Schrödinger dynamics of a two-state system under measurement

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Spontaneous collapse models use non-linear stochastic modifications of the Schrödinger equation to suppress superpositions of eigenstates of the measured observable and drive the state to an eigenstate. It was recently demonstrated that the collapse of the wave function under observation can be modeled by the linear Schrödinger equation with a Hamiltonian represented by a random matrix from the Gaussian unitary ensemble. The matrices representing the Hamiltonian at different time points throughout the observation period are assumed to be independent. Instead of suppressing superpositions, such Schrödinger evolution makes the state perform an isotropic random walk on the projective space of states. The probability of reaching a particular final state is then given by the Born rule. Here, we apply this method to study the dynamics of a two-state system undergoing measurement. It is shown that in this basic case, the state undergoes the gambler's ruin walk that satisfies the Born rule, providing a suitable representation of the transition from the initial state to an eigenstate of the measured observable.

I. POPULAR SUMMARY

The superposition principle of Schrödinger mechanics is foreign to Newtonian mechanics. Macroscopic objects are not observed in two different places, and the cat is not alive and dead simultaneously. However, such states are commonplace in the microworld. The debate on reconciling quantum and classical physics has continued since the early days of quantum mechanics. Here, we propose a new approach to the problem that allows us, against common wisdom, to derive Newtonian behavior of macroscopic particles and establish a connection between measurement in quantum and classical physics without violating Schrödinger dynamics.

In our model, Newtonian motion emerges from Schrödinger evolution by constraining the state of the particle to a certain part of the space of all its possible states. Mathematically, this part includes the usual 3-dimensional space of possible positions of the particle. On this 3-space, the Born rule, which gives the probability of finding the particle at a certain point in quantum theory, is equivalent to the classical probability law. Conversely, the classical probability law on the 3-space implies the Born rule on the space of states. Moreover, the Schrödinger evolution that accounts for random fluctuations of the state of the measured particle becomes the Brownian motion of the particle, modeling the process of measurement in classical physics. In this setting, the superposition principle does not create a problem because superpositions of states follow the same evolution and satisfy the same Born rule. We provide details of the evolution of the state in the case of the double-slit experiment, where the position of the particle by the slits is or is *not* measured. The wave and corpuscular properties of the particle in the model are then explained.

The model prompts an important paradigm shift. Namely, the transition from the classical 3-space of our everyday experience to the space of states not only allows us to combine classical and quantum under one roof, but it also enables us to demystify intricacies of the famous double-slit experiment.

II. PREREQUISITES

The Newtonian dynamics of a mechanical system can be identified with Schrödinger dynamics under a constraint. The latter bears resemblance to the dynamics of a constrained classical system, like a bead on a wire. However, given that Schrödinger dynamics is the dynamics of a quantum state, the constraint is now applied directly to the system's state. For instance, consider a single-particle system in $\mathbb{R}^3$ described by the Hamiltonian $\widehat{h} = -\frac{\hbar^2}{2m}\Delta + \widehat{V}(\mathbf{x}, t)$. The variation of the functional

$$S[\varphi] = \int \overline{\varphi}(\mathbf{x}, t) \left[i\hbar \frac{\partial}{\partial t} - \widehat{h} \right] \varphi(\mathbf{x}, t) d^3\mathbf{x}dt \qquad (1)$$

yields the Schrödinger equation for the state function φ of coordinates and time. Let $M^{\sigma}_{3,3}$ be the submanifold of the space of states CP^{L_2} of the particle formed by the states

$$\varphi(\mathbf{x}) = g_{\mathbf{a},\sigma}(\mathbf{x}) e^{i\mathbf{p}\mathbf{x}/\hbar}. \qquad (2)$$

Here

$$g_{\mathbf{a},\sigma} = \left(\frac{1}{2\pi\sigma^2} \right)^{3/4} e^{-\frac{(\mathbf{x}-\mathbf{a})^2}{4\sigma^2}} \qquad (3)$$

is the Gaussian function of a sufficiently small variance σ^2 centered at a point $\mathbf{a}$ in the Euclidean space $\mathbb{R}^3$, and $\mathbf{p}$ is a vector in $\mathbb{R}^3$. For the states φ constrained to the manifold $M^{\sigma}_{3,3}$, the functional (1) reduces to the classical action for the particle

$$S = \int \left[\mathbf{p} \frac{d\mathbf{a}}{dt} - h(\mathbf{p}, \mathbf{a}, t) \right] dt. \qquad (4)$$

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Here $h(\mathbf{p},\mathbf{a},t)=\frac{\mathbf{p}^2}{2m}+V(\mathbf{a},t)$ is the Hamiltonian function for the system. It follows that the variation of the functional (1) subject to the constraint that the state function φ is in $M^{\sigma}_{3,3}$ yields Newtonian equations of motion.

The Fubini-Study metric on CP^{L_2} provides a Riemannian metric on $M^{\sigma}_{3,3}$. The map $\Omega:(\mathbf{a},\mathbf{p})\longrightarrow g_{\mathbf{a},\sigma}e^{i\mathbf{px}/\hbar}$ serves as an isometry between the Euclidean space $\mathbb{R}^3\times\mathbb{R}^3$ and the Riemannian manifold $M^{\sigma}_{3,3}$. If desired, a linear structure on $M^{\sigma}_{3,3}$ can be induced by Ω from the one on $\mathbb{R}^3\times\mathbb{R}^3$. The restricted map $\omega:\mathbf{a}\longrightarrow g_{\mathbf{a},\sigma}$ acts as an isometry between the Euclidean space $\mathbb{R}^3$ and the Riemannian submanifold M^{σ}_3 of CP^{L_2} formed by the states $g_{\mathbf{a},\sigma}$ [1, 2]. The relationship between action functionals (1) and (4) enables us to identify classical particles, i.e., particles that satisfy Newtonian dynamics, with quantum systems whose state is constrained to the manifold $M^{\sigma}_{3,3}$. The map Ω identifies the Euclidean phase space $\mathbb{R}^3\times\mathbb{R}^3$ of positions and momenta $(\mathbf{a},\mathbf{p})$ of a classical particle with the manifold $M^{\sigma}_{3,3}$ of states φ in (2). Imposing the constraint amounts to making the components of the velocity of state $\frac{d\varphi}{dt}=-\frac{i}{\hbar}\widehat{h}\varphi$ orthogonal to the manifold $M^{\sigma}_{3,3}$ vanish. The components tangent to $M^{\sigma}_{3,3}$ correspond to the Newtonian velocity and acceleration of the particle. Commutators of observables become Poisson brackets, transforming the Schrödinger dynamics of the constrained state into the Newtonian dynamics of the particle [3].

The Fubini-Study distance $\rho(g_{\mathbf{a},\sigma},g_{\mathbf{b},\sigma})$ between points $g_{\mathbf{a},\sigma}$ and $g_{\mathbf{b},\sigma}$ in M^{σ}_3 is related to the Euclidean distance $\|\mathbf{a}-\mathbf{b}\|$ between $\mathbf{a}$ and $\mathbf{b}$ in the following way:

$$e^{-\frac{(\mathbf{a}-\mathbf{b})^2}{4\sigma^2}}=\cos^2\rho(g_{\mathbf{a},\sigma},g_{\mathbf{b},\sigma}). \tag{5}$$

The Fubini-Study distance between points in $M^{\sigma}_{3,3}$ is related to the Euclidean distance between the corresponding points in the classical phase space $\mathbb{R}^3\times\mathbb{R}^3$ by a similar formula [3]. The metric relationship (5) establishes a connection between the normal probability distribution of a position random variable in $\mathbb{R}^3$ and the Born rule for the transition between states on CP^{L_2}. To achieve this, the identification between a classical particle and the quantum system whose state is constrained to $M^{\sigma}_{3,3}$ is employed. Specifically, consider a scenario where the measurement of the position of a classical particle yields the typical normal probability distribution of the position radius-vector in $\mathbb{R}^3$. Suppose that, under a similar measurement on a quantum system, the probability of transition between states depends solely on the distance between them. In this case, the relationship (5) implies the Born rule for the transition between arbitrary states in CP^{L_2}. Conversely, the Born rule on CP^{L_2} yields the normal distribution on $M^{\sigma}_3=\mathbb{R}^3$ [3].

The correspondence established between classical and quantum systems, and between normal probability distribution and the Born rule was leveraged in [3] to put measurements performed on classical and quantum systems on an equal footing. To achieve this, the following proposition, based on Wigner's work [4] and the Bohigas-Giannoni-Schmit conjecture [5], and further expounded upon in [3], was introduced:

> **(RM)** *The dynamics of the state of a particle, whose position is measured, can be modeled by a random walk of the state on the space of states. The steps of the random walk, without drift, satisfy the Schrödinger equation, with the Hamiltonian represented at any time by a random matrix from the Gaussian unitary ensemble (GUE). The matrices representing the Hamiltonian at different moments in time are independent.*

The Gaussian unitary ensemble consists of Hermitian matrices whose entries on and above the diagonal are independent random variables. The entries above the diagonal are identically distributed normal complex random variables with mean 0 and variance d^2. The diagonal entries are real normal random variables with mean 0 and variance $2d^2$. Such matrices can be expressed in the form $\frac{1}{2}(A+A^*)$, where A is a square matrix whose entries are independent, identically distributed complex normal random variables, and A^* is the Hermitian conjugate of A. The central characteristic of the Gaussian unitary ensemble is that the probability density function P on matrices $\widehat{h}$ within the ensemble remains invariant under unitary transformations: $P(U^*\widehat{h}U)=P(\widehat{h})$ [6].

A small step in the walk of state driven by the Hamiltonian in **(RM)** is a random vector in the tangent space to the space of states CP^{L_2}. As demonstrated in [3], the distribution of steps in the walk is normal, homogeneous, and isotropic. In particular, the orthogonal components of a step at any point are independent normal random variables. The probability of transition between two states connected by the walk depends solely on the Fubini-Study distance between them and satisfies the Born rule. Under the condition that the steps of the walk occur on M^{σ}_3, the probability of transition is determined by the normal probability distribution function. In this case, the random walk of the state approximates Brownian motion on $\mathbb{R}^3$, which can be used to model classical measurement [3]. Consequently, both the normal probability distribution valid for classical measurements and the Born rule for the probability of transition between general quantum states arise from the Schrödinger evolution with a Hamiltonian satisfying **(RM)**.

The wave-function collapse models [7, 8] utilize a non-linear stochastic modification of the Schrödinger equation to guide the state towards an eigenstate of the measured observable, typically position or energy. The underlying assumption is that non-linearity is necessary to eliminate superpositions of eigenstates. In contrast, the Schrödinger equation with the Hamiltonian in **(RM)** does not eliminate superpositions but causes the state to meander through the space of states. The conditional probability of reaching an eigenstate, given that the state reached one of them, is determined by the Born rule. The conservation of energy in the system induces a drift of the state, ensuring that the system reaches one of the eigenstates. Importantly, these two approaches are not in conflict [3].

Here, we apply the conjecture **(RM)** to analyze a "which-way" type of measurement in the double-slit experiment. We re-derive the Born rule, which, in this case, reduces to a simple "gambler's ruin" random walk. We then provide physical details of the evolution of the state driven by the Hamiltonian in **(RM)**. The path of the state between the source, the screen with the slits, the detector, and the backstop screen is traced. It is demonstrated that the space of states and the Fubini-Study metric on it provide a suitable framework for the experiment, enabling us to demystify its intricacies. The general results of [3] are made more tangible and useful for understanding the process of collapse in this fundamental case.

III. THE DOUBLE-SLIT EXPERIMENT WITH A MEASUREMENT

Consider the double-slit experiment with a microscopic particle of mass m whose motion is adequately described by the Schrödinger equation. Let us choose the Z-axis on the screen with the slits, orthogonal to the slits. Suppose the z-coordinates of the lower and upper slits are a and b, respectively. Let the horizontal axis run along the particle's path from left to right. At a point immediately to the right of the slits, the particle is in a superposition of states g_a and g_b, representing the particle passing through one of the slits with the other slit closed. The state of the particle at that point can be identified with a function $\varphi = \alpha g_a + \beta g_b$, where α and β are complex constants. For the purpose of this paper, the functions g_a and g_b immediately to the right of the slits can be approximated by Gaussian functions of z of a certain "width" δ, peaked at a and b respectively. Interaction of the particle with the screen is described by the usual Schrödinger equation. Thus, the Schrödinger evolution takes the initial state of the particle at the source to the two-dimensional space of states $\mathbb{C}^2$ of linear combinations of g_a and g_b, or, more precisely, to the projective space $CP^1 = S^2$ formed by the unit states in $\mathbb{C}^2$ modulo the phase factor.

Let us now insert a particle detector by one of the slits on the right. By measuring the particle's position, the detector provides information about the slit near which the particle is located at the time of measurement. This is an example of what is called the "which way" measurement. To make the measurement successful, we need to assume that $|a - b| \gg \delta$, so that the states g_a and g_b are nearly orthogonal. In fact, if the "overlap" of g_a and g_b is significant, no detector will be able to identify the slit by which the particle is located. In particular, the detector should be placed sufficiently close to the screen, before g_a and g_b spread and start interfering. With this in place, the measurement causes the collapse of the wave function and results in a transition from wave to particle properties of the system. The common view is that the measurement tells us which slit the particle went through.

For simplicity and to be specific, let us assume the detector is a small scintillation screen positioned near the slit at $z = b$. The detector's role is to confirm or deny the particle's location by the slit at the time of observation. Let the state function of the particle detected at a point of the scintillation screen be denoted by η. Realistically, η cannot be the Dirac's delta function; its support must be at least the size of the scintillator material's atom on the screen. We divide the screen into cells of the corresponding small size d_η and identify the state of the particle detected in the k-th cell by the normalized characteristic function η_k of the cell. An ideal detector would detects the particle in a state η_k with probability 1. The probabilities $P_b = \Sigma_k |(g_b, \eta_k)|^2$ and $P_a = \Sigma_k |(g_a, \eta_k)|^2$ characterize the effectiveness of an ideal detector in the experiment. These probabilities depend on the functions g_a and g_b as well as the position, size, and the "granularity" parameter d_η of the detector. Note that P_b must be sufficiently high, and P_a must be sufficiently small to identify the slit by which the particle was found. We then say that the particle in state φ is near the slit $z = b$ if $\Sigma_k |(\varphi, \eta_k)|^2 \geq P_b - \epsilon$ for some $\epsilon > 0$, sufficiently small for the state to identify the slit. This condition is met by a range of states φ that are all identified with g_b in the experiment. The resulting equivalence class of states will be called the *physical eigenstate* of the position operator on the Z-axis. In this case, we will also say that the state φ is *measurable without dispacement* by the detector. Note that the term $\Sigma_k |(\varphi, \eta_k)|^2$ in the definition of physical eigenstate is approximately the squared norm of the "part" φ_D of φ that is cut from it by making it vanish outside the interval D occupied by the detector. If η_D is the characteristic function of D, then $\varphi_D = \varphi \cdot \eta_D$. The state φ is in the equivalence class of g_b if the "tails" of φ outside D are sufficiently small, i.e., the norm of $\varphi - \varphi_D$ is small. For an arbitrary value of c in Z, the equivalence class $\{g_c\}$ of the state g_c is defined in the same way as for the class $\{g_b\}$, by translating the interval D. Note that a state in the equivalence class $\{g_a\}$ of g_a is approximately orthogonal to a state in the equivalence class $\{g_b\}$ of g_b. In what follows, we will assume that such orthogonality of states is fulfilled.

Let us define the Fubini-Study distance between a state φ and the equivalence class $\{g_b\}$ by

$$\rho(\varphi; \{g_b\}) = \inf_{g_b \in \{g_b\}} \rho(\varphi; g_b), \tag{6}$$

where $\rho(\varphi; g_b)$ is the Fubini-Study distance between states. In particular, for the distance between $\varphi = \alpha g_a + \beta g_b$ and $\{g_b\}$ under the accepted conditions, we have $\cos \rho(\varphi; \{g_b\}) = |\beta|$. For the state φ to reach the physical eigenstate $\{g_b\}$, it is necessary and sufficient that $\rho(\varphi; \{g_b\}) = 0$. Note that the equivalence class $\{g_b\}$ of the eigenstate g_b is rather "large". In particular, it contains functions with support in the interval D occupied by the detector, provided their total variation is not too large. It follows that $\{g_b\}$ contains many orthogonal states, i.e., states at the Fubini-Study distance equal to the maximal possible value of $\pi/2$ from each other.

To clarify the role of the equivalence class during a measurement, let us consider a few examples. We assume a slit separation of $10^{-5}m$, a slit-width of $10^{-9}m$, and that the width parameter δ of the states g_a and g_b is comparable to the slit-width. These values are typical for a successful experiment of this sort. The length of the detecting scintillation screen by the slit is taken to be about half the slit separation. Suppose the initial state g_b, denoted as $g_{b,\delta}$ here, moves to the point represented by the Gaussian state $g_{b,100\delta}$ with a width of 100δ. We have $|(g_{b,\delta}, g_{b,100\delta})| = \cos\rho$, where $\rho = \rho(g_{b,\delta}, g_{b,100\delta})$ denotes the Fubini-Study distance between the states. We then have $\rho \approx 1.43$ radians or about $82°$. Because $100\delta = 10^{-7}m$, the width of the state $g_{b,100\delta}$ is less than the size of a scintillation screen. In particular, the condition $\Sigma_k|(g_{b,100\delta}, \eta_k)|^2 \geq P_b - \epsilon$ is satisfied for a small ϵ. It follows that the state $g_{b,100\delta}$ is still within the equivalence class of g_b, and thus, it represents the same physical eigenstate. On the other hand, we also have $|(g_{a,\delta}, g_{b,100\delta})| < \exp(-10^4)$, which is an extremely small number. So, by any measure the states $g_{a,\delta}$ and $g_{b,100\delta}$ can be considered orthogonal, as needed for the experiment.

For the second example, consider that the state $g_b = g_{b,\delta}$ is displaced by a distance of $10\delta = 10^{-8}$ along the Z-axis. We then have $|(g_{b,\delta}, g_{b-10^{-8},\delta})| < \exp(-12)$, corresponding to a Fubini-Study distance of about $89.999°$. So, the states are nearly orthogonal. However, because 10^{-8} is much smaller than the size of the detector, the condition $\Sigma_k|(g_{b-10^{-8},\delta}, \eta_k)|^2 \geq P_b - \epsilon$ is satisfied for a small ϵ. It follows that the states $g_{b,\delta}$ and $g_{b-10^{-8}}$ belong to the same equivalence class. At the same time, the states $g_{a,\delta}$ and $g_{b-10^{-8}}$ remain orthogonal to a very high degree of accuracy, as required for successful measurement.

Suppose now that the initial state is a superposition $\varphi = \alpha g_a + \beta g_b$ with moduli $|\alpha|$ and $|\beta|$ that are away from zero, for example, $\varphi = \frac{1}{\sqrt{2}}g_a + \frac{1}{\sqrt{2}}g_b$. Unlike the states g_a and g_b, the state φ cannot be "measured without displacement" by the detector capable of resolving the slits. In other words, such a state does not satisfy the condition $\Sigma_k|(\varphi, \eta_k)|^2 \geq P_b - \epsilon$ with a small ϵ or a similar condition for the detector located at $z = a$. In other words, the superposition φ is far from the physical eigenstates of the measured particle. The measurement happens only if and when the initial state φ is moved to the equivalence class of either g_a or g_b. The Fubini-Study distance from the state $\varphi = \frac{1}{\sqrt{2}}g_a + \frac{1}{\sqrt{2}}g_b$ to $\{g_b\}$ is

$$d(\varphi; \{g_b\}) = \frac{\pi}{4} rad. \tag{7}$$

So, the initial state φ traveling the distance of $\pi/4$ along the shortest geodesics towards the physical eigenstate $\{g_b\}$ will reach the physical eigenstate and become directly measurable by the detector. At the same time, the state $\varphi = \alpha g_{a+10^{-8},\delta} + \beta g_{b-10^{-8},\delta}$ based on the earlier example travels almost twice the distance from the initial state $\alpha g_{a,\delta} + \beta g_{b,\delta}$ but is still at the same distance from the physical eigenstate $\{g_b\}$. The reason for the difference between the first two and the last example is due to the fact that the detector stretches along interval D in the Z-axis. This makes displacements within D or relatively small changes in the width parameter of g_b possible without affecting the distance of the resulting state to the equivalence class $\{g_b\}$.

Let us return to the double-slit experiment with both slits open and the detector near the slit $z = b$. According to **(RM)**, the observed state φ will be acted upon by the Hamiltonian represented by a random matrix and will perform a random walk on the space of states. As a result of this walk, the state may be able to reach one of the physical eigenstates of the measured observable. Our main goal is to find the probability of transition of the initial state to physical eigenstates $\{g_a\}$ and $\{g_b\}$ for this experiment. Additionally, because the distribution of steps of the random walk of the state is isotropic and the space of states CP^{L_2} is infinite-dimensional, we need to explain why the probability of reaching an eigenstate is non-vanishing to begin with.

To achieve these goals, let us utilize the expected value μ_z and the standard deviation δ_z of the z-coordinate to identify a submanifold of CP^{L_2} helpful for describing the measurement and to establish a coordinate system on it. We have:

$$\mu_z = \int z|\varphi(z)|^2 dz, \tag{8}$$

and

$$\delta_z^2 = \int (z - \mu_z)^2 |\varphi(z)|^2 dz. \tag{9}$$

These two measures are defined on functions φ in a dense subset of the space of states, which is sufficient for our purposes. Given an initial state φ with an expected value μ_z and standard deviation δ_z, consider the two-dimensional manifold M_φ parametrically defined by

$$\varphi_{\tau,\lambda}(z) = \sqrt{\lambda}\varphi(\lambda(z - \mu_z - \tau) + \mu_z). \tag{10}$$

The numeric parameters τ and λ serve as coordinates on the manifold. Along the path $\varphi_\tau = \varphi_{\tau,\lambda}|_{\lambda=\lambda_0}$ with a fixed value of λ, the expected value changes from μ_z to $\mu_z + \tau$, while the standard deviation remains constant. Similarly, along the path $\varphi_\lambda = \varphi_{\tau,\lambda}|_{\tau=\tau_0}$ with fixed τ, the standard deviation changes from δ_z to δ_z/λ, while the expected value stays the same.

The motion along φ_λ "squeezes" or "stretches" the state function without altering its shape or translation. This motion can relocate the state from its initial position in the space of states CP^{L_2} to the Z-axis represented by the family of equivalence classes $\{g_c\}$ with $c \in Z$.

Similarly, motion along φ_τ translates the state along the Z-axis. This motion can bring the "squeezed" state to the detector. The role of the equivalence class is crucial in this process: squeezing a state may not move it closer to a g_c-state by itself, but it will bring it closer to an equivalence class $\{g_c\}$.

Let us prove that the steps of the random walk of the state $\varphi = \alpha g_a + \beta g_b$ along the paths φ_τ and φ_λ on M_φ are independent random variables. As we know, the probability distribution of the random vector representing a step in **(RM)** is a normal isotropic distribution. The orthogonal components of such a vector are independent random variables. Therefore, we need to check that the steps along these paths take place in the projective space of states and that they are orthogonal in the Fubini-Study metric. Let us first check that this is true for the steps originating at the initial state $\varphi = \alpha g_a + \beta g_b$. First of all, because the norm of the state along the paths φ_τ and φ_λ is preserved, the paths take values on the unit sphere S^{L_2} in the space of states. In particular, the vectors $\frac{d\varphi_\lambda}{d\lambda}$ and $\frac{d\varphi_\tau}{d\tau}$ are tangent to the sphere. Also, $\left.\frac{d\varphi_\tau}{d\tau}\right|_{\tau=0} = -\frac{d\varphi}{dz}$ and $\left.\frac{d\varphi_\lambda}{d\lambda}\right|_{\lambda=1} = \frac{1}{2}\varphi + \frac{d\varphi}{dz}(z-\mu_z)$, and for the state $\varphi = \alpha g_a + \beta g_b$ we have

$$\mathrm{Re}\left(i\varphi, -\frac{d\varphi}{dz}\right) = 0 \tag{11}$$

and

$$\mathrm{Re}\left(i\varphi, \frac{1}{2}\varphi + \frac{d\varphi}{dz}(z-\mu_z)\right) = 0 \tag{12}$$

by the properties of states g_a and g_b. It follows that the vectors $\frac{d\varphi_\lambda}{d\lambda}$ and $\frac{d\varphi_\tau}{d\tau}$ are orthogonal to the fibre of the fibration $S^{L_2} \longrightarrow CP^{L_2}$. In particular, they can be thought of as vectors tangent to the projective space of states CP^{L_2}. Now,

$$\mathrm{Re}\left(\left.\frac{d\varphi_\lambda}{d\lambda}\right|_{\lambda=1}, \left.\frac{d\varphi_\tau}{d\tau}\right|_{\tau=0}\right) = -\mathrm{Re}\left(\frac{d\varphi}{dz}, \frac{1}{2}\varphi + \frac{d\varphi}{dz}(z-\mu_z)\right). \tag{13}$$

Using the orthogonality of φ and $\frac{d\varphi}{dz}$, the equality of the inner products $(g_a, \frac{d^2 g_a}{dz^2})$ and $(g_b, \frac{d^2 g_b}{dz^2})$, the expression for φ, and the definition of μ_z, the obtained expression (13) can be written and evaluated as follows:

$$\mathrm{Re}\left(\varphi\cdot(z-\mu_z), \frac{d^2\varphi}{dz^2}\right) = \left(|\alpha|^2(a-\mu_z) + |\beta|^2(b-\mu_z)\right)\left(g_a, \frac{d^2 g_a}{dz^2}\right) = 0. \tag{14}$$

This proves the orthogonality of steps from the initial state φ along the paths φ_τ and φ_λ. The application of the chain rule demonstrates that the preceding calculations remain valid for steps from any point on M_φ.

The established orthogonality confirms that steps of the random walk from any state ψ in M_φ along the direction tangent to paths φ_τ and φ_λ through ψ are independent random variables. Furthermore, it is possible to re-parametrize the paths φ_λ to make the Fubini-Study metric on M_φ in the new coordinates explicitly Euclidean. Specifically, by setting $s = \ln\lambda$, we obtain the new parametrization of φ_λ in the form $\varphi_s(z) = e^{\frac{s}{2}}\varphi(e^s(z-\mu_z-\tau_0)+\mu_z)$. We can see that the norm of the tangent vector $\frac{d\varphi_s}{ds}$ is preserved along the path. The same is true for $\frac{d\varphi_\tau}{d\tau}$, which, together with the orthogonality of these vectors, signifies that the induced metric is Euclidean. The coordinates τ and s are then Cartesian coordinates on $M_\varphi = \mathbb{R}^2$.

An arbitrary state on M_φ has the form $\psi = \alpha\widetilde{g}_c + \beta\widetilde{g}_d$, where $\widetilde{g}_c$ and $\widetilde{g}_d$ are Gaussian functions with equal width, and $(\widetilde{g}_c, \widetilde{g}_d) = (g_a, g_b)$. The expected value of the z-coordinate for an arbitrary state ψ in M_φ is given by

$$\mu_z = \int z|\alpha\widetilde{g}_c + \beta\widetilde{g}_d|^2 dz = |\alpha|^2 c + |\beta|^2 d. \tag{15}$$

The variance is given by

$$\delta_z^2 = \int z^2|\alpha\widetilde{g}_c + \beta\widetilde{g}_d|^2 dz - \mu_z^2 = |\alpha|^2|\beta|^2(c-d)^2. \tag{16}$$

Provided the coefficients α and β do not vanish, equations (15) and (16) can be solved for c and d. If one of the coefficients is 0, the state is an eigenstate of z. In either case, we see that the pair (c,d) for the states on M_φ can be represented by the pair (μ_z, δ_z), identified in this context with coordinates τ and s. It follows that the Fubini-Study distance from a state in M_φ to the eigenstates g_a and g_b can be expressed through the values of μ_z and δ_z. Likewise, the Fubini-Study distance $d\rho$ between two neighboring points of M_φ can be expressed through the differentials $d\mu_z$ and $d\delta_z$ for the points. In fact,

$$d\rho^2 = d\rho_1^2 + d\rho_2^2, \tag{17}$$

where $d\rho_1$ and $d\rho_2$ are obtained by the following rotation in the tangent plane to M_φ:

$$d\rho_1 = |\alpha|d\mu_z - |\beta|d\delta_z \tag{18}$$

$$d\rho_2 = |\beta|d\mu_z + |\alpha|d\delta_z. \tag{19}$$

Unlike the Fubini-Study distance between states, the expected value μ_z and standard deviation δ_z have the advantage in being familiar spatial quantities. Moreover, the condition that the initial state φ has reached the detector or, equivalently, that it became a physical eigenstate of z can be expressed in terms of the corresponding change in the variables μ_z and δ_z of φ. Specifically, for this to happen, it is sufficient that the interval $(\mu_z - r\delta_z, \mu_z + r\delta_z)$ for a proper value of the parameter r for the final state φ_f is contained in the interval D occupied by the detector. First, for the given values of δ_z and μ_z of the initial state φ, the parameter $r > 0$ is selected to ensure that the tails of φ outside the interval $D_r = (\mu_z - r\delta_z, \mu_z + r\delta_z)$ are small enough to satisfy the condition $\Sigma_k |(\varphi, \eta_k)|^2 \geq P - \epsilon$ on the interval. Then, the coordinates τ and s (i.e., the corresponding values of μ_z and δ_z) are selected to make sure that the interval $(\mu_z - r\delta_z, \mu_z + r\delta_z)$ is in D. The range of possible values of μ_z and δ_z that satisfy this condition determines the end-states φ_f in M_φ that are elements in the corresponding physical eigenstate of z.

Suppose first that the random walk of the initial state $\varphi = \alpha g_a + \beta g_b$ generated by the Hamiltonian $\widehat{h}$ in **(RM)** takes place on the manifold M_φ. That is, we select only those steps of the walk generated by $\widehat{h}$ that begin and end on M_φ. We will address the known isotropy of the distribution of steps later. Note that the eigenstates g_a and g_b are the points of M_φ where $\mu_z = a$ or $\mu_z = b$ and $\delta_z = \delta$. Furthermore, in the considered approximation, the states g_a and g_b are orthogonal, which means that they lie at the opposite points in the space of states. It follows that in this approximation, the expected value μ_z of the coordinate z cannot exceed the value b or be smaller than a. It also follows that there is a maximum possible value of the standard deviation δ_z of z. According to (16), this value is equal to $\frac{1}{2}|a-b|$. These constraints simply mean that there is a very small probability for the particle to be found beyond a small neighborhood of the interval $[a, b]$ separating the slits, which we know to be true for a proper set up of the experiment.

To find the probability that the initial state has reached an eigenstate of z is to find the probability P_a or P_b of μ_z having the value near a or b and δ_z to be near δ at the same time. We know that τ and s represent orthogonal coordinates on $M_\varphi = \mathbb{R}^2$. We also know that the steps in τ and s are independent, identically distributed normal random variables. In the absence of the boundary conditions, the probability density function of the random vector of the final state φ_f at the time of observation is normal, circularly symmetric function of τ and s on $M_\varphi = \mathbb{R}^2$. Therefore, the probability density function is a product of functions of τ and s. Using absorbing boundaries at $\tau = a$ and $\tau = b$, and a reflecting boundary at $\delta_z = \frac{1}{2}|a-b|$, we preserve the product form of the probability density function. It follows that the probability we are looking for is the product of the probability of μ_z to be near a or b and the probability of δ_z to be near δ. However, for a given initial state the probability of δ_z to be near δ is just a constant coefficient, which is the same for convergence of the initial state to g_a or g_b. This is because the change in δ_z from the initial value $|\alpha||\beta||a-b|$ to δ (or, equivalently, the change in s from 0 to $\ln(|\alpha||\beta||a-b|/\delta)$) is the same for both eigenstates. In other words, the probability we are looking for is proportional to the probability of μ_z being near a or near b. It follows that the problem of finding the probability of transition of the initial state to g_a or g_b can be solved by studying the random walk in the coordinate τ of the state φ under the action of $\widehat{h}$ on φ.

The steps of the random walk along the τ coordinate line are given by

$$\left(-\frac{i}{\hbar}\widehat{h}\varphi, \widehat{\frac{d\varphi}{d\tau}}\right), \tag{20}$$

where the hat over the derivative means that the vector is unit-normalized. It was shown earlier that $\frac{d\varphi}{d\tau}$ is tangent to the space of states CP^{L_2}. Because the distribution of the vector $\widehat{h}\varphi$ is homogeneous, isotropic and normal, the steps (20) are identically normally distributed for all φ along the coordinate line. From $\varphi_\tau(z) = \varphi(z-\tau)$ we know that $d\tau = d\mu_z = -dz$. So, we are dealing with the random walk with Gaussian steps on the Z-axis, where μ_z takes values. When the number of steps is large, the obtained walk with Gaussian steps can be approximated by the walk whose steps have a fixed length. The end-points of the interval $[a, b]$ are absorbing and correspond to the particle being absorbed by the detector. The probability of reaching the point $\mu_z = b$ for the state $\varphi = \alpha g_a + \beta g_b$ is then given by the usual gambler's ruin formula that yields in this case

$$P_b = \frac{\text{number of steps from } \mu_z \text{ to } a}{\text{number of steps from } a \text{ to } b} = \frac{\mu_z - a}{b-a} = |\beta|^2. \tag{21}$$

Here the definition (15) together with normalization $|\alpha|^2 + |\beta|^2 = 1$ were used. Similarly, the probability P_a for the initial state φ of reaching the state g_a (equivalently, reaching $\mu_z = a$) is given by $P_a = |\alpha|^2$. The Born rule for the state is thus derived.

The random walk of the state was conditioned so far to stay on the manifold M_φ. This contradicts the isotropy of the distribution of steps of the state driven by the Hamiltonian in **(RM)**. The isotropy allows the state to propagate into the space of states CP^{L_2}. The resulting conditional probability of reaching the eigenstates given that the state has reached the Z-axis is consistent with the Born rule [3]. However, the unconditional probability of transition between states is vanishingly small in this case. To address this problem, note that we have disregarded an additional physical process that takes place during measurement. Namely, the interaction of the particle with the screen results in a decrease of its energy. The energy is deposited to atoms of the scintillation material of the screen. To be specific and for simplicity, suppose that the incident particle is charged and distinguishable from the particles of the screen participating in the interaction. With this in place, we can follow the particle throughout its evolution. Inelastic collisions between the incident particle and bound electrons in atoms of the scintillation material produce a stream of low-energy photons whose quantity is proportional to the energy transferred by the particle. The outgoing low-energy particle is then trapped in the potential of the system. It continues to lose energy as its state goes down the ladder of discreet energy levels towards the ground state.

Although complicated, the process can be described by the standard Schrödinger dynamics with elements of quantum electrodynamics. When the potential of the system, composed of the weakened particle and atoms of the screen it interacts with, is harmonic, the energy levels of the trapped particle are given in natural units by $E_n = \frac{1}{2} + n$. In this case, a calculation yields the expression for the variance as $\delta_{zn}^2 = \frac{1}{2} + 4n$. As the excited state descends the ladder and approaches the ground state, the standard deviation δ_z for the state decreases to a small value, comparable to the size of a molecule of the screen. A similar process takes place for a potential that is approximately harmonic or quartic near the stable point, or, more generally, is U-shaped. We conclude that the interaction with the screen and the resulting trapping of the particle in the potential well are responsible for generating a drift of the particle's state towards the set of equivalence classes g_c representing the Z-axis. Moreover, assuming that the ground state is attained in the process, the drift is directed towards the set of Gaussian states g_c.

It follows that there are two types of motion of the state participating in the collapse in the model. The first one is the random walk of state without drift generated by the Hamiltonian $\widehat{h}$ in **(RM)**. The second is the drift of the state towards the Z-axis. With the state guaranteed to reach a neighborhood of the Z-axis, the random walk of the state reduces to a gambler's ruin process on the Z-axis and results in the Born rule. Assuming, for example, that the drift towards the Z-axis happens approximately along the shortest line, we conclude that the entire process can be modeled within the manifold M_φ. Recall that M_φ is isometric to the Euclidean space $\mathbb{R}^2$ and the τ and s-coordinate lines are orthogonal. The steps of the random walk generated by the Hamiltonian $\widehat{h}$ in **(RM)** in these two directions are independent identically distributed normal random variables. The walk of the initial state $\varphi = \alpha g_a + \beta g_b$ on M_φ consists of the random walk without drift along the τ-coordinate line and the random walk with the drift in the positive direction of the s-axis. In other words, the walk is represented as follows:

$$\tau_k = \tau_{k-1} + \xi_k \tag{22}$$

and

$$s_k = s_{k-1} + a + \eta_k, \tag{23}$$

where ξ_k and η_k are independent identically distributed normal random variables, and a is a positive number equal to the step of the drift. Using $s_0 = 0$, we have, for the N-th step of the walk in s:

$$s_N = a \cdot N + \sum_{k=1}^{N} \eta_k. \tag{24}$$

Given that $\lambda = e^s$ and $\delta_z = \lambda^{-1}\delta_{z_0}$, we see that $\delta_z = e^{-s}\delta_{z_0}$. Therefore, the variance exponentially approaches zero with an increase in s. In this case even a few steps of the walk of the state may be sufficient to reach a neighborhood of the Z-axis. The gambler's ruin process in the variable τ is then guaranteed to take the state to $\{g_a\}$ or $\{g_b\}$ with the probability satisfying the Born rule, as derived in (21). The expected time interval of collapse in the model depends on the frequency and the distribution of steps of the walk, the value of the parameter a, and the parameters in the definition of the equivalence classes $\{g_a\}$ and $\{g_b\}$.

There is an interesting geometric interpretation that relates the considered walk with a walk of a spin-state $[\alpha, \beta]$ on the sphere $S^2 = CP^1$. Namely, by a proper choice of the unit and the origin on the Z-axis, one can always ensure that $a = -1$ and $b = 1$. With this, we have for the initial state $\varphi = \alpha g_a + \beta g_b$:

$$\mu_z = |\beta|^2 - |\alpha|^2 \tag{25}$$

and

$$\delta_z^2 = 1 - \mu_z^2 = 4|\alpha|^2|\beta|^2. \tag{26}$$

Expressions (25) and (26) are intimately related to the expressions for Cartesian coordinates of the spin-state $[\alpha, \beta] \in \mathbb{C}^2$ under the usual bundle projection $\pi : S^3 \longrightarrow CP^1 = S^2$. These coordinates are given by

$$x = \alpha\overline{\beta} + \overline{\alpha}\beta, \tag{27}$$

$$y = i(\alpha\overline{\beta} - \overline{\alpha}\beta), \tag{28}$$

$$z = |\beta|^2 - |\alpha|^2. \tag{29}$$

From these equations, we see that $\mu_z = z$ and $\delta_z^2 = x^2 + y^2$. The coefficients α and β of φ may also have a phase difference θ. Adding the variable θ to the pair (μ_z, δ_z), we obtain cylindrical coordinates on the sphere S^2.

We would like to use the triple $(\mu_z, \delta_z, \theta)$ to describe the walk of state φ in the model as a motion on the sphere. Namely, given a state $\alpha\widetilde{g}_c + \beta\widetilde{g}_d$ evolving on M_φ, we could identify its coordinates $(\mu_z, \delta_z, \theta)$ and then find the corresponding point (x, y, z) on the sphere with the help of equations (15,16) and (27-29). In such a way, we would identify the change in the values of c and d with the corresponding change in the coefficients α and β of the initial state $\varphi = \alpha g_a + \beta g_b$. In this case, the basis states g_a and g_b would remain fixed during the evolution while the value of the coefficients α and β would be obtained from the equations (15,16). The problem with this geometric realization of the evolution is that it imposes a relationship between μ_z and δ_z. Namely, it requires that $\delta_z^2 = 1 - \mu_z^2$, which is not valid in general. Furthermore, the parameter θ cannot be determined from equations (25,26). However, imposing the relationship $\delta_z^2 = 1 - \mu_z^2$ without changing the walk in τ preserves the probabilities of reaching the eigenstates. Furthermore, it makes reaching the values $\mu_z = a$ of $\mu_z = b$ equivalent to reaching the eigenstates, which is similar to what the drift in s has achieved. Although imposing this relationship is rather arbitrary, the change in $\mu_z = z$ and $\delta_z^2 = x^2 + y^2$ when the state approaches the poles of S^2 gives us a nice illustration of collapse in the model. Note that the actual random walk of state studied in the paper does not happen on $CP^1 = S^2$, which does not even include the Z-axis. In particular, the walk does not converge to Brownian motion on the sphere.

IV. ELECTRONS VERSUS BULLETS

In a version of Feynman's experiment with bullets, a machine gun shoots a stream of bullets into a screen with two slits. Behind the slits, there is a wooden screen that absorbs bullets. A small movable sandbox in front of the screen is used as a detector of bullets along the Z-axis on the screen. The setup of this experiment is, therefore, very similar to the one with a microscopic particle such as electron considered in the paper. Furthermore, we saw that classical space $\mathbb{R}^3$ is isometric to the submanifold M_3^σ of the space of states CP^{L_2}. A point a in classical space $\mathbb{R}^3$ is represented by the state $g_{\mathbf{a},\sigma}$ in M_3^σ, defined in (3). Similarly, the classical phase space $\mathbb{R}^3 \times \mathbb{R}^3$ for a particle is isometric to the submanifold $M_{3,3}^\sigma$ of the space of states of the particle. Most importantly, it was verified that Newtonian motion of a particle is equivalent to the Schrödinger evolution of its state, provided the state is constrained to the manifold $M_{3,3}^\sigma$. Based on that, we can identify the path of a classical particle with the corresponding path in $M_{3,3}^\sigma$ in a physically meaningful way. In particular, neglecting other coordinates in $\mathbb{R}^3$, the path $z = c(t)$ of a particle going through point a is represented by the path $\varphi = g_{c(t)}$ of its state going through the point g_a. This mathematically rigorous and physically valid identification, together with the conjecture **(RM)**, give us a perfect setup for analyzing and comparing the double-slit experiments with electrons and bullets.

Let us consider the experiment with electrons first. The electron's spin properties in the experiment will be neglected. At the beginning of the experiment, an electron gun fires electrons one by one. We may assume that state of the initial electron is a Gaussian wave packet moving towards the screen with the slits. In particular, the state is near the manifold $M_{3,3}^\sigma$ in the space of states CP^{L_2}. That is, the Fubini-Study distance from the state to $M_{3,3}^\sigma$ is small. During this time, the state propagates by the usual Hamiltonian $\widehat{h} = \frac{\widehat{\mathbf{p}}^2}{2m} + \widehat{V}(\mathbf{x})$, where $\widehat{V}(\mathbf{x})$ is an external potential including the one associated with the screen with the slits. Interaction of the electron with the surrounding matter in the experiment can be neglected. Upon interaction with the screen, the wave packet splits into a superposition of two wave packets. That means that the state is no longer on the manifold $M_{3,3}^\sigma$. In fact, assuming, for example, that $\varphi = \alpha g_a + \beta g_b$ with $|\alpha| \leq |\beta|$, the cosine of the smallest distance between the state and $M_{3,3}^\sigma$ is given by

$$|(\alpha g_a + \beta g_b, g_b)| = |\beta|. \tag{30}$$

It follows that the state is close to $M_{3,3}^\sigma$ only when α is close to 0. This is not the case immediately to the right of the screen with both slits open.

Note that nothing special has happened to the state at this time. It simply moved away from the classical phase space submanifold $M_{3,3}^\sigma$ into CP^{L_2}. In particular, the path of the state did not go through the points g_a or g_b, or any other point g_c with c on the Z-axis. It passed in the space of states "over" the Z-axis and the screen. However, for the electron to have any position in $\mathbb{R}^3$ at all, the electron's state must be in M_3^σ, which is not the case when the electron interacts with the screen. So, the electron position is not defined at this time. It is not given by a or b on the Z-axis, or by any other point in $\mathbb{R}^3$. At the same time, whenever the electron's state *is* in M_3^σ, it identifies the electron's position in $\mathbb{R}^3$ correctly, as a dynamical variable, in a way consistent with Newtonian dynamics. In this sense, the state variable φ is an *extension* of the classical position variable of the particle. Instead of saying that the electron's position is not defined when the particle interacts with the screen, we can say that the electron's path takes off the classical space and passes "over" the screen in the space of states. Its position along the path is well-defined but requires additional dimensions provided by the space of states CP^{L_2}. In particular, the electron's path does not "split" to go through two slits at once. It is only when we insist that the electron's state must always be on $M_{3,3}^\sigma$ that we run into this paradox.

What happens to the right of the screen, when the particle interacts with the detector? The Born rule for the probability density function for the particle's position, in the considered approximation, yields $P(z) = |\alpha g_a(z) + \beta g_b(z)|^2 = |\alpha|^2 |g_a(z)|^2 + |\beta|^2 |g_b(z)|^2$. Integrating this over the area occupied by the detector near point a, we get approximately $|\alpha|^2$. The probability of being near b is then $|\beta|^2$. This result is identical to the one obtained from the conjecture **(RM)** in the paper. According to **(RM)**, the state φ is driven by the Hamiltonian represented by a random matrix. The random walk of state brings it back to the classical space submanifold M_3^σ to the equivalence class of one of the eigenstates g_a or g_b by the process described in the previous section. The electron is then positioned near the point a or point b with the probabilities $|\alpha|^2$ and $|\beta|^2$ respectively.

Suppose now that the detected particle is able to continue its motion towards the screen on the right of the detector. It will then arrive at the screen as a spread-out version $\widetilde{g}_a$ (or $\widetilde{g}_b$) of the detected Gaussian state g_a

(or g_b). The probability density function for the electron's position on the screen is then given by either $P(z) = |\widetilde{g}_a(z)|^2$ or $|\widetilde{g}_b(z)|^2$ and no interference pattern is observed on the screen. The resulting "corpuscular" properties of the detected electron are due to the closeness of its "post-detector" state to the classical phase space manifold $M^{\sigma}_{3,3}$ during its motion from the detector to the backstop screen. As we know, when the electron's state is *on* $M^{\sigma}_{3,3}$, it satisfies Newtonian dynamics and behaves like a particle.

If the experiment is repeated without the detector, the state $\varphi = \alpha g_a + \beta g_b$ obtained to the right of the slits will continue its motion towards the backstop screen along a path that is away from $M^{\sigma}_{3,3}$. Interaction of the particle with the backstop screen happens in the same way as its interaction with the detector. However, this time the spread-out states $\widetilde{g}_a$ and $\widetilde{g}_b$ may not be considered orthogonal. As shown in [3], the conjecture **(RM)** applied to this case yields the Born rule as before. The loss of energy and trapping of the particle bring the state to the screen. Provided the particle has been detected by the screen, the probability density function for the position is given by $P(z) = |\alpha\widetilde{g}_a(z) + \beta\widetilde{g}_b(z)|^2$. The interference term is now present. The observed "wave" properties of the electron are caused by its state being distant from the classical phase space submanifold $M^{\sigma}_{3,3}$ during its motion from the screen with the slits to the backstop screen. That is, the state arrives at the backstop screen as a superposition $\alpha\widetilde{g}_a + \beta\widetilde{g}_b$, and such a superposition is away from $M^{\sigma}_{3,3}$. When the state of the particle in the experiment moves away from the classical phase space submanifold $M^{\sigma}_{3,3}$, the standard deviation δ_z increases and the particle demonstrates its wave properties. When the state is brought back to the manifold $M^{\sigma}_{3,3}$, the standard deviation decreases, and the particle demonstrates classical corpuscular properties.

What is different about the experiment with bullets? Measuring the position of a small electron in the experiment requires a detector or a backstop screen that the electron interacts with. On the other hand, the bullet interacts randomly and continuously in time with particles of the surroundings even before it reaches the sandbox or the backstop screen. Because of this continuous interaction, the surroundings (particles of air, radiation) contain information about the bullet's position at all times. In other words, the bullet's position is constantly measured by the surroundings. It follows that the conjecture **(RM)** needs to be applied to the entire motion of the bullet in the experiment.

As shown in [3], the state driven by the Hamiltonian in **(RM)**, and conditioned to stay to the manifold $M^{\sigma}_{3,3}$, describes the Brownian motion of the particle. When the particle is sufficiently large, the diffusion coefficient for the Brownian motion vanishes, and the particle is at rest in the lab system. The isotropy of the probability distribution of steps of the random walk of the state signifies that the state of the particle in the space of states CP^{L_2} must then be at rest as well. If an external potential is applied to such a system, the particle will move in accord with Newtonian dynamics [3]. A bullet is large enough for its Brownian motion in natural environment to be trivial. It follows that the state of the bullet is confined to $M^{\sigma}_{3,3}$. Accordingly, the dynamics of the bullet is described by Newton's equations of motion. Thus, if accepted, conjecture **(RM)** has the potential to elucidate why the bullet does not exhibit wave properties but instead moves in accordance with Newtonian dynamics.

V. WHY RANDOM MATRICES?

The conjecture **(RM)** provides us with a model of measurement that works for macroscopic and microscopic particles alike. The constraint that relates measurement on macroscopic and microscopic particles is identical to the one that relates Newtonian and Schrödinger dynamics (see section II). The usual translational and rotational symmetries of measurement in the macro-world are preserved. The irreversibility of measurement is tied to the fact that Hamiltonians in the Gaussian unitary ensemble are not invariant under time reversal [3]. The model yields the Born rule and explains what happens in the double-slit experiment with and without a detector. These groundbreaking results validate the conjecture, albeit indirectly. However, the question remains: why would the Hamiltonian during measurement be represented by a random matrix?

Random matrices were introduced into quantum mechanics by Wigner [4] in a study of excitation spectra of heavy nuclei. Wigner reasoned that the complexity of the motion of nucleons in the nucleus could be handled by modeling the Hamiltonian of the system with a random matrix. The ensemble of matrices only had to respect the symmetries of the system. The correlations in the spectrum of random matrices that Wigner discovered turned out to be applicable to a remarkably large number of quantum systems with many as well as few degrees of freedom. Experimental evidence suggests that all quantum systems whose classical counterpart is chaotic demonstrate random matrix statistics, as proposed in the Bohigas-Giannoni-Schmit (BGS) conjecture [5]. On another note, classical measurement can be modeled by Brownian motion. It is known that Brownian motion can be characterized as a chaotic process [9–11]. The intricate nature of the interaction between the measured particle and atoms of the detector, coupled with the chaotic features of Brownian motion, suggests that the system's Hamiltonian can be effectively represented by a random matrix.

Decoherence theory [12] seeks to explain the process of position measurement based on the Schrödinger evolution of the system interacting with the environment. A typical Hamiltonian modeling this situation would describe a particle linearly coupled to a set of harmonic oscillators. Alternatively, the scattering matrix can be used to determine the effect of the collective scattering of particles on the particle whose position is measured. The evolution of the density matrix of the measured particle would then exhibit a damping of interference terms in the matrix. The theory has been successful in explaining the emergence of classical probabilities. However, it falls short in explaining how the observed quantum state arises as a result of measurement and does not lead to the Born rule. Loosely speaking, the derivation of evolution equations for the density matrix in decoherence theory is akin to attempts to derive Brownian motion from Newtonian dynamics of a system of particles. Both attempts provide a proof of concept but require several important assumptions and fall short of providing a fundamental explanation of the phenomena. Ultimately, these attempts can be regarded as useful models. Additional statistical or symmetry-based assumptions, such as those made by Einstein in the theory of Brownian motion or by Wigner in the study of spectra of heavy nuclei, are still needed to gain deeper insight into the phenomena. Similarly, the universal applicability of random matrix theory to fluctuations in quantum systems, together with the results derived here, suggests that random matrices may offer the missing insight into the process of measurement.

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