Chapter 3
Not Even Ideal. Kant on Absolute Time and Gödel's Rotating Universes

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Abstract In this Chapter, the concept of time is analyzed in a particular set of solutions of Einstein Field Equations that was proposed by Kurt Gödel (1906-1978) in 1949 and 1952. The rotating universe model has been largely inspired by Gödel's reading of Kant's *Metaphysical Foundations of Natural Science* (1786) in which Kant provided an argument for the relativity of motion of reference frames and for the rejection of absolute time in cosmology. After presenting Gödel's analysis of Kant's notion of time in his manuscripts, I shall show which aspects of Kant's philosophy had an impact on Gödel's view. Finally, I shall conclude by showing that Gödel's critical reading of Kant's theory knowledge did not take into account his late work and therefore could not comment on core ideas of Kant's late cosmology embodied by the *dictum* 'forma dat esse rei'. Indeed, in the *Opus postumum* Kant appealed to the concept of cosmic aether as hypostatized space and provided an argument for distinguishing the notion of cosmic local time, which is necessary for the description of the observable universe, from an a priori account of the cosmos as a whole.

3.1 Introduction: Gödel's Appraisal of Kant's *Phenomenology*

In a volume devoted to time and timelessness in fundamental physics and cosmology, it seems natural to mention Kurt Gödel's contribution to debates in both physics and philosophy. Together with Luitzen E. J. Brouwer (1881-1966), Henri Poincaré (1854-1912) and Hermann Weyl (1885-1955), Kurt Gödel (1906-1978) might well be considered one of the greatest polymaths of the 20th century, who heavily contributed to the foundations of logic, mathematics and physics. He had robust philosophical views that exerted great influence on his scientific and mathematical work. In particular, the notion of time always attracted Gödel's attention

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because it is a subject that connects philosophy with the sciences and he dealt with it in several occasions during his life.[1] Gödel's model of a rotating universe and his reflections upon Kant's philosophy highlight interesting questions surrounding time and timelessness in relativistic physics and cosmology.[2] Thus, this Chapter investigates Gödel's interpretation (even if it would be more correct to define it as an application) of Kant's philosophy of space and time.[3] In *The modern development of the foundations of mathematics in the light of philosophy* (1961), Gödel concludes:

> I believe that precisely because in the last analysis the Kantian philosophy rests on the idea of phenomenology, albeit in a not entirely clear way, and has just thereby introduced into our thought something completely new, and indeed characteristic of every genuine philosophy – it is precisely on that, I believe, that the enormous influence which Kant has exercised over the entire subsequent development of philosophy rests. Indeed, there is hardly any later direction that is not somehow related to Kant's ideas. On the other hand, however, just because of the lack of clarity and the literal incorrectness of many of Kant's formulations, quite divergent directions have developed out of Kant's thought - none of which, however, really did justice to the core of Kant's thought. This requirement seems to me to be met for the first time by phenomenology, which, entirely as intended by Kant, avoids both the death-defying leaps of idealism into a new metaphysics as well as the positivistic rejection of all metaphysics. But now, if the misunderstood Kant has already led to so much that is interesting in philosophy, and also indirectly in science, how much more can we expect it from Kant understood correctly?" (Gödel 1961, 385; 387)

As we shall see, whereas Gödel's appraisal of Kant's 1786 *Metaphysical Foundations of Natural Science* (MAN)[4] is clear, the same cannot be said for the analysis he provided of Kant's theoretical philosophy in general. This depends on Gödel's dissatisfaction with the idea that a progress in the knowledge of the essence of the world is limited to phenomena and marked by the impossibility of knowing the thing in itself as the *correlatum* affecting our senses.[5] Nevertheless, in the passage quoted above from Gödel (1961), the reference to Kant's phenomenology is not to be understood in Husserlian terms or at least not directly and simply as it stands. Gödel is rather alluding to Kant's *Phenomenology*,[6] which is represented as the most genuine legacy of Kant's mature philosophy of nature discussed in the MAN. In what follows, I shall first resume Kant's arguments for relative motion, absolute space and relative

[1] For a detailed overview of Gödel's philosophical notes, see Crocco and Engelen (2016).

[2] For a recent discussion of Gödel's universe and conception of time, see Kiefer (2023).

[3] Kant is to be mentioned among the most important philosophers that had an impact on Gödel's scientific production. Gödel was engaged with the study of Plato, Aristotle, Augustine of Hippo, Thomas of Aquinas, René Descartes, Gottfried Wilhelm Leibniz and Georg Wilhelm Hegel, as well as with contemporaries, such as Gottlob Frege, Giuseppe Peano, Bertrand Russell, Luitzen Brouwer, and Rudolf Carnap (see Crocco and Engelen 2016).

[4] References to Kant's texts follow the pagination of the Academy edition (AA). References to the *Critique of pure Reason* are abbreviated as CpR, followed by the A/B editions pagination, whereas those to the *Opus postumum* are abbreviated as OP. Translations are from the Cambridge Edition of the Works of Immanuel Kant unless stated otherwise.

[5] For a detailed discussion of Gödel's idealism, see Parsons (2010).

[6] The *Phenomenology* is the fourth and last Chapter of the MAN in which Kant wants to harmonize the mathematical and philosophical representation of time, space, and motion with both the foundations of cosmology and the end of natural science in general (see MAN, AA 04:477).

time as spelled out in 1786. In Section 3.3, I shall briefly present Gödel's solution to Einstein's equation and comment on the influence of Kant's philosophy of time. In Section 3.4, I shall present the 'new' cosmological picture advanced by Kant in the *Opus postumum* and present notions of time and timelessness that he used in natural science. Before concluding, in Section 3.5, I shall confront Gödel's interpretation of Kant's philosophy and provide evidence for the fact that Kant shaped his late philosophy to make the universe as a whole an indirect object of experience thereby asserting that the thing in itself is – at least with respect to the form – determinable from a further standpoint than that described in the 1780s.

3.2 Kant's Critique of Absolute Time (1786)

Let us now take a step back and resume Kant's results obtained in the MAN with regard to space, time and motion. Surprisingly, the "Phenomenology" is one of the most underestimated parts of Kant's system (De Bianchi 2022b). Gödel had the merit of understanding well before Michael Friedman's analysis that this section of the work deserved careful attention since it presented an argument for the relativity of motion of inertial frames (Friedman 2013) that, as Gödel underlined, represents an instantiation of an embryonal form of Einstein's relativity principle and more importantly offers a clear account of a relational time when conceptions of absolute time were dominating pre-relativistic physics. In recent work, I have argued that Kant's notion of relational time has been employed to make sense of Euler's equations of rigid bodies (De Bianchi 2022b). The study of the latter together with the reflection upon the notion of force occupied Gödel's philosophical reflections that he explicitly connected to Kant's philosophy (see Gödel 1949b, 238-239). Indeed, in 1786 Kant aimed at grounding a metaphysics of nature devoid of fallacies to account for any development of Newtonian mechanics by means of the assumption of the law of antagonism in all community of matter through motion based on the principle of reciprocal interaction (*Wechselwirkung*). The latter has the relevant function of reducing and unifying all appearances under an idea of reason (De Bianchi 2022b). This law embodies the great advancement in Kant's ontology and idea of simultaneity that is reported in the second edition of the *Critique of pure Reason* (CpR). As Kant states in the *Introduction* to the second edition of the CpR: "the simultaneity of substances in space cannot be cognized in experience otherwise than under the presupposition of an interaction among them; this is therefore also the condition of possibility of the things themselves [*Dinge selbst*] as objects of experience" (CpR B258). Kant's argument for this conclusion is that just as empty or absolute time cannot be perceived and thus used to determine succession or simultaneity, neither can absolute space be "an object for our possible experience" (CpR A214/B261) and thus either the simultaneity of two objects, or the specific place of a single object, can only be determined through the category of community and the dynamic community (*commercium*) or reciprocal influence of all substances. [7] Let us now consider more in detail the implications that this law has in the MAN. In the *General Remark to Phenomenology* Kant comments on the general use of three concepts in natural science: motion in relative (movable) space; motion in absolute (immovable) space; relative motion in general as excluding absolute motion. The basis (*Grund*) of these three concepts is the idea of absolute space. The relationship between these concepts of motion and the idea of absolute space should be understood as a ground-consequence (*Grund-Folge*) relation. Thus, absolute space, as an idea of reason can never be a possible object of experience, but it is necessary to ground the three kinds of motion:

> Absolute space is therefore necessary, not as a concept of an actual object, but rather as an idea, which is to serve as a rule for considering all motion therein as merely as relative; and all motion and rest must be reduced (*reducirt*) to absolute space, if the appearance thereof is to be transformed into a determinate concept of experience (which unite all appearances). (MAN AA 04:560)

The argument relies on the ideality of space set forth in the first Critique, since for Kant external absolute space can only be generated *ad infinitum*.[8] From this follows the general rule that all physical motion or rest can only be relative and never absolute, that is, matter can be thought as in motion or as at rest only in relation to another portion of matter and never with respect to mere empty space. Therefore, not only absolute motion is impossible without any relation of one matter to another but even the notion of absolute rest loses any meaning (see MAN, AA 04:559–60). Thus, the unconditioned idea of absolute space becomes the reference frame through which not only all relative motions, but also reciprocal empirical interactions are determined. In MAN, Kant claims that "absolute space is *in itself* nothing and no object at all," but refers to an indefinite process of considering ever more extended relative spaces (MAN, AA 04:481–82). Moreover, Kant portrays absolute space as the rule for considering all motion and rest therein merely as relative (MAN AA 04:560). This conception is closer to Euler's (1752, 186) rather than Newton's. Indeed, differently from the latter, Kant applied the concept of central force to the universe as a whole and in particular to its internal structure by building up a hierarchy of systems starting from our solar system and further including galaxies and clusters of galaxies. As a second point of departure from Newton, Kant clearly attributes only a transcendental status to absolute space, which cannot neither be taken as a substance nor being physically intuitable. Finally, Kant claims that gravitation is only one particular instance of the concept of central force and that the universal law that Kant has in mind is the one from which Newton's third law is derived, i.e. the law of

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[7] As we shall see, this position will constitute the core argument for the development of Kant's late cosmology and natural science (see Sections 3.4 and 3.5).

[8] For Kant, both space and time are defined as *quanta continua*, and they are infinite given magnitude in intuition, e.g. their parts always presuppose a greater whole in which they are contained. Gödel strongly criticized the view of space and time as infinite *quanta continua*, but his critique is made from the logical standpoint and it does not fit in the Kantian scheme. Indeed, for Kant, general logic has nothing to do with space and time but only with pure functions of the superior faculties, e.g. Understanding, Reason and so forth.

antagonism in all community of matter through motion (see MAN, AA 04:563). In order to obtain experience of matter through the empirical concept of motion, one needs a double determination of time (as form of intuition and as formal intuition). Indeed, to determine acceleration in agreement with Euler's equations of motion of rigid bodies, other two elements which give us a necessary universal determination of bodily motion are required, and they are the translation of the center of mass and the rotation about the center of mass. In other words, to prove that the Earth is revolving upon its axis or completing its revolution around the Sun we do not have to observe the starry heavens, but rather the measurement apparatus must be put within the systems of which we want to determine the relative motion. To prove the Earth's revolution about the Sun, one has to consider the Sun-Earth system as two body problem:

> There is thus no absolute motion, even when a body in empty space is thought as moved with respect to another; their motion here is not considered relative to the space surrounding them, but only to the space between them, which considered as absolute space, alone determines their external relations to one another and is in turn only relative" (MAN, AA 04:562).

It is worth mentioning that the Newton-Euler equation fits nicely with the observations made by Kant in the MAN. In classical mechanics, the Newton–Euler equations describe the combined translational and rotational dynamics of a rigid body. This formalism relates the motion of the center of gravity of a rigid body with the sum of forces and torques (or moments) acting on the rigid body. In other words, it embodies in a very sophisticated way the scope of objectively determining material bodies through actual relative motions, namely actual relative motions extend Newton's laws to all appearances, such as comoving bodies, be they elastic, rigid or fluid. Therefore, Kant's *Phenomenology* aimed at encompassing the scope of Euler's Mechanics. Differently from Euler, however, Kant dismissed the idea of absolute time and removes it from the picture, even if he clearly thinks that the objective representation of time as a formal intuition is fundamental to give meaning to the measurement of acceleration and therefore to connect the plurality of comoving bodies. Indeed, Kant supports the empirical reality of time, i.e. its objective validity in view of all objects that may ever be given to our senses, but according to transcendental idealism it is not an absolute reality, and if one abstracts from the subjective conditions of sensible intuition, it is nothing at all (see CpR B52-54). Time is not even ideal without our intuition. This aspect deserves more careful attention. In the second edition of the *Critique of pure Reason*, Kant draws an important distinction between space as formal intuition and as form of intuition:

> Space, represented as object (as is really required in geometry), contains more than the mere form of intuition, namely the comprehension of the manifold given in accordance with the form of sensibility in an intuitive representation, so that the form of intuition merely gives the manifold, but the formal intuition gives unity of the representation. In the Aesthetic I ascribed this unity merely to sensibility, only in order to note that it precedes all concepts, though to be sure, it presupposes a synthesis." (CpR, B160–161 footnote)

Contrary to the geometrical representation of space, time must undergo an indirect objective representation through motion (see CpR, B154–56). In other words, only by means of measurement the subjective representation of time acquires an objective meaning, it is so to speak made 'tangible' in physics, e.g. by measuring acceleration. The *Phenomenology* shows us that time can be represented as object through comoving reference frames rather than by means of an abstract motion of a space. However, if comoving reference frames are at stake, it means that their velocity can be measured relatively to one another and that their relative magnitude can be established with respect to the observer. In other words, time can be objectively represented, but not, for instance, as a single cosmic time. Time is always relative and represented as effective only relatively to the observer measuring other objects, i.e. reference frames, astrophysical objects and so forth. This move conforms Kant's theory of matter to Euler's equations but creates an asymmetry between space and time. This is due to the fact that an idea of absolute time would lead to a concept of absolute acceleration and the impossibility of comparing reference frames. In connection to the results of the *Phenomenology* regarding the three types of motion this would also lead to consequences contrary both to the universal law of antagonism in all community of matter through motion and to Kant's own ontology. We have now the necessary tools to compare Kant's and Gödel's views in the next Section.

3.3 Gödel's Rotating Universe(s) and Kant's Concept of Time

Gödel's solution of Einstein's field equations describing a rotating universe has been subject of a vast literature and studies that engaged, among others, Sir Roger Penrose, George Ellis and Stephen Hawking (see Hawking and Ellis 1973). The solution was published in July 1949 under the title "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation" and described a non-static stationary universe that is rotating upon its axis endowed of a specific metric dubbed "Gödel metric". The model provided a solution that did not include cosmological time that on the ground of Weyl's postulate was rather introduced by Robertson (1933):

> All cosmological solutions with non-vanishing density of matter known at present have the common property that, in a certain sense, they contain an "absolute" time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter. It is easily seen that the non-existence of such a system of three-spaces is equivalent with a rotation of matter relative to the compass of inertia. In this paper I am proposing a solution (with a cosmological term 0) which exhibits such a rotation." (Gödel 1949a).

Gödel's solution served as toy model to show that Einstein's field equations included solutions of a non-static universe, thereby enlarging the analysis of relativistic cosmology. Gödel also provided an analysis of time in relativistic cosmology. In particular, he enumerates the properties of the solution, which are properties of the four-dimensional space defined by the solution itself: it is homogeneous, there exists a one-parametric group of transformations of this space into itself which carries each world line of matter into itself (any two world lines of matter are equidistant), and

it has rotational symmetry. The fourth characteristic is of extreme interest because it consists in the fact that the totality of time-like and null vectors can be divided in positive and negative vectors so that a positive direction of time can be introduced in the solution and "a temporal orientation can be defined for the world line of every (real or possible) particle of matter or light" (Gödel 1949a). This means that there is *a rule for determining* for any two neighboring points on the world line which one is *earlier than the other*. Nevertheless, there no exist any uniform temporal ordering of all point events agreeing in direction with all these individual orderings. This straightforwardly leads to the fifth property of Gödel's model according to which it is impossible to assign a time coordinate t to each space-time point in such a way that t always increases when one moves in a positive time-like direction. The sixth characteristics is the famous implication, which I shall not discuss in this contribution, of closed time-like lines admitted by Gödel's model and implying the theoretical possibility to travel into the past or influence the past in specific world lines of matter.[9] The seventh property excludes the existence of three-spaces that are everywhere space-like and intersect each world line of matter in one point. The following characteristic is more intriguing for the scope of this Chapter. Indeed, Gödel defines Σ as "any system of mutually excluding three-spaces each of which intersects every world line of matter in one point" (Gödel 1949a). Then he concludes that it must exist a transformation which carries the four-dimensional space and the positive direction of time into itself, but that does not carry Σ onto itself. The upshot of all this is that absolute time does not exist, namely there no exist a time definable without reference to individual objects, i.e. a galactic system etc. Finally, the ninth property states that matter everywhere rotates relative to the compass of inertia with a defined angular velocity defined by the mean density of matter ρ and Newton's gravitational constant κ, reading $2(\pi\kappa\rho)^{\frac{1}{2}}$.
The latter is a property that further stimulated the analysis of the function of the cosmological constant in analogy with inertia and is still used for pedagogical purposes in current physics. However, Gödel's model is not considered a faithful representation of the expanding universe, rather it is taken as a powerful cosmological toy-model and a solution of the field equations. Einstein himself agreed with its elegance, but immediately reacted to it by underlying the necessity of empirically testing it. Without entering in the physics of the solution, one can easily see that it has several conceptual and philosophical implications that are worth being considered in the light of both his reflection and Kant's. In particular, it is worth underlying that Gödel not only dismissed the notion of an absolute cosmic time, but he also endorsed an account of relativistic time that echoes the definition of time that Kant used in the MAN.[10] This emerges in the work titled "Rotating Universes in General Relativity

[9] Gödel never thought that these curves were geodesics with necessary physical meaning. They are a mathematical possibility, see Audureau (2021).

[10] Dorato (2002a) devotes careful attention to a comparative analysis of Kant's and Gödel's views of time, but does not consider the *Phenomenology*, whereas Dorato (2002b) argues that arguments supporting the ideality of time based on Gödel's own 'rotating' solution to Einstein's field equation fail.

Theory" (1952) and in "Some observations about the relationship between theory of relativity and Kantian philosophy" (1949), in which he writes:

> [. . .] The impossibility of defining any absolute time (among or besides the various relative ones) in the empty space-time scheme of special relativity theory (upon which the foregoing considerations have been based) does not exclude that matter and the curvature of space-time produced by it, *if the structure of the world as a whole is taken into account*, may enable us to determine some objectively distinguished ordering of all events to which the properties contained in our intuitive idea of time could consistently be attributed, and compared to which the various observed times would appear as something like systematic errors due to motion of the observers. This view is supported by the fact that in all known cosmological solutions (i.e., relativistically possible structures in the large of non-empty worlds) such an "absolute world time" really can be defined. But nevertheless the conclusions drawn above can be maintained because there exist other cosmological solutions for which a definition in terms of physical magnitudes of an absolute world time is demonstrably impossible. If, however, such a world time were to be introduced in these worlds as a new entity, independent of all observable magnitudes, it would violate the principle of sufficient reason, insofar as one would have to make an arbitrary choice between infinitely many physically completely indistinguishable possibilities, and introduce a perfectly unfounded asymmetry. Therefrom it follows that the possibility of a determination of an absolute world time, where it exists at all, is certainly not due to the laws of nature (which are satisfied in all cosmological solutions), but only to the special distribution and motion which matter has in those instances. A lapse of time, however, would have to be founded, one should think, in the laws of nature, i.e., it could hardly be maintained that whether or not an objective lapse of time exists depends on the special manner in which matter and its impulse are distributed in the world. (Gödel 1946/9 B-2, 237-238, emphasis is mine)

Gödel is here referring to his work on rotating universes in which he defines the conditions for his solutions to allow for the representation of cosmic time or "world time". The latter is by no means to be considered as the result of an absolute slicing of "nows", but it is a cosmic relativistic time that is admitted by Gödel's solutions for an isotropic but *not homogenous* rotating universe. More importantly, Gödel's argument relies on considerations involving symmetry of the laws of nature and, in this respect, he is using symmetry in a transcendental sense: the a priori condition for the possibility of cosmic time must derive from the symmetric laws of nature and must respect logical a priori principles, e.g. the principle of sufficient reason. This is what I dub "Gödel's Kantian take" on cosmology. Indeed, his cosmological models 'exploit' the symmetries of the laws of nature as a priori principles determining the condition of possibilities and the modality (in a Kantian sense) that we can express in our judgments regarding entities, such as cosmic time. These symmetries of physical laws by themselves do not imply any cosmic absolute time nor an absolute independent time interval between events. Before analyzing in more detail Gödel's debt to Kant in 3.2, let me first introduce his view on Kant's philosophy of time.

3.3.1 Gödel in Between Kant and Einstein

Gödel's interest in comparing Einstein's relativity and Kant's philosophy of space and time is mentioned in the letter he wrote to his mother on November 7th, 1947:

> I was asked to write a paper for a volume on the philosophical meaning of Einstein and his theory; of course, I could not very well refuse. I am also not sorry that I have accepted and chosen this theme [on the relationship of Kant to relativity theory], because the problem has always interested me and its fundamental investigation has in addition led to purely mathematical results (Wang, 1987, 38).

In order to spell out the terms of "the problem", let us consider the pages containing reflections upon Einstein's theory and Kant. In "Some observations about the relationship between theory of relativity and Kantian philosophy" (1946/9-B2) Gödel immediately clarifies that he is not an adherent of Kantian philosophy in general,[11] but that he finds remarkable that there are several aspects of Kant's view of space and time (especially time) that are compatible with relativity and that it would be a mistake to conflate, as it happened in the past, Kant's notion of time with Newton's. I want now to highlight a second point that Gödel considers relevant, that is the Kantian conception of space and time in the treatment of rigid bodies. In *Some observations* he writes:

> Kant however denied the objective existence not only of space and time, but also of spatial and temporal relations (B42a), and therein exactly consists the novelty of Kant's view. That time and space have no existence independent of and beside the things was asserted already by Leibniz (Gödel 1946/9-B2, 238)

This point is extremely important for Gödel, since it vindicates in the most appropriate terms the subjective nature of time, time intervals and time relations in general. We have already seen in Section 2 that for Kant there is no absolute time in the universe, time can only be objectively represented when associated to relative motions, but a regulative idea of absolute space must be presupposed to generate a model of nature in agreement with Euler's equations of motion. The upshot of all this is an asymmetric treatment of space and time in cosmology. In other words, cosmic timelessness must be admitted, since the relative time of reference frames is never independent from the observer,[12] whereas the ideality of space means that absolute space cannot be ruled out at all from the picture. It is an ideal of absolute space, a regulative idea of reason that must be considered *as if* it were objectively existing independently from the subject and as a container of absolute spatial relations for any pair of material objects. Gödel notices this asymmetry in treating space and time even in relativistic physics. Indeed, he states:

> But for space the situation is different. The spatial relations directly observed and measured, it is true, also have no absolute meaning, since they are different for different observers. But, in contradistinction to time, there also exist for any two material objects (i.e., things persisting in time, not events) absolute spatial relations and they lead to a structure (see below) not so very different from that of intuitive spatial relations. (Gödel 1946/9-B2, 238-239)

[11] Gödel's text states: "In order to prevent any misunderstanding, I wish to say right in the beginning that I am not an adherent of Kantian philosophy in general".

[12] Gödel writes: "Now as to time this view of Kant's has been verified to a large extent by relativity theory insofar as, in general, for some of the pairs of events no absolute temporal relation whatsoever exists in relativity theory, and for the remaining ones [there exist relations of an entirely different nature and structure]" (Gödel 1946/9-B2, 238).

Contrary to temporal relations, spatial relations among two things or material objects can exist in terms of "true spatial distance" of two points of these two material objects. The rules of these absolute spatial relations are the schema of all possible motions of rigid bodies and this schema is independent of both the observer and the reference frame:

> These movements can be described by the world lines of all points of these bodies, and these world lines in their turn are described by their coincidences with or relative positions to the material points of the actually existing bodies. This schema is what may be called the objective physical geometry of the world, of which the various geometries existing for various observers are only different aspects. (Gödel 1946/9-B2, 239)

This passage is extremely interesting, since in it we not only find Gödel's interpretation of Kant's *Phenomenology*, but also its application and translation in the framework of relativistic physics. There is however one point to clarify and that concerns Kant's doctrine of space and time as spelled out in the *Prolegomena* and the *Critique of pure Reason*. Indeed, Gödel notices, "these absolute spatial relations, it is true, are not strictly isomorphic with the corresponding ones of intuitive space (insofar as, e.g., they are non-Euclidean)" (Gödel 1946/9-B2, 240), given that the concept ("finitness") of straight line in Riemannian geometry is completely in disagreement with our spatial intuition. Gödel assumed that for Kant spatial relations of pure intuition correspond to Euclidean space. This interpretation naturally led him to stress the difference between 'Kant and us', namely for Gödel relativity marks a step beyond the ontology and the epistemology of transcendental philosophy because we are closer than Kant in grasping the real structure of the world. In other words, we make things in themselves more intelligible than previously thought and advance in the knowledge of the real structure of the world. This move necessarily includes a "kinematic" of scientific progress and is compatible with the idea of a multi-layered epistemology:

> In the present imperfect state of physics, however, it cannot be maintained with any reasonable degree of certainty that the space-time scheme of relativity theory really describes the objective structure of the material world. Perhaps it is to be considered as only one step beyond the appearances and towards the things (i.e., as one "level of objectivation", to be followed by others). Quantum physics in particular seems to indicate that physical reality is something still more different from the appearances than even the four-dimensional Einstein-Minkowski world. T. Kaluza's fifth dimension points in the same direction. (Gödel 1946/9-B2, 240).

However, Gödel's interpretation is not entirely correct. Kant never identified the pure form of outer intuition with Euclidean space. He only stated that it has three-dimensions, whereas the pure form of inner intuition, i.e. time, has one dimension. The reason why we cannot identify space as form of intuition with Euclidean space is trivial: for Kant, Euclidean geometry, like any conceivable geometry, represents space as object and is the result of what Kant called a synthetic "construction" of concepts. In other words, the form of outer intuition alone can never be identified with Euclidean space because the latter is the result of a complex construction of concepts in intuition that imply that the principles of the pure understanding are at work and generate through imagination suitable schemas to apprehend the manifold of inner sense according to rules, e.g. through algebra or geometry, depending on the synthesis involved (see Rechter 2013). Euclidean geometry and its objects or axioms cannot be directly intuited, they can only be constructed, according to *a priori* rules of synthesis. This in turn shows that Kant's view of geometry is far more sophisticated and to think that all that can be said with regard to space and time is contained in the *Prolegomena* and the *Transcendental Aesthetic* of the first Critique is a mistake that we can forgive to Gödel, but that we should avoid after 200 years of Kant scholarship.

3.3.2 On the Impact of Kant's Notion of Time on Gödel's Solution

We are now in a position to evaluate the impact of Kant on Gödel's work on the solution to Einstein field equations as presented in Gödel (1949a). Recall that for Kant geometric construction presents concepts *in* intuition and the latter has two pure forms: space and time. The forms of intuition can be represented objectively in geometry, in particular it is the case of space, and it can be done via construction of concepts according to schemas and in agreement with the principles of the pure understanding. By rephrasing Leibniz's and Wolff's rules of ontological ordering, Kant embeds spatial and temporal relations within a new framework. In particular, in the *Critique of pure Reason* and late works, Kant describes them as the rule of positioning "beside and after one another" (*neben und nach einander - iuxta ac post se invicem ponendo*) the manifold in apprehension.[13] To order beside one another parts of space, e.g. adjacency, juxtaposition and so forth, is what characterizes a priori relations of the parts of the pure form of outer intuition: principles of action or principles of motion must always assume and express this relationship of adjacency between two points and then describe how it is conserved or under which condition it is not the case. This is translated in Gödel's solution when he states that:

> After a direction of time has been introduced in this way, a temporal orientation is defined for the world line of every (real or possible) particle of matter or light, i.e., it is determined for any two neighboring points on it which one is earlier. On the other hand, however, no uniform temporal ordering of all point events, agreeing in direction with all these individual orderings, exists (Gödel 1949a).

Thus, Gödel is here translating Kant's philosophy in the construction of his solution: temporal orientation for the world line of each particle can be introduced, but it

[13] Note that for Kant "before" is something thinkable but not givable in terms of temporal relations in intuition. Furthermore, this means that the representation of space grounds the temporal apprehension and that simultaneity is always relative to the subject's apprehension and includes backwards and forwards relationships: "But it is not possible to cognize simultaneity [*Zugleichseyn*] except insofar as, beyond my action of grasping it together, I can apprehend (not merely think) the multiplicity as given both forwards and backwards. There must thus be given in perception an intuition in which the manifold is represented outside and beside each other [*ausser einander und neben einander*], i.e., the intuition which makes possible the representation of space [*Raumesvorstellung*]" (Kant, *Handschriftlicher Nachlaß - Metaphysik Zweiter Teil*, AA 18:616).

cannot imply a uniform temporal ordering of all point events. What is left to us is just the determination of the ordering between two points and this ordering is spatial in the first place. Then, one can interpret this spatial relation in terms of temporality and thus construct a temporal direction in apprehension, but this is possible only because a representation of space is assumed. Gödel (1952) offers an even subtler analysis of Kant's philosophy and silently vindicates it when it comes to the denial of closed time-like lines. In it, Gödel writes:

> For these solutions, also, the nonexistence of closed time-like lines is equivalent with the existence of a "world-time", where by a world-time we mean an assignment of a real number t to every space-time point, which has the property that t always increases if one moves along a time-like line in its positive direction (Gödel 1952).

The world-time is a number assigned to each (but not all) space-time point and has a direction. Recall that for Kant time is the form of ordering or the mode in which we relate the content of inner intuition to apperception. The rule of ordering temporal relations is to be *after* one another (*nach einander*). This point is crucial. The ordering does not determine which element comes before and after the other, but only after. In other words, the form of pure intuition of inner sense has a direction, it points towards the positive direction that we call "future". The upshot of all this is that there exist solutions of Einstein field equations that not only can spell out the rules of spatial but also of temporal ordering that pertain to intuition. In Kantian terms this means that they are givable rather than thinkable. Therefore, not only cosmic time, but also time relations are not even ideal, where by "ideal" I mean something that can become a concept of reason, an idea, in Kantian terms. Finally, another important aspect that characterizes Kant's legacy for Gödel's work concerns the definition of time intervals relative to astrophysical moving bodies. This is something that permeates not only Kant's transcendental philosophy but his entire cosmology as set forth from 1755 onward. Indeed, Kant generalized the application of Newton's law of gravitation to structures far beyond the solar system, including galaxies and nebular clusters. In Kant's view, the universe must be modelled as a system of systems, more precisely as a system of comoving systems. This is also in line with his treatment of relative motion in the MAN and with the necessity of presupposing a regulative idea of absolute space when treating the universe as a whole as constituted by different densities of matter. This in turn led him, just like Gödel, to imagine the geometry of the universe: Kant used a spherical geometry for the expanding universe because he only admitted three-dimensional space. Gödel, on the contrary, considering the different geometry and metric of spacetime only admitted an elliptic shape as the one compatible with the homogeneous non-isotropic rotating and expanding universe. Nevertheless, not only both denied the existence of absolute time, but they also converged in defining relative time with respect to moving bodies such as galaxies that were not homogeneously distributed.[14]

[14] For Kant the hydrodynamic of matter in the universe depended on a quasi-static law (see De Bianchi 2013).

3.4 Time and Timelessness in Kant's Late Natural Science and Cosmology

In Section 3.2, I reminded that Kant draws a distinction between formal intuition and form of intuition. The distinction between a subjective and an objective representation of space and time is fundamental to understand how Kant meant to apply his transcendental philosophy to the scientific enterprise. The fact that one can objectively represent space, as it happens in geometry, naturally poses the question regarding the objective representation of time. More in general, the question arises when one wants to find the *pendant* (*Gegenstück*) of such objective representation in processes or entities other than motion. It must be noticed that Kant never attained a final answer to this question and in a sense the notion of time as formal intuition constituted a "problem" in his system more than a solution. We witness Kant's attempts at reconstructing a refined view until the last days of his life. In the late 1790s he effectively merged space and time and talked about them as *absolute unities*:

> All phenomena of matter and their moving forces are connected to the entire universe because space and time are absolute units. One can therefore assume a general principle of their reciprocal interaction, which exists in real relationships to one another, and experience is not possible other than insofar as every object is thought of with every other in this interaction and is assumed to be given a priori in appearance.[15] (OP 22:340, translation is mine)

This passage echoes a sort of transcendental version of Mach principle. However, the justification for making inertia dependent on such a universal principle relies on Kant's theory of space and time. The latter are seen as a unique whole and are defined as absolute units. A caveat is in order here: to talk about absolute *units* is not the same thing as talking about absolute space and absolute time. Again, Kant here is referring to his doctrine set forth in the CpR according to which spaces or times can always be intuited as embedded in *one* greater space and time.[16] Now, the interesting point is that in the *Opus postumum*, not only spaces and times are thought together, but also the greater units space and time are unified and together characterize the aether as the realization of a unit of all possible interactions in the universe that can be given a priori in intuition. Kant defined the aether as hypostatized space (OP 22:221) and he did not represent it as an absolute ideal space as he did in the MAN, but he wanted to talk about it in terms of a postulate that makes the reciprocal effective interaction among phenomena possible. Kant attempted at giving several proofs and definition of the aether as the all-pervading and self-oscillating original cosmic

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[15] The original text reads: "Alle Erscheinungen der Materie u.[nd] ihre Beweg.[enden] Kräfte sind mit dem ganzen Vnivers.[um] verbunden weil Raum u.[nd] Zeit absolute Einheiten sind. Man kann daher ein allgemeines Princip ihrer Wechselwirkung aufeinander annehmen was in realen Verhaltnissen zu einander besteht u.[nd] Erfahrung ist nicht anders möglich als in so fern jeder Gegenstand mit jedem andern in dieser Wechselwirkung gedacht und a priori in der Erscheinung gegeben angenommen wird."

[16] This is justified by Kant's doctrine of representation (*Vorstellung*) and in particular by his definition of intuition as singular representation. This trait of uniqueness is proper of spatiotemporal intuition.

matter filling the universe and making the latter empirically perceivable (OP 21:219-220). It is also defined as the original material (*Urstoff*) endowed with attraction and repulsion, namely he understood it as the receptacle of the dynamical forces of matter in contrast with the mechanical ones (centrifugal and centripetal forces). In the late period (1796-1804), we witness once again Kant's denial of cosmic absolute time, but this does not prevent him to admit a systematic treatment of the universe and its content: a world time can be admitted in Kant's system and it is when an account of the origin of the universe is needed. In order to provide a cosmogonic description, one must consider the beginning of the (relative) motion of matter in time, which in turn signifies the condition of possibility to locally define a chronological order (OP 22: 195-198).[17] However, the cause of the first motion of the original matter is to be found in a virtual agitation (a sort of quantum fluctuations we would say today) that expresses a priority that cannot be understood in terms of temporal succession, but as a transition that is thought in analogy with the concept of an end (*Zweck*): an internal ground for the generation and growth of organisms has driven the universe to evolve and the aether to expand, and since internal structures were created and moved one with respect to the other the idea of cosmic time toward the future or local world line can be admitted in cosmology (OP 22:272). The transition to the physical evolving universe, in turn, is not just virtual but also temporal. In the manuscript of the *Opus postumum*, indeed, there is a sense of temporality that Kant describes and that is worth mentioning: the instant. Contrary to Plato, who in the *Parmenides* tried to underscore how atemporality admits a form of instant that is out of the temporal series (see De Bianchi 2022a), Kant "temporalizes" the notion of instant (*Augenblick*) and thinks of it as limit (*Grenze*). In particular, Kant thinks that this notion of temporality describes phenomena like phase transitions in crystallization and the generation of matter. However, this amounts to the denial that time intervals are something objective of/in natural processes and phase transitions (OP 21:270; 22:275), because Kant claims that the transitions or the switches do not happen in a time lapse of which we can measure a duration by comparing relative motions, rather we must represent the transition as a limit that we calculate through the method of fluxions. Thus, precisely because time is something subjective, instants can still acquire a meaning in so far as they can signify infinitesimal intervals to be measured and calculated by means of fluxions.

[17] From this and other passages in the OP it emerges that Kant's view of a description of the beginning of the world confronted the Platonic account such as that presented in the *Timaeus* where it is stated that time emerged together with the motion of the global sphere of fixed stars, see OP 21:33. However, it must be noticed that Plato's *Timaeus* offers a "second" version (or at least I interpret it as a second version) of the cosmogonic account. In the dialogue an alternative discourse regards the *chôra*, the all-encompassing receptacle that is self-moved and moves, producing oscillations that generate imperfect configurations of the geometrical shapes and mathematical entities dictating the fundamental structure of matter.

3.5 Kant's Hypostatized Space

Before concluding, I would like to draw attention on the fact that in the 1790s Kant added several notes and reflections surrounding the scholastic dictum "forma dat esse rei" (the form makes the thing real). The dictum represents pretty well the metaphysical take that Kant endorsed to justify his late cosmology. Indeed, not only he believed that one can *a priori* determine and classify the moving forces of matter according to the formal logical rules of the understanding, but one can even *a priori* postulate the material form, i.e. the aether as hypostatized space that allows one to divide the universe in spatio-temporal regions possessing an internal dynamics. This move is not only compatible with making the idea of local world time possible together with the reciprocal interaction among bodies, but it also enables one to think of the universe as an evolving totality in analogy with natural ends, e.g. organisms. What is givable (*dabile*) and thinkable (*cogitabile*) with respect to the form is completely determined a priori and this seems to be a natural outcome of Kant's universal law of reciprocal interactions and the necessity of representing systematic unity of reason in natural science. Nevertheless, there is a further point to be highlighted: since the whole of experience is made possible through the postulate of the aether, Kant found (or he believed so) the ground for defining *a priori* the thing in itself, i.e. the whole of experience from a different perspective (*Gesichpunkt*). This perspective was extremely relevant for him because it was the formal and material condition of possibility for unifying the objective and subjective representations of space and time: the aether or hypostatized space is the material ground for any possible outer and inner intuition of our sensibility, the form according to which our senses can be affected by the external world and the formal and material condition of perception. Thus, in the *Opus postumum*, we notice a further step towards the unification of Kant's forms of intuition of space and time together with the attempt at showing that time is subjective, but it can be embedded in a superior objective representation together with space: the aether. When saying that we could have had further interpretation in store for Kant's philosophy, nobody is more correct than Gödel in having understood the great intuition that Kant had in denying the objective and absolute nature of time. However, for Kant, time is not an illusion. For to say that something is subjective does not equate to say that it is not real, rather time is not existent as an absolute substance and it is not an independent physical object. Furthermore, the mathematical construction of times allows one to define time operationally as formal intuition in physics, and it does so in many different ways, as it is evident even in physics today.

3.6 Closing Remarks

This Chapter provided a link between the historical and the systematic part of this volume by showing the impact of Kant on Gödel's rotating universe model. In particular, it discussed the denial of absolute time in both Kant and Gödel, by including Kant's *Phenomenology* within the picture. This in turn led us to show that a notion of local world time is admitted by both of them, in a classical and in a relativistic framework, respectively. Yet the common denial of the idea of absolute time does not prevent us from showing that Kant admitted a further standpoint according to which time can also be considered real (*wirklich*) when associated to space, and objective as formal intuition. In the 1790s, indeed, Kant embraced a view according to which the notion of hypostatized space, the aether, unifies space and time into a whole. The metaphysical stance advanced by Kant is sum up by the scholastic dictum 'forma dat esse rei', according to which the total complex of spatio-temporal phenomena must be ontologically grounded on the complete system of moving forces of matter to make the idea of the evolution of the universe possible. This in turn offers a different and richer reading of Kant's philosophy of time.

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References

Audureau, É. (2021). Gödel: From the Pure Theory of Gravitation to Newton's Absolute. In G. Cocco, and E.-M. Engelen (Eds.), *Kurt Gödel: Philosopher-Scientist* (pp. 57–79). Presses Universitaires de Provence.

Crocco, G., Engelen E.-M. (2016). Kurt Gödel's *Philosophical Remarks (Max Phil)*. In G. Cocco, and E.-M. Engelen (Eds.), *Kurt Gödel: Philosopher-Scientist* (pp. 33–54). Presses Universitaires de Provence.

De Bianchi, S. (2013). The Evolution of the Sphere. Kant's Theory of Matter and the Expanding Universe. In S. De Bianchi (Ed.), *The Harmony of the Sphere: Kant and Herschel on the Universe and the Astronomical Phenomena* (pp. 17-45). Cambridge Scholars Publishing.

De Bianchi, S. (2022a). Eternity, Instantaneity, and Temporality: Tackling the Problem of Time in Plato's Cosmology. In A. Ross, and D. Vázquez (Eds.), *Time and Cosmology in Plato and the Platonic Tradition* (pp. 156–178). Brill.

De Bianchi, S. (2022b). How do we transform appearance into experience? Kant's Metaphysical Foundations of Phenomenology. In B. McNulty (Ed.), *Cambridge Critical Guides* (197–214), Cambridge University Press.

Dorato, M. (2002a). Kant, Gödel and Relativity. In P. Gärdenfors, K. Kijania-Placek, and J. Wolenski (Eds.), *The Scope of Logic, Methodology and Philosophy of Science: Proceedings of the Invited Lectures for the 11th International Congress of Logic Methodology and Philosophy of Science* (pp. 331–348). Kluwer.

Dorato M. (2002b). On Becoming, Cosmic Time and Rotating Universes. *Royal Institute of Philosophy Supplement*, *50*, 253–276.

Euler, L. (1752) Découverte d'un nouveau principe de mécanique. *Memoires de l'Academie Royal des Sciences*, *6*, 185–217.

Friedman, M. (2000). Geometry, construction, and intuition in Kant and his successors. In G. Sher, and R. Tieszen (Eds.), *Between logic and intuition: Essays in honor of Charles Parsons* (pp. 186-218). Cambridge University Press.
Friedman, M. (2013). *Kant's construction of nature: a reading of the metaphysical foundations of natural science*. Cambridge University Press.
Gödel, K. (1949a). An example of a new type of cosmological solutions of Einstein's field equations of gravitation. *Reviews of Modern Physics*, *21*(3), 447.
Gödel, K. (1946/9 B2). Some observations about the relationship between theory of relativity and Kantian philosophy. In S. Feferman, J. W. Dawson, Jr., W. Goldfarb, C. Parsons, and W. Siegin (Eds.), *Kurt Gödel, Collected Works* (Vol. 3, pp. 230–246). Clarendon Press.
Gödel, K. (1952). Rotating Universes in General Relativity Theory, In L. M. Graves et al. (Eds.), *Proceedings of the International Congress of Mathematicians* (Vol. I, pp. 175–181), American Mathematical Society.
Gödel, K. (1981 [1961]). The modern development of the foundations of mathematics in the light of philosophy. In S. Feferman, J. W. Dawson, Jr., W. Goldfarb, C. Parsons, and W. Siegin (Eds.), *Kurt Gödel, Collected Works* (Vol. 3, pp. 375–387). Clarendon Press.
Hawking, S. W., and Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time*. Cambridge University Press.
Kiefer, C. (2023). What Does it Mean for Our World-View If We Assume with Gödel the Nonexistence of Time?. In O. Passon, C. Benzmüller, and B. Falkenburg (Eds.), *On Gödel and the Nonexistence of Time – Gödel und die Nichtexistenz der Zeit: Kurt Gödel essay competition 2021 – Kurt-Gödel-Preis 2021* (pp. 69–78). Springer.
Parsons, C. (2010). Gödel and Philosophical Idealism. *Philosophia Mathematica*, *18*(2), 166–192.
Patton, L. (2011). The Paradox of Infinite Given Magnitude: Why Kantian Epistemology Needs Metaphysical Space. *Kant-Studien*, *102*(3), 273–289.
Rechter, O. (2013). On Kant on Arithmetic, Time, and Irrationals. In S. Bacin, A. Ferrarin, C. La Rocca and M. Ruffing (Eds.), *Kant und die Philosophie in weltbürgerlicher Absicht: Akten des XI. Kant-Kongresses 2010* (pp. 209–222). De Gruyter.
Robertson, H. P. (1933). Relativistic cosmology. *Reviews of Modern Physics*, *5*(1), 62.
Wang, H. (1987). *Reflections on Kurt Godel*. The MIT Press.
Wang, H. (1995). Time in Philosophy and in Physics from Kant and Einstein to Gödel. *Synthese*, *102*, 215–234.