 1936, p. 118). This quotation reveals two features relating to the use of the word *virtual* in the field of neutron scattering in the mid-1930s. First, it shows that *virtual* was used to specify the concept of a state of a system and by extension, to that of energy level. Second, since the term was introduced without further consideration, Bethe's Bible also suggests that it was widespread and had a clear meaning in problems related to neutron

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scattering. A broad survey of the literature of the 1930s confirms this impression. It even reveals a rather large linguistic extension for the concept of virtual state (or level) since it was used during the 1930s not only in English, but also in German (Beck 1930), Russian (Gamow 1932), French (Perrin and Elsasser 1935), or even Italian (Fermi 1936).

This extension is indicative of the fruitfulness of a concept and the success of a formulation that had only recently appeared with the first applications of quantum mechanics. More precisely, the physical concept of virtual state had emerged in contributions related to Friedrich Hund's work in 1927 on the theory of molecular spectra, which provided the first discussion of the phenomenon of quantum mechanical barrier penetration, better known nowadays as the quantum tunneling effect (Hund 1927a; 1927b; 1927c).<sup>7</sup> While Hund's approach was limited to the ground state energy in a double potential well, later developments by Lothar Nordheim, Ralph Fowler, and Robert Oppenheimer quickly extended the relevance of the phenomenon to unbound states with continuous energy *eigenvalues* (Nordheim 1927; Fowler and Nordheim 1928; Oppenheimer 1928). In 1928, George Gamow explained  $\alpha$ -decay by building on these results (Gamow 1928). Ronald Gurney and Edward Condon arrived independently at the same conclusion (Gurney and

<sup>&</sup>lt;sup>7</sup> The phenomenon was also discovered independently by Leonid Mandelstam and Mikhail Leontovich (1928). On the history of the quantum tunneling effect, see: Merzbacher (2002).

Condon 1929). All of these fundamental contributions to quantum physics gradually outlined the concept of the virtual state, but it was not until 1930 that it was formally defined and named as such by Guido Beck (Beck 1930).

Beck was interested in applying quantum mechanics to the problem of scattering of  $\alpha$ -particles by a nucleus.<sup>8</sup> To this end, his initial reasoning was based on the model of a simple one-dimensional potential well and its associated time-independent Schrödinger equation:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E-U)\psi = 0$$

For E < 0, we obtain a discrete energy spectrum. According to Beck, a quantum state is defined by such an *eigenfunction* and its associated energy *eigenvalue*. For  $E \ge 0$ , the *eigenvalue* spectrum is continuous. However, in this second case, Beck inferred from the wave equation solutions that there are discrete energy values for which the particle density in the potential well reaches a maximum and becomes equal to the density outside. By analogy, he attached these energy values, for which particles like to be in the potential well, to the concept of quantum state: "we can therefore say that the discrete quantum states continue in a certain sense into the continuous spectrum"

<sup>&</sup>lt;sup>8</sup> More on Beck's scientific interests and trajectory in: Darrigol (1995).

(Beck 1930, p. 333).<sup>9</sup> Previously identified by Gurney as "quasi-discrete levels in the atomic nucleus" for the case of  $\alpha$ -decay, these states, described as no longer sharp but with some width, were then newly labeled as virtual by Beck: "we want to refer to these states as 'virtual quantum states' in the following" (Gurney 1929, p. 565; Beck 1930, p. 333).<sup>10</sup>

Based on this, Beck established that "the phenomena of particle scattering can be understood as resonance phenomena of the incident particles with virtual quantum states in the continuous spectrum" (Beck 1930, p. 331).<sup>11</sup> In particular, his discussion of the Ramsauer effect and of  $\alpha$ -rays anomalous scattering provided important insights into the generalization of theories of particle scattering by a Coulomb field to a wider range of fields. The wording *virtual state* or *virtual level* then began to be used by other physicists, such as Nevill Mott in his work on artificial disintegration, Ettore Majorana in his theory of incomplete P' triplets, or Walter Bothe and Herbert Becker in the results of their nuclear experiments in which they mistook the not yet discovered

<sup>&</sup>lt;sup>9</sup> Unless otherwise indicated, all translations are ours. "Wir können daher sagen, daß sich die diskreten Quantenzustände in gewissem Sinne ins kontinuierliche Spektrum fortsetzen [...]."
<sup>10</sup> "Wir [...] wollen diese Zustände im folgenden als 'virtuelle Quantenzustände' bezeichnen."
<sup>11</sup> "Die Erscheinungen der Teilchenstreuung lassen sich als Resonanzerscheinung der einfallenden Teilchen mit virtuellen Quantenzuständen im kontinuierlichen Spektrum auffassen [...]."

neutron for γ-rays (Bothe and Becker 1930; Majorana 1931; Mott 1931). But in reality, it was the actual discovery of the neutron by James Chadwick in 1932, followed by the observation by Enrico Fermi and his collaborators in Rome of anomalously large cross-sections of nuclei for the capture of slow neutrons in 1934, that led to the wide dissemination of the concept of virtual state as named by Beck (Chadwick 1932; Fermi et al. 1934).

Used by Francis Perrin and Walter Elsasser, as well as by Bethe before the publication of his Bible, in their discussion of the theory of scattering of slow neutrons, this terminology was remarkably brought to the fore by Fermi whose experimental results in 1936 "would indicate that the level <sup>1</sup>S of the deuteron is virtual" (Bethe 1935; Perrin and Elsasser 1935; Fermi 1936, p. 439).<sup>12</sup> In parallel to this long-debated special case,<sup>13</sup> the concept of the virtual state was also widely disseminated by the influential work of Gregory Breit and Eugene Wigner in 1936 on the capture of slow neutrons (Breit and Wigner 1936). Thanks to an approach close to the concomitant development of Bohr's compound nucleus model—which assumed in nuclear reactions the formation of an unstable nucleus composed of the incident particle and the target nucleus—they established the standard formulas for both neutron-capture

<sup>&</sup>lt;sup>12</sup> "La seconda relazione dà  $\tau = 2,6.10^{-4}$  in accordo col valore sperimentale, il che indicherebbe che il livello <sup>1</sup>S del deuterio sia virtuale."

<sup>&</sup>lt;sup>13</sup> See notably its discussion by S. T. Ma (1953).

and neutron-scattering cross-sections.<sup>14</sup> Nevertheless, it is worth mentioning that by alternately using the terminologies of *quasi-stationary* and *virtual* energy levels, Breit and Wigner also established a form of inconsistency in the way physicists, up to the present day, refer to the concept discussed here.

# 2. Theories of nuclear forces: "virtual transitions"

As mentioned in the introduction, in addition to the virtual state, the other use of virtuality in Bethe's Bible concerns the concept of the virtual particle, which is now fundamental to quantum field theory. In reality, a direct link with the conceptual framework of this field was established by Bethe thanks to a precision hitherto eluded: "Each of two neutrons emits 'virtually' (<u>intermediate state</u>, cf. 228a) an electron and a neutrino" (Bethe and Bacher 1936, p. 204).<sup>15</sup> Indeed, the concept of "intermediate state" originates from Paul Dirac's 1927 first contributions to quantum electrodynamics, which provided a mathematical description of emission and absorption processes. More precisely, it results from second-order terms in a perturbative treatment of the radiation field and designates the transition of the studied system from an initial state to a final state via an "intermediate" state (Dirac 1927). Here

<sup>&</sup>lt;sup>14</sup> On Bohr's compound nucleus, as well as the work of Breit and Wigner, see: Stuewer (2018, chap. 13).

<sup>&</sup>lt;sup>15</sup> The emphasis is ours.

again, the quote from Bethe's Bible, in the context of considerations on the strong nuclear forces, reveals two characteristics of this specific recourse to the notion of virtuality in the mid-1930s. First, it shows that it was used to describe physical processes, such as transitions and particle emissions or absorptions, and not to the intermediate state itself. Second, the authors' mention of the latter, as well as the quotation marks, suggest that this terminology was not as common in 1936 for nuclear forces as was *virtual state* for neutron scattering.

In a recent study of the early history of the virtual particle concept, Markus Ehberger traced its tortuous beginnings in quantum electrodynamics and established how transitions to intermediate states began to be referred to as virtual from the very end of the 1920s (Ehberger 2020). In particular, he identified that Chandrasekhara Raman was the first user of the term *virtual* in this context at a symposium on molecular spectra and molecular structure held in Bristol in September 1929 by the Faraday Society (Raman 1929). In his discussion of the Raman effect, the Indian physicist argued that the energy of the visible light used in experiments was insufficient to allow the molecule under study to transit through the intermediate state determined theoretically by Dirac's dispersion theory. He therefore concluded that "the transition of the molecule assumed for the purpose of the calculation is a purely virtual one which cannot actually occur" (Raman 1929, p. 790). A few months later, Robert Wood, who attended the symposium in Bristol, and Gerhard Dieke examined the Raman effect with direct references to "virtual transitions" as well as to "virtual absorption and emission act" (Wood and Diecke 1930). Nevertheless, from this point on, such a use of the terminology of the virtual did not meet with immediate success and remained limited to rare occurrences (Hulme 1932; Peierls 1934). It was not until the mid-1930 that it began to gain popularity in quantum electrodynamics and was also transferred to nuclear physics, thanks notably to the contributions of physicists working in Leipzig.

Werner Heisenberg was the central figure in this process. As a starting point, on February 5, 1934, he mentioned in a letter to Wolfgang Pauli "infinitely many virtually possible transitions" to discuss the problem of the electron's self-energy.<sup>16</sup> His interest in this phraseology then found its way into quantum electrodynamics through the influential work on light-by-light scattering of Hans Euler and Bernard Kockel, which the founder of matrix mechanics supervised in Leipzig (Euler and Kockel 1935; Euler 1936;

<sup>&</sup>lt;sup>16</sup> Letter from Heisenberg to Pauli, February 5, 1934. Quoted in: von Meyenn (1985, p. 273). "Die Selbstenergie rührt in der bisherigen Theorie davon her, daß die unendliche Anzahl der Freiheitsgrade des Systems durch die im Prinzip unendlich vielen virtuell möglichen Übergänge zu anderen Zuständen auf divergente Summen für diese Energie führt."

Heisenberg and Euler 1936).<sup>17</sup> At the same time, Heisenberg was also deeply involved in research on the origin of nuclear forces.<sup>18</sup> After the discovery of the neutron in 1932, he had applied the methods of quantum mechanics to nuclear physics to introduce a first model that consisted of a force caused by an exchange of electrons between protons (Heisenberg 1932a; 1932b; 1933).<sup>19</sup> In 1934, however, Enrico Fermi's successful use of the quantum perturbative approach to formulate his theory of  $\beta$  decay in terms of an electron-neutrino field led many physicists to reject Heisenberg's original model and to consider the electron-plus-neutrino system as the conveyor of nuclear forces (Fermi 1934). Although it soon proved insufficient to account for the values of the binding energies of neutrons and protons,<sup>20</sup> it was in this very context that Heisenberg first transferred the terminology of the virtual, as related to transitions to an intermediate state, from quantum electrodynamics to nuclear physics:

> If one carries out the perturbation calculation [...] then additional terms of the second approximation arise [...],

<sup>&</sup>lt;sup>17</sup> For more details on the notion of virtuality in the work of Euler and Kockel, see: Ehberger (2020, pp. 267–273).

<sup>&</sup>lt;sup>18</sup> On the theories of nuclear forces in the 1930s see: Brown (1981).

<sup>&</sup>lt;sup>19</sup> Heisenberg conceived the neutron as a composite particle made of a proton and an electron.

<sup>&</sup>lt;sup>20</sup> It was notably shown by Igor Tamm (1934) and Dmitri Ivanenko (1934).

which correspond to the virtual emergence and disappearance of a positron-neutrino pair in the case of the proton, or electron-neutrino in the case of the neutron (Heisenberg 1935, p. 113).<sup>21</sup>

The following year, Carl Friedrich von Weizsäcker—a close collaborator of Heisenberg in Leipzig—made extensive use of this wording in a work on the spin dependence of nuclear forces (von Weizsäcker 1936). As observed before, it was also adopted by Bethe in his influential Bible (Bethe and Bacher 1936).

None of these authors was aware of the essential, but still largely ignored, work of Hideki Yukawa.<sup>22</sup> Using the formalism of quantum electrodynamics, he had postulated in late 1934 a new particle—later called a meson and known today as a pion—as responsible for nuclear forces (Yukawa 1935). In the first article dealing with this "meson theory"—which was considered fundamental until further developments were made in quantum chromodynamics in the 1970s—the word *virtual* was not used. Nonetheless, Yukawa was an attentive reader of Heisenberg, and it was probably from him, if not from Bethe's Bible,

<sup>&</sup>lt;sup>21</sup> "Führt man die Störungsrechnung [...], so entstehen Zusatzglieder zweiter Näherung, [...] die dem virtuellen Entstehen und Wiederverschwinden eines Paares Positron-Neutrino beim Proton, bzw. Elektron-Neutrino beim Neutron entsprechen."

<sup>&</sup>lt;sup>22</sup> More details on Yukawa's work in: Brown (1981).

that he took the initiative to introduce it in a later paper published in 1937 with Shoichi Sakata:

> The quantum [meson] of negative or positive charge thus emitted virtually can be absorbed by 2 or 1, which in turn changes into the neutron or the proton, so that the state of the system, in which the particle 1 is in a proton state, *q* say, and 2 in a neutron state, *m* say, is linked together with the initial state through the intermediate states above considered (Yukawa and Sakata 1937, p. 1088).

By the time this article was prepared with Sakata, the context for the reception of Yukawa's theory had changed considerably. It had been thrust into the limelight after Carl Anderson and Seth Neddermeyer experimentally discovered the muon in cosmic rays in 1936 (Anderson and Neddermeyer 1936). This new particle was initially considered as the one predicted by Yukawa and the meson theory of nuclear forces began to be studied in depth in the late 1930s. With it, the notion of virtuality, understood as related to transitions to an intermediate state, spread widely in the nuclear physics community. Later, in 1949, Richard Feynman partially relied on these developments, in addition to other fundamental considerations in quantum electrodynamics, to develop the idea of his famous diagrams. This work greatly

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generalized and popularized the concept of the virtual particle as it is still understood and used today (Feynman 1949).

#### 3. A potential common influence: the fleeting BKS theory

Richard Arthur has recently discussed the possibility of extending the conceptual analysis of quantum tunneling to virtual processes (Arthur 2012). In a study of the paradigm shift from energy levels to scattering that quantum field theory experienced in the 1930s and 1940s, Alexander Blum has pointed out the conceptual similarity between the cases of optical dispersion and the problem of scattering of neutrons off heavy nuclei (Blum 2017). This was suggested as early as 1936 by Breit and Wigner, who considered that the problem of neutron capture "resembles closely the problem of absorption of light from a level a to a level c which is strongly damped by radiation in jumps to a third level b" (Breit and Wigner 1936, p. 520). Such and other elements suggest points of convergence between the two cases developed above.<sup>23</sup>

<sup>&</sup>lt;sup>23</sup> This convergence has probably not escaped the notice of the informed reader, especially if one considers the identification of particles with resonance phenomena in accelerator physics. More specifically to our work, we can also mention the introduction in 1933 by Beck of the term *virtual* to designate electrons of negative kinetic energy in the framework of the Dirac hole theory (Beck 1933). Nevertheless, we have not integrated this last point in our considerations because such a use of the word *virtual* had little success in the scientific community in the mid-1930s and remained outside the development paths discussed here.

However, a proper historical account of the conceptual similarities between virtual states and virtual processes requires further investigation and remains outside the scope of this work.

What really matters here is to consider how Raman and Beck almost simultaneously opened two independent development paths for the use of the notion of virtuality in modern physics. Because, despite the possibility of an a posteriori reconstruction based on conceptual points of convergence, there was, in the 1930s, no direct and clearly established correspondence between virtual states and virtual transitions in the discourse and practices of physicists. In this respect, it is necessary to underline the independent character of the initial reasoning of our two protagonists. Indeed, if we cannot exclude that Beck was aware of Raman's Bristol lecture, published a few months before his own article, there is no reason to think that formal elements could have influenced his terminological choices in return (Raman 1929; Beck 1930). Their phenomenological approaches at the time of the introduction of the word virtual were too different: while Raman focused on the whole light scattering process, Beck relied on a simple analysis of the properties of the wave function. Nevertheless, the possibility of a common influence with the BKS theory, formulated in 1924 by Niels Bohr, Hendrik Kramers and John Slater, must be considered (Bohr, Kramers and Slater 1924).

As one of the last semi-classical attempts to develop a theory of radiation before the advent of quantum mechanics, the BKS theory was indeed the first work of the modern physics era that introduced the term *virtual* to specify new physical concepts. As its genesis, content, and fate have already been discussed at length in the literature,<sup>24</sup> we limit ourselves here to a brief and simplified presentation of such concepts, namely the "virtual oscillator" and the "virtual radiation field." The first one belonged to a stream of research which, since the late 1910s, aimed at defining a coherent state-transition representation within the framework of the Bohr atomic model of 1913, using the correspondence principle.25 The "virtual" oscillators were therefore conceived as systems whose frequencies would cover all possible quantum jumps in an atom. The genuine novelty in 1924 was that such oscillators were coupled to a "virtual" radiation field, as they continuously emitted radiation while the system was in a stationary state. As a whole, the BKS theory then defended a classical wave description of such a field, which, allowing the atoms to "communicate" with each other, guaranteed a purely statistical conservation of energy and momentum.

<sup>&</sup>lt;sup>24</sup> Among others: Hendry (1981); Stolzenburg (1984); Darrigol (1992, chap. 9), Blum and Jähnert (2022). See also the latter's contribution to this volume, which sheds particular light on the use of the notion of virtuality in the BKS theory.

<sup>&</sup>lt;sup>25</sup> More in: Blum and Jähnert (this specific issue).

Within a year or so, such an approach to quantum phenomena was discarded. Bothe and Hans Geiger provided experimental evidence for the conservation of energy in individual Compton scattering processes, which led to the abandonment—despite isolated revivals—of the inadequate concepts of virtual oscillator and virtual radiation field (Bothe and Geiger 1925). Regardless of its short lifespan, it must nevertheless be underlined that the BKS theory was widely discussed and established itself as a form of obligatory passage point towards matrix mechanics. With it, the notion of virtuality has been brought to the fore in the community of theoretical physicists. Thus, in the absence of direct references, different elements suggest that the virtual oscillator—which, after all, somehow provided a sort of extension of the concept of state to include that of transitions—served as an inspiration for both Beck and Raman.

Born in 1903, Beck most likely came across the BKS theory when, after a doctorate on the theory of relativity in Vienna, he turned in 1926 to radiation theory and a discussion of the Compton effect (Beck 1926). Subsequent work related to the application of quantum mechanics then proved that he was willing to rely on analogies and conceptual transfers to propose original

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ideas.<sup>26</sup> Bohr and Heisenberg's discussions of resonant radiation in their applications of the BKS theory to the problem of polarization of fluorescent light, as well as the former's attempts to extend his work to collision problems, may therefore have served as sources of reflection for Beck's work on particle scattering (Bohr 1924; Bohr 1925; Heisenberg 1925). By analogy, they even give meaning to his use of the term *virtual*. After all, the blurred nature of virtual states, expressed by a width "generally of the same order of magnitude as the distance between two states themselves," is somewhat reminiscent of the shape of virtual oscillators (Beck 1930, p. 333).<sup>27</sup>

The case of Raman has already been discussed by Ehberger, who noted its inclusion in follow-up considerations of the BKS theory (Ehberger 2020). However, we want to add briefly that the physicist, born in 1888, was in fact reluctant to accept the latest developments in quantum theory (more in: Brand 1989). In the late 1920s, Raman remained strongly committed to semiclassical approaches, leaving little doubt that he was familiar with the work of Bohr, Kramers, and Slater. Hence, the non-conservation of energy in the

<sup>&</sup>lt;sup>26</sup> In particular, Beck attempted to develop a complete analogy between light quanta and electrons (Beck 1927). He also used considerations on the shell structure of atoms to discuss nuclear structures (Beck 1928). See: Darrigol (1995).

<sup>&</sup>lt;sup>27</sup> "Diese virtuellen Zustände [...] haben eine gewisse Breite, welche im allgemeinen von derselben Größenordnung ist wie der Abstand zweier Zustände selbst."

individual transitions to intermediate states could have naturally triggered in his mind an analogy with virtual oscillators.

From a modern point of view, one can nevertheless see a form of irony in Beck and Raman's reuse of the notion of virtuality, historically considered as strongly constitutive and symbolic of the BKS theory. On the one hand, if it seems counterproductive for Beck to be associated with clearly disproven positions, it is also notable that his 1930 paper discusses the Ramsauer effect at length. Indeed, its unsuccessful approach in the frame of the BKS theory was nothing more than one of the main arguments for Bohr's abandonment of the ideas he had developed with Kramers and Slater (see: Stolzenburg 1984; Darrigol 1992). On the other hand, despite his proximity to semi-classical methods, Raman's use of the virtual had no positive connotations. It served as an argument to undermine Dirac's work and deny the physical possibility of transitions to intermediate states. This ensemble suggests that, despite the convenience of possible conceptual analogies, what really mattered in using the notion of virtuality went far beyond the physical insights of the BKS theory. Although elements such as non-conservation of energy or lack of sharpness may have played a decisive role in the individual terminological choices, they are not equally found in the works of our various actors. Therefore, they cannot be thought of as essential properties of the virtual, for which a necessary look at its semantics allows us to reconsider its introduction into modern physics, as well as its diffusion.

#### 4. The meaning(s) of the virtual

As Blum and Martin Jähnert argued, the BKS theory was at the end of a chain of developments, one of the last links of which was the *ersatzoszillatoren* (substitute oscillators) (Blum and Jähnert 2022). This concept, named as such in 1923 by Rudolf Ladenburg and Fritz Reiche in their theory of quantum dispersion, was identical in many respects to virtual oscillators (Ladenburg and Reiche 1923).<sup>28</sup> But Bohr, Kramers, and Slater may have been dissatisfied with the notion of "ersatz" (substitute; replacement).<sup>29</sup> Not only it is physically unclear what the *ersatzoszillatoren* would be the substitutes for, but it is also likely that they did not find it representative of the specifics of the BKS theory taken as a whole.<sup>30</sup> In order to provide an adequate English designation, the

<sup>&</sup>lt;sup>28</sup> This similarity was acknowledged by Bohr, Kramers, and Slater themselves, who, after introducing the virtual oscillator in their 1924 paper, mentioned that "[s]uch a picture has been used by Ladenburg" (Bohr, Kramers and Slater 1924, p. 790).

<sup>&</sup>lt;sup>29</sup> It should be noted that the word *ersatz* in German does not have the pejorative connotation inherited from the Second World War that it has today in English.

<sup>&</sup>lt;sup>30</sup> More in Blum and Jähnert (this specific issue) who notably argue that the virtual entities of the BKS theory had the role of avoiding a manifest tension resulting from Bohr atomic theory, that of introducing a coupling mechanism between distant atoms through radiation.

three men agreed on *virtual*.<sup>31</sup> Bohr's interest in the semantics of this word is beyond doubt: in a letter to Pauli written before the publication of the BKS article, the Danish physicist asked his colleague to comment on the use of the words *virtual* and *communicate*, which had become, "after lengthy consideration [...], basic pillars of the exposition."<sup>32</sup>

An important English definition of the virtual in the early 20<sup>th</sup> century was that of Charles Sanders Peirce, as stated in the 1902 *Dictionary of Philosophy and Psychology*: "A virtual X (where X is a common noun) is something, not an X, which has the efficiency (virtus) of an X" (Peirce 1902, p. 763).<sup>33</sup> Its application in the various contexts of physics previously described turns out to be full of meaning. In the early 1920s, virtual oscillators and radiation fields

<sup>&</sup>lt;sup>31</sup> Our hypothesis is that the notion of virtuality in optics (virtual images, sources, rays, etc.) may have served as an inspiration. The quantum theories of radiation were an extension of previous optical theories, and the actors involved in our developments had an excellent knowledge of classical approaches. During the subsequent diffusion of the term *virtual*, it is also worth remembering that the Raman effect is an optical phenomenon and that Beck, in his 1930's paper on particle scattering, built at length an analogy with thin-film optics.

<sup>&</sup>lt;sup>32</sup> Letter from Bohr to Pauli, February 16, 1924. Reproduced and translated from German into English in: Stolzenburg (1984, pp. 408–410).

<sup>&</sup>lt;sup>33</sup> More on Peirce in Friedrich Steinle's contribution to this volume.

were hypothetical entities with unusual properties.<sup>34</sup> Also, in Beck's potential well analysis, the lack of sharpness of virtual states differentiated them from properly defined quantum states. Finally, due to a lack of energy to physically perform quantum jumps, virtual transitions were processes that could not be considered as standard. This is only one part of the reasoning, the one that puts forward that a virtual X is not an X. The part that brings us to efficiency requires a complementary element, which is notably revealed by Raman's phrasing in his introduction of the term virtual in Dirac's quantum electrodynamics. Indeed, the transitions being "assumed for the purpose of calculation" appear in reality only as mathematical (theoretical) constructs allowing the approximate description, but efficient to account for experimental results, of a physical phenomenon—here, the Raman effect. This approach also applies to virtual oscillators and radiation fields within a qualitative theory built to represent radiation interactions. Despite the classical essence of the BKS theory, they were conceived primarily as an efficient way to describe quantum effects. Ditto for the virtual states, which, inferred from an analysis of the particle density in a potential well and not from the direct determination of energy *eigenvalues*, made it possible to explain

<sup>&</sup>lt;sup>34</sup> Our considerations on virtual oscillators and radiation fields in this paragraph directly echo the contribution in this volume by Blum and Jähnert, who highlight the adequacy of the notion of virtuality in the BKS theory with Peirce's definition of the virtual.

resonance phenomena in particle scattering. To sum up, a comparative analysis of the origins of different uses of the notion of virtuality in modern physics in the 1920s and 1930s leads us to understand the different physical concepts to which it has been applied as follows: a virtual X (where X is a common noun attributed to a physical entity or process) is a theoretical construct, similar to X but not an X, which, in its associated theoretical framework, has the efficiency (virtus) of X in describing physical phenomena.<sup>35</sup>

Nevertheless, references to virtual states and virtual transitions as theoretical constructs remained rare in the 1930s. To our knowledge, besides Raman, only Breit emphasized this aspect by mentioning that virtual levels (states) "are introduced only in order to simplify the calculations," that they have an "arbitrary" character and that they "must be decided using some physical rather than [...] formal mathematical consideration[s]" (Breit 1938, p. 68; see also: Breit, Thaxton, and Eisenbud 1939). The last comment, in fact, points to an explanation for this rarity. Indeed, if the updated definition of the virtual that we have given reflects well the motivations of the introduction of this notion to specify new concepts, in contrast, it does not appear fully appropriate in terms of practices since it does not carry a definite physical meaning when used in context. For physicists, what remains the most interesting are the physical features of virtual entities and processes, as well

<sup>&</sup>lt;sup>35</sup> Note that this also fully applies to older concepts, such as the virtual image.

as the way they play a role in their theories. Therefore, when deemed necessary, they started to justify the use the word *virtual* primarily in terms of the physical deviation of a concept from a certain norm.

In the case of states, alongside some references to the continuous spectrum as opposed to its discrete counterpart, the notion of binding energy became one of the main justifications for the use of the term *virtual* when applied to nuclei. It comes not from Beck, but from Fermi, who in 1936 stated that the deuteron's <sup>1</sup>S level would be "real or virtual depending on whether its binding energy is positive or negative" (Fermi 1936, p. 437). In the case of transitions, scientists relied on nothing more than the physical attributes of intermediate states, such as energy non-conservation and unobservability, which Dirac originally presented in 1927 (Dirac 1927).<sup>36</sup> In the continuity of Raman's discourse, the non-conservation of energy became the main criterion to account for the virtual nature of transitions. This was notably the case in quantum electrodynamics with Euler, but also for the nuclear forces with

<sup>&</sup>lt;sup>36</sup> More on the physical attributes of the intermediate state in: Ehberger (2020). It should be noted that a conception of "virtual" as synonymous with "potential" also emerged in the mid-1930s for transitions and processes, notably under the impulse of Heisenberg. While such an approach shifted the emphasis from the notion of efficiency to that of possibility, it did not prevent physicists from considering virtual entities in terms of physical deviations from a norm. The impact of this specific meaning, which results from linguistic aspects, is currently being studied by us and will be the subject of a future publication.

Yukawa (Euler 1936, p. 415; Yukawa and Sakata 1937, p. 1088). In fact, in the late 1930s, it was in the framework of the meson theory that physicists most explicitly invoked physical characteristics to justify the use of the word *virtual*. Among others, in 1938, Giancarlo Wick stated: "[...] these are not, of course, actual emission and absorption processes, which would be contrary to the energy principle; they are called, therefore, virtual transitions" (Wick 1938, p. 994). Homi Bhabha, for his part, even linked it directly to the notion of observability: "Of course the emission or absorption of a *U*-particle [meson] only takes place observably when it is consistent with the conservation of energy. In other cases the emission or absorption is merely virtual [...]" (Bhabha 1938; p. 117).

During their respective developments in the 1930s, the concepts of virtual state and virtual transition were thus characterized by physical deviations from the norm of very different natures. As a result, the notion of virtuality in modern physics progressively took a pluralist turn, from Peirce's initial meaning to its various applied meanings. It became indeed common for some actors to consider that virtual means "unobservable" or "non-conservative of energy," while for others it means "negative binding energy" or "part of the continuum." In spite of this, it is clear from our previous developments that none of these meanings is essentially representative of the full extension of

the notion of virtuality in physics. They have value only if they are applied in specific contexts to characterize appropriate entities and processes.

The historical highlighting of the incipient polysemy of the term *virtual* in modern physics must therefore draw our attention to the risk of abusive generalization of one of its applied meanings. On the one hand, it would be misleading to consider in the first place that different virtual entities and processes necessarily have common physical features. On the other hand, it would be wrong to consider that virtuality would be appropriate to qualify other (or future) concepts simply because they share a specific physical attribute with current virtual entities or processes. In other words, to summarize, virtual entities and processes cannot be characterized by certain necessary and sufficient physical conditions.

# Conclusion

By way of conclusion and opening, we would like to briefly elaborate on the lessons of our findings for the debate on the reality of virtual particles.<sup>37</sup> In the 1930s, the norm from which virtual transitions deviated was naturally designated as "real" or "actual." But in the frame of our developments, it would be fallacious to reduce these formulations to a univocal ontological statement against the reality of virtual processes, and by extension virtual

<sup>&</sup>lt;sup>37</sup> See note n°4.

particles. Of course, it is true that Raman, in speaking of transitions that "cannot really occur," suggested strong implications for realism. Still, such a position was also quickly countered, notably by Yakov I. Frenkel, who responded to the "usual assumption, which does not regard the [intermediate] state *n* as 'really' occurring," by arguing about the "definite duration" of such states (Frenkel 1929, p. 758).<sup>38</sup> As a matter of fact, from that moment on, what really mattered to most physicists was simply the application of the concept of virtual transition in virtue of its efficiency in describing physical phenomena.<sup>39</sup> Until the 1960s, fundamental ontological questions were not directly addressed and neither "unobservability" nor "non-conservation of energy" were in this context presented as direct denials of realism. In parallel, it should be noted that "virtual" states were also contrasted to "real" ones, and ontological issues never became topical in this case. Ultimately, in the absence of necessary and sufficient physical conditions for virtual entities and processes, it follows from our developments that the adjective real applied as opposed to virtual could only denote something "being precisely what its name

<sup>&</sup>lt;sup>38</sup> Note that in this context the "usual assumption" also includes considerations of a protoconcept of intermediate state developed in the mid-1920s by Adolf Smekal. More details on Frenkel's argument in: Ehberger (2020, pp. 265–266).

<sup>&</sup>lt;sup>39</sup> Ehberger also emphasized the "pragmatic attitude" of physicists toward Dirac's quantum electrodynamics (Ehberger 2020, p. 266).

implies."<sup>40</sup> After all, in Peirce's sense, a virtual X is not an X, but that does not mean that the former is inexorably unreal.

Certainly, as for any other entity of modern physics, it remains legitimate to wonder to what extent virtual particles are part of reality. But we hope to have shown that the answer to this question cannot be based on a mere opposition that would suggest fundamental ontological differences between "virtual" entities or processes and others, supposedly "real." Such an orientation would be misleading because it would not only be unreflective of some of the initial implications of the notion of virtuality, but also rather reductive. Indeed, by considering that virtual particles are original constructs resulting from a theoretical process, they are only elements among others, usually not qualified as virtual, within a more general problematic: that of knowing to what extent we accept that our theories—considered as efficient to describe physical phenomena—are an accurate representation of the reality of the external world.

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<sup>&</sup>lt;sup>40</sup> This is one of the meanings of the word *real* according to the Merriam-Webster dictionary.

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