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Mach on Analogy in Science

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Introduction

Many nineteenth-century natural philosophers and scientists employed analogy, and some (e.g. James Clerk Maxwell, John Herschel) discussed it as a subject in its own right, too. Analogy plays a role in Ernst Mach's philosophy as well as in his scientific work. What I want to do in this chapter, however, is examine Mach's views on how analogy is used in natural science. I think the uses Mach saw for analogy in natural science is nothing short of majestic and that, when properly understood, his views on analogy help us to see the roles appropriately played by logic, psychology, and scientific principle when analogy is used in natural science. Unfortunately, some reprints and translations of Mach's essay on the topic contain omissions whose presence and placement are crucial to understanding the surrounding text. These textual inadequacies have undoubtedly contributed to a lack of understanding – and thus of appreciating – Mach's views. (I describe and discuss the associated textual corrections in Footnotes 1 and 2 to this chapter for the benefit of the reader who wants to get a clear idea of the nature and significance of these textual inadequacies.)

Until it is fully understood, what Mach says about analogy can come across as unexceptional and, at times, even contradictory. Neither is true. In fact, some points that are today regarded as discoveries about, or recent advances in, the study of analogical reasoning are seen to be not only already articulated by Mach, but taken to a higher level of sophistication. Inasmuch as is possible, though, I aim to avoid using anachronistic terminology to express Mach's points about analogy, and I will focus simply on sorting out and presenting Mach's view.

The Distinctiveness of Analogy

Mach says that analogy is a *special case* of *similarity* ('*Die* Analogie *ist jedoch ein* besonderer *Fall der* Ähnlichkeit'[1]), and that there are good reasons for

[1] 'Die Aehnlichkeit und die Analogie als Leitmotiv der Forschung' in the journal *Annalen der Naturphilosophie* (Mach 1902). The third sentence of the paper is '*Die* Analogie *ist*

regarding analogy that way. Analogy is a special case of similarity in that what matters in the kind of similarity we call an analogy are 'conceptual relations': both the conceptual relations that concepts have to each other and the relations that concepts have to the objects between which the analogy is drawn. One way in which the kind of similarity that is an analogy differs from other cases of similarity, he notes, is as follows: 'Not a single immediately perceptible feature of one object need be found to be a feature of the other object' ('*Nicht ein einziges unmittelbar wahrnehmbares Merkmal des einen Objektes braucht mit einem Merkmal des anderen Objektes übereinstimmender, identischer Weise wiedergefunden werden*' (Mach 1902, p. 5). This is, I think, meant to be arresting, for it was fairly common to portray an analogy as requiring – or even as consisting in – the fact that two objects have some of the same immediately perceptible (or at least observable) features. This idea is still commonly found today in introductory textbooks on logic and scientific reasoning as well (e.g. Salmon 2012).

The way Mach presents analogy in the outset of the essay is thus as a contrast case to other cases of similarity. That is, first he characterises similarity in terms of how it compares to identity as far as having features in common: 'Similarity is partial identity: the characteristics of similar objects are in part identical and in part different' (Mach 1976, p. 162). Mach then emphasises that two objects related by analogy might not have *any* immediately perceptible features in common. So analogy is an extreme case – we might say it is a limiting or degenerate case – of similarity; Mach says just that it is a special, or very particular (*besonder*) case of similarity. Analogy, he is highlighting here, is about sameness (identity) of *relations between*, not sameness (identity) of *features of* objects. Further, recognition of an analogy consists in determining that the same relations exist between the features of one object as exist between the homologous features of another object. We shall see that when it comes to proffering his own definition of analogy,

jedoch ein besonderer *Fall der* Aehnlichkeit'. This sentence appears in both the version in the journal (1902) and in the German-language edition of the anthology *Erkenntnis und Irrtum* (1905). It does not appear in the English translation by Thomas J. McCormack in *Knowledge and Error* (Mach 1976), which was translated from the 5th edition of *Erkenntnis und Irrtum*, 1926. Since Mach also indicates later in the paper that his view is that analogy is a special case of similarity, there is no question that he endorses the statement. Excising the statement from the essay, however, as in the English translation, or even moving it from its position as the third sentence in the paper, changes the emphasis of the sentence that follows it. In the original version published in the journal in 1902 and the anthology in 1905, the statement that begins 'Not a single observable mark . . .' is clearly meant to be bringing out a point about analogy; in the English translation (Mach 1976), where the sentence '*Die* Analogie *ist jedoch ein* besonderer *Fall der* Aehnlichkeit' has been excised, it appears to be a comment about similarity. My thanks to Karin Krauthausen for urging me to consult the version that originally appeared in *Annalen der Naturphilosophie* and providing me with a copy of it.

though, Mach does not speak in terms of objects or, even, features of objects. Rather, he speaks of an analogy holding between systems of concepts. Why is he doing so here, then, at the very outset of the essay?

As I see it, the reason that Mach is highlighting the fact that these two different objects between which an analogy is drawn might not have any (immediately perceptible) features in common is not so much to endorse a definition of analogy as holding between objects, but rather to draw attention away from the *sharing of features* and towards what is essential to an analogy on his account of analogies in natural science: *relations between concepts*. At this point of the discussion in the essay, he does so from *within* the presuppositions and terminology of the existing discourse about similarity, which is similarity of objects. Discussions later in Mach's 'Similarity and Analogy' may appear inconsistent with this early portion of the essay, unless one takes account of that.

It is the common usage of the term 'analogy', on which an analogy is drawn in terms of objects and their features, that is under discussion when, later in the same essay, Mach points out that the expectations generated by analogies are not logically justified. ('*Diese Erwartung ist logisch nicht berechtig . . .*'; Mach 1902, p. 9) On this kind of characterisation, 'Inferences from similarity and analogy are not strictly matters of logic, at least not of formal logic, but only of psychology' (Mach 1976, p. 166). Analogy is distinguished from similarity here, too, and on the same general basis that we will see Mach emphasise on his own account, on which analogy holds between systems of concepts: relations rather than relata. Considering the case of an analogy between two objects M and N where 'an object M has marks a, b, c, d, e, and another object N agrees with it as regards a, b, and c,' he writes that 'If a, b, c, d, and e above are directly observable, we speak of similarity; if they are conceptual relations between marks, analogy is closer to normal usage' (Mach 1976, p. 166). If d and e are 'indifferent', the analogy merely makes us *associate* d and e with the object N. If d and e are especially 'useful or noxious' properties, he says, we may go on to investigate further. But Mach does not say that those investigations are carried out using analogy: it seems that they would be carried out using whatever methodologies for finding or figuring things out one would normally use: 'by simple sense observation or by means of complex technical or scientific conceptual reactions' (Mach 1976, p. 166). This discussion seems to be reviewing and describing contemporaneous accounts of analogy, on which analogies are heuristics for suggesting possibilities to investigate. However such investigations turn out, our knowledge is extended, he says. Therefore, this use of analogy results in extending our knowledge, even though the inference from the analogy itself is not logically justified.

In what follows, I will lay out Mach's account of analogy as well as his views about the significance of analogy in many advances in natural science. Some apparent inconsistencies may arise from what he says about analogy in that context and what he says about analogy in the context just quoted above in discussing the term in common usage. Hopefully, what I point out about the two uses will show that the inconsistency is merely apparent. The uses of the term 'analogy' in the different contexts are actually referring to different kinds of analogy: in the common use of the term, an analogy is drawn between two *objects* and is feature-based. In the powerful use made of it in the historical case studies he has in mind when talking about analogy in natural science, an analogy is drawn between two *systems of concepts*. The latter kind of analogy opens up a role for scientific laws and principles to play in analogies.

Mach's Account of Analogy

Since Mach characterises similarity as 'partial identity' and includes analogy as a special case of similarity, not all cases of similarity are cases of analogy, and so some things can be said of analogies that are not true in general for cases of similarity. Both identity and analogy are special cases of similarity. They are extreme cases: at one extreme (identity), all of the features *must* be the same; at the other extreme (analogy), none of them *need* be the same. To put it another way: what we can say about every case of identity that we cannot say about every case of similarity is that all of the features between the two things being compared are the same. What we can say about every case of analogy that we *cannot* say about every case of similarity is that a specified set of connections or relations – connections or relations that may not be immediately perceivable – are the same.

When highlighting how analogy differs from other kinds of similarity, Mach spoke of the marks (*Merkmale*), or features, of objects, but, as noted above, I think the terminology (marks, or features) used there when discussing analogy was for consistency with the terminology already in use when talking about similarity, of which analogy is a special case. It makes sense to distinguish between those contexts in which he is using the terminology in common use when discussing analogy and similarity in order to contrast his view with it from contexts in which he is presenting his own definition of analogy.

When it comes to proffering *his own* definition of analogy, Mach defines analogy as holding between 'systems of concepts' rather than between objects or their features. It is a significant difference that, in the first place, the things that an analogy holds between are *systems*, because a system consists of, or contains, interrelated items. Secondly, he says they are systems of *concepts* – thus, the relations are between interrelated concepts (rather than features, or marks, of objects). They are *logical* relations.

More specifically, Mach defines analogy as 'a relation between *systems of concepts*, in which both the *difference between* two homologous concepts, and the *identity of the logical relations* of each pair of homologous concepts come to clear consciousness' (emphasis added).[2] That is, in an analogy, we can see clearly that the concept in one system and the homologous concept in the analogous system of concepts are different (when they do differ), while, at the same time, we are also aware that, for each pair of homologous (corresponding) concepts in the two analogous systems, the logical relations associated with one are the same as with the other. What are identical in an analogy are these logical relations between concepts.

I think it is worth noting that Mach does not say that more is required regarding the logical relations than just being conscious *that they are identical*. Often a determination of identity can be made on partial information, as when we can see that the levels in two glasses are the same without having to measure what those levels are, or showing that two shapes are the same without having to quantify or identify the shape of either by overlaying each over the other. Likewise, it seems that what is required is just to show that the logical relations in one of the systems of concepts work in the same way as the logical relations in the analogous system of concepts work with the homologous concepts.

I pointed out above that, on Mach's definition of analogy in natural science, there are two aspects that distinguish an analogy from other cases of similarity: (1) an analogy is drawn between *systems of concepts*, whereas this is not true for all cases of similarity and (2) the logical relations in each of the two systems between which the analogy is drawn are identical. We might wonder what each of these two aspects contributes to his definition of analogy independently.

Although Mach defines analogy as holding between systems of *concepts*, the provision in his definition about identity of relations could be illustrated with systems of spatially interrelated things rather than systems of logically interrelated concepts. I find it helpful to consider how this provision would go for a concrete example before considering how it would go for a system of concepts. This might be illustrated, it seems to me, by the following simple example: consider a sketch that has been constructed to record and display observations of the location of a system of things. Here, a sketch made of astronomical observations provides a good example. Each of the points in the sketch bears spatial relations to other points on the sketch. Each mark on the sketch is unlike the body in space that it is homologous to, though, in terms of what is immediately perceptible about it. Yet, we can point out that certain spatial relations that each mark on the sketch has to other marks on the sketch are *the same as* the relations that the observations in the sky that a certain mark is

[2] The original in German reads: '*eine Beziehung von Begriffsystemen, in welcher sowohl die Verschiedenheit je zweier homologer Begriffe als auch die Uebereinstimmung in den logischen Verhältnissen je zweier homologer Begriffspaare zum klaren Bewusstein kommt*'.

homologous to has to the other observations in the sky that the other marks on the sketch are homologous to.

Now, as I have described it, the above example of a sketch and some observations in the sky is an example of a similarity that illustrates aspect (2) of Mach's definition of analogy in natural science, but it would *not* count as an analogy on Mach's view, since it is not a case of a similarity holding between systems of *concepts*. This is important to keep in mind so as to fully appreciate what he is saying in discussing examples of the use of analogy in mathematics, especially ones that involve geometrical figures.

What does Mach cite to show the use of analogy in mathematics, then? First, his general remarks on the physical application of mathematics involve analogies of *operations* rather than *objects*: it is not just the correspondence of *mathematical entities* or marks with *physical entities* that makes the application of mathematics an analogy. Mach writes, 'Every physical application of mathematics rests on taking note of analogies between facts and mathematical operations' (Mach 1902, p. 7). When Mach says that Hermann Grassman's mechanics or vector theory makes use of an analogy between 'lines and forces, areas and torques, and so on', he makes it clear that he would spell this out as an analogy between operations defined on lines and areas, as well as facts about forces and torques. I take him to be saying that there is an analogy between two systems: one a system containing lines and areas and an operation that relates them; and the other a system containing forces and torques and some facts that relate them. In addition, the logic of the operation in the system relating the lines and areas is identical to the logic of the facts in the system relating the forces and torques. Algebra can be used to formulate the relation in such a way that the identity is clear (i.e. 'the logical relations come clearly to consciousness').

Note that Mach's analogy is not the only kind of mathematical analogy possible; other mathematicians opted for other ones. Others writing contemporaneously on Grassman's mechanics sometimes spoke of analogies in terms of objects and their properties. One explains the fact that 'the summation of sects . . . corresponds completely to the discussion of the resultant of a system of forces in a plane' as following from the observation that 'a sect possesses the exact geometrical *properties* of a force, namely, magnitude, direction, and position' (Hyde 1905, p. 31, emphasis added). Mach's choice to pick out *operations* rather than the *things* being operated on or *features* of the things being operated on in the analogies he draws in the physical application of mathematics is due to his view of analogy as a relation between *systems of concepts*. The operations in Grassman's abstract treatments of mechanics were often spatial transformations (rotations, translations, etc.).

Secondly, the analogy that Mach chooses in order to illustrate 'the great value of geometry in cognition' is more involved and contains subtleties, but in it we see the same point. The case deals with the 'optical properties' of

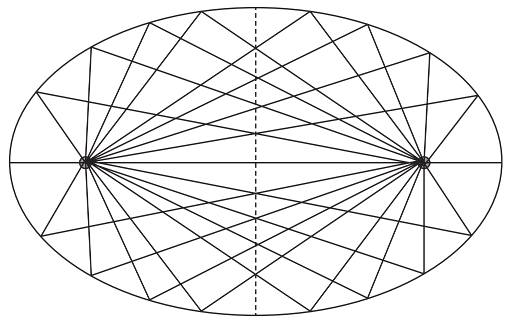

Figure 4.1 Foci of an ellipse. The two points of intersecting rays shown are called foci. Every light ray sent out from one of the foci will be reflected off the surface of the ellipse on a path that intersects the other focus. The same holds for sound waves, too, and this is the phenomenon behind a 'whispering gallery' – a room with an elliptical surface in which a whisper spoken at one of the ellipse's foci can be heard by someone located at the other focus of the ellipse

curves – specifically, 'logical properties' of conic sections. 'Optical properties' of conic sections arise from the study of how light rays are reflected off and travel within spaces bounded by surfaces shaped like those conic sections: probably the most familiar 'optical property' of a conic is the pair of foci of an ellipse. In a space bounded by a surface shaped like an ellipse, light rays emanating from one focus, no matter how they are orientated, will pass through the other focus (see Figure 4.1).

Considered *with respect to the optical properties* of a conic section, the *focus (of an ellipse)* is a concept bearing a logical relation to the concept of *ellipse*. It was in studying the *conceptual relations* of foci (of various particular conic sections) to particular conic sections that Johannes Kepler employed the reasoning Mach cites here. Thus, on my reading of Mach, this *would* count as an analogy, and Mach does refer to it as an analogy, too. But he is *not* referring, as above, to the physical application of the concepts of conic section, but rather to an analogy that Kepler finds among conic sections. As we will see in what follows, it is a case of analogy used within mathematics itself to generate a result in abstract mathematics.

Kepler explains how the concepts of *foci* and *conic section* figure in the characterisations of a circle, ellipse, parabola, hyperbola, and point (the degenerate case). Mach quotes him as saying (in Latin in the original): 'The one focus of a circle is A, namely at the centre; in the ellipse there are two foci, A and B, equidistant from the centre of the figure in the more pointed part.' Kepler then examines how the location of a focus or foci with respect to the curve formed by the outer surface of the cone and the cutting plane varies: 'In the circle, the focus is thus at the centre, as far from the circumference as possible, in the ellipse already less and in the parabola much less, and finally in

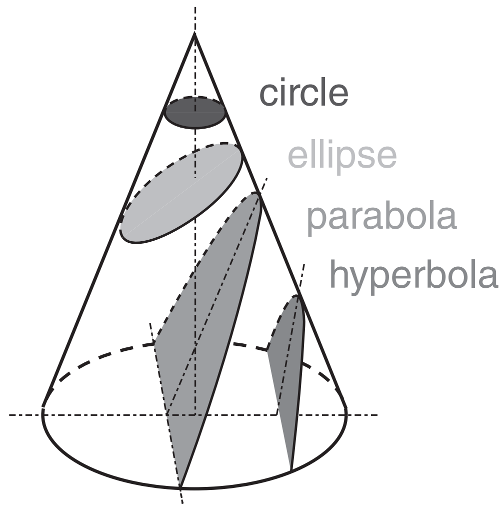

Figure 4.2 Conic sections. The conic sections are the various shapes formed by the intersection of a solid cone and a plane cutting through it at various angles

the straight line it is at minimal distance, that is, it falls on the line.' The straight line is not obvious in a progression of conic sections (indeed, it is not generally included), but Kepler remarks on the apparent misfit in a parenthetical remark: '[W]e speak of straight lines not so much in the ordinary way but rather to complete analogy.' Using the graphical means of a plane intersecting a conic, though, as is seen in Figure 4.2, a straight line can be seen as being at the other end of the progression that begins with a horizontal plane intersecting a cone to form a circle. A line would in fact result from the intersection of the plane and the cone when the plane's orientation coincides exactly with the angle of the cone. How to treat the *focus* of the conic when the conic is a straight line? This is a matter of using the analogy: 'It follows by analogy [with the hyperbola] that in a straight line either focus . . . falls on the line: there is but one [focus], as in the circle' (Mach 1976, p. 164). The circle and the line are the two extremes. Mach attributes Kepler's ability to grasp these 'deep-seated' analogies to his use of 'the principle of continuity', which I take to refer to how Kepler has organised the progression of conic sections from circle to line. The circle can be continuously transformed into an ellipse, the ellipse into a parabola, the parabola into a hyperbola, and then the hyperbola into a line, and the foci continuously change their locations accordingly. In fact, thinking of these figures – circle, ellipse, parabola, and hyperbola – as conic sections is associated with imagining a cutting plane being continuously rotated from a beginning position in which it cuts out a circle through cutting out an ellipse, parabola, hyperbola, and line. Significantly, Kepler's explanation of his reasoning shows that the foci associated with each of these conic sections figures in the comparisons and transformations. In regarding them as 'optical properties' of the conic sections, the foci here are characterised in terms of *facts about how rays of light behave*, so this example illustrates how Mach's approach to analogy provides a role for scientific principles to play.

Mach on Analogy in Natural Science

We have seen that Mach's definition of analogy ('a relation between systems of concepts, in which both the difference between two homologous concepts, and the identity of the logical relations of each pair of homologous concepts come to clear consciousness') provides a role for scientific principles and laws to play in analogy. When it comes to explaining the significance of analogy in natural science, Mach specifies no set way in which analogy is used. In the historical cases he discusses, there is always some area of science about which a lot is already known, but analogies are generated and contribute to extending knowledge in a variety of ways. What comes through in reading his discussions of cases in the history of science is the need for agility and flexibility in recognising the possible analogies one can draw upon and being able to adapt them to the needs of the situation at hand.

As might be expected considering the era, Mach describes cases of analogies between different kinds of waves, different kinds of currents, and different kinds of fields. But he also mentions Galileo's discovery of the moons of Jupiter as an important case of analogy in natural science, writing that Galileo's discovery was 'more powerful than any other arguments [by analogy][3] in supporting the Copernican system'. We might ask what kind of analogy that is, or how analogy is involved in that case, as it seems to concern two particular *physical* systems (i.e. the system of the planet Jupiter and its moons and the system of the sun and the planets in our solar system). Galileo's discovery of the moons of Jupiter, reported in *The Starry Messenger*, arose from his handwritten sketches of their positions at successive points in time: he came to see that the bodies he was observing must have been moving in orbits around Jupiter. Mach's succinct comment about the kind of analogy between the system of Jupiter and its moons and the solar system he is referring to as so powerful an endorsement of the Copernican view is telling: '[W]e have here a small scale model of the solar system' (Mach 1976, p. 167). The reason that the phrase 'small scale model' is telling is because Mach certainly knew something about methodologies of modelling (what we would

[3] The journal version of Mach's essay seems to say this is the strongest of all the arguments by analogy, rather than the strongest of all arguments. '*Die Entdeckung der Jupiter Trabanten durch Galilei hat das Copernikanische System mächtiger als alle anderen Argumente durch die Analogie gestutzt. Das Jupitersystem stellte ein verkleinertes Modell des Planetensystems dar*' (Mach 1902, p. 11). Hence, I have indicated the qualification in brackets in my English rendition here.

call) kinematic similarity used to model one physical system with another. In fact, he wrote about them, though he did not use the terminology of kinematic similarity or physical similarity. In *The Science of Mechanics*, Mach discusses Newton's notion of similar systems, which Newton uses to indicate when the motions of the bodies in one physical system of bodies will be homologous to those in another. As I wrote on an earlier occasion in discussing the history of the concept of physical similarity:

> After generalizing one of his own conclusions, Mach remarks: 'The considerations last presented [on similarity and similar systems] may be put in a very much abbreviated and very obvious form by a method of conception first employed by Newton.' He does not quite accept Newton's use of the term similar system there, though:
>
> 'Newton calls those material systems *similar* that have geometrically similar configurations and whose homologous masses bear to one another the same ratio. He says further that systems of this kind execute similar movements when the homologous points describe similar paths in proportional times' (Mach 1960, p. 203).
>
> Mach admires Newton's methodology here, but he points out an issue with Newton's use of the term *similar* . . .'
>
> (Sterrett 2017, p. 379)

Mach's care in distinguishing the use of geometrical similarity from uses of other closely allied kinds of comparisons is evident in this discussion in *The Science of Mechanics* (Mach 1960) – as it is in his essay on similarity and analogy (Mach 1902, 1976). I went on to describe Mach's attempt to rescue Newton's analysis here:

> However – and what is significant and interesting – Mach does not say that Newton is wrong here; rather, what he says is that what Newton was doing is better understood in Mach's day in terms of affine transformations:
>
> 'The structures might more appropriately be termed *affined* to one another. We shall retain, however, the name phoronomically [kinematically] *similar* structures, and in the consideration that is to follow leave the masses entirely out of account (Mach 1960, p. 204).'
>
> (Sterrett 2017, p. 380)

Now, I suggest, the way in which Mach proposes Newton's work should be understood fits Mach's definition of an analogy in natural science, for it turns out that 'phoronomically similar' structures relate homologous concepts (rather than bodies) and that the relations between them are identical. As I explained previously:

> . . . Mach shows how to understand phoronomically [kinematically] similar structures for the topic of oscillation he has been discussing:

> 'In two such similar motions, then, let
> the homologous paths be s and αs,
> the homologous times be t and βt;
> whence the homologous velocities are $v = s/t$ and $\alpha v = \alpha/\beta\ s/t$,
> the homologous accelerations $\varphi = 2s/t^2$ and $\varepsilon\varphi = \alpha/\beta^2\ 2s/t^2$
>
> Now all oscillations which a body performs under the conditions above set forth with any two different amplitudes 1 and α, will be readily recognised as *similar* motions (Mach 1960 p. 204).'
>
> (Sterrett 2017, p. 380)

So, if path, time, velocity, and acceleration can be considered concepts, what Mach describes above are two systems of interrelated concepts. He states the logical relations between them in the excerpt shown above: we can immediately see that the relations between the interrelated concepts in the two systems of concepts are identical. Hence, it is a relation between two systems of concepts that meets Mach's definition of analogy in natural science. His assessment of the value of such methods of investigation is thus an assessment of the value of analogy in natural science, too, and about this he is positively effusive: 'After showing how elegantly theorems about centripetal motion can be obtained by such means', Mach remarks:

> It is a pity that investigations of this kind respecting mechanical and phoronomical *affinity* are not more extensively cultivated, since they promise the most beautiful and most elucidative extensions of insight imaginable (*The Science of Mechanics*, p. 205).
>
> Thus Mach sees the great power of the notion of similar systems.
>
> (Sterrett 2017, p. 380)

As Mach made a point about the crucial importance of Kepler's use of continuity along with his use of analogy in developing his account of conic sections, it is noteworthy that Mach's discussion about this 'scale model' kind of analogy in *The Science of Mechanics* also employs continuity. The transformations he describes between the two systems of bodies (which is what would be relevant to the case of seeing the system of Jupiter's moons as a scale model of the solar system) are *continuous* transformations, and many of the concepts related by the analogy likewise include continuity (e.g. time is continuous, paths and velocities are at least part-wise continuous). What is significant about that with respect to scale models and similar systems is that, when these ideas were later formalised in the landmark paper of 1914 by Edgar Buckingham, the idea of a similar system (and so of a scale model) was presented in terms of one system undergoing continuous changes to create the other, all the while obeying applicable physical laws and relations at every point in the transformation (Buckingham 1914). Thus, continuity is important in the case of the kind of analogy that underwrites use of a scale model – even in the later formulation of it that appeared in a physics journal.

Mach also cites analogies between different kinds of waves in illustrating how important analogies have been in the development of natural science: surface (water) waves, sound waves, and light waves. A favourite theme of his in both his popular lectures and his scientific publications is the analogy between sound and light; in his essay 'Similarity and Analogy' he writes, 'As to light, the appropriate ideas were developed from the case of sound' (Mach 1976, p. 167). This is the kind of statement that might make Mach look like an uncritical user of wave analogies, or at least analogies that are out of date and irrelevant to the knowledge of today. Yet that judgement is mistaken, for as I pointed out in 'Sounds Like Light: Einstein's Special Theory of Relativity and Mach's Work on Acoustics and Aerodynamics':

> Commentators on Einstein and special relativity tend not to look in the direction of work in acoustics for conceptual precursors to the special theory of relativity; expositions on special relativity that compare light and sound tend to associate the insight of special relativity with the contrast between light and sound, and the similarities between them with the (discredited) classical wave theory of light.
>
> . . . Mach was exceptional here in that, in the very context of drawing an analogy between sound and light, he explicitly freed the notion of a wave from the necessity of having a mechanical basis.
>
> (Sterrett 1998, p. 2)

The occasion for Mach's own substantive scientific work on wave analogies was determining the correct explanation of the Doppler effect. He used acoustic experiments to prove that the observed effects (difference in pitch of sound) arose from the relative motion of the sound source and the observer of the emanating sound wave, and then concluded that, based on the analogy between light waves and sound waves, the explanation of the Doppler effect for light (difference in colour of light) was likewise due to the relative motion of light source and observer. To develop the analogy, he looked at many different kinds of waves and identified what he thought they had in common that also captured what was essential to waves. This included being propagated in time, having spatial periodicity, having temporal periodicity, and being able to be algebraically summed. Mach says something very bold and striking about what his experiment showed: 'It is of absolutely no significance for the question of whether Doppler's principle applies to light', he says, 'whether or not light is a mechanical wave motion [like sound]. One could just as well think of light as chemical oscillation, for many of the appearances, such as anomalous dispersion and fluorescence, can be better understood, in many respects, under such a notion of light' (Sterrett 1998, p. 22). In that earlier work on Mach, I commented that Mach

> confidently states that the Doppler principle can be applied to light in the same way as for sound, on the basis that light and sound are propagated in time, have spatial and temporal periodicity, and can be algebraically summed. This is in keeping with his earlier remarks on the Doppler effect for sound, in which he kept clear of appealing to any causes arising from the mechanical nature of sound waves, and stuck to kinematical considerations.
>
> (Sterrett 1998, p. 22)

The above quote indicates Mach's use of an analogy between light and sound; it is drawn in terms of an analogy between different systems of (interrelated) concepts, in that homologous concepts such as *pitch* (frequency of sound) and *colour* (frequency of light) are different, yet their relations to other concepts such as (sound/light) wave velocity and (sound/light) wavelength are the same between homologous concepts.[4] It certainly fits Mach's definition of an analogy in natural science.

There is another noticeable pattern, a trend we see as we consider more and more of Mach's discussions of the use of analogy in historical examples: a trend towards eliminating the more material-laden aspects of a situation. Often, material aspects of an area are responsible for a point of disanalogy between two areas of science, but they make no difference to many other points of analogy. Leaving the material medium out of the account of waves seems to be part of this pattern, just as, in Mach's discussion of Newton's work on 'similar systems' above, Mach proposed to 'leave masses out of the account'. In applying his 'method of physical analogy', James Clerk Maxwell likewise would leave masses and materiality of a fluid out of his account. Mach shows his admiration for Maxwell's successful investigations in natural science, and especially for his 'method of physical analogy' throughout his essay on similarity and analogy. Maxwell, Mach says, 'describes analogy as that partial similarity between the laws in one field and those in another, so that each illustrates the other' – a view, he says, from which his own view of analogy is 'not different' (Mach 1976, p. 162). Later, he uses Maxwell as an illustration of the fact that the use of analogy tends to lead to being able to use abstraction.

From his ability to see an analogy between different kinds of physical phenomena in electrostatics and electrodynamics with fluids, Maxwell is able to use both abstraction (i.e. finding features common to various phenomena in physics and imagining a fluid that has just those and not other features such as mass) and physical intuition: '[W]e do not take it as real and we know precisely how it coincides conceptually with the facts to be represented' (Mach 1976, pp. 168–169). Mach, too, employed analogy in somewhat the same way in investigating different kinds of waves, in that one goal of his

[4] This is explained in greater detail in Sterrett (1998).

comparisons was to separate off certain facts about waves in order to use them in reasoning what could be deduced more generally about any given wave.[5]

In saying that his view of analogy is not different from Maxwell's, and throughout the entire essay, Mach seems to be supportive of other scientists and natural philosophers writing on analogy. His general approach in this essay seems to be to find common ground, rather than to find fault with the views of other natural philosophers and scientists on the topic. Yet we may wonder what points of disagreement there might be; philosophers often find it illuminating to identify them in order to better understand someone's views. Here, I would suggest there is a point of disagreement with Herschel, in spite of Herschel clearly articulating some of the most significant points about analogy that Mach wished to emphasise. Mach certainly recognised Herschel's strength in identifying and reasoning with analogies; he mentions Herschel's striking success in predicting experimental results on the basis of analogy before there was direct experimental evidence for them (e.g. the case of polarisation of light (Mach 1902, p. 225), among others), but he does not endorse Herschel's philosophical views on the matter. Yet Herschel's views in *Preliminary Discourse* seem at times akin to Mach's, such as when Herschel writes of the 'general resemblance between the two sciences of electricity and magnetism' and notes that 'many of the chief phenomena in each were ascertained to have their parallels, mutatis mutandis, in the other' (Herschel 1845, section 85, p. 94), which he explains as follows: 'If we encounter the same elementary phenomena in the analysis of several composite ones, it becomes still more interesting, and assumes additional importance: while at the same time we acquire information respecting the phenomenon itself, by observing those with which it is habitually associated . . .' (Herschel 1845, section 85, p. 93). There is a basic difference, though, in that Herschel consistently conceives of and applies analogies in terms of a 'cause' that they have in common, whereas Mach's definition and use of analogies is in terms of systems of concepts and relations. The difference between Mach's and Herschel's uses and conceptions of reasoning by analogy shows up strikingly in how they conceive of and make use of the analogy between sound and light, as Herschel writes that 'an analogy between sound and light has been gradually traced into a closeness of agreement, which can hardly leave any reasonable doubt of their ultimate coincidence in one common phenomenon, the vibratory motion of an elastic medium' (Herschel 1845, section 85, p. 94). In contrast, the value of Mach's application of the analogy between sound and light in explaining the Doppler effect is that it *did* allow for agnosticism regarding the existence of a medium for the transmission of light – which,

[5] This is laid out in Sterrett (1998), which explains the momentous significance for twentieth-century physics of Mach's theoretical and experimental work in his explanation of why the Doppler effect and the phenomenon of shock waves hold for waves of any kind.

I have argued (Sterrett 1998), was later crucial for the development of the special theory of relativity. Thus, the difference between Mach and Herschel on the use of analogy is significant, not just philosophically, but in terms of the scientific conclusions that their methods underwrite. It is not surprising, therefore, that the enthusiasm and admiration Mach shows Maxwell for his method of physical analogy does not extend to Herschel.

Conclusion

I have not done justice here to all of the aspects of Mach's rich and subtle discussion of analogy. There is much more he says to open his readers' eyes to uses of analogies in natural science. We now take stock of a few basic points we have been able to touch upon in this chapter.

In his short essay on similarity and analogy, Mach explores the use of analogy in natural science, while at the same time recognising a more common usage of the term. He distinguishes analogy from similarity for the more common usage of the term on the basis of an emphasis on identity of relations (between features of objects) rather than relata (features and objects). This basis for identifying what is distinctive about analogy is formulated in such a way that it later carries rather smoothly over into his own account of analogy in natural science. Mach discusses limitations of the common usage of analogy (i.e. inferences are not logically justified), as well as virtues (it can stimulate investigation, extend our knowledge, and give a biological and physiological account of why we value analogies) (Mach 1976, p. 166). These points about the value of analogy are being rediscovered. An especially striking example of a discipline in which analogy is indispensable, in spite of being recognised as not providing logical justification of inferences, is ethnographic archaeology.

Alison Wylie (1985) and Mads Ravn (2011, 2018) each give a historical narrative of how views of the use of analogy in ethnographic archaeology have changed. At first, the power of analogy to extend knowledge was embraced: knowledge about certain present societies that were considered representative of past ones in other places, it was argued, could be used to extend the sparse evidence from those past societies. When the bases for making such comparisons between present and past societies were later recognised as faulty, and thus the conclusions obtained by the use of analogy untrustworthy, the use of analogy was disavowed, even disparaged. However, Wylie showed that even the methods proposed as alternatives to analogy by those who openly disavowed analogy were actually analogical methods of reasoning, too. Instead of disavowing the use of analogy, she argues, legitimate critiques of the use of analogy can help guide more appropriate use of analogy in ethnographic archaeology. These critiques direct our investigations into both the source-side domain and the subject-side (target) domain of knowledge. These points are in line with Mach's views on what I have called the common view of analogy above.

However, Mach went further than the question of how analogies can be licensed and talked about the value of analogy even when the analogy is shown to be undermined by a negative analogy (i.e. when we find features that the two objects being compared do not have in common). Mach said the cases where there is such a negative analogy are equally important (Mach 1902, p. 10). A recent in-depth study by Ravn argues this, too. In 'Roads to Complexity: Hawaiians and Vikings Compared' (Ravn 2018), Ravn explicitly highlights the role that negative analogies play in making the point that complexity in a society does not necessarily depend on certain specific features. Ravn's study is a very detailed use of analogy in which comparisons and analogies are drawn using the 'long view' of societies, rather than snapshots of them at only certain points in time, and in which processes, rather than only features, of societies are considered. What makes Ravn's study notable here is that archaeology is often cited for its use of feature-based analogies. Here, it is fitting to recall Mach's admiration of Kepler's use of continuity in drawing analogies between the conic sections. Thus, we see in these case studies in archaeology that delving seriously into valuing and making use of analogy in the more common sense of the term eventually leads towards the kind of analogy that Mach meant when talking about its use in the natural sciences: the analogy is between systems of concepts, and what is being equated are logical relations between concepts.

Mach's real interest in this essay lies in the notion of analogy as used in natural science. On Mach's account, the notion of analogy in natural science is a relation between *systems of (interrelated) concepts*, and what makes the relation an analogy is the identity of logical relations between homologous concepts. Mach saw analogies used in a variety of ways in natural science, so that even though he said his account was 'not different' from Maxwell's view of analogy, he had a lot to say about how and where analogies had been, and could be, used in natural science. Maxwell's description of analogy as a 'partial similarity between the laws in one field and those in another', Mach said, brought to light what was most valuable about analogy for scientific enquiry: each of the two laws illustrates the other. He discusses how powerful this method can be in extending knowledge, and he gives some unusual and surprising examples from the history of science.

We have also seen that Mach's appreciation of the power of analogy is evident in other works, albeit not always described explicitly as an analogy. For Mach, natural science is more variegated than just scientific enquiry into the unknown. His discussion indicates that sometimes the use of an analogy is about understanding one known thing in terms of another known thing (Mach 1976, pp. 167–168), but he also said that sometimes new analogies and questions arise that were not the target of enquiry (Mach 1976, p. 165). He noted that there may be many different areas one could draw upon in using analogy: 'Several equally known areas M, N, O, P may enter into analogy, in groups of two or more', so that there may be 'different analogies, each justified in its setting' (Mach 1976, p. 167).

Mach felt that there was much to be gained in using analogy, and that opportunities to do so were being wasted. He warned about how much could be lost in not seizing the opportunity to do so, as evidenced by his closing anecdote in the essay. The anecdote is meant to have a moral of epic significance: Newton's failure to consider analogies other than the one he was using in planning his experimental investigations – analogies that were easily available to him – meant that he ignored easily available but crucial observations, and this failure left him clinging to the wrong theory of light (ibid., p. 169).

Mach clearly had a mission in writing about analogy: I hope that the work I have done here, although just touching on a few of Mach's key ideas about analogy, will help us better understand what that mission was.

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