On the reality of the quantum state once again: A no-go theorem for ψ-ontic models?

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Abstract

In a recent paper (Found Phys 54:14, 2024), Carcassi, Oldofredi and Aidala concluded that the ψ-ontic models defined by Harrigan and Spekkens cannot be consistent with quantum mechanics, since all pure states of these models must be orthogonal to each other according to their information theoretic analysis, in clear violation of quantum mechanics. In this paper, I argue that this no-go theorem for ψ-ontic models is false.

The reality of the quantum state has been a hot topic of debate since the early days of quantum mechanics. Is the quantum state real, directly representing the ontic state of a physical system, or epistemic, merely representing a state of incomplete knowledge about the underlying ontic state? In recent years, a rigorous approach called ontological models framework has been proposed by Harrigan and Spekkens (HS) to distinguish the ψ-ontic and ψ-epistemic views [1]. Moreover, several important ψ-ontology theorems that establish the reality of the quantum state have been proved in the framework, two of which are the Pusey-Barrett-Rudolph (PBR) theorem [2] and Hardy's theorem [3, 4]. In this background, Carcassi, Oldofredi and Aidala's (COA) recent no-go theorem for ψ-ontic models [5] is unexpected and surprising. If it is correct, it will be a very important new result. In this paper, I will examine the COA theorem and argue that it is false.[1]

Before presenting my critical analysis of the COA theorem, I will briefly introduce the ontological models framework in which the theorem is proved.

[1]A referee of my recent paper [6] asked me to evaluate COA's paper in his/her report. This paper can be regarded as my fulfillment of this task.

The framework has two fundamental assumptions [1, 2]. The first assumption is about the existence of the underlying state of reality. It says that if a quantum system is prepared such that quantum mechanics assigns a pure state to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. This assumption is necessary for the analysis of the ontological status of the quantum state, since if there are no any underlying ontic states, it will be meaningless to ask whether or not the quantum states describe them.

Here a strict ψ-ontic/epistemic distinction can be made. In a ψ-ontic ontological model, the ontic state of a physical system determines its quantum state uniquely, and the quantum state represents the ontic state of the system. While in a ψ-epistemic ontological model, the ontic state of a physical system can be compatible with different quantum states, and the quantum state represents a state of incomplete knowledge – an epistemic state – about the actual ontic state of the system. Concretely speaking, the quantum state corresponds to a probability distribution $p(\lambda|P)$ over all possible ontic states when the preparation is known to be P, and the probability distributions corresponding to two different quantum states may overlap.

In order to investigate whether an ontological model is consistent with the empirical predictions of quantum mechanics, we also need a rule of connecting the underlying ontic states with the results of measurements. This is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is only determined by the ontic state of the system, along with the physical properties of the measuring device. More specifically, the framework assumes that for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the predictions of quantum mechanics then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|P) = p(k|M, P)$, where $p(k|M, P)$ is the Born probability of k given M and P.

COA's proof of their theorem based on the ontological models framework is simple and clear and thus it can be readliy examined. COA analyzed the information entropy of a mixed state in both ψ-ontic models and QM, and argued that since they are different, ψ-ontic models are not consistent with QM according to their information theoretic analysis. The key then is to examine if the information entropy of a mixed state in these two theories are really different.

COA considered the mixed state $\rho = \frac{1}{2}(|\psi\rangle \langle\psi| + |\phi\rangle \langle\phi|)$, where $|\psi\rangle$ and $|\phi\rangle$ are two pure states. Its quantum information entropy is given by the von Neumann entropy, namely

$$H_{QM}(\rho) = -\frac{1+p}{2}ln\frac{1+p}{2} - \frac{1-p}{2}ln\frac{1-p}{2}, \tag{1}$$

where $p = |\langle\psi|\phi\rangle|$ is the absolute value of the inner product of the two states.

In ψ-ontic models, the two pure states $|\psi\rangle$ and $|\phi\rangle$ correspond to two δ probability distributions of ψ-related ontic states or two ψ-related ontic states λ_ψ and λ_ϕ. That the von Neumann entropy of a pure state is zero in QM requires that a pure state corresponds to a δ probability distribution of ψ-related ontic states or a unique ψ-related ontic state in ψ-ontic models;[2] otherwise the Shannon entropy for a pure state in ψ-ontic models already disagrees with the von Neumann entropy of the pure state in QM. Then the information entropy of a uniform mixture of these two states, λ_ψ and λ_ϕ, in ψ-ontic models is given by the Shannon entropy:[3]

$$H_{OM}(\rho) = -\frac{1}{2}ln\frac{1}{2} - \frac{1}{2}ln\frac{1}{2} = 1. \tag{2}$$

Now it can be seen that the information entropy of a mixed state in ψ-ontic models and in QM can be the same only when the pure states in the mixture are orthogonal, namely the inner product of the two pure states such as p in Eq. (1) is zero. Since the information entropy of a general mixed state (in which the pure states in the mixture are non-orthogonal) in ψ-ontic models and in QM are different, COA concluded that the ψ-ontic models cannot be consistent with quantum mechanics.

In order to find where the above proof goes wrong, we need to understand why the quantum information entropy of a mixed state is given by the von Neumann entropy, which, unlike the classical information entropy, relates to the inner product of the pure states in the mixture. First, for a mixture of orthogonal states $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, where $|\psi_i\rangle$ are certain orthogonal bases, the von Neumann entropy is just the Shannon entropy for a classical mixture. That is, the von Neumann entropy is $H(\rho) = -\sum_i p_i ln p_i$. Next, for a mixture of non-orthogonal states whose density matrix is equal to $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ (where p_i are determined by the inner product of the non-orthogonal states[4]), its von Neumann entropy is equal to the Shannon entropy of the above mixture of orthogonal states.

Then, Why? The reason is as follows. First, in QM orthogonal states can be distinguished by experiments with certainty, while non-orthogonal

[2]Even if there are hidden variables besides the quantum state (in which case a pure state will correspond to a general probability distribution of ontic states that include both the unique ψ-related ontic state and other hidden variables), they cannot be measured in principle and thus no information about them can be obtained. This means that the existence of hidden variables does not change the information entropy of a pure state as given by the Shannon entropy for the unique ψ-related ontic state.

[3]Note that COA gave a different formulation of the Shannon entropy in their paper [5]. But their result is the same as the one given here. In my view, the formulation given here is simpler and clearer.

[4]The appendix of COA's paper gives a clear illustration of this result [5].

states cannot be distinguished by experiments with certainty. Thus, for a mixture of orthogonal states, the von Neumann entropy is the same as the Shannon entropy, since the orthogonal states in QM can be distinguished with certainty, just like the classical states in a classical mixture. Next, since the non-orthogonal states, unlike the classical states, cannot be distinguished with certainty, the von Neumann entropy for a mixture of non-orthogonal states is different from the Shannon entropy for a classical mixture. In other words, the Shannon entropy requires that the states in the mixture should be distinguishable in experiments with certainty. Finally, since two mixed states in QM which have the same density matrix cannot be distinguished by experiments (which means that the information we can obtain from them must be the same), the von Neumann entropy of a mixture of non-orthogonal states, whose density matrix is equal to the density matrix of a mixture of orthogonal states, is equal to the von Neumann entropy of the mixture of orthogonal states, which is also the same as the Shannon entropy of the mixture of orthogonal states.

Based on the above analysis of the origin of the von Neumann entropy in QM, we can see where COA's proof of their no-go theorem for ψ-ontic models goes wrong. The issue lies in that COA implicitly assumed that in ψ-ontic models, the two ontic states λ_ψ and λ_ϕ, which are represented by two non-orthogonal states $|\psi\rangle$ and $|\phi\rangle$, are classical states that can be distinguished by experiments with certainty. Only by this assumption, can the information entropy of a uniform mixture of these two states be given by the Shannon entropy. It is this result that contradicts QM, in which the information entropy of a uniform mixture of two non-orthogonal states is given by the von Neumann entropy that is different from the Shannon entropy. However, this assumption is not part of the ψ-ontic models defined by HS, and as argued above, it is not consistent with QM either. In order that an ψ-ontic model is consistent with QM, it should additionally assume that the ontic states represented by non-orthogonal states cannot be distinguished with certainty (which is an assumption about the dynamics), and thus the information entropy of a uniform mixture of these states should be given not by the Shannon entropy, but by the von Neumann entropy (when further considering the fact that two mixed states that have the same density matrix cannot be distinguished by experiments). Therefore, COA did not successfully prove that the ψ-ontic models are inconsistent with quantum mechanics. What they proved is merely that the ψ-ontic models with an additional wrong assumption about the distinguishability of non-orthogonal states are inconsistent with quantum mechanics. When replacing this wrong assumption with a correct assumption about the indistinguishability of non-orthogonal states, the ψ-ontic models can agree well with quantum mechanics in giving the same information entropy of a mixed state.

To sum up, I have argued that Carcassi, Oldofredi and Aidala's recent no-go theorem for ψ-ontic models is false. However, the reality of the quantum state still deserves to be studied once again.

References

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