The Eye Stays in the Picture. Virtual Images in Early Modern and Modern Optics

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Abstract

In optics, real images can be projected onto a screen, while virtual ones always remain behind mirrors. This apparently straightforward distinction is based on complex premises which emerged in the Early Modern period, and its development went hand in hand with a transformation of the notion of image, which became detached from sensual perception. In this article I will outline this historical process, and argue that the distinction between a real and virtual image still implies a reference to visual perception which makes it problematic, yet useful for didactic purposes.

1. Introduction

The distinction between real and virtual images in today’s optics is based on a geometrical definition of images: A real image lies at the convergence of light rays (Figure 1)[2], a virtual image at that of their prolongations (Figure 2). This definition is well suited to explain the workings of artificial optical systems, where it makes no difference whether rays actually emanate from the same point or simply appear to do so. However, this equivalence is not generally valid for visual experiences: As anyone who has tried knows, it is quite difficult to clearly see a real image produced by a lens unless we project it onto paper or other surfaces. Mirror images, on the other hand, can be clearly seen even though they are virtual. The gaps and tensions between geometrically defined images and visually perceived ones will be

2 All figures can be found in the appendix.

a recurring theme on the following pages, which are devoted to outlining the emergence, development and significance of the distinction between real and virtual images. I will argue that this development can be understood as a process in which experiences of quite different kinds, such as seeing images apparently hanging in the air, watching distorted reflections in mirrors or witnessing how a lens focuses sunlight, came to be conceptualized as different instances of the same phenomenon. This was done by means of geometrical constructions defining a new notion of image, and the distinction between real and virtual images helped ease the passage from a concept of image based on perception to one that was geometrically defined. Once the new notion of image was established in the early 18th century, the distinction between real and virtual images was set aside, only to reemerge in the 19th century as a didactic tool.

2. Real and virtual images in today's geometrical optics

Today, geometrical optics belongs to the field of physics, while visual perception is the subject of physiology and psychology. Moreover, geometrical optics does not deal with light propagation in general but only in so far as this phenomenon can be modeled in terms of light rays propagating along straight lines. Under this assumption, the discipline provides mathematical descriptions of how light is reflected and refracted when passing through optical systems, such as a surface separating air from water or a telescope (Landsberg 1989; Katz 2002). Optical-geometrical computations usually take the form of analytical formulas or, more recently, of computer programs. However, when simplifying approximations are possible, as is the case for thin lenses and monochromatic light, rules of geometrical image construction can be derived from the general framework. Historically, these simple geometrical rules were often formulated before the more complex mathematical apparatuses from which they are derived today emerged, and they played an important role in helping connect perceived optical phenomena to the developing mathematical models of light propagation. It was in this context that geometrically defined notions of images started emerging and were classified as "real" or non-real (i.e., "imaginary," "virtual," or "fictitious").

A standard definition of real and virtual images in diagrams and words can be found in Wikipedia (Figure 1, Figure 2). In the diagrams, the lines with an arrow represent light rays and the dotted lines represent their prolongations. The real image is upside-down with respect to the object, while the virtual image is right side up. The text explains: "A real image is the collection of focus points actually made by converging rays, while a virtual image is the collection of focus points made by extensions of diverging rays" (https://en.wikipedia.org/wiki/Real_image, accessed on March 30, 2022). Similar figures and definitions can be found in most optics textbooks (e.g. Hecht 1975, pp. 63-64; Landsberg 1989, pp. 214-215; Katz 2002, p. 18; Meschede 2010, pp. 498-499). Although geometrically defined, real and virtual images are often later connected to visual experiences by means of verbal statements, further diagrams or even photographs. The most usual statement about the difference between real and virtual images is that "a real image can be projected directly onto a screen, while a virtual image (the kind you see in a mirror) cannot be."[3] Such statements implicitly suggest that the geometrical features of real and virtual images, including their position, can be perceived by the eyes, a view at times strengthened by inserting an eye in the diagram (e.g. Katz 2002, p. 18). Although it is usually not explicitly stated that the eye sees the image in the position where it is drawn, a connection is established to the everyday experience of seeing images beyond a mirror at the same distance as the object. The author of the widely used German-language physics manual *Gerthsen Physik* even explicitly states that a single eye can determine the position of an image: "The eye automatically assumes that the rays that it receives have proceeded along straight lines. When a pencil originating from L is deviated, for example by a mirror, we position its origin along the prolongation backwards of the ray and we see a virtual image in L' " (Meschede 2010, pp. 498-499).[4] Here it seems that the single eye performs a triangulation to find the origin of the rays, an idea which, as we shall see, has a long and distinguished historical pedigree. Nonetheless, it is today generically accepted that the visually perceived position of both objects and their images is determined

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3 (Hecht 1975, p. 64), with similar statements in (Landsberg 1989, p. 215, Meschede 2010, p. 498).

4 "Das Auge setzt automatisch voraus, die Strahlen, die es empfängt, seien immer geradlinig gelaufen. Wenn ein von L ausgehenden Büschel z.B. durch einen Spiegel abgelenkt wird, verlegen wir den Ausgangspunkt in die rückwärtige Verlängerung der Strahlen und sehen ein virtuelles Bild in L' " (Meschede 2010, pp. 498-499).

by a combination of binocular vision and psychological factors (Wade 2021). In the most general case, there is no certainty that a single eye, or even both of them, will see an image at the point in space where the optical geometrical construction locates it. A very interesting example are images seen when looking through a converging lens or a glass sphere (Figure 3). What one sees is an inverted image of the objects on the other side, but this image is not located between the sphere (or lens) and the eye, as the geometrical construction of the real image requires, but rather appears to lie on the refracting surface. This can be seen for example in Figure 3, which Wikipedia presents as an example of an inverted, real image, without noticing the incoherence between what is shown by the photograph and the geometrical construction. As we shall see, Early Modern scholars were instead aware of the tension between geometry and vision, and in particular Kepler devoted some thought to the images seen when looking through a glass sphere.

3. Optical diagrams in antiquity and the Middle Ages

Diagrams have been used since antiquity to try and grasp some aspects of the process of human vision and, accordingly, these constructions always included the eye.[5] Geometrical constructions were also used early on to explain the formation of pinhole images as well as to formulate quantitative theories of burning mirrors. The methods developed in these three areas were eventually combined and further

5 If not otherwise stated, the following discussion is based on: (Lindberg 1968, 1970, 1996; Darrigol 2012; Smith 2015; Goulding 2018).

developed first in the Arabic Middle Ages and then in medieval and Renaissance Europe. It is of course not possible to describe here all these long and complex developments, and in the following I will only outline the main features relevant to the topic at hand.

The most important issue for this topic is the way in which the perception of reflected or refracted images was described in ancient and medieval times with the help of geometrical diagrams. These constructions had a structure which remained stable in time, although their natural philosophical interpretations greatly varied. A simple example is the diagram in Figure 4, which represents vision by an eye E of an object O as reflected by a plane mirror (horizontal surface). The line going from the object O to the surface and from there to the eye E follows the path today attributed to a ray of light, but in premodern times this line was usually referred to as a "visual ray." Depending on the natural philosophical context of the author, the visual ray could be considered as emitted by the eye or as coming from the object, and there was a broad variety of views on how it could give rise to visual impressions. In the Scholastic tradition, which shaped most late medieval Latin theories of vision, the eye received a "species" of the object, a term that for our purposes may be understood as a representation, or form (Smith 2015, pp. 245-277). What is important is that the "species" was not composed of different points, as we conceive an optical image today, but was a single unit somehow communicated along the visual ray. Independently from philosophical interpretations, authors assumed that the eye E would see the object along the prolongation of the visual ray. To establish where along the prolongation the object would be perceived, one drew a line passing through the object and perpendicular to the reflecting (or refracting) surface (Figure 4). This line was called the “cathetus,” and the rule formulated by Euclid was that the object would be seen as if lying at the intersection between the cathetus and the prolongation of the ray reaching the eye, that is at the point O’ (Smith 2015, pp. 59-62).

The cathetus rule delivers a plausible estimate of visual impressions from reflection and refraction from plane surfaces, and it was generally accepted until the Renaissance, although the question of its possible natural philosophical significance remained open. For our purposes it is important to note that the function of this geometrical construction was not to model physical processes of light, or "species," propagation, but to at least partially make sense of visual perception. In this context, the modern distinction between a geometrically constructed optical image and what is perceived by the eye simply did not exist. The two lines in Figure 4 were qualitatively different: The visual ray was generally assumed to be present in space, but the cathetus line was not. Accordingly, their intersection point did not mark the position of a physical phenomenon, and it was only through the eye and for the eye that an image appeared to be located there. This situation displayed some similarity to the one obtaining for virtual images, which only exist for the eye looking at them. In this sense, historian A. Mark Smith stated that in premodern times “all images were virtual” (Smith 2015, p. 309, p. 318).

In the Renaissance, diagrams involving the cathetus were combined with those explaining burning mirrors and pinhole images. An author whose work deeply shaped these developments was the Arabic scholar Ibn al-Haytam (ca. 965-1040), whose writings on reflection, refraction and pinhole images were translated into Latin in the Middle Ages and widely read (Smith 2015, pp. 181-227). Ibn al-Haytam built upon the previous tradition, developing quite refined geometrical optical treatments of light propagation and vision in which the lines usually seen as visual rays were interpreted as rays of light propagating from all point of visible objects, penetrating the eye and giving rise to visual impressions. Although this view may seem equivalent to the modern one, al-Haytam considered light as a means to transmit formal properties of visible objects which the mind then somehow turned into vision. Moreover, he still made use of the cathetus rule to explain reflection and refraction. Ibn al-Haytam's theories were very influential in medieval and Renaissance Europe, where they were usually combined with the notion of "species" propagation (Smith 2015, pp. 228-277).

4. Optics in the Renaissance: new instruments and images "hanging in the air"

From the 16th century onward, progress in glass-making and glass-working delivered both material and motivation for rapid development of geometrical optics (Smith 2015, pp. 323-333). Transparent, homogeneous glass ("crystal glass") was produced and crafted into lenses and mirrors with increasingly precise curvature. The new artifacts and their combination into optical systems, most notably telescopes, enabled production of a broad range of optical effects, such as sharp and much enlarged likenesses of objects lying far away, that had rarely, if ever, been observed before that time.

Among the optical phenomena fascinating Renaissance audiences were the so-called "images hanging in the air," i.e. images which observers perceived as floating in space, as opposed to those formed in mirrors or projected onto a surface.[6] Images hanging in the air usually appeared in dark surroundings and more often than not took the form of monstrous, deformed figures with demonic character. Some authors claimed that charlatans used the effect to trick gullible audiences, yet its production was no secret: It could already be found in the anonymous text *Secretum Philosophorum* (13th or 14th c.), which explained how, looking into a concave mirror placed in a box, "you see your image outside the box, in the air between you and the mirror" (quoted from Dupré 2008, p. 231). According to contemporary literature, images in the air could also be created using glass spheres or cylindrical and conical mirrors.

Can we grasp this phenomenon in today's optical terms, or was it only a figment of Renaissance imagination? In the case of the concave mirror or the glass sphere, we could interpret it as due to a real image which, although not projected onto a surface, may still be visible under suitable conditions, for example in a darkened room with a very bright source, or in a Keplerian telescope. Such an image, if visible, would

6 My discussion of images hanging in the air is based on Dupré 2008. For more information on Dupré's interpretation of Della Porta's discussion of these images, see Borrelli 2014.

indeed appear to float in space. Cylindrical or conical mirrors, on the other hand, produce virtual images we see located not in the air, but beyond the mirror's surface, although presumably a person standing in the dark near the mirror might perceive the image as floating in the air. Indeed, images hanging in the air are a paradigmatic example of how Renaissance optics inextricably combined what we would call geometrical optics with psychological effects: when using the cathetus rule, all images were ultimately formed not in space, but in the mind, and images floating in the air only made this fact particularly evident. Nonetheless, optical diagrams could help explain the perceived location of the image, as we shall see.

The study of new instruments and experiences by scholars and artisans led to the extension and adaptation of the geometrical methods described in the previous section. However, developing coherent geometrical optics that encompassed all the new experiences was no trivial step. As we saw above, the cathetus rule had been the main, and indeed the only, means to explain the perceived position of reflected and refracted images since antiquity. That rule had also been successfully applied to the study of spherical mirrors, but extending it to surfaces of other shapes, or to lenses with two spherical surfaces, was a challenge, as became clear to the Neapolitan scholar Giovan Battista Della Porta (ca. 1535-1615) in the last years of his life. Della Porta had become famous in Europe thanks to his *Magia naturalis*, a collection of old and new recipes and experiences, some useful, some imaginary and some based on the latest technological developments, among them optical instruments. Della Porta's work helped generate greater interest in optics in Europe.

In it he discussed among other things the analogy between the eye and camera obscura (Smith 2015, p. 344). In his later treatise *On refraction*, Della Porta successfully extended the cathetus rule to refraction in a glass sphere and in a manuscript draft "On the telescope" he tried to do the same with lenses (Borrelli 2014, 2017). Unsurprisingly, the latter attempt was not successful, yet Della Porta's constructions, though flawed, did qualitatively demonstrate how glass spheres and biconvex lenses work. As his use of the cathetus rule shows, for Della Porta optical geometrical constructions were aimed at explaining what the eye sees, yet in discussing glass spheres and lenses he noted how they could also produce images "hanging in the air" which appeared "behind the eye" (Borrelli 2014, p. 56). As noted above, these latter kind of images was quite new at the time and very different from other optical effects: The fact that Della Porta showed how both kinds of images could be systematically explained by the same geometrical descriptions can be seen as a first step towards detaching the geometrically constructed image from the visual impression. The creation of images hanging in the air was particularly appreciated in Renaissance courts, where experiences were expected to combine entertainment and education. At the Dresden court, Kepler witnessed such a demonstration and, as we shall see, he included it in his optical discussion as a starting point for his definition of "imago" and "pictura."

5. Kepler's "imago" and "pictura": the origin of virtual and real images?

The distinction between real and virtual images is usually traced back to Johannes Kepler's (1571-1630) definitions of "imago" and "pictura" in his optical treatise *Ad Vitellionem paralipomena* (1604) (Shapiro 2008, pp. 270-271; Darrigol 2012, p. 30).[7] However, historians have underscored how Kepler's notions cannot be regarded as equivalent to the modern ones, and I will argue that they are best interpreted as a step towards the (re)conceptualization of different optical phenomena as instances of an emerging concept of image defined in optical-geometrical terms. Kepler's very individual approach to optics was partly due to the route by which he had come to study the field, as like earlier astronomers he employed a camera obscura to observe sun eclipses and other celestial phenomena. Pinhole images, as we saw, had been described geometrically since antiquity in terms of rays propagating from a luminous source (a candle, the sun), passing through a pinhole, and forming an (inverted) image of the source on a screen placed in front of the hole. In his *Magia Naturalis* Della Porta had drawn an analogy between the camera obscura and the eye, and in Kepler's *Paralipomena* the camera obscura became a template to diagrammatically grasp optical phenomena using constructions involving only light rays, without the cathetus. After a general discussion of the properties of light, Kepler described the camera obscura, using the term "pictura" to indicate the figure projected on the screen in the back of the camera (Kepler 1604, p. 58). Later on, he explained vision as involving a "pictura"

7 My discussion of Kepler's optical work is based on the primary sources and on the treatment by (Shapiro 2008; Darrigol 2012; Smith 2015).

of an object projected onto the retina (Kepler 1604, p. 153). These and all other instances of use of the term show that, for Kepler, a "pictura" always implied the existence of a material surface supporting it, just like a painting.

In contrast, Kepler defined the "imago" seen through reflection or refraction according to tradition: It was the visual perception of an object in a place and with features different from the actual ones. He stated that the "imago" was "the vision of an object in conjunction with an error of the faculties involved in seeing," and added: "the image in itself is therefore almost nothing, and should rather be called imagination."[8] Kepler explained that, due to its subjective nature, the apparent position of an "imago" was determined by a combination of psychological and physiological causes, of which the latter could partly be understood with the help of geometrical constructions which used the paths of light rays entering the eye, where they projected a "pictura" of the object seen, just like in a camera obscura. The cathetus rule, in contrast, was seen by Kepler as inconsistent and he believed that it should be abandoned. Accordingly, all his optical-geometrical constructions involved only light rays.

From today's perspective, rejecting the cathetus rule may appear an obvious decision, but at the time that rule was essential for explaining perception of the apparent position of an image. Therefore, once Kepler had rejected it, he had to find some substitute for it. Beside the already mentioned psychological factors, he

8 "Breviter, imago est visio rei alicuius, cum errore facultatum ad visum concurrentium coniuncta. Imago igitur per se penè nihil est, imaginatio potius dicenda" (Kepler 1604, p. 64).

invoked binocular vision, which he claimed allowed for some kind of triangulation (Shapiro 2008, pp. 274-276). However, he further argued that position could also be perceived by a single eye, since the pupil had a finite dimension, and rays with different inclinations could enter it, again allowing for some kind of triangulation (Kepler 1604, p. 67; Shapiro 2008, p. 275). As Shapiro noted, although this argument is incorrect from today's perspective, it came to constitute an important starting point for further work on geometrical optics, and in particular, as we shall see, for the construction of an analogy between "pictura" and "imago" (Shapiro 2008, pp. 274-277).[9] After having been introduced separately to describe quite different experiences, the terms "pictura" and "imago" were connected when discussing what can be seen with the help of a transparent sphere filled with water (Kepler 1604, pp. 163-164). Using a water sphere, Kepler explained, one could generate a "pictura" on paper or on a wall and its position could be determined geometrically. However, other experiences could not be explained with diagrams. For example, an object seen through the sphere appeared inverted, smaller and

9 As Robert Goulding has recently shown, Kepler's critique of the cathetus rule and his proposal for an alternative were essentially the same as those expanded in more detail by Giovan Battista Benedetti in 1585, and Kepler may well have taken them from there, though he did not quote Benedetti (Goulding 2018, pp. 512-527). Benedetti, too, referred both to binocular vision and to the possibility of determining distance with a single eye. For our question here, however, it is not of primary relevance whether Kepler's idea was original or not, as his writings were most certainly the main vehicle for its diffusion.

located on the surface of the sphere (Kepler 1604, pp. 162-163). This is the experience depicted in the photograph shown in Figure 3, which today does not seem to raise any questions. Kepler instead found it problematic and asked: How is this possible, how does it fit with the geometrical construction? To answer this question, he referred to the "image hanging in the air" described by Della Porta in the *Magia Naturalis* ("pendula imago in aere", Kepler 1604, p. 164). These images were seen at the geometrically constructed location, but could only be perceived with some difficulty: "As Della Porta says, the image ("imago") in the naked air is always badly or even barely visible [...] but if you put a piece of paper in front of it, between the lens and the eye, then the image ("imago") will be seen not hanging in the air, but fixed on the paper."[10] After the word "imago" Kepler added in parenthesis: "here Porta, like me, speaks of 'imago,' and not yet of 'pictura,' of which this statement is true, as will become evident later on."[11] From these and the following passages it appears that, for Kepler, the image hanging in the air is not a "pictura," but rather an "imago" the apparent location of which is not determined by geometry alone, but by a number of causes. Kepler explains that the image appears to be hanging in the air only if the surroundings are very dark and if you

10 "Nempe malignè et vix videbitur imago, fatente PORTA, in ipso nudo aereo. At si papyrum obiicias, si inquam interponas papyrum inter lentem et visum [...] iam non pendula in aere, sed fixa in papyro videbitur imago" (Kepler 1604, p. 164).

11 "[N]am hic PORTA mecum adhuc de imagine loquitur, nondum de pictura, de qua verum hoc est, ut infra patebit" (Kepler 1604, p. 164).

look very carefully. Otherwise, the luminosity of the sphere attracts the eye, and the image is seen on its surface. Here he corroborates his thesis by explaining how, in a presentation at the Dresden Kunstkammer, he was the only one to actually see the image hanging in the air, because he was used to such optical demonstrations (Kepler 1604, pp. 164-165; Dupré 2008, pp. 232-237).

Kepler summarizes the distinction between "imago" and "pictura" as follows: "While 'imago' was so far an entity of the mind ("ens rationale"), we will call 'picturae' the visual representation of things ("figura rerum") which really exists on paper or on other surface."[12] As Shapiro noted, this is not the modern opposition between real and virtual image. I suggest to understand this passage not as the statement of a distinction, but as a remark underscoring that, despite their differences, the "imago" produced by lenses and mirrors and the "pictura" seen in the camera obscura have something in common. Thanks to Kepler's rejection of the cathetus rule, the two visual phenomena could be connected to the same kind of geometrical construction involving only light rays and their prolongations. By understanding the eye as a camera obscura and leaving out the cathetus line, optical geometrical diagrams could be interpreted as showing the path of light from a source, through the optical system, to the "pictura" in the eye which triggered vision. However, while these diagrams could explain the perceived magnitude and orientation of the image, they could not always account for its perceived position,

12 "Cum hactenus Imago fuerit Ens rationale, iam figurae rerum verè in papyro existentes, seu alio parete, picturae dicantur" (Kepler 1604, p. 174).

as Kepler had shown with the example of the water sphere. Accordingly, the suggestion that the position could, at least under certain circumstances, be determined by triangulation with a single eye was necessary to argue that geometrical optics could also explain visual experiences, at least in a first approximation.

Later authors developed more rigorous methods of geometrical-optical image construction than Kepler's semi-qualitative ones, and at the same time moved the focus of their analysis towards geometrically defined images and the functioning of optical instruments, separating this topic from the question of what the eye and the mind actually perceive. In this context, the distinction between an image hanging in the air and one projected onto paper eventually disappeared, while the one between real and virtual image emerged.

6. The emergence of virtual and real images in the late 17th century

A first step in this direction is found in the second book of the treatise on "Optics and Catoptrics", which was published posthumously 1651 under the name of Marin Mersenne (1588-1648) but was in fact written by the mathematician Gilles Personne de Roberval (1602-1675) (Shapiro 2008, p. 293). Roberval's discussion of reflection is as semi-qualitative as Kepler's and, although he provides diagrams of a few geometrical constructions, most of them were described only verbally. Roberval explained that, to see the image of a point, it was necessary that a pencil of light rays (apparently) proceeding from a single point should enter the eye and be made to converge again to a point, so as to form what Roberval, following Mersenne, called the “interior image” (“image interieure” Mersenne 1651, p. 115). The “exterior image” (“image exterieure” Mersenne 1651, p. 109), on the other hand, was the image that the eye perceives as located somewhere outside of it. This term was used to indicate both images seen as lying beyond a mirror and those located in front of it hanging in the air. The two phenomena were thus presented as optically equivalent, although Roberval acknowledged that images hanging in the air were much admired, while the others were seen as “common” (Mersenne 1651, p. 120).

Roberval stated that the position of an external image apparent to the eye was the point from which the light rays appeared to diverge and supported this claim with a verbal description of the same constructions Kepler had used to make the argument for triangulation, first for both eyes and then for a single one (Mersenne 1651, pp. 109-114). At the end of his verbal description of the geometrical constructions of exterior images generated by spherical mirrors, Roberval summarized: “In a few words, the apparent position of the external image of a point of an object, in all kinds of vision, direct, reflected or refracted, by a single eye or by both of them, is the point where the rays which fall on the eyes converge in effect or in potency.”[13] Shapiro interpreted the distinction between “in effect” and “in

13 “En deux mots, le lieu apparant de l'image exterieure d'un point d'un obiect, en toutes sortes de veuës, droite, reflechie, et rompuë; tant pour un oeil seul, que pour les deux, estant le point ou

potency" as that between real and virtual images (Shapiro 2008, p. 295),[14] whereas I see it as a verbal description of an absent diagram, with the aim of making clear to the reader that the convergence either of the rays or of their prolongations could lead the eye to perceive an image in that position. The two points were posited as geometrically and optically equivalent, although physically they were not: It was a step in a process of assimilation of the "common" images seen beyond the mirror and the surprising images hanging in the air. Thanks to this assimilation a notion of (exterior) image emerged which was subjectively perceived and geometrically located at the same point in space. Roberval's exterior image overcame the Keplerian distinction of "imago" and "pictura" to combine the entity of the mind to the one geometrically positioned in space, independently of whether the latter could or could not be projected on paper. In short, I believe that Roberval did not distinguish between virtual and real images, as later authors would, but still contributed to the emergence of this distinction by promoting a notion of image shaped by geometrical constructions which blurred the Keplerian distinction between "imago" and "pictura."

A quite explicit distinction between real and virtual images, though with a different terminology, was drawn by James Gregory (1638-1675), a mathematician,

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les rayons qui tombent sur les yeux concourent en effet ou en puissance" (Mersenne 1651, p. 121).

14 Such a reading would exactly not fit Peirce's understanding of "virtual," as presented by Steinle in this special issue.

astronomer and inventor of the Gregorian telescope, in his treatise *Optica promota* (1663). Gregory defined an image as follows: “An image is the likeness [“similitude”] of radiating matter which results from the divergence or convergence from individual points or towards individual points of the same surface of rays from the individual points of the radiating matter.”[15] He distinguished between images “in front of the eye” (“ante oculum”) and “beyond the eye” (“post oculum”), corresponding to the distinction between real and virtual images (Gregory 1663, p. 1). However, as the terminology chosen clearly expresses, Gregory did not see the two kinds of images as in any way physically different and did not mention the fact that one could be projected onto paper while the other could not. Later, he posed the problem: Given a visible point (“punctum visibile”), an eye and a reflective or refracting surface, find the position of the point’s image (“locum imaginis puncti” Gregory 1663, pp. 46-47). What is meant here with “position of the image”: the geometrical one or the perceived one? Gregory did not distinguish between the two and used the triangulation with one eye to argue that even when the rays did not really meet, the perceived position of the image was the point of convergence of the prolongations of the reflected rays (Figure 5, Gregory 1663, p. 47). Neither here nor elsewhere did he discuss whether the geometrical and perceived positions were always the same: The position of an image was simply defined geometrically.

15 “Imago est similitudo materiae radiantis, orta ex divergentia, vel convergentia radiorum, singolorum materiae radiantis punctorum, a punctis singulis, vel a puncta singula unius superficiei” (Gregory 1663, p. 1).

Gregory presented all his constructions as valid when the visible thing ("visibilis") was a material source, an image "in front of the eye" ("ante oculum") or one "beyond the eye" ("post oculum") (Figure 6, Gregory 1663, p. 58): To him, images were geometrically defined, an attitude that fit his primary interest in instrument construction.

While Gregory put the geometrically defined image at the center of his optics, the Jesuit Francesco Eschinardi (1623-1703) presented an interesting mixture of traditional and new views. In his *Centuria problematum opticorum* (1666-1668) Eschinardi stated that visible objects emit "visual rays" ("radii visuales"), yet also explained that the eye sees because an inverted image of objects is formed within it (Eschinardi 1666, pp. 3-4). When determining the apparent position of the reflection of an object in a plane mirror, he invoked the cathetus rule, but anticipated that the rule would encounter difficulties in other cases (Eschinardi 1666, pp. 5-6). Later, in his discussion of lenses, mirrors and optical instruments, Eschinardi treated visual rays like other authors treated light rays, so that the procedures for image construction were in effect the same. He used a specific terminology to indicate optical-geometrical constructions. A "focus" was any point of convergence of rays or their prolongation: A "real focus" was at the intersection of rays, and an image of it could be projected on paper, while the crossing point of rays' prolongations was a "fictive or imaginary focus" (Eschinardi 1666, pp. 31). The term "basis" had the same meaning as focus, but while the focus was point-like, a basis was extended. A "basis" could also be real or fictive/imaginary and can be interpreted as a real or virtual image. Yet the fact that Eschinardi spoke of focus and basis, and not of image ("imago"), can be seen as a sign that he was distinguishing between the perceived image and the geometrically constructed "basis." When discussing the apparent position of a perceived image, Eschinardi followed Kepler in considering it as depending on a number of causes in the mind ("intellectus," Eschinardi 1666, p. 72). However, if no other causes were in force, the image would be perceived to be at the focus or basis. To support this view, he also used the construction employed by Kepler, Roberval and Gregory, in which a single eye was able to triangulate the position of the (apparent) origin of the rays entering it (Figure 7, Eschinardi 1666, p. 72).

In Gregory's and Eschinardi's work, a geometrically defined concept of image had emerged that could be applied to phenomena as varied as magnifying lenses, images hanging in the air or burning mirrors. It could also be connected to visual perception, albeit in a rather problematic way by means of triangulation with a single eye. The physical differences between these phenomena were conceptualized by distinguishing between an image in front of or beyond the eye, or between imaginary and real "basis" – distinctions that can be seen as equivalent to today's oppositions of virtual and real images. The term "virtual" was, however, introduced only later on by Claude-François Milliet Dechales (1621-1678) in his *Cursus seu mundus mathematics* (1674) (Shapiro 2008, pp. 301-302). Dechales used the terms "focus" and "basis" in the same way as Eschinardi did and distinguished between a real ("realis") and a virtual ("virtualis") focus. He neither asked about nor tried to prove that images are seen by the eye in their geometrically defined position: The only definition he gave was the geometrical one. As Shapiro notes, the distinction between real and virtual (or imaginary) image was taken up by other authors of the time, while the geometrical definition of image became fully established (Shapiro 2008, p. 303).

7. Real and virtual images disappear from view in the 18th century

In the previous section I argued that the definition of real and virtual images was not the discovery of a difference, but rather a means of conceptual unification which helped establish the primacy of the geometrical definition of image above the one based on visual perception. To that aim, images hanging in the air and those seen beyond mirrors were presented as two manifestations of the same phenomenon: the optical image, which could be real or virtual. Once the new, unified notion of optical image was established, the distinction between real and virtual images appeared much less crucial. Therefore, it is hardly surprising that the real/virtual distinction started losing importance between the late 17th and the early 18th century, especially among those authors most interested in further developing mathematical approaches to ray-tracing and optical instrument construction and those scholars focusing mainly on general questions of the nature and properties of light, like Isaac Barrow (1630-1677), Christian Huygens (1629-1695) or Isaac Newton (1642-1727) (Darrigol 2012, pp. 60-107). Shapiro, on the other hand, regards the disinterest in the real/virtual distinction notable in these and other authors as surprising and asks about its possible motives (Shapiro 2008, pp. 308-312). He tentatively finds an answer in the mechanical philosophy that these authors followed, which allegedly led them to give priority to the perceived image over the geometrically constructed one. From this point of view, so Shapiro, all geometrical images appeared equal and it was therefore unnecessary to distinguish between real and virtual ones.

I believe that this interpretation is hardly tenable in light of the fact that the real/virtual distinction was also not underscored by 18th century authors, who very clearly defined images in geometrical terms. A significant example is Robert Smith (1689-1768), a mathematician and astronomer who in the early 18th century published *A Compleat System of Opticks: A Popular, a Mathematical, a Mechanical and a Philosophical Treatise* (1738). The first, "popular," book presented optical diagrams but did not introduce any distinction corresponding to the one between real and virtual images. For example, when discussing the diagrams in Figure 8 on reflection of point-sources from concave or convex mirrors, Smith wrote that the diagrams show how "all the reflected rays should converge and convene pretty close together about some certain point T of the direct ray QC, if the reflecting surface be concave, or else diverge from it, if the surface be convex" (Smith 1738, p. 8). Although the diagram graphically distinguished between rays and their dotted prolongations, the text did not: no matter whether lines converged or diverged, the optical constructions were equivalent.

When dealing with these constructions, the eye was never mentioned and the process of vision was only discussed later, explaining the optical structure and functioning of the eye (Smith 1738, pp. 25-31). The apparent position of an object was presented as depending on many different factors, of which optical-geometrical features were only one aspect (Smith 1738, pp. 49-52). In the second book of the treatise, mathematical rules to determine the focus of lenses and image properties were discussed, but no distinction made between real and virtual image. In conclusion, Smith drew a clear distinction between geometrically defined images and what the eye perceives, and his geometrical optics only dealt with the former topic. According to Shapiro, therefore, Smith should have regarded the real/virtual image distinction as important, but this was not the case. If my thesis is correct and the distinction between virtual and real images became less relevant when optical images were defined in strictly geometrical terms, then the question to ask is instead: When and why did this distinction make a comeback to become a staple feature of today's introductions to optics? Based on an admittedly incomplete survey of optical literature from the 19th and early 20th century, I will suggest that real and virtual images proved useful as a didactic means of introducing the geometrical notion of image to young students and to a broader public.

8. The comeback of real and virtual images in the early 19th century

During the 19th century optics became a topic of interest not only for astronomers and instrument-makers, but also for the general public, thanks to a growing range of optical apparatuses such as laterna magica, stereoscopy and dioramas, which possibly even contributed to reshaping visual perceptions (Crary 1990; Hankins and

Silverman 1999, pp. 148-177; Stafford, Terpak and Poggi 2001, pp. 301-364; Schiavo 2003). This led to the publication of popular introductions to the topic, and in these texts the distinction between real and virtual images appeared increasingly often.

Already in 1807 Thomas Young (1773-1829) had discussed virtual images in his natural philosophical lectures aimed at a broad public (Young 1807), but extensive use of the distinction, and of the term “virtual,” is found in David Brewster’s (1781-1868) *Treatise on Optics*, published in 1831 as part of the *Cabinet Cyclopedia*, a series of self-improvement books aimed at the general public. Brewster was at the time already a prominent scientist and had contributed to the development of photography and to research and popular interest in optics, among other things with the invention of the stereoscope and kaleidoscope. Probably because of the didactic aim of the text, images were not defined in purely geometrical terms, but rather introduced by referring to visual impressions in words probably inspired by Kepler: “The image of any object is a picture of it formed either in the air, or in the bottom of the eye, or upon a white ground, such as a sheet of paper” (Brewster 1831, p. 22). When discussing images in convex mirrors, Brewster used diagrams and spoke of a virtual image, explaining: “[the image] is called virtual because it is not formed by the actual union of rays in a focus, and cannot be received on paper” (Figure 9, Brewster 1831, p. 25). Interestingly, the figure also contains an eye, and the text explains how the apparent position of the virtual image is determined by using the triangulation with a single eye of finite dimension, as done by Kepler and other authors of the 17th century. This might at first appear surprising since, as we saw above, Smith had already not felt the need to use that argument. Moreover, the construction is flawed, as Brewster might have known, given his interest in stereoscopy. Nonetheless, triangulation with a single eye was both diagrammatically represented and verbally explained in his treatise. In the text he stated that, of all the rays emitted from the object “a few only can enter the eye” after reflection and used the diagram to argue that “if we continue backward the rays DE, FE they will meet at *m* and will therefore appear to the eye to have come from the point *m* as their focus. For the same reason the rays GE, HE will appear to come from the point *n* as their focus, and *mn* will be the virtual image of the object MN” (Brewster 1831, pp. 24-25). Why did Brewster use this argument? Having defined images not geometrically but as something visually perceived thanks to the “painting” on the back of the eye, he had to use that “painting” to justify using the geometrical construction for determining the position of the image. This is in effect the same reason why the argument at times still appears in modern textbooks.

Another prominent scientist who wrote a treatment of optics accessible to a general public was the astronomer and pioneer of photography John Herschel (1792-1871), who contributed the article “Light” to the 4th volume of the *Encyclopaedia Metropolitana* (1845). In contrast to Brewster, Herschel did not introduce images in terms of vision but by first discussing light ray propagation and only afterwards explaining how an “image” of the sun could be projected onto the back of a camera obscura (Herschel 1845, p. 342). Later, he defined images “in Optics” in geometrical terms: “The image of an object, in Optics, is the locus of the focus of a pencil of rays diverging from, or converging to, every point of it, and received on a refracting surface” (Herschel 1845, p. 392). He also distinguished between real and virtual images: “If the lens used to form the image be a concave one, or if a convex reflector be used, [...] the rays, after refraction or reflection, diverge, not from any actual points in which they cross, but from points in which they would cross if produced backwards. There is in this case, then, no real image formed capable of being received on a screen, but what is called a virtual one, visible to the eye if properly situated” (Herschel 1845, p. 394). However, Herschel never again addressed this distinction, which thus appears to only play a marginal role for him. He also never suggested that a single eye can estimate distances. In short, like Smith, Herschel used a purely geometrical notion of image and did not attempt to link its properties to vision.

The distinction between real and virtual images is instead prominent in the series of optical-geometrical drawings executed by Ferdinand Engel (1805-1866) under the direction of the mathematician and pedagogue Karl Heinrich Schellbach (1805-1892), who also wrote the accompanying booklet explaining the images (Engel and Schellbach [1849]1856). The aim of the work was didactical, as stated right at the beginning: “The aim of these drawings is to make easier the difficult study of optics and more specifically to give a clear idea of the functioning of optical instruments. The most competent physicists witness that even the best known manuals of optics and the most precisely explained computations do not fulfill this aim so well as the constructions presented here.”[16] The tables presented image constructions for different configurations of mirrors and lenses, prisms, and finally also for telescopes. Although the constructions were purely geometrical, eyes were drawn on them in various positions to show how the image’s shape, dimensions and position may appear different depending on where the observer is. In describing the drawings, the author spoke of “objective” (“objectiv”) and “"subjective” (“subjectiv”) images, which correspond to real and virtual ones (Engel and Schellbach [1849] 1856, p. 12). The notions of real and virtual image were used as didactical tools to connect visual experience to geometrical optics. The eyes in the picture helped achieve that aim, but they did not stand for a triangulation with a single eye, as was the case in Brewster’s text. Once again, authors defining images right from the beginning in geometrical terms had no need of that construction.

9. Real and virtual images in late 19th and early 20th century

Once they had been reintroduced, real and virtual images remained a frequent component of introduction to optics, and I will conclude my brief overview by presenting two examples from the late 19th and early 20th century. The first one is

16 “Der Zweck dieser Zeichnungen ist, das schwierige Studium der Optik zu erleichtern und namentlich eine klare Vorstellung der Wirkungsweise optischer Instrumente zu erwecken. Dass die bekanntesten Lehrbücher der Optik und selbst sorgfältig angestellte Rechnungen diesen Zweck nicht in dem Masse erfüllen, als die vorliegenden Constructionen, dafür können wir das Urteil der sachkundigsten Physiker anführen” (Engel and Schellbach [1849] 1856, p. 1).

the textbook *Geometrical Optics Adapted to the Use of Higher Classes in Schools*, which was published in 1870 by Osmund Airy (1845-1928). Airy begins the Preface with the remark: “This is, I imagine, the first time that any attempt has been made to adapt the subject of geometrical optics to the reading of the higher classes in our good schools” (Airy 1870, p. iii). This fact is surprising to him, since optics seems very fitting for both interesting and educating college students, introducing them to new ideas: “The conception of a virtual image, to take an early instance, is probably an entirely new one to the reader’s mind” (Airy 1870, p. iii). This remark is particularly interesting, as it supports my thesis that the notion of virtual image is on the one hand not trivial to grasp when starting from visual experience, but on the other hand appropriate to introduce to geometrical optics. Airy started his introduction by referring to everyday visual experiences, described pinhole images and defined basic notions like light rays and their linear prolongations, but not images (Airy 1870, pp. 1-5). He then explained the basic laws of reflections and refraction and presented an optical-geometrical diagram of reflection from a plane surface which showed a single eye establishing the position of the image by some kind of triangulation, as Brewster had done (Figure 10). The text explained how the eye, in this way, could see the image of the object exactly in the position determined by the geometrical construction, and that in this case the image was virtual (Airy 1870, pp. 9-10). Airy realized that the geometrically constructed image was difficult to reconcile with the one perceived by the eye or with those projected on a screen. He noted that it might feel wrong to speak of an image being formed by reflection, if the rays did not really meet, and explained: “A virtual image may be described as an image that does not exist until there is an eye to receive the rays. The eye calls the image, though not the rays, into existence” (Airy 1870, p. 13, 17). This statement expresses well a concept of optical image as something which is geometrically defined and ready to be seen by the eye, independently of whether it can be projected onto a screen or not. In the early 20th century, the same construction appeared in *Elementary Geometrical Optics* (1914) written by Arthur Stanley Ramsey (1857-1954) for students preparing the for Mathematical Tripos.

The context was, once again, reflection from a plane surface, with Figure 11 showing a cone of reflected rays entering the eye and allowing it to establish the position of the reflected image, as explained in the text: “Hence if PQ be the pupil of an eye conveniently placed to receive some of the rays, it will receive a cone of rays of vertex A’ and base PQ; that is, the eye will see the point A’. The point A’ is called the *image* of A in the mirror XY. The image is in this case called *virtual image*, because the rays do not pass through it, but only their prolongations” (Ramsey 1914, p. 8).

While some physicists were trying to connect optical-geometrical constructions to visual experiences, physicist and philosopher Ernst Mach (1838-1916) sharply criticized these attempts. In his monograph on *Die Prinzipien der physikalsichen Optik, historisch und erkenntnispsychologisch entwickelt* (1921) Mach did not discuss real and virtual images and, in a chapter devoted to vision, criticized the confusion between physical and physiological properties of vision which in his opinion had begun with Kepler, namely the notion that the eye is capable of perceiving the direction a ray is coming from and the exact position of the source (Mach 1921, pp. 62-64). He wrote: “On the basis of its perceptions the eye knows no geometry, and geometrical notions cannot be applied to these perceptions, they make no sense for them. The sense of vision sees only the position of objects with respect to each other and orients itself using for example objects that are always present (nose, eyebrows etc.) [...] We cannot speak of any exact, quantitative, geometrical determination of place in direction and distance.”[17] Mach referred to recent physio-psychological discoveries to criticize not only the idea that one eye can triangulate but also that binocular vision can be reduced to a purely geometrical process. This topic cannot be pursued further here, but it deserves mention to underscore how, with the further development of different branches of optics, the relationship between optical-geometrical constructions and what is seen by the eye (or eyes) appeared increasingly complex. Presenting the virtual image as not only geometrically but also physically analogous to the real one could be didactically useful, but also scientifically problematic. Nonetheless, in the 20th century virtual

17 “Das auf seine Empfindungen angewiesene Auge kennt keine Geometrie, ja die geometrischen Begriffe finden auf diese Empfindungen keine Anwendung, haben für diese gar keinen Sinn. Der Gesichtssinn sieht nur die Anordnung der Sehobjekte gegeneinander und orientiert sich etwa nach immer vorhandenen Objekten (Nase, Augenbrauen usw.) [...] Von einer genauen, quantitativen, geometrischen Fixierung des Ortes nach Richtung und Entfernung kann keine Rede sein” (Mach 1921, p. 64).

and real images became established as central tenets of (geometrical) optics and, as we saw in the beginning of this paper, they today feature prominently in most didactical treatments of the discipline.

10. Conclusions

The distinction between real and virtual images occupies a prominent place in today's introductions to optics. In the previous pages I have sketched its development and use since the Renaissance, arguing that today it is not primarily a notion necessary to the practice of geometrical optics but rather an expression of the multiple tensions between geometrical-optical constructions and experiences of vision. Due to its special place at the interface between geometry and experience, the distinction of images in real and virtual ones, though problematic, is nonetheless useful in connecting the two, both with didactical aims and more generally to qualitatively grasp the principles of geometrical optics, for example in the study of optical instruments.

My research built upon and partially reassessed the results presented by (Shapiro 2008), who placed the origin of the distinction in Kepler's definitions of "imago" and "pictura." The "imago" was for Kepler an impression of the mind, while the "pictura" was the likeness of an object actually projected by light onto a material support like paper or the back of the eye. Subsequent authors, such as Shapiro, took up and reworked Kepler's distinction into what we today call virtual and real image. In contrast to Shapiro, I have argued that Kepler's "imago" still comprised both what we would call real and virtual images, while the “pictura” was a very specific kind of phenomenon whose reality was ensured only by the existence of a material support for it. Neither the “imago” nor the “pictura” were defined geometrically in Kepler’s writings, although some of their properties could be geometrically determined. In order for the distinction between real and virtual image to emerge, it was first necessary that a primarily geometrical definition of optical image should become established. On the basis of such a definition, substantial phenomenological differences such as those between mirror likenesses, images hanging in the air and pictures projected on a wall could be presented as distinctions between subtypes of the same category: the optical image. In other words, the distinction real/virtual was a byproduct of a shift in optics in which what may now be called psycho-physiological aspects of vision came to be studied separately from physical and optical-geometrical issues of light propagation in different media.

I further argued that, when the geometrically defined concept of image became established in the 18th century, mention of the distinction between real and virtual images steadily decreased. It was only in the early 19th century that authors writing on optics again underscored the real/virtual distinction for images, and they did so in introductory or otherwise didactic contexts, to help students or non-experts to connect their experiences with mirrors and lenses to the more abstract concepts of geometrical optics. I have suggested that these developments were linked to a broader interest in optics due to the spread of new optical apparatuses and illusions, as well as of photography. From then on, the distinction between real and virtual images became a central tenet of introductions to optics and remains so to this day. Some of today’s introductions to optics even contain a problematic optical-geometrical statement which extends back to Kepler’s time, namely the idea that a single eye can estimate the distance of an image by some kind of triangulation procedure. I believe this example serves well to illustrate how the tensions between geometrical and perceptual notions of image and vision persisted and that, though in principle excluded from geometrical optics, the eye still stays in the picture.

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Figure captions

Figure 1 Real image in a concave mirror. Image by Krishnavedala - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=20365945.

Figure 2 Virtual image in a concave mirror. Image by Krishnavedala - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=20378054.

Figure 3 Inverted image seen through a biconvey lens. Image by AntanO - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=64673937

Figure 4: The cathetus rule: the eye E sees the reflected image of the object O at the point where the perpendicular to the reflecting surface passing though the object (the cathetus line) crosses the prolongation of the visual ray entering the eye.

Figure 5: Reflection from a curved mirror, and triangulation with a single eye (Gregory 1663, p. 47)

Figure 6: Images in concave and convex mirrors (Gregory 1663, p. 59)

Figure 7: Virtual image and triangulation with a single eye (Eschinardi 1666, p. 72)

Figure 8: Reflection in a convex mirror: no mention of a distinction between real and virtual image (Smith 1738, p. 8)

Figure 9: Virtual image and triangulation with a single eye (Brewster 1831, p. 24)

Figure 10: Virtual image and triangulation with a single eye (Airy 1870, p. 9)

Figure 11: Virtual image and triangulation with a single eye (Ramsey 1914, p. 8)

Appendix: Figures

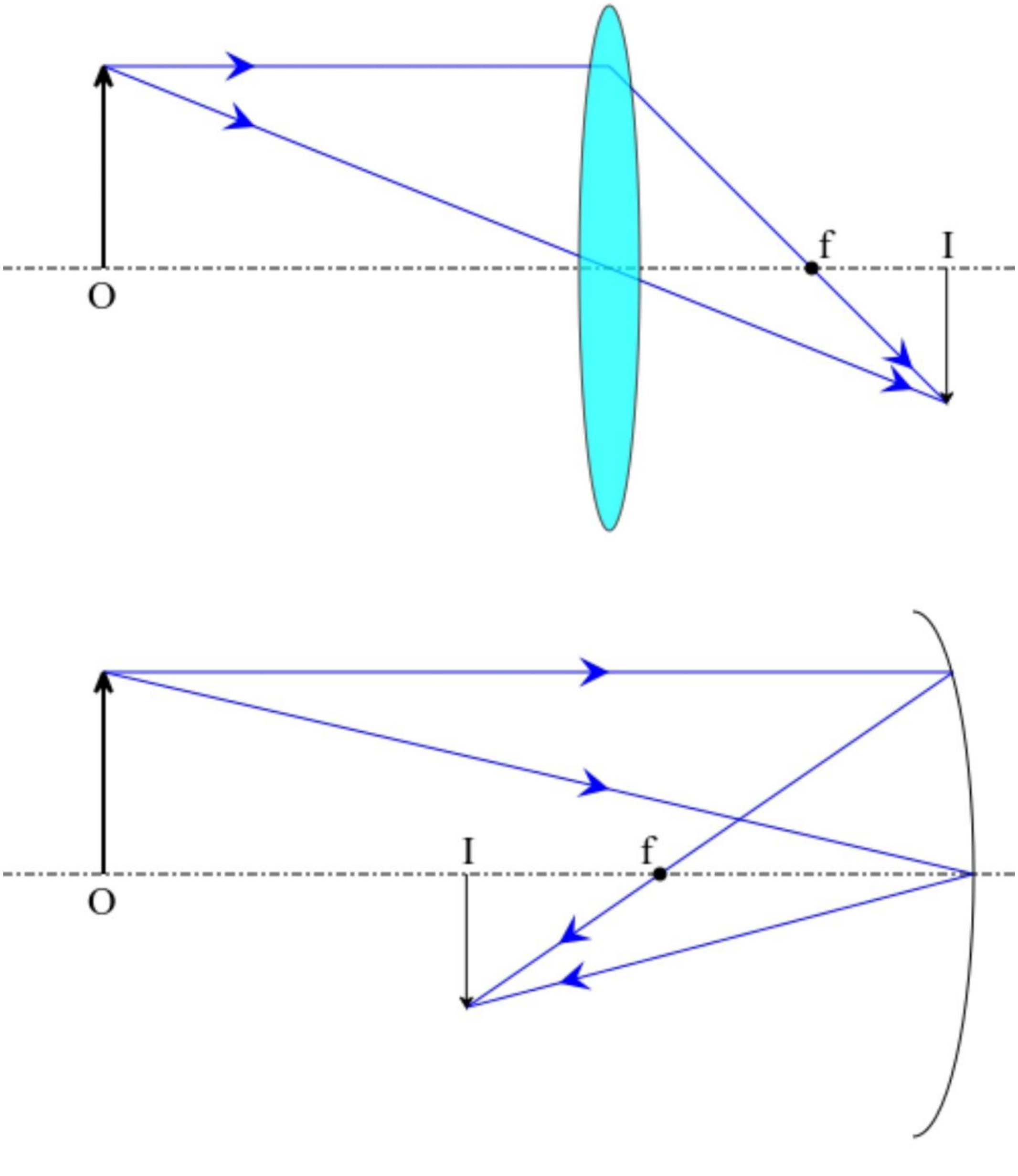

Figure 1: Real image in a concave mirror. Image by Krishnavedala - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=20365945

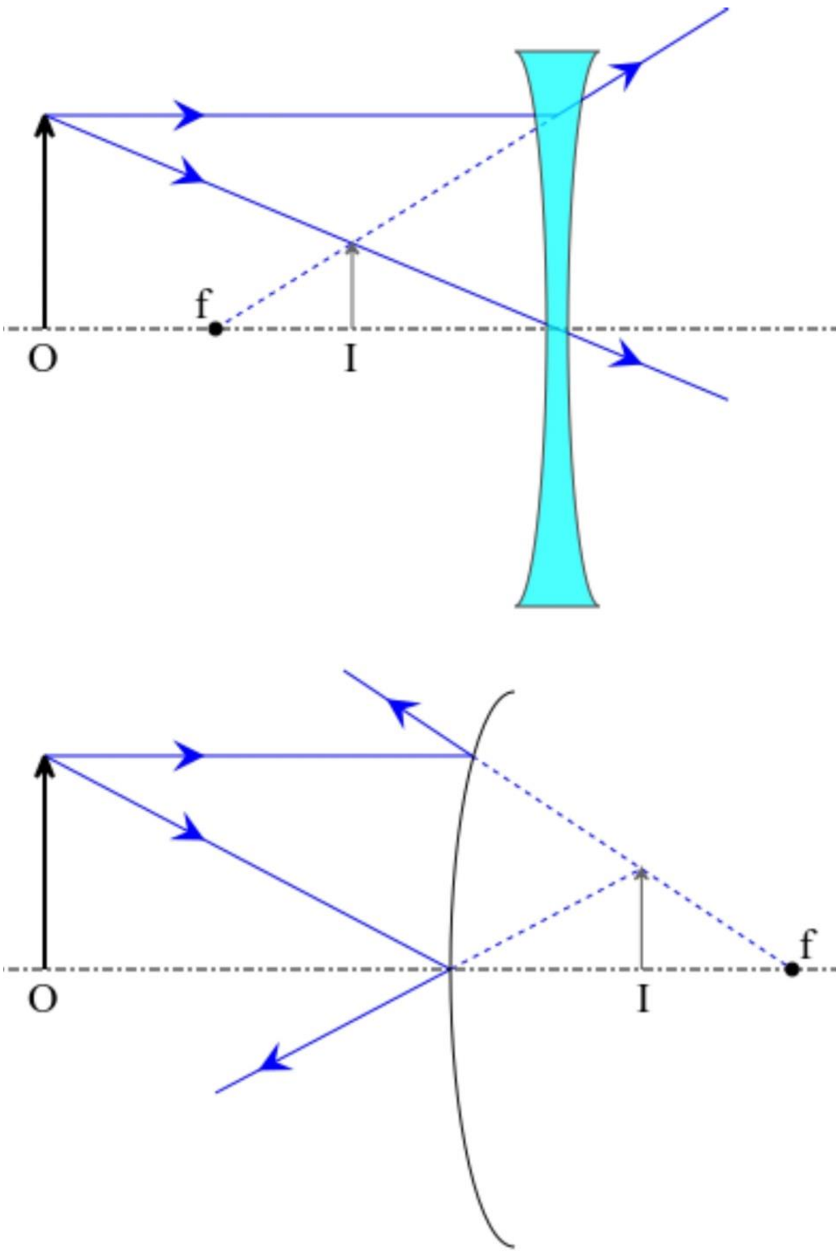

Figure 2: Virtual image in a concave mirror. Image by Krishnavedala - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=20378054

Figure 3: Inverted image seen through a biconvey lens. Image by AntanO - Own work, CC BY-SA 4.0

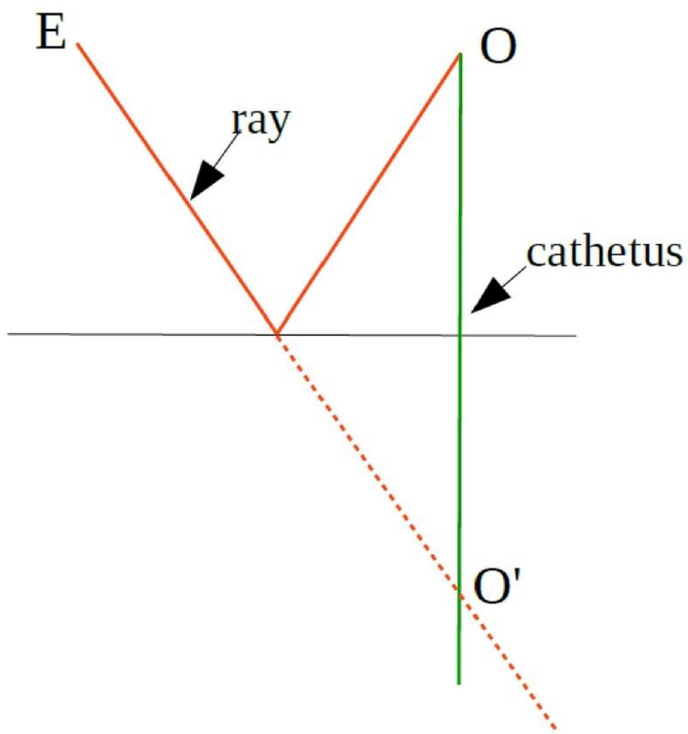

Figure 4: The cathetus rule: the eye E sees the reflected image of the object O at the point where the perpendicular to the reflecting surface passing though the object (the cathetus line) crosses the prolongation of the visual ray entering the eye.

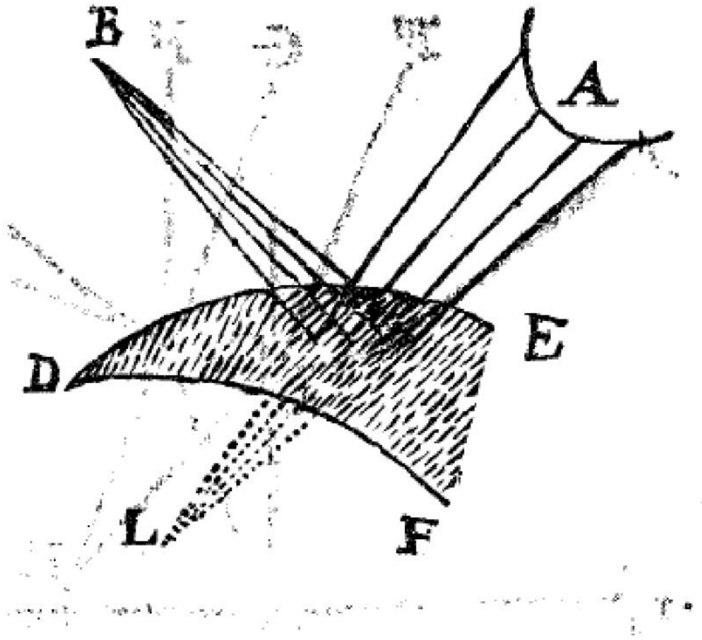

Figure 5: Reflection from a curved mirror, and triangulation with a single eye (Gregory 1663, p. 47).

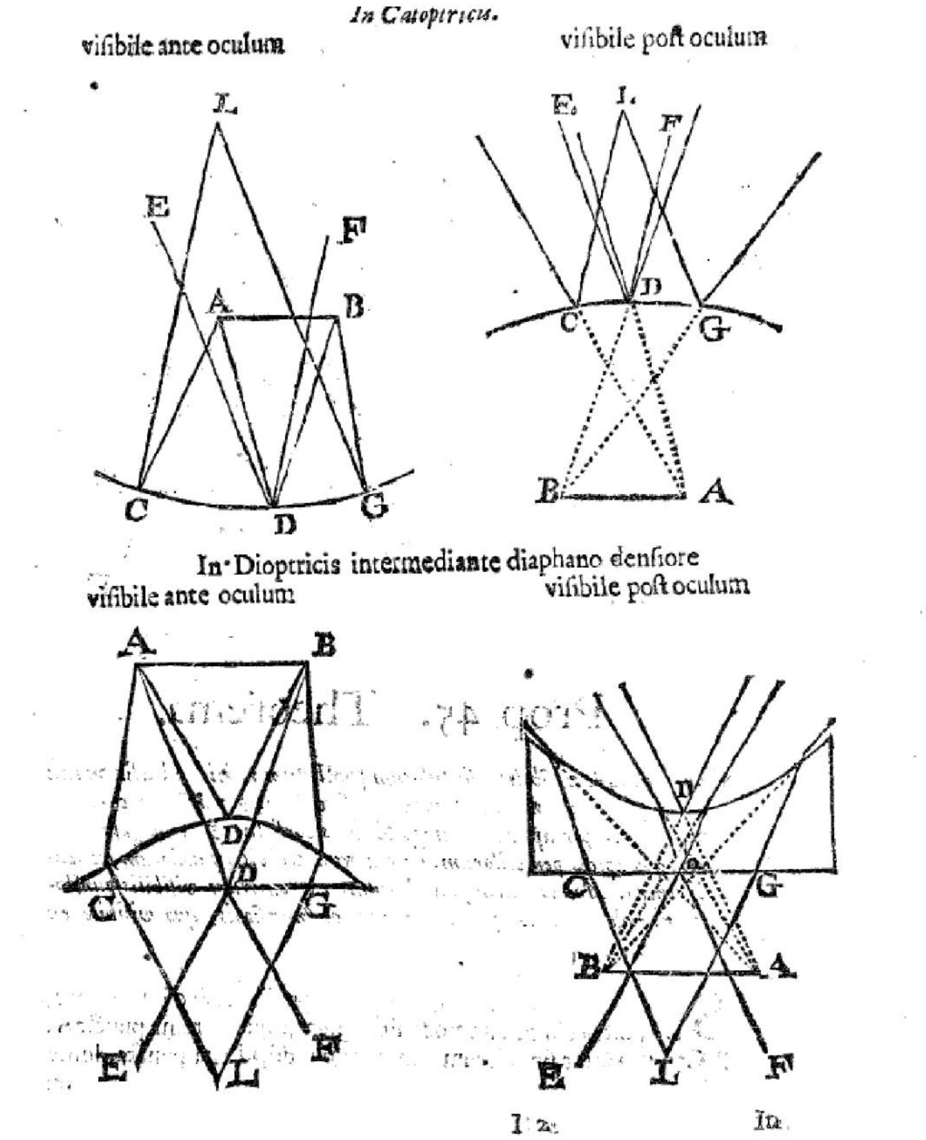

Figure 6: Images in concave and convex mirrors (Gregory 1663, p. 59).

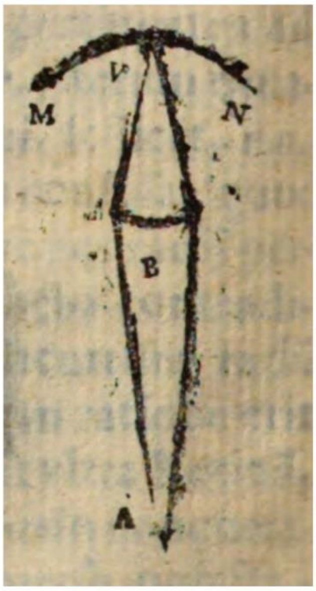

Figure 7: Virtual image and triangulation with a single eye (Eschinardi 1666, p. 72)

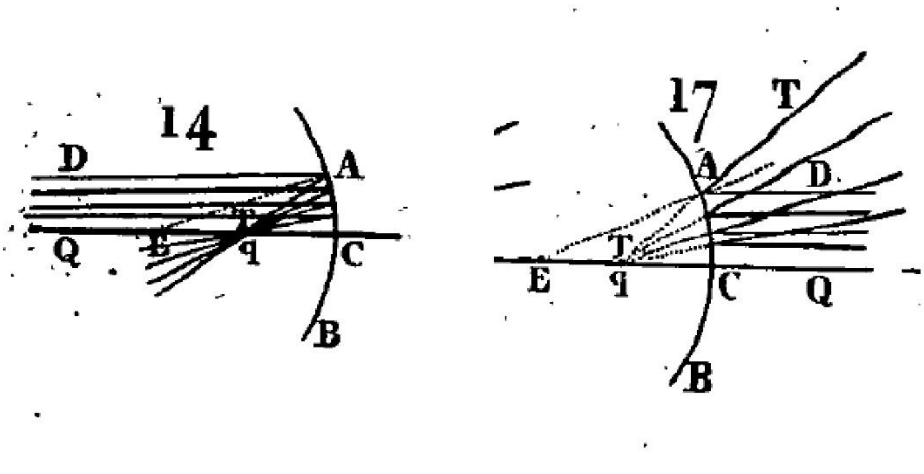

Figure 8: Reflection in a convex mirror: no mention of a distinction between real and virtual image (Smith 1738, p. 8).

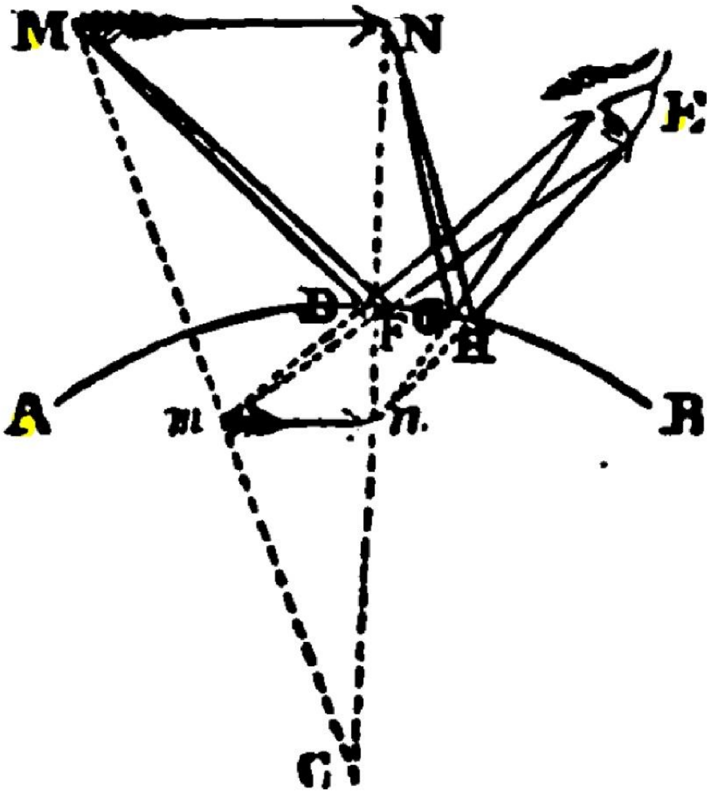

Figure 9: Virtual image and triangulation with a single eye (Brewster 1831, p. 24).

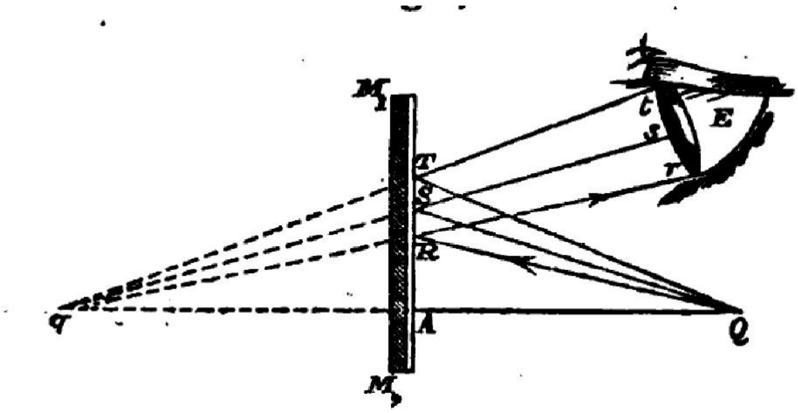

Figure 10: Virtual image and triangulation with a single eye (Airy 1870, p. 9).

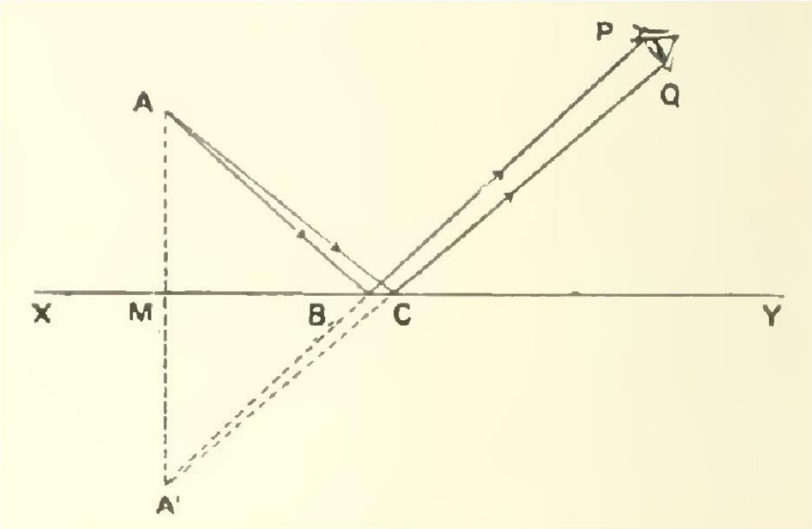

Figure 11: Virtual image and triangulation with a single eye (Ramsey 1914, p. 8).