**Explicating quantum indeterminacy**

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**1. Quantum indeterminacy: the challenges**

The dispute about whether there is indeterminacy in the world is long and inconclusive. At first glance, it seems like quantum mechanics ought to provide a quick, empirical resolution to the debate: prima facie, a photon in a superposition of right-polarized and left-polarized states has an indeterminate polarization. But quantum mechanics has not provided any such resolution; the controversy drags on. In this paper, I suggest some reasons for this impasse, and lay out a path forward.

 The typical argument that quantum mechanics exhibits genuine indeterminacy goes something like this: First, a general account of indeterminacy is constructed, and second, it is argued that quantum mechanics is an *instance* of that general account (e.g. Calosi and Wilson 2019). But this strategy runs into problems at each step. At the first step, the general accounts of indeterminacy are controversial, both individually (Darby 2010; Glick 2017) and because they compete with each other (Wilson 2013). And at the second, even though quantum mechanics is arguably our best case for an instance of indeterminacy in the world, it remains a deeply problematic theory. There is no consensus over what, if anything, quantum mechanics tells us about the nature of the physical world (Lewis 2016), and so any proposed argument from quantum mechanics to indeterminacy faces the objection that it rests on a mistaken interpretation of quantum mechanics (Glick 2017).

 Of course, this might just be the nature of philosophical debate. But I want to suggest a strategy that might avoid some of these controversies. The strategy is grounded in a broadly Carnapian methodology. As long as we see the issue of indeterminacy as one of *discovery*, analogous to scientific discovery of empirical facts, the impasse over its existence seems inevitable. Both the description of what it is we are aiming to discover in the world, and the claim that in quantum mechanics we have discovered it, remain controversial. But once we realize that the issue turns in large part on *constructing* an appropriate conceptual structure to *use*, we can make progress.

 In his *Logical Foundations of Probability*, Carnap suggests that constructing an appropriate concept is a matter of *explication*. Explication consists of taking an imprecise “prescientific” concept and transforming it into a precisely-defined “scientific” concept (Carnap 1950, 3). Of course, a concept can be precisified in a number of different ways; Carnap suggests that we should judge potential explications by how *fruitful* they are likely to be for the scientific enterprise, weighing this against the potentially competing considerations of simplicity and similarity to the prescientific conception (1950, 5). The targets of explication for Carnap in this work are the concepts of *confirmation* and *probability*: they are used extensively but without precision in the sciences, and by paying attention to the *way* they are used, we can see how to make them precise is a scientifically fruitful way. My contention here is that the same is true of *indeterminacy*. That is, by looking at quantum mechanics, we find widespread appeal to an imprecisely defined concept of indeterminacy, and we can also see how that concept can be made more precise in a way that is likely to be fruitful within physics.

 In the next section, I argue that indeterminacy plays an important role in quantum mechanics. In section 3, I argue that constructing a precise version of this concept is straightforward: the structure needed to represent indeterminacy precisely is already present in the Hilbert-space formalism, and indeed a canonical interpretation of the formalism by von Neumann already provides the required precisification. In section 4, I defend the resulting account of indeterminacy: the road to this account passes a number of conceptual choice-points leading to different interpretations of quantum states, but I argue that the canonical one is superior as an explication. In section 5, I consider some more general questions about the nature of the concept of indeterminacy explicated here: is it of the same kind as everyday instances of indeterminacy, e.g. concerning the extent of the Australian Outback, and is it genuinely *metaphysical* indeterminacy?

**2. The role of indeterminacy in quantum mechanics**

Open any textbook on quantum mechanics, and you will find a passage something like this:

Let us summarize our description of the *z* component of the angular momentum of a photon. Corresponding to this physical quantity we have an *operator*, i.e. a matrix, $ℏS$. Photons that are in an *eigenstate*, $\left.|R\right⟩$ or $\left.|L\right⟩$, of this operator can be assigned a definite value of the *z* component of angular momentum… Any other photon state $\left.|ψ\right⟩$ cannot be assigned a definite value of angular momentum. (Baym 1969, 17)

If we read “definite” as “determinate”, this says that in many states (in fact, in all states except for a few special ones), the photon’s angular momentum is indeterminate.

 Similar language can be found in the experimental literature. For example, Juffmann et al. (2009) produce multi-slit self-interference between C60 molecules, and then image the individual molecules making up the resulting interference pattern. They describe the imaged individual molecules as “localized”, but the molecules passing through the slits as “delocalized” (Juffmann et al. 2009, 3); again, we can read the positions of the former as determinate and the positions of the latter as indeterminate.

 Of course, the fact that physicists use a concept of indeterminacy doesn’t tell us what *kind* of concept they are using. The textbook passage is closely followed by a description of the Born rule, via which a state fixes the *probabilities* that the various possible values of angular momentum would be revealed by an appropriate measurement. Since probabilities are often interpreted epistemically, this is naturally read as suggesting that a photon whose state is not an eigenstate nevertheless *has* a perfectly determinate angular momentum property, but one that we do not *know* prior to measurement. In that case, the concept of indeterminacy would apply to our *knowledge* of the world, not to the world itself.
 However, an epistemic understanding of the quantum state faces well known difficulties, in the form of various no-go theorems (Bell 1964; Kochen & Specker 1967; Pusey, Barrett & Rudolph 2012). These theorems each show that, subject to plausible assumptions, an epistemic understanding of the quantum state is incompatible with the predictions of the theory. This does not mean that an epistemic understanding of the quantum state is absolutely precluded; the assumptions underlying each theorem can be denied (Leifer 2014). But for present purposes I will set epistemic interpretations of the quantum state aside. I also assume that we take a form of *realism* for granted; that is, we don’t take the quantum state to be merely a guide to what we should *believe* (Healey 2012). In other words, for present purposes I assume that the quantum state is a representation of the *world* rather than of our knowledge.

 Given that assumption, the conclusion that a concept of *worldly* indeterminacy plays an important role within quantum mechanics apparently follows straightforwardly. The above-mentioned photon, when its state is not $\left.|R\right⟩$ or $\left.|L\right⟩$, has a non-zero probability, on measurement, of being found to be right-polarized and a non-zero probability of being found to be left-polarized. If these probabilities are not to be understood epistemically, it looks like we must say that the photon has *neither* polarization prior to measurement. If the state is not to be understood instrumentally, it looks like a non-eigenstate *describes a state* in which the photon has neither polarization. But since, due to quantization, right-hand and left-hand polarization are the only polarization properties the photon can (determinately) have, the natural thing to say is that the photon’s polarization is *indeterminate* prior to measurement.

 Furthermore, this kind of indeterminacy arguably plays a role in the kind of explanations quantum mechanics gives. Consider self-interference of C60 molecules again. The “delocalization” of each molecule as it passes through the slits is apparently required to explain interference; a molecule that passes determinately through one slit cannot exhibit self-interference. If the superposition is understood in terms of molecules with indeterminate locations, an explanation of interference is at least possible, insofar as the set of trajectories between which each molecule is indeterminate includes trajectories passing through both slits and converging at the screen (Calosi and Wilson 2021). To the extent that physicists think of interference phenomena in terms of overlap of delocalized trajectories of a single molecule, it looks like they are helping themselves to the notion that molecule trajectories can be indeterminate. Indeed, part of the *motivation* of researchers like Juffmann et al. (2009) in exploring interference effects in larger and larger molecules is to find the *extent* of this indeterminacy: where does the indeterminacy characteristic of the quantum micro-world give way to the determinacy of the observable macro-world?

 It would be going too far to assert that quantum mechanics *requires* a concept of indeterminacy: as we shall see in section 4, it is possible to *eliminate* the indeterminacy from quantum mechanics instead of explicating it. But for the moment, let us proceed under the assumption that indeterminacy is a concept worth retaining, and consider how it should be made precise.

**3. The structure of quantum indeterminacy**

In fact, such a precisification has largely been done for us, and indeed is almost as old as quantum mechanics itself. In the first systematic account of the structure of quantum mechanics, von Neumann (1932) constructs an interpretation that incorporates indeterminacy. We can think of his interpretation as an explication of quantum indeterminacy in Carnap’s (1950) sense—a way of making the concept *precise* in such a way as to be *fruitful* for the fledgling discipline of quantum mechanics.

The state of a system is represented in quantum mechanics by a vector in a Hilbert space. An observable—a set of experimentally accessible, mutually exclusive properties of the system—is represented by a Hermitian operator on the Hilbert space that takes a vector as input and produces a vector as output. And, as noted in the previous section, a system (determinately) has one of the properties that make up the observable if, and only if, its state is an eigenstate of the corresponding operator—a vector that is mapped to *itself* by the operator.[[1]](#footnote-1) The eigenstates of a Hermitian operator are mutually orthogonal vectors. A system whose state is not one of the eigenstates of the operator does not determinately have any property from among those that make up the observable, and is, to that extent, indeterminate.

But here we need to be a little more careful. In the Juffmann apparatus, a C60 molecule that is “delocalized”—that is not determinately approaching any specified slit—can still be described as determinately “encountering” the slits, since it is determinately located in the tube containing the slits and not anywhere else (Juffmann et al. 2009, 2). The molecule does not determinately have a location at sufficiently fine grain, but nevertheless if we consider a *disjunction* of those fine-grained locations, the molecule determinately has the disjunctive property. To understand indeterminacy, then, we need to understand its *logic*.

This is precisely what von Neumann (1932, 130–134) provides us with. Consider an operator with *n* mutually orthogonal eigenstates φ1 through φ*n*, corresponding to *n* mutually exclusive properties *p*1 through *pn*. Let P*i* be the proposition that the system has property *pi*. Then P*i* obtains (the system has property *pi*) iff the state is φ*i*. The disjunction P*i* ∨ P*j* obtains (the system has *pi* or *pj*) iff the state lies in the subspace spanned by φ*i* and φ*j*—if it lies somewhere in the plane formed by arbitrary weighted sums αφ*i* + βφ*j*. The negation ¬P*i* obtains (the system lacks *pi*) iff the state is orthogonal to φ*i*. More generally, for propositions Q*i* and Q*j* that may themselves be compound, Q*i*∨ Q*j* obtains iff the state lies in the subspace formed by arbitrary weighted sums of the vectors in the subspaces corresponding to Q*i* obtaining and to Q*j*obtaining. The negation ¬Q*i* obtains iff the state lies in the *orthocomplement* of Q*i*—the subspace formed by arbitrary weighted sums of vectors orthogonal to every vector in the subspace corresponding to Q*i* obtaining.

This logic has become known as *quantum logic* (Birkhoff and von Neumann 1936). Quantum logic is non-classical: bivalence fails, as does the compositionality of disjunction. In particular, since the state of a system can be such that it is neither *in* Q*i* nor *orthogonal* to Q*i*, the system can be *indeterminate* with respect to the corresponding property, in the sense that it neither has it nor lacks it. Indeed (as Torza (2021) suggests[[2]](#footnote-2)), we can *define* indeterminacy in this way:

**Definition 1:** A system exhibits indeterminacy iff there is at least one property *p* such that the system neither has *p* nor lacks *p*.

The eigenstates of a Hermitian operator form a *complete* basis: they span the vector space, so that any vector in the space can be expressed as a linear combination of eigenstates. This means that the set of mutually exclusive *properties* {*p*1, *p*2, … *pn*} picked out by an operator is complete in a corresponding sense: the disjunction P1 ∨ P2 … ∨ P*n* necessarily obtains.[[3]](#footnote-3) But it does not follow that the system has any one of the basic properties: it can be indeterminate whether the system has *pi* for each *pi*. This gives us an alternative (and equivalent[[4]](#footnote-4)) definition of indeterminacy:

**Definition 2:** A system exhibits indeterminacy iff it fails to have each of a non-empty complete set of mutually exclusive properties{*p*1, *p*2, … *pn*}, where a set of properties is complete iff the disjunction P1 ∨ P2 … ∨ P*n* necessarily obtains.

Given this precisification of the concept of indeterminacy, we can see how the claims of physicists about the properties of quantum systems make sense. Consider C60 interference again, and consider a *location* observable whose eigenstates correspond to the molecule being in a particular small region of space, say a 100 nanometer cube. A C60 molecule passing through the slits is not in an eigenstate of this location observable, and hence cannot be said to determinately pass through any particular slit. Nevertheless, the state of the molecule does lie within the subspace corresponding to the large disjunction of these small regions that makes up the tube leading to the slits; hence the molecule can correctly be described as encountering the slits. The location of the molecule is indeterminate at one scale, but not at another.

**4. The conceptual landscape**

The account of indeterminacy in quantum mechanics presented so far has historical precedence: it is the earliest, and has become canonical. But that doesn’t mean that it is the *best* account. In laying out the account, I have breezed past some conceptual choice-points. In particular, there are choices to be made concerning the association of properties with quantum states, the logic obeyed by those property ascriptions, and the definition of indeterminacy. So let us revisit these choices a little more systematically to better examine whether the original explication of indeterminacy is the one we should retain.

 First, I followed the textbooks in asserting that, for a given observable, a system determinately has a property from among those making up the observable when its state is an eigenstate of the corresponding operator, and otherwise is indeterminate regarding those properties. So, for example, the *z*-spin operator for a spin-1/2 particle has two eigenstates, spin-up and spin-down. The particle has the *z*-spin-up property when its state is the spin-up eigenstate, the *z*-spin-down property when its state is the spin-down eigenstate, and otherwise has indeterminate *z*-spin. Call this *non-classical* property ascription.

 This is not the only way to describe these states. One could instead say that, for a given observable, a system determinately has a property from among those making up the observable when its state is an eigenstate of the corresponding operator, and otherwise *determinately lacks* that property. So, for example, a spin-1/2 particle has the *z*-spin-up property when its state is the spin-up eigenstate, the *z*-spin-down property when its state is the spin-down eigenstate, and otherwise *lacks* both the *z*-spin-up property and the *z*-spin-down property. Call this *classical* property ascription.

 The choice regarding property ascription is clearly connected to the choice of logic. Non-classical property ascription allows that a system can neither have nor lack a given property, which is most naturally accommodated by a three-valued logic such as quantum logic: for a non-eigenstate of *z*-spin, it is neither true nor false that the system has *z*-spin-up. Classical property ascription allows the adoption of a fully classical logic: for every property, a system either has it or lacks it, and for a non-eigenstate of *z*-spin, it is false that the system is *z*-spin-up and false that it is *z*-spin-down.

 This pair of choices is also connected to the choice of a definition of indeterminacy. The former pair is naturally associated with Torza’s (2021) definition of indeterminacy: a system exhibits indeterminacy iff there is at least one property *p* such that the system neither has *p* nor lacks *p*. The latter pair is more naturally associated with Wilson’s (2013) definition of indeterminacy: a system exhibits indeterminacy iff it lacks every determinate property for a given *determinable* property. So, for example, a system that is not in an eigenstate of z-spin lacks the z-spin-up property and lacks the z-spin-down property, and these are the only two determinate properties for the z-spin *determinable*.[[5]](#footnote-5)

 So we have two packages of conceptual choices: Torza’s (2021) and Wilson’s (2013). Both yield the conclusion that there is indeterminacy in quantum mechanics, via different routes. But there are other combinations of choices that do not yield indeterminacy. If we combine classical property ascription and classical logic with Torza’s definition of indeterminacy, we get a conceptual structure according to which non-eigenstates determinately lack the relevant property, so that there is no property such that a system neither has it nor lacks it, and hence no indeterminacy.[[6]](#footnote-6) Similarly, if we combine non-classical property ascription and quantum logic with Wilson’s definition of indeterminacy, we get a conceptual structure according to which non-eigenstates do not determinately lack the relevant property, hence there is no state that determinately lacks every property for a determinable, and hence no indeterminacy.

 How should we judge these competing conceptual structures? Recall Carnap’s (1950, 5) four criteria for evaluation of a potential explication: the explicated concept should be *exact* and *fruitful*, and it should also be *simple* and *similar* to the imprecise concept on which it is based. These criteria may compete with each other; it is the best overall package that should be adopted.

 I submit that explications that do away with indeterminacy altogether fare poorly under these criteria. Elimination of an imprecise prescientific concept is typically an option in explication projects; one can replace the imprecise concept with *nothing*, and let other concepts do the work. Sometimes elimination is a *reasonable* option. For example, it is reasonable to hold that the only biologically respectable explication of *race* is one that does away with the concept altogether (Spencer 2018).

 But elimination is clearly a *radical* option, faring very poorly on the *similarity* criterion—the *null* concept is clearly very different from any substantive prescientific concept. It is only a reasonable option when retaining the concept in question would be *worse*. In the case of race, a precise, biologically respectable definition arguably fares poorly on the *simplicity* criterion—it would have to be gerrymandered and disjoint in order to bear any relation to our ordinary race concept—and also on the *fruitfulness* criterion—generalizations over races may fail precisely because of this disjoint structure. In the case of indeterminacy, there are no such downsides to retaining the concept. The conceptual choices that eliminate indeterminacy are no simpler than those that retain it: in both cases, you need to pair a particular rule for property ascription and its associated logic with a particular definition of indeterminacy, so simplicity is on a par. And the concept of indeterminacy, as noted in section 2, has been quite fruitful in the development of quantum mechanics, appearing in standard expositions and arguably motivating a good deal of experimental work.

 So while skeptics like Glick (2017) are surely right that we *can* do without a concept of indeterminacy in describing the quantum world, this doesn’t address the question of whether we *should*. Glick’s “sparse view” of properties holds that a system that lack each determinate property for a determinable also thereby lacks the determinable, and hence does not exhibit indeterminacy according to the letter of Wilson’s definition. This has the effect of combining classical property ascription with an account of indeterminacy according to which a system that lacks each determinate property for an observable does not constitute a case of indeterminacy. This combination of conceptual choices was considered above; there is nothing wrong with it as a conceptual structure, but since the concept of indeterminacy is a fruitful one, it is not one we should feel motivated to adopt.

 The remaining question, then, is how we should judge the two packages of views that endorse quantum indeterminacy: Torza’s and Wilson’s. This is a trickier issue, and perhaps less important—either will probably do fine. My endorsement of Torza’s scheme above largely follows from considerations of *similarity*: it sticks closer to the way physicists have understood the theory—e.g. through the work of von Neumann—and this mode of understanding has also been extremely *fruitful*. To be more specific: according to Wilson’s scheme, a particle that is not in a location eigenstate (at a certain degree of fine-grainedness) determinately lacks each location, and hence lacks their disjunction. Torza (2020, 4262) interprets this as meaning that the particle fails to have a position—that it is *nowhere in space*. Calosi and Wilson (2021) correct this interpretation: being located in space, they say, is a matter of having a *determinable* property, which cannot be reduced to a disjunction of determinates. But the disjunctive account of coarse-grained location and its associated non-classical logic are part and parcel of the way physicists think about quantum systems: they are encapsulated in von Neumann’s interpretation, which is, after all, *canonical*. One could *replace* this familiar conceptual structure with an unfamiliar structure of determinables and determinates extracted from the metaphysics literature, but it takes quite a lot of philosophical work to spell out this alternative, and such a revision seems ultimately unnecessary. It is worth noting, though, that a Carnapian approach embraces conceptual *pluralism*. I have defended a particular approach to indeterminacy based on fruitfulness *for physics* and similarity to concepts already used *by physicists*. But there may be other goals, and other constituencies, relative to which Wilson’s approach is preferable.

 Two further views of indeterminacy are worth commenting on. First, I have been concentrating on Wilson’s “gappy” account of indeterminacy, according to which a system exhibits indeterminacy iff it lacks each determinate property for some determinable. But she also constructs a “glutty” account of indeterminacy (2013, 367), and promotes it as an account of quantum indeterminacy (Calosi and Wilson 2021). According to the “glutty” account, a system exhibits indeterminacy if it has (to some degree) *more than one* determinate property for some determinable. It is natural to combine this with classical logic and an account of property possession according to which a system possesses a property (to some degree) unless its state is orthogonal to the relevant eigenstate. Again, we get a serviceable account of indeterminacy. This account has the advantage that it allows us to take at face value physicists’ claims that e.g. a particle can be “in two places at once”, whereas if we adopt Torza’s account have to interpret this as loose talk that should be eliminated from precise science. But accepting a “glutty” account adds a further complication in addition to an appeal to the determinate/determinable distinction: it needs an account of relativized or partial property possession, and this is neither simple nor familiar.

 Second, there is another prominent account of indeterminacy in literature, namely the precisificationist approach of Barnes and Williams (2011). According to this account, a system exhibits indeterminacy iff there are two or more precisificational possibilities—complete, classical property ascriptions—each of which does not misrepresent reality. I don’t think this account fits well with the kind of indeterminacy we find in the quantum world. As Darby (2010) and Skow (2010) argue, the precificational possibilities violate quantum no-go theorems: a system that satisfies a complete, classical property ascription could not reproduce quantum mechanical empirical predictions. If the precificational possibilities don’t reproduce well-confirmed quantum mechanical predictions, it looks like they *do* misrepresent reality: a quantum system couldn’t be represented by any of the precisificational possibilities and still behave like a quantum system. While a precisificationist account may apply well to cases of *vagueness*, it doesn’t apply so well to quantum indeterminacy, where the indeterminacy does not (obviously) result from any kind of vagueness. The relationship between quantum indeterminacy and indeterminacy due to vagueness is taken up in the following section.

**5. Remaining issues**

I have argued for a particular account of quantum indeterminacy over others. In particular, I have argued that there is no reason to reject the canonical precisification of quantum indeterminacy based on the work of von Neumann in favor of an account based on concepts drawn from metaphysics. But there are further important issues that I can only touch on briefly here and will have to remain for future work. The first concerns the relationship between quantum indeterminacy and common or garden indeterminacy. Is the indeterminacy one finds in quantum mechanics of the same kind as the indeterminacy one finds in the everyday world, such as the extent of the Outback or of my lawn? If not, this might lead to a kind of skepticism: although we have used the word “indeterminacy” to describe the concept we are explicating, if it is not the same as everyday indeterminacy, perhaps we are not explicating *indeterminacy* at all, but rather some *sui generis* quantum concept that we might better call *superposition*.

 There are clearly important differences between quantum indeterminacy and ordinary indeterminacy. As just noted, ordinary indeterminacy involves vagueness, whereas there is no obvious vagueness in the quantum case: under either classical or non-classical property ascription, the line between having a property and lacking it, or between having it and being indeterminate, is perfectly precise. Furthermore, ordinary indeterminacy generally disappears when one moves to a more fundamental description: while the extent of my lawn may be indeterminate, the exact proportion of grass to other plants at any location is not indeterminate, or at least, not in the same way. In the quantum case, on the other hand, it is not clear whether any more fundamental description is available. Admittedly, various underlying ontologies for the quantum world have been posited, for example by Bohmian, spontaneous collapse, and many-worlds theories of quantum mechanics (Lewis 2016, 179). Indeed, Glick (2017) argues against quantum indeterminacy in part based on the claim that these underlying ontologies are always fully determinate. But all of these posited underlying accounts face serious problems (Lewis 2016, 70), and even setting these problems aside, it is not obvious that reducibility to a determinate underlying ontology undermines claims to indeterminacy at the higher level (Calosi and Mariani 2021).

 But even though there are important differences between quantum indeterminacy and ordinary indeterminacy, there are also significant similarities, in particular regarding property ascription. Consider a region in the vague border between my lawn and the surrounding native plants. It seems natural to say that it neither determinately has the “lawn” property nor determinately lacks it, in line with the definition of indeterminacy advocated in section 3 for quantum systems. That is, despite the differences between quantum indeterminacy and ordinary indeterminacy, the same *concept* of indeterminacy seems to be applicable. There is clearly a lot more work to be done here, but it looks initially plausible, at least, that quantum indeterminacy and ordinary indeterminacy can be conceived under the same general conceptual structure.

 Given that quantum indeterminacy really deserves the name, a further question concerns how we should understand the resulting indeterminacy. A common question about indeterminacy is whether it is *metaphysical* or *merely semantic* (Barnes and Williams 2011, 104; Wilson 2013, 360; Torza 2021). In Carnapian vein, my inclination is to duck the question. Consider the question of whether there are human *races*. In part, this is an *empirical* question: it depends on facts about the morphology and phylogeny of certain creatures in the world. In part, it is a terminological question: when do we count two humans as being of the same race? Once we answer the empirical and terminological questions, we determine whether there are human races in the world. If there is a *further* question concerning whether what we are doing counts as *metaphysics*, that question needs to be spelled out.

 Similarly for the question of whether there is *indeterminacy*. In the quantum case, this depends on both empirical facts concerning the behavior of the physical world, and terminological choices concerning what kinds of property structure we count as exhibiting indeterminacy. Once we answer these questions, we determine whether there is indeterminacy in the world. Again, if there is a *further* question concerning whether what we are doing counts as *metaphysics*, that question needs to be spelled out. Clearly, though, such a stance raises deep metaontological questions, for example concerning *naturalness* (Torza 2021). I have not attempted to address such questions here.

**Acknowledgments**

I would like to thank Claudio Calosi, David Glick, Christian Mariani, Amie Thomasson, Alessandro Torza, Jessica Wilson, and two anonymous referees for extensive and very helpful comments on earlier drafts of this paper.

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1. From hereon I will generally drop the word “determinately”: to say that a system has a property (lacks a property) means that it determinately has that property (determinately lacks that property). [↑](#footnote-ref-1)
2. Torza (2021) defines indeterminacy in terms of *facts* rather than property possession. The definition here can be regarded as a special case of Torza’s for facts about the possession of a property; this is the only kind of fact considered in this paper. [↑](#footnote-ref-2)
3. The nature of the necessity here is an interesting question. It is not a *logical* necessity that a disjunction of this form is true. Rather, the source of the necessity is the property structure of the world. Does this make it a *metaphysical* necessity? This question is briefly take up in section 5. [↑](#footnote-ref-3)
4. Suppose a system satisfies Definition 1, and let *q* be the property of lacking *p*. Then the set {*p*, *q*} is a non-empty complete set of mutually exclusive properties, and the system fails to have each of them, hence satisfying Definition 2. Conversely, suppose a system violates Definition 1: for each property *pi* in{*p*1, *p*2, … *pn*}, it either has *pi* or lacks *pi*. Since there is no vector that is orthogonal to every eigenstate φ*I*, the system cannot lack every property in the set. Hence it must have some property in the set, in violation of Definition 2. [↑](#footnote-ref-4)
5. See also Bokulich (2014) and Wolff (2015) for discussion of a determinable-based definition of indeterminacy. [↑](#footnote-ref-5)
6. For this reason, Torza (2020, 4261) argues that adopting a non-classical logic is essential to an account of quantum indeterminacy. But as Calosi and Wilson (2021) note, this is only so if one also adopts Torza’s preferred definition of indeterminacy; it does not follow under Wilson’s definition. [↑](#footnote-ref-6)