Believing in the objects: the shift to faith*

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Abstract

In this paper we analyze object-talk, some of its uses and misuses, more closely. Our main claim is that object-talk should be understood in the context of faith instead of that of rational argumentation. Hence, object-talk is compared with God-talk and the way of understanding the so-called 'abstract entities' from logic and mathematics turns out to be essentially linked with how the contact with the Divine is conceived. Moreover, we discuss the particular case of the independence phenomenon and some of the proposals that have been made in order to deal with it. The conclusion is that, even if we succeed in granting such status to object-talk, different accounts of its nature may arise. We finally present our view as one of these alternatives.

This is the animal that does not exist.
They never knew it and yet, nonetheless,
they loved the way it moved, its suppleness,
its neck, its very glaze, mild and serene.
It never was. *But, because they loved it, it was*
a pure animal.

Rainer Maria Rilke, *Sonnets to Orpheus*, Part II, Sonnet IV.

1. It is natural to rise suspicions regarding radical views that do not apparently provide reasonable alternatives to the phenomenon to which they oppose. The view that I will try to defend in this article, that 'object-talk' is meaningless, is perhaps one of these. But let me make this slogan more precise. What I succinctly mean by 'object' should be understood as '*abstract* object', whereas the discourse I intend to examine is specifically restricted to logic and, by similarity, to mathematics. Hence, the view I endorse can be rephrased as: ontological discourse has nothing to do with logic or mathematics; the so-called formal sci-

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ences are not ontologically-charged[1].

The careful reader is probably wondering: if you claim that discourse surrounding abstract objects has no meaning, how should your own reasons be taken? I will address this fishy feeling at the end of the text. It is precisely the reader who has to decide whether this problem can be put on hold until then or if it constitutes an absolute menace for the article as a whole invalidating it from the beginning.

2. The idea that I wish to defend encapsulates three main notions: 'object', 'abstractness' and 'existence'. Note that all of these are common ground for both the realist and the anti-realist, since they accept a similar account of abstract objects but disagree on the ontological domain that should be accepted in each case. I am aware that such views may be held in very diverse ways by many different individuals. I tend to identify *realism* with some form of the indispensability argument and *anti-realism* as a whole with the negation of at least one of its premises[2]. The usual form of the indispensability argument runs as follows: (1) we ought to be ontologically committed to all and only the entities that are indispensable to our best scientific theories, (2) (logico-)mathematical entities are indispensable to our best scientific theories, (C) we ought to be ontologically committed to (logico-)mathematical entities. In particular, what I understand by *fictionalism* can be identified with an argument of the following sort: (1) our mathematical sentences and theories are about abstract mathematical objects, (2) there are no such things as abstract objects, (C) our mathematical theories are not true.

Here, the lack of coherence of object-talk faces the problem of being identified with a form of anti-realism[3]. Since the anti-realist seems to use object-talk just for selecting the entities they claim to be non-existent it could seem that, as a consequence of their view, the object-talk will be ultimately substituted by, say, the individual-talk. Hence, the claim that formal sciences do not bear any ontological value would be implicit for the anti-realist. But it should be also observed that the anti-realist *first* accepts the ontological charge of mathematical theories that the realist posits and *only then* goes against the ontological view that the other defends. In other words, the anti-realist is prone to modify some logico-mathematical

[1] During the preparation of this article, I learnt about Putnam's proposal in [18], quite similar to the stated thesis. On the other hand, I have tried to occupy myself with more tangential problems as those regarding faith and revolutions. What is clear to me now is that the unsatisfied reader should give a chance to this wonderfully-written book. Nevertheless, divergences are inevitable and we have pointed them out through the text.

[2] This shortcut is not at all justified: see the addendum for a brief explanation.

[3] I do admit, however, that the strategy of identifying some view with an already established set of beliefs is simply more economical than arguing against some view in particular, with all its caveats. In the present case, the reader should be aware of my oversimplifications, but keeping in mind that, without any of them, the length of the article in pages would be an exponential function of the number of cited authors.

theories because they have already identified some of them with the realist ideals[4].

This is not what I defend here. What I intend to argue is simply that the first supposition is, by itself, unjustified. The anti-realist articulates their view as a reaction to the initial intention of the realist and, while doing so, leaves the fundamental supposition untouched. The fictionalist approach is also unsatisfactory because it implies a renounce of a strong sense of the word 'truth', even though in order to articulate their own view, they implicitly accept that object-talk is coherent. My question to the anti-realist, in general, is: why conceding *that much* to the realist?

3. But the relevance of the ontological charge is, in some cases, secondary. Dummett's view is a perfect example that illustrates the shift from the shady ontological problem to certain epistemological features of logic. According to Dummett, two principles may be attributed to realism: (1) if a statement is true, there must be something that makes it true, and (2) if a statement is true, it must be in principle possible to know that it is true[5]. The principle (1) is clearly related to the object-talk problem, but Dummett is more interested in (2)[6]. He argues that theism entails realism, being the main idea behind his argument that an omniscient being like God will surely be able to establish the bivalence principle for any particular (mathematical) statement. Hence, the realist implicitly assumes the existence of an omniscient being like God. Although this superficial form of the argument is circular[7], what is clear is the moral of it: we, as finite beings, should stick to intuitionistic logic; God, on the other hand, has enough power to use a different kind of logic[8] and, as a consequence, realism is a Prometheic project, as it stands in denial of our own nature.

Nevertheless, the realist may easily overcome what Dummett finds problematic. At a first glance, they will claim that the reduction of (1) to (2) is not sound: after all, the point of realism *is* that knowledge and existence are, at least in some degree, to be understood independently from each other. Moreover, even if the theological insight of Dummett were to be accepted, the realist would feel that the reference to God is misguided.

[4] I am not totally sure that this view includes Putnam, since he favors some kind of modal conception of mathematics, as one can read in [18]. This is for sure a piece of mathematics that can only be appealing for philosophers or, at best, logicians. Mathematicians use, *in general*, set-theoretical language. The problem is then how to conceive the relation between such language and the philosophies of mathematics that vindicate some kind of mythical status for sets. Also, the same mythical status could be attributed to, say, structures.

[5] For more details, see [20].

[6] In fact, he claims that (2) implies (1) since 'if it were in principle impossible to know the truth of some true statement, how could there be anything which *made* that statement true?'

[7] See the previous footnote. Dummett provides another argument that relies on the reflection of knowledge, but in the cited article it is argued that the circularity prevails.

[8] Dummett seems to accept that God's logic would be 3-valued and decidable. The reason for this is that God knows whether a gap occurs in nature, so that He can assign no classical truth-value (true or false) in such situations.

Despite the fact that all of this should be acknowledged, one of my main claims in this article will be that there is a tendency, at least within modern realism, that should be understood in terms of *faith*. If we inspect (1) through the theological lens of Dummett, it seems more convenient to assert that the object of faith is not God but, rather, the 'something' of which this principles talks, as instantiated by abstract objects. In other words, the theological commitment is not to be placed on the knowledge but on the object of that knowledge: the special faculty of accessing this object is not divine anymore; the attempts at explaining how this faculty naturally belongs to human beings is for sure the trademark of modern realism, as we will later see.

4. Having reduced the problem of abstract objects to the competition between intuitionistic and classical logic, Dummett accepts that intuitionistic logic is less expressive than classical logic; moreover, it is precisely *this* what later allows him to derive his theological insight for favoring the first one: classical logic simply enables us to derive more than what is expected for human beings. This picture certainly presupposes that intuitionistic and classical logic can be compared in a (crucial) sense and, despite being the usual attitude in modern logic, such a view is quite far away from the original desire of intuitionism as a radical philosophy of mathematics. Let me be clear: I do not have sympathy for the intuitionist project, but this does not undermine my desire to defend that some of its consequences, taken face value, differ deeply with a conception that it is today quite extended, even among philosophers.

This tendency, rooted in the contemporary so-called foundations of mathematics, consists in the use of the *to-really-talk-about* slogan. The set theorist is inclined to say that working mathematicians, from the present and the past, are *really talking about* sets. The modern logician is inclined to say that every intuitionist, or at least the mathematically serious ones, are *really talking about*, say, a concrete reduct of some classical system. After all, they argue, few use metalogical systems following their own rules (this applies, by the way, for every exotic non-classical logician)[9]. The first use of the slogan shows the reductionism that later on constitutes a clear temptation for establishing a simple ontology of mathematics and logic. The second use shows how this reductionism severely simplifies a historical tendency in a modern, appealing way. Why conceding this kind of reductionism in the first place? Is it not intuitionism, as an special historical tradition, different from the acceptance of such-and-such logical rules?

I am not vacuously attacking a straw man. The epitome of this conception is to do math-

[9]There are exceptions. The more radicals, like Brouwer, would simply eliminate the apparent need for metamathematics and, additionally, the need for formal systems in the first place. The paradigm of exotic non-classical logician is, certainly, Graham Priest.

ematics pretending to do philosophy (of course, the other possibility is usually regarded as the clumsy work of a bad metaphysician[10]): consider, for example, the famous 'squeeze argument' of Kreisel. This approach works as if philosophical, informal, historical concepts were of the same nature as formal ones. Still, it can even be heard today that their difference is only in degree of precision or demarcation. But seriously believing that mathematics can solve philosophical problems because the (mathematical) work in the foundations of mathematics has made some (philosophical?) revisions necessary is equivalent to believing that one can solve Riemann's hypothesis with the only means of the synthetic-analytic distinction.

5. It is my desire to have made this clear: the Dummettian approach of reducing the object-talk problem to an epistemological one ultimately relies on some obscure shift from the ontological domain that both the realist and the anti-realist concede to some kind of 'neutral language'[11] in which their –more concrete– beliefs regarding logic may be easily compared. This conception goes directly against any hint of *incommensurability* regarding different traditions of mathematics; conception that, as announced before, I will embrace here[12]. The common object-talk simply induces, assuming Dummett problem-shifting, a conception which is, under our point of view, equally pernicious, namely, the idea of a common, historically-independent language that guarantees a crystalline possibility of demarcation and scrutiny of two competing views[13].

Despite all of this, it may still be defended that this historical nature is not a worry for the set theorist and the modern logician. Perhaps, it can be argued, the history of logic and mathematics, through its accumulating progress (of progressive accumulation of truths) makes object-talk rationally justified. If we are growing gradually closer, towards the warm light of truth, it seems natural to seek the abstract objects that logic and mathematics analyze: the legitimization of object-talk is then guaranteed by historical considerations; the rational delimitation of the formal sciences will ensure a rational delimitation of the corresponding ontological charge, at least if one is not to fall for skepticism. If object-talk ultimately depends on the elements of the scientific practice we are dealing with (i.e. to which one is willing to attribute a degree of ontological charge), any argument regarding its historical naturalness must rely on a similar one for the delimitation of such practice: how

[10]Just take the opinion of any 'analytical' philosopher on Badiou. I agree on the use of adjective 'clumsy' here, but I am willing to use it for the cases I am considering above as well.

[11]I borrow this terminology from Kuhn, see [14].

[12]Kuhn leaves the possibility of translators between different linguistic communities open, but keeping in mind that translating is not the same as adapting, subsuming or altering another conception to one's own. See [15].

[13]One may think that, since the syntax of both intuitionistic and classical logic is the same, 'neither party knows what he is talking about', as Quine believed. Kuhn, on the other hand, can be read as admitting the possibility that their use of language, though it superficially seems to be the same, hides the very nature of the radical incompatibility of their views and, hence, their incommensurability. See [14] and [15] for more details.

else could anyone even point at what *there is*? But maybe it is the case, in fact, that history had just to develop as it has done until now so that, actually, mathematics *really talks about* sets and non-classical logicians *really talk about* some funny situations in which our mind is troubled by strange arguments. This notion of 'naturalness' is what we will now analyze with more detail.

6. In [2], Ferreirós argues that first-order logic is far from being a *natural unity*, that is, a 'system the scope and limits of which could be justified solely by rational argument'[14]. He later provides a counterfactual characterization of this same concept: a natural unity is 'a system that sooner or later had to be adopted, once an interest in logical matters had arisen'[15]. Against this, he confronts the conception of first-order logic as 'a compromise between the natural and the contingent'. This alternative is what we may take as Ferreirós' own point of view:

> The reader should not, however, take the preceding statement as a denial that first principles may have played any role along the road to modern logic. Being contingent is not, by any means, the same as being purely contingent – a maxim that seems particularly applicable in the case of logic. In my opinion, the development of modern logic was partially guided by (relatively clear) guidelines or principles, partially by a peculiar Occidental tradition, and partially of course by subsequent events[16].

To the descriptive tool provided by Kuhnian exemplars, Ferreirós adds the mentioned guidelines 'that help define the realm of logic and its scope' such as: (a) logic is the analysis of deduction, (b) logic is universal and (c) logic is formal[17]. Later on, he includes both the principles of 'immediacy' and of 'calculus'[18], the second of which is characterized as follows:

> This is the Leibnizian ideal of submitting logic to algorithmic mathematical treatment: it was regarded as possible, and of course desirable, to express all kinds of so-called 'logical' notions and principles by means of calculi (similarly to the new arithmetic and algebra), so that 'logical' deductions could be mechanically computed. [...] The first clear success in implementing the principle of the calculus came with Boole's algebra of logic; Frege made an impressive step forward, and Gödel established the crucial limitations for its implementation.

The point that Ferreirós wishes to make ultimately relies, however, on arguing against the *partisans* of first-order logic[19] and defending that 'no clear-cut argument can establish that logical theory must embrace both sentential logic and quantification theory'[20]. Through the

[14] See p. 441 in [2].
[15] See p. 449 in [2].
[16] See p. 455 in [2].
[17] See p. 456 in [2].
[18] See pp. 458-459 in [2].
[19] See p. 448 in [2].
[20] See p. 457 in [2]. Here, one could quote Kant on how the mathematical judgement deals with some kind of intuition (not sensory but as a *repraesentatio singularis*) of some kind, something that seems far away from logic, at least if we consider it to be purely formal. But for establishing some statement for,

text, it is also shown how the schema of exemplar-shifting applies for the particular case of modern mathematical logic[21]. But it is less clear the tradition to which this historical phenomenon belongs. Regarding Carnap's identification of intuitionism with the mere adoption of the restricted functional calculus (instead of the extended one), Ferreirós admits that

> Carnap's reference to the intuitionists is puzzling, since at the time Brouwer's school avoided completely reliance on formal systems of logic and mathematics. Still, it is quite true that some authors of a *constructivist* tendency [Weyl, Skolem] had proposed FOL as the adequate system for foundational work[22].

So we can see how the, at first, scarcely mentioned principle of calculus has acquired more eminence in the ongoing discussion. He later adds:

> As is well known, the most convincing reasons for a restriction of modern logic to FOL emerged in 1930 and 1931, with Gödel's metalogical results. Such reasons can only be compelling if one focuses on formalized systems for classical mathematics and on Hilbert-style metamathematics; they were of no weight in the eyes of Brouwer and other heterodox authors. But the intriguing historical fact is that years before 1930 a few authors started to advance the thesis that FOL is the natural system to use in the foundations of mathematics. They did so on the grounds of conceptual reasons of principle, not on the basis of technical results; indeed, Carnap identified them as constructivists. [...] Brouwer and his followers opposed the formal trend, but the same does not apply to constructivists such as Skolem[23].

The problem of 'natural unities' is that it seems that they trivially exist or do not exist at all. According to the first definition above, first-order logic would count as a natural unity just in case it could be defended (as a set of defining features) only in virtue of a rational argument or, as the counterfactual characterization tells us, just in case it would turn to be accepted *sooner or later*. But the existence of 'heterodox' views (like the crude version of intuitionism mentioned before) denies, at least, that the acceptance of first-order logic could be universal even when our interests in logical matters had arisen and, hence, the counterfactual account. One could try restricting the tradition we are examining to, say, the one characterized by *all* the guidelines provided by Ferreirós. But then, it seems that the field of possibilities is so reduced that the rational justification of such natural unity can be derived from the the guidelines themselves or, even worse, that the latter is implicit to their adoption.

In other words: if the set of guidelines for the development of some subject is *too small*, then there will not be natural unities at all and, if it is *too big*, we will have then actually

say, all triangles (a quantified statement), one has to take this intuition as *schema* and then verify that the statement holds in general. Thus, it is dubious that the demarcation of the logical and the mathematical for quantified statements has been historically invariable.

[21] In particular, the shift from the theory of quantification, classes and relations towards type theory and, from here, to first-order logic.

[22] See p. 446 in [2].

[23] See pp. 470-471 in [2].

specified the natural unity through such principles. But philosophy is not the science of measurement. Hence, it seems more reasonable to simply state that one can not even find a relevant example of natural unity in the history of logic (or mathematics).

7. We have seen, therefore, that the Dummettian approach for eluding the problem of object-talk leads to new problems and surprising prejudices regarding some historical features belonging to the development of knowledge[24]. The reader is probably asking: and now, *what*? Well, the turn to faith seems at this point more clear than before, at least. The same man who believed that

> But what experience and history teach us is this, that nations and governments have never learned anything from history, or acted according to rules that might have derived from it. Every period has such peculiar circumstances, is in such an individual state, that *decisions* will have to be made, and *decisions* can only be made, in it and out of it.

also thought that morals and religion occupy a special place, that they play indeed a radically different role for us[25].

In the quoted lines, the italics are mine. I wish to put emphasis on the notion of *decision*[26]. There is a wide range of possible decisions that, once taken, shape the history of logic and mathematics. Every revolution in these fields consists in a concrete active decision-making, regarding concrete topics[27]. Examples of these can be found in instances of axiom-adding, definition-making or, more generally, in the exemplar-shifting discussed by Ferreirós. What is clear is that, in each generation, there is an active and chaotic decision-making, which later leads to the constitution of the *statu quo*, the tradition, for the future ones[28]. Trying to identify the structure of a historical fact in order to apply it to a present phenomenon is

[24] One can also see the parallelism with Gómez-Torrente's view in [9] with the principle vs. pragmatically-based approaches in problem of demarcation of the logical constants: 'a logical constant may be just an expression which satisfies some appropriate basically pragmatic principles. The complexity of the principles, and also their somewhat vague nature, does much work to avoid any easy refutation of a proposal of this sort –either of the claim that it provides necessary conditions for an expression to be logical, or of the claim that it provides sufficient conditions. The problem with typical characterizations is that they have no place for that sort of complexity, as they try to unearth necessary and sufficient *mathematical*, *semantic* and *epistemic* properties for an expression to be logical; it is a characterization of this sort that seems hopeless' (p. 34).

[25] This quote belongs to Hegel but I read it for the first time in Feyerabend's *Against Method*.

[26] For me, this term has some deep Wittgensteinian connotations, but I will not talk about this here. See [17].

[27] It is my opinion that revolutions may occur in logic and mathematics, but this belief is far from being obvious: see [5].

[28] Of course, I am not implying that learning the history of logic and mathematics in this way would be productive for future logicians and mathematicians: the *statu quo* is a reading of history or, even better, a way of articulating a reading of such history. What is more clear is, however, that the philosopher is the one that should investigate these topics further. Kuhn recognizes that the textbooks omit this revolutionary nature in order to efficiently articulate knowledge for the professional. Otherwise, logico-mathematical work, as we know it, would be compromised. See [14].

insufficient. The *statu quo* is at best understood when a competing, incommensurable view reveals deep disagreement with tradition, as we have seen above. One of the tasks for (an already moonlighting) philosophy should be this: articulating the tradition in a systematic way so that the historical flow of revolutions is not blocked up by its influence. And this articulation is only provided by making explicit the decisions that have already been taken.

But perhaps the term 'decision' is too light for the realist (or even the anti-realist) that has had enough willpower to have followed us here. They do not fully agree with us, for a decision seems, rather, a mere consequence of an *imposition*: abstract objects impose *themselves* on us and guide (even when we are not fully aware of their status) our activities[29]. Note that this view is compatible with our conclusions above. Even when this realist concedes the Hegelian picture in which the individual subject is expected to *act* without any definite deep principle rooted in knowledge (ultimately historically-dependent) they still hold that such an activity is implicit to the self-imposition of abstract objects upon us. This is the more precise sense in which faith 'escapes' history, and this is what we will restrict ourselves to study during the rest of the text: whether it makes sense to talk about what *lies behind* the flow of our rational activities.

8. Until now, I have been arguing for my conception of object-talk as the sign of some kind of faith in a *negative way*, that is, through arguments against alternative proposals that wish to elude it without accepting the relevance that it may bear for the philosophy of logic and mathematics. Now, let me try the *analogical way*. The reader should note that trying to trace the use of the terms 'object', 'abstractness' and 'existence' back to its origins is surely an impossible task. Moreover, I seriously believe that every mathematician, philosopher or even natural scientist has a more or less complex idea of its usual behaviour. Therefore, it is not at all unreasonable to begin with the modern tradition that we have been referring to during the previous pages. In other words: let me directly borrow from the opposition, instead of starting from the Greeks.

First, we have to talk about the *definition of the object of faith*. Otherwise, how could be faith properly directed, after all? Frege was probably the first modern logician to implicitly link logic and ontology. As it is widely known, he came to define the fundamental distinc-

[29]It has been pointed out that this term is misleading: the realist may argue that, following the search of truth, the imposition would rather be found in the evaluation of proposals, but not necessarily in the proposals themselves. And I agree that how abstract objects may 'guide' us is not as simple as I have presented here, as a mere negation of the proliferation of proposals. In fact, one may consider the following reasoning embodying such a form of realism: (1) any consistent formal system can be associated with a domain of mathematical objects, to the existence of which we are committed, (2) mathematics is about the study of mathematical objects *in general*, so (C) every consistent mathematical theory is worthy of study (leaving pragmatical grounds aside). Hence, if I want to keep using the term 'imposition' in a strong sense, I agree that I have to apply it, rather, to the evaluation of methodological proposals.

tion between 'function' and 'argument' as a difference between saturated and unsaturated expressions that, consequently, refer differently. On the one hand,

> I am concerned to show that the argument does not belong with a function, but goes together with the function to make up a complete whole; for a function by itself must be called incomplete, in need of supplementation, or unsaturated [*ungesättigt*][30].

And, on the other,

> When we have thus admitted objects without restriction as arguments and values of functions, the question arises what it is that we are here calling an object. I regard a regular definition as impossible, since we have here something too simple to admit of logical analysis. It is only possible to indicate what is meant [*gemeint*]. Here I can only say briefly: an object is anything that is not a function, so that an expression for it does not contain any empty place[31].

Of course, such a criterion is only possible in virtue of Frege's so-called 'linguistic turn', which implicitly assumes a form of realism[32]. It is particularly interesting that 'object' is here defined negatively, since a positive approach would instantly violate its simplicity. This calls to mind the position held by negative theologians, who stated that we may only grasp the Divine by stating what the Divine is *not*. We will return to this later. But it should be noted that the lack of possibility of definition of 'object' does not undermine the overall ontological enterprise initiated by Frege.

Now, the reader could argue here that Frege is not taking the word 'object' to pick *every* element of his ontology and that, therefore, we have no right to include him in the tendency we are attacking: an object is, instead, a kind of *entity*, just as a function is another. However, the use that we are making of the word 'object' differs from the Fregean one, as do their meanings; we may simply repeat all of our arguments above replacing it by the favorite expression purported by the reader[33]. The important fact here is, rather, that these entities or objects are to be understood, as *being*, as bearing some features that *have to* be labeled as ontological[34]. For the case of, say, physical objects, this is out of question, there is no

[30] See [4].

[31] *ibid.*

[32] Compare with the two principles described above by Dummett.

[33] Perhaps the reader is, as it seems fashionable nowadays, some kind of eliminativist structuralist or of deflationary platonist. In the second case: why conceding a bit of the story but not the satisfying part of it? In the first case: is it not a structure some kind of entity, after all? It seems strange to make even some *ante rem* defence of something that is pointless to talk about. Note that this makes our view to be divergent with that of Putnam, since he favors a modal structuralist view as a solution to the overall ontological mess. Again, the reader is redirected to [18] for more details.

[34] It may be argued that Frege leaves the status of functions as essentially differing from the one of objects. If one is totally committed to the saturation criterion (whatever that is), one may argue that a physicist probably regards numbers as needing further completion while a mathematician may regard them as saturated, i.e. objects. Hence, we think that it is convenient to consider the Fregean function as an *entity*, that is, as the Fregean appropriate substitute corresponding to the so-called *umbrella* view.

strange component at all (is there?). But then, the adjective 'abstract' makes its entrance.

Hence, our next stop will be the *ways in which one may grasp the abstract object of faith*. The concatenation 'abstract object' has, surely, intriguing consequences. Not only we have agreed in certain application of the term 'object', we now intend to justify such an application through a set of *ways*, that is, thought-experiments, that may help to elucidate its results. That we are not be able to properly define 'object' does not imply that we cannot succeed in describing some properties verified by 'abstract' objects and falsified by 'concrete' ones (or reciprocally)[35]. Thus, these ways have a distinction as a goal: the distinction between the Divine and the mundane, between the abstract and the concrete.

In [3] one may find some examples of these ways, namely: the way of *example*, *conflation*, *negation*, *abstraction*, etc. Certainly, it would be an error to take all of them as essentially similar, for each one of these demarcation criteria relies indeed on a different thought-experiment. But what is present in all of them is a spirit of *predication*, of *property-attribution*, be it positive or negative. The way of 'accessing' what is demarcated also describes further properties of the object of faith. Of course, God is not finite, God may be said to be omnipotent, etc. in the same fashion. These conceptions can later constitute divergent traditions, as some philosophers of mathematics may point out[36].

But we are not dealing with theological arguments yet, even if the comparative with the Thomistic *ways* is attractive. For a *theological argument* to be carried out properly, one should first accept some basic common-ground. Otherwise, the argument by itself would not be able to establish anything, since the truth of its conclusion could not be evaluated in an explicit way (i.e. in terms of the truth of its premises). This background is provided by (at least) one of the ways above, that is, by means of the acceptance of some basic properties of abstract objects and, therefore, of the consequent object-talk inherent to any predication. But something else is needed.

If the reader checks out the realist's argument mentioned in the first page, they will come across the term 'indispensable': this is precisely where the link between the object of faith and the concrete *practice* of the logicians and mathematicians receives its apparent clarification. In other words, this is what ultimately enables the indispensability argument to impel us to ontologically commit to abstract entities. Thus, possibly the most famous of Ferreirós' first-order logic partisans claimed that

> A theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the

[35]In this way, Frege described a 'third kingdom' of entities different from physical objects and psychological entities.

[36]For example, as we will see later, in [16] Maddy takes sets as abstract *but* spatio-temporal.

theory be true[37].

Note that this way of selecting *entities*, namely, by examining *what is quantifiable*, is specially attractive, for it seems quite specific in fixating the ontological domain to which one ought be committed in each case: looking up for the quantified variables in a theory seems, for the working mathematician, a task as natural as any other. It also provides a principle of economy: if one theory can be expressed in terms of another we then can restrict our ontological commitment to the latter[38].

From here, the indispensability argument runs as smoothly as usual, now with its full force as a theological argument. The ontological enterprise of Frege is finally reinforced with the help of Quine. As a consequence of Quine's analysis in [19], as Putnam says, ontology received a renovated status in 'analytical' philosophy, specially in the philosophy of mathematics and logic[39]. It is not at all surprising, then, that object-talk flourished within the discourse of realist and anti-realist philosophers: further demarcations and ways were to be discovered and new arguments regarding the subject-matter of logic and mathematics emerged as belonging to some romantic and crucial project.

The problem is, as we have argued above, that a hidden premise is needed in order to enable ontological commitment to abstract entities[40], that is, that formal sciences bear some kind of ontological-charge or that, equivalently, we *are able* to apply the term 'object' in a strangely similar way to the one in which we do with mundane things[41]. Is there or is there not a distinction between the abstract and the concrete, after all?

9. I would like to recycle this ongoing analogical reasoning one last time. It seems to me that the best way to understand the comparative that I have been pursuing arises when dealing with questions belonging to the field usually named as *phenomenology of the religious experience*. I am far from being an expert in such topic, but I wish to, at least, point

[37] See [19].

[38] Of course, this economy is in fact a sign of reductionism, as Benacerraf and other have been devoted to argue and as we have succinctly explained before through the *really-talk-about* slogan. As Zalta says in [22], '[i]n general, the fact that mathematical theory T_1 can be reduced to theory T_2 doesn't imply that the quantifiers of T_1 range over the same domain as the quantifiers of T_2'. Again, Zalta concedes to Quine more than what is needed.

[39] See [18].

[40] Putnam also claims in [18] that Quine's indispensability argument makes use of a hidden premise: the uniqueness of a 'real' and 'literal' sense of the terms 'identity' and 'existence'. Thus, Quine's rejection of modal constructions was due to the fact that their corresponding ontological commitments were unclear, whereas this did not happen in the case of sets. (Here one can see why Ferreirós' label of partisan is very adequate.) Our claim is, then, a bit more radical than Putnam's, since we reject the very same applicability of the mentioned terms in some contexts.

[41] For example, Russell: 'I shall use as synonymous with ['term'] the words unit, individual and entity. The first two emphasize the fact that every term is one, while the third is derived from the fact that every term has being, i.e. is in some sense. A man, a moment, a number, a class, a relation, a chimera, or anything else that can be mentioned, is sure to be a term'.

out the similarity that struck me originally and led me to the considerations that we are facing right now[42].

Let me return to the beginning of this article, namely, to Dummett's discussion on bivalence. This problem, far from being just philosophically convoluted, bears a serious mathematical content, at least at a first glance. For the contemporary mathematician, independence is a common –yet undesirable, at least in some sense– phenomenon. This situation is prominent in the jewel in the crown of the modern (so-called) foundations of mathematics: set theory. (Remember that, for Quine, following the economy principle above, set theory is the paradigm of a fundamental mathematical theory whose ontological domain is *everything what is needed to be ontologically committed to*, so we are in the right path!) The most famous example of such independent set-theoretical statements is Cantor's Continuum Hypothesis, hereafter, CH. What is of interest here is not this (mathematical!) fact by itself but, rather, the reactions that it brought forward. Briefly, set theorists were divided in two groups: those who thought that the problem posed by CH found the (maybe unfortunate) end of independence and those who insisted in regarding that very same problem as still open. One of the most famous proponents of the latter position was Gödel himself, and his view has being respectably continued by Maddy[43].

Gödel's turn to phenomenology has been widely studied and discussed[44]. It has, as a matter of fact, influenced not only a current methodological but also a philosophical trend which is, so to speak, quite vigorous. Gödel, for all I know, could already be labeled as a realist during his 'Leibnizian phase', before this turn even took place. For example, he would maintain that

> Classes and concepts may, however, also be conceived as real objects, namely classes as "pluralities of things" or as structures consisting of a plurality of things and concepts as the properties and relations of things existing independently of our definitions and constructions[45].

Could one imagine a more perfect prototype for the notion of 'object' that has been occupying us? I don't think so. He later continues by establishing a comparative which has been later developed by Maddy in [16]:

> It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence.

[42]Note that the problem of *sacred language* is very close to what I intend to cover right now; perhaps later we will also examine some phenomena belonging to this field, if we have enough space.

[43]In fact, our title is a reference the two articles by Maddy, *Believing the axioms* I and II. Maddy's view can also be understood as a naturalist answer to Quine's indispensability argument, since she holds that it is rational to believe the axioms (of set theory) without appealing to Quine's scientism (i.e. the reference to 'our best scientific theories').

[44]See, for example, [11] and [21].

[45]See [6].

> They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions and in both cases it is impossible to interpret the propositions one wants to assert about these entities as propositions about the "data", i.e., in the latter case the actually occurring sense perceptions[46].

Regarding the failure of late Russell's 'no-class' approach, which he qualifies as a form of anti-realism, he concludes:

> This seems to be an indication that one should take a more conservative course, such as would consist in trying to make the meaning of the terms "class" and "concept" clearer, and to set up a consistent theory of classes and concepts as objectively existing entities. [...] Many symptoms show only too clearly, however, that the primitive concepts need further elucidation[47].

In my opinion, this spirit of 'further elucidation', together with the previous account of the perception of the abstract realm is what ultimately led Gödel to try the phenomenological path. The other component that one needs to add to the equation is the already mentioned independence phenomenon. Thus, one can read that

> the axioms of set theory by no means form a system closed in itself, but, quite on the contrary, the very concept of set on which they are based suggests their extension by new axioms which assert the existence of still further iterations of the operation "set of". [...] the axiomatic system of set theory as known today is incomplete, but also that it can be supplemented without arbitrariness by new axioms which are only the natural continuation of the series of those set up so far[48].

And, in a footnote for this very same paragraph, Gödel writes:

> Similarly also the concept "property of set" (the second of the primitive terms of set theory) can constantly be enlarged, and furthermore concepts of "property of property of set", etc. be introduced whereby new axioms are obtained, which, however, as to their consequences for propositions referring to limited domains of sets (such as the continuum hypothesis) are contained in the axioms depending on the concept of set.

This is the *locus classicus* for the so-called *dream solution* of CH[49]. Clearly, the most problematic element –the core of Gödel's view– is the notion of '[natural] supplementation without arbitrariness', for here the question on how to achieve such an important feature is left open. This idea is connected to the clarification of meaning mentioned above in an essential way. Now, the turn to phenomenology can be read in the following lines:

> Now in fact, there exists today the beginning of a science which claims to possess a systematic method for such a clarification of meaning, and that is the phenomenology

[46] Again, see [6]. He adds, in a footnote: 'The "data" are to be understood in a relative sense here, i.e., in our case as logic without the assumption of the existence of classes and concepts. In other words, he is probably thinking about a 'uninterpreted' approach to logic and mathematics. He also confronts this same idea in [8].

[47] *ibid.*

[48] See [7].

[49] As it has been named by Hamkins in [10].

> founded by Husserl. Here clarification of meaning consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our own acts in the use of these concepts, onto our powers in carrying out our acts, etc.[50]

What does Gödel understand as 'phenomenology'? He provides some hints:

> it is or in any case should be a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other basic concepts hitherto unknown to us[51].

'New state of consciousness': how could one not read here some kind of *mystical ecstasy* caused by the *contact* with the Divine? Despite this, and similarly to Dummett, Gödel tries to reduce object-talk to another problem. Here, the Husserlian Gödel shifts from some form realism (that one may take, without over-complicating too much, as platonism) to what may be called 'epistemological platonism'[52]:

> [...] the question of the objective existence of objects of mathematical intuition (which, incidentally, is an exact replica of the question of the objective existence of the outer world) is not decisive for the problem under discussion here. The mere *psychological fact* of the existence of an intuition sufficiently clear to produce the axioms of set theory and an open series of extensions of them suffices to give meaning to the question of the truth or falsity of propositions like Cantor's continuum hypothesis[53].

This phenomenological analysis of the faculty of accessing the abstract realm or, in other words, the contact with the Absolute[54], eliminates any sign of mysticism by means of a restriction to some kind of psychological setting[55]. Thus, for example, in [12], one can read: 'I cannot see anything irrational in the proposal to investigate this 'givenness as real' that we encounter in perception as well as in mathematical intuition' and, in the same spirit, Maddy's whole article [16] is devoted to introduce a common faculty of intuition of the abstract realm. But the faith itself is left unchanged: the primacy of (self-)*imposition*, against *decision*, of the abstract realm, be it spatio-temporal or not, be it causally efficient by psychological or metaphysical means is preserved; the faculty of mathematical intuition is just the way in which this imposition is enabled on us. To put it in simpler terms: what is accomplished here is the shift from object-talk to faculty-talk while, on the other hand, the assumed ontological charge of logico-mathematical theories (in particular, of set theory)

[50] See [8].
[51] *ibid.*
[52] See [12].
[53] See [12] for this quote of Gödel.
[54] Which has to be understood as something different from the absolute that Cantor apparently had in mind and, of course, from any kind of insight that the reflection principles may bear. This opinion, by the way, is not that popular among the modern treatment of the ineffable and its contact with, say, mathematics.
[55] I would not even go that far. One could then ask Maddy and Gödel about how such an intuition could be empirically detected. I am pretty sure that their answer would amount to saying that mathematicians (at least the normal ones) talk about abstract objects, so that one of these subjects would count as some kind of abstract object detector (similarly as the way in which one can talk about machines detecting subatomic particles).

remains untouched.

10. Can the Divine be causally efficient in the mundane? The authors of the previous article on Dummett's arguments claim that

> The assumption of omniscience, then, offers no help in settling any debate between these two competing positions. Omniscience will not secure a new kind of logic, nor will it allow us to decide debates over which is the correct logic for us to use; it will simply conform to whatever logic we assume prior to our assumption of an omniscient being[56].

Now, we can translate this very same debate, around 'the correct logic', to the one about 'the natural extension of ZFC' or, in general, 'of a formal system'. But it should be observed that, unlike in a competition between incommensurable theories, Gödel's intuition is likely intended to fulfil some kind of creative development, whatever that is. Still, it is clear that this intuition would play a very similar role to that of God's omniscience in Dummett's arguments. Let us briefly examine, as we did before in that case, the historiographical consequences of such view.

As a matter of fact, set theorists are nowadays working with CH *and* its negation. Near the end of [10], one can read:

> My challenge to anyone who proposes to give a particular, definite answer to CH is that they must not only argue for their preferred answer, mustering whatever philosophical or intuitive support for their answer as they can, but also they must explain away the illusion of our experience with the contrary hypothesis. [...] Before we will be able to accept CH as true, we must come to know that our experience of the ¬CH worlds was somehow flawed; we must come to see our experience in those lands as illusory.

Of course, Gödel's account of mathematical intuition as non-arbitrary would immediately (and *a priori*) provide a ground for debunking such an illusion, at least *once the required axioms would be found.* This means that his faculty of intuition exceeds any possible (historical, contingent) experience that we may have with the set-theoretical frameworks in which CH or its negation are assumed. How could we possibly settle a choice between such frameworks? For sure, workers in those fields would need much more than a epistemological grounding of such decision[57], let alone one that reminds of Descartes thoughts on the pineal gland[58].

[56] Again, see [20].

[57] Let us not forget that Maddy herself defends such a maxim: in a debate between philosophy and mathematics, the later will more likely win. Nevertheless, one is left wondering if such a clash may occur (or has indeed happened) in the history of logic and mathematics. Well, it *does* occur in the case we are examining, although I am pretty sure that Maddy will not concur!

[58] This remark is completely serious: Gödel did talk about 'organs' at some point, without falling to some kind of physiologist explanation of abstract objects (which is a view that I have not found anywhere but maybe is as reasonable as any other). Maddy's account is intertwined with her naturalism, so it is not worthy to examine it here in a very exhaustive way.

Other sympathisers of *Gödel's program* have tried to confine his psychological mysticism to a more modest set of well-defined criteria that every new candidate for axiom of ZFC should satisfy in order to count as *natural*[59]: note the similarity with Ferreirós' unities and sets of guidelines! But then, the faith in the transcendental faculty is translated into the faith in these meta-axioms. In fact, the problem with this set of meta-axioms is completely analogous with the one posed above for Ferreirós' guidelines. If they are too specific then they will sanction a concrete axiom that it is believed from the start to be natural. If they are too general then, as Hamkins argues in [10], it seems that the present situation could not be *improved* in any possible sense.

The problem, thus, replicates itself. Hence, the reference to something *outside* or, better, *behind* history is needed. As happened with Dummett, once Gödel's suppositions are placed outside the sea of faith and under the light of rational debate, some surprising consequences arise.

11. I hope that the reader is more open now towards our faith-based approach to the problem of the object-talk. Nevertheless, they may be thinking, my point of view consists in giving up this very same object-talk: why expending, then, so many words in explaining the tradition that I intend to undermine? Let me explain myself. First, let me get fancy and state a surreptitious shift in our current argumentation. The real point that I want to make is that, even when we understand object-talk inside our faith-based approach, one can maintain our original, radical insight intact. In other words, I claim that one can defend another *way of faith* that enables the 'no-ontological-charge' view or, equivalently, that this very same claim is tenable while simultaneously conceding that the whole problem has to do with faith.

The example of Gödel's account of the faculty of mathematical intuition characterizes, under our point of view, what we will call the *direct faith approach*. Here, the subject is allowed to grasp the abstract through some kind of transcendental[60] –yet human– sensory organ and, moreover, the abstract will actively impose itself on the passive work of the subject through such an organ. Here, object-talk is to be taken at face value, since one can talk about, say, sets as one talks about this table[61]. Thus, from this perspective, there is no Prometheic

[59]See [1].

[60]This adjective corresponds to the Husserlian *epoché*, also called 'transcendental reduction', which we have seen roughly presented in the Gödel's quotes above. Perhaps its use is somewhat vague but I only wanted to exhibit the contrast between the human and the abstract realm which, under my point of view, is problematic for Gödel's approach.

[61]Now it comes to my mind an assertion made by a famous philosopher of set theory: 'for me, ω_2 exists as does this chair'. It seems like a Moorean argument: 'Where could we find an abstract entity? Take this

claim at all. As we have seen, this viewpoint is completely idle if we regard it as something purely human, since it will not enable us to decide in a *practical* situation[62], and completely mystical if we regard it as something divine. Of course, we have nothing against the second option, but one should not try to maintain both at the same time or to just take one as implying the other.

What I favor here is a *liturgical faith approach.* The problem regarding object-talk is, in my opinion, that once the faith-based explanation is conceded, it seems that only a conception of faith, that is, of the human-divine link, must be possible. Remember that the ways of explaining the differences between the concrete and the abstract relied upon some kind of *positive* or *negative* predication. This is what, in Medieval ages, was customary of the *Cataphatic* and the *Apophatic* theological traditions, respectively. Maddy's view in [16] is, for example, that sets are abstract *and* spatio-temporal, that one may claim that *here* there is an apple and *here* there is its singleton. I have already expressed my concerns surrounding this kind of discourse, but I will be more specific here. What if God were *ineffable*? For sure one could not predicate anything about Him *at all.* In the same sense, any object-talk is meaningless for us because it is a sign of faith and this faith is to be understood as a link with the Ineffable. The dizzy reader is perhaps wondering if we have lost our mind. 'What is then your proposal, to simply shut up following the Wittgensteinian code?' Well, something certainly *remains.*

One of the may ways of undermining the whole object-talk problem is to simply regard such a talk as a certain *inclination*[63] of mathematicians, without any philosophical (or even qualitative) identifiable content[64]. It seems true that, when *doing* mathematics, one cannot freely claim, for example, to have a proof of such-and-such theorem (at least not *within* certain rules of a formal system). This sensation of objectivity[65], which is more or less clear in

$\aleph_{\epsilon_0}$ over here'.

[62]What I understand by 'decision' is surely fishy. Here what I have in mind is the concrete methodological decision of accepting some set of axioms in order to *decide* (in the mathematical sense) CH or its negation. One would also like to include definition-making under the range of this term. Roughly, if we accept logico-mathematical pluralism, axiom-adding and definition-making will generate a myriad of, at least, equally interesting formal systems. If one is to take logico-mathematical truth at face value, much as Gödel does, one is then left with the task of *deciding* between these alternatives (of course, one may also talk about imposition as we have done before). This situation, namely, the 'how to continue [axiom-adding or definition-making]' problem, can be seen as an instance of Krikpe's rule following problem. See [17] for more details.

[63]As, by the way, Wittgenstein would say: mathematicians are allowed to have inclinations induced by their work but the work of the philosopher is to actually *treat* any such inclination. Compare this with the Moorean argument in the footnote above. Perhaps the problem is that set theorists like to consider themselves as philosophers and mathematicians at the same time which, in the end, is equivalent to being a mathematician with a special status (let us not forget Maddy's view that *mathematics is not subjected to any philosophical tribunal*). From here, the set theorist feels legitimized to ignore any philosophical warning regarding their use of the really-talk-about slogan.

[64]I may include here what Putnam calls 'value judgements' in [18].

[65]Which is, then, the source of such sensation? See the addendum for more details regarding my proposal. Here, what I understand by 'sensation' is an instance of the aforementioned *inclinations* that mathematicians

this context, may easily lead to object-talk (and I am not meaning that the mathematician simply takes the *object* from *objectivity*). The ontological enterprise of Frege and Quine is to be understood as much more than this deflationary exposition of what counts as object-talk: such a discourse is here taken, in fact, at face value, and the same applies for Gödel.

However, other way of regarding object-talk is closely connected to the idea of logic and mathematics as some kind of *game*[66], in the sense that, when logicians and mathematicians talk about so-and-so they are merely *inside* a practice with its own language use, standards, etc. and, hence, meanings. This point of view is almost co-extensive with the concluding observation that I want to make here. My use of the adjective 'liturgical' is very similar to that of 'game', but some theological flavour is added through the reference to the Ineffable.

If we return to Gödel's characterization of the phenomenological method, i.e. 'focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our own acts in the use of these concepts, onto our powers in carrying out our acts', etc., we may regard his proposal as following the Husserlian conception of *act* and *epoché*:

$$\text{Act(Content)}\rightarrow\text{[Object]}^{67}$$

where the bracketing of the object corresponds to the phenomenological reduction of which Gödel is talking about. Of course, the object at which we direct our acts is not to be thought in terms of causality, like in the *natural attitude*[68]. It does not necessarily belong to the usual context of human practices, for we may be talking about some kind of *categorial intuition*, like Gödel seems to argue[69]. Rephrased in these terms, our conception would look more similar to the Rilkean schema:

$$\text{Act(Content)}$$

In other words, how we act, including how we talk, when we are doing logic or mathematics is better conceived as a form of liturgy. The act is, by itself, worthy. The reference to some realm of objects is as needed as it is a God when praying. But note that we are not favoring any concrete mathematical or logical practice. In fact, identifying one of these with the *only way* of acting in such fields is dogmatic and assumes a theological character that human activities, on their own, do not have. Thus, the ineffability conception turns out to be an instrument of philosophical *criticism*. This is what we have been trying to make clear through this article.

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may share. And, additionally, objectivity can be also conceived while eluding any reference to objects, like Putnam wishes to argue in [18].

[66] Again, Wittgenstein and Putnam have championed this view.

[67] As it is explained in [21]. It is not surprising that the use of the phenomenological reduction made by Gödel is not free of disputes, see [13].

[68] For more details, again, see [21].

[69] See [13] for a different opinion on Gödel's use of this Husserlian term.

We started with Rilke and we have ended with Rilke. As a coda, it seems to me appropriate to include the following fragment from the second of the *Duino Elegies*:

> *Have you not been amazed at the discretion*
> *with which all human gestures were portrayed*
> *in Attic steles? Were not love and parting*
> *laid on their shoulders with so light a touch*
> *as though they knew them in a different guise?*
> *Recall how their hands touch without insistence,*
> *despite the strength that in their torso sleeps.*
> *They, in their mastery of self, were wise:*
> *so far are we,* this *is ours,*
> so *to caress each other; for the gods*
> *press heavier on us. Yet this concerns*
> *the gods.*

Further topics. The possible continuations of our view that we can conceive at the moment could be classified in two kinds: (i) The practical character of mathematics (and logic) following the maxim *if you are hungry and you use a knife to cut the bread, you won't say that the knife has removed your hunger* for the case of logico-mathematical truth and 'objects'; (ii) The ineffable and its language or, in general, God-language: is affirming the existence of entities a predication about those entities? What is to count as such predication? (Note the similarity with the problem of logical constants and their demarcation.)

Addendum. There are two main problematic features of my previous presentation of ideas. The first one is perhaps more crucial because it can be raised at the beginning of the article: why dealing only with forms of realism and anti-realism derived from the indispensability argument? The other concern has to do with the conclusion at which I finally arrived: what does the 'liturgical faith approach' exactly mean? I have left both of these worries for this appendix because they are deeply connected and affect the overall purpose of the paper.

When I introduced the various approaches towards object-talk, I wanted to point out how all of these views intended to make use of *rational* arguments in order to justify some belief. Against that, I have argued that object-talk is best understood when linked with the notion of faith. Therefore, if I had presented realism and anti-realism in the usual fashion, my view would have to be introduced as just pointing out that these positions bear no rational source at all, which is perhaps a more obscure way of putting the whole problem. Also, I think that the point remains, at least when realism is identified with 'trying to provide a rational argument for realism' (and, as always, the same goes for anti-realism).

Therefore, if reason alone is insufficient in order to clarify the claims surrounding object-talk, we are left with the alternatives that I have tried to elucidate above. But even the faith in the abstract realm is susceptible of bifurcating proposals. Another issue with my preceding explanation is that these are not carefully distinguished. At first, we may separate the positive or negative approaches to the object of faith (let me label them as *descriptive*) from the ineffability one, the one that I defend here. Then, we may also differentiate the ways in which the link between faith and practice is understood. Gödel's view is certainly descriptive and, additionally, favors the practical consequences of the phenomenological process (or of the faculty of intuition, if we follow Maddy) and, in this last sense, it is *direct*.

On the other hand, my (brief sketch of a) proposal is anti-descriptive and goes against any direct link between faith and the rational contexts considered. Thus, object-talk lacks of any rational meaning in the first place and, nevertheless, it is to be recognized as worthy. The sensation of objectivity that we mentioned before is merely *contextual*, it arises from our acts alone (say, within a language game). But how does the usual discourse of mathematicians and logicians fit in this picture? Is it not 'object-talk'? Of course it is, but not in the sense that we are criticising here. Rather, this narrow form of discourse (which constitutes an inclination as any other when it is intended as a quick and appealing philosophical explanation, i.e. when it is employed outside its usual context) arises *within* logico-mathematical practice and its traits are those of a *tradition*. I hope to have properly shown why this explains the adjective 'liturgical' that accompanies our view.

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