Creative and geometric times in physics, mathematics, logic, and philosophy

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We propose a distinction between two different concepts of time that play a role in physics: *geometric time* and *creative time*. The former is the time of deterministic physics and merely parametrizes a given evolution. The latter is instead characterized by real change, i.e. novel information that gets created when a non-necessary event becomes determined in a fundamentally indeterministic physics. This allows us to give a naturalistic characterization of the present as the moment that separates the potential future from the determined past. We discuss how these two concepts find natural applications in classical and intuitionistic mathematics, respectively, and in classical and multivalued tensed logic, as well as how they relate to the well-known A- and B-theories in the philosophy of time.

I. INTRODUCTION

There is almost nothing that we perceive so ubiquitously than the passage of time. And yet our most successful physical theories still struggle to make sense of this concept in an unequivocal way. Actually, modern physics has relegated time to play a less and less special role [1]. However, in the words of I. Prigogine, "no formulation of the laws of nature that does not take into account this constructive role of time can ever be satisfactory" [2].

To address this fundamental problem, we propose here two different concepts of time, which we call *geometric time* and *creative time*, respectively. We show that those stem from our fundamental assumption of physics as being either deterministic or indeterministic at the fundamental level, and that they both seem to contribute to our understanding of natural phenomena. In particular, we will see that geometric time is the parametric time appearing in the (deterministic) equations of motions of physics, just a coordinate that, together with the three spatial directions, labels fix events in a geometrical block-universe. On the other hand, creative time originates from the assumption that certain events which are fundamentally indeterminate become determinate. Our general approach is admittedly highly influenced by quantum mechanics, which has brought the concept of fundamental indeterminacy into the domain of physics. But we abstractly conceive the possibility that there is ontic indeterminacy in the world (also at the classical level, see [3, 4]). This idea of creative time resonates to some extent with the positions of the philosopher H. Bergson, well-known for having confronted Einstein about the nature of time, who stated: "Time is what prevents everything from being given all at once. [...] It must therefore be development. Wouldn't it then be the vehicle of creation and choice? Would time's existence not prove that things are undetermined? Would time not be this very indeterminacy?" [5]. We somewhat twist this view around by assuming that it is indeterminacy that defines a different fundamental type of time. Indeed, if there is ontic indeterminacy in nature (i.e., physical pure states do not fully determine the future), this leads to multiple potential future states, i.e., to indeterminism and an open future. It is the actualization of these potentialities, i.e., when new information previously not existing is created, that makes this creative time tick.

Starting from physics (Sect. II), we apply these two concepts of time to different disciplines, showing that this distinction gives rise to natural parallels in mathematics (classical versus intuitionistic; Sect. III), helps clarify the logic of scientific propositions (Sect. IV), and it contributes further to the metaphysics of time (so-called A-theory versus B-theory; Sect. V). More specifically, the approach based on consecutive actualizations of potentialities, that is when creative time is at work, leads to a naturalistic characterization of the difference between past, present and future.

II. TWO TIMES IN PHYSICS

In order to illustrate different concepts of time in physics, let us start from an example. Let us ask the following scientific question: what will be the weather like, say on the field hockey pitch of Geneva, exactly in one month from now? One can envision several procedures to reply to this question. On the one hand, it is possible to use the equations of motion of all the involved molecules of air, water vapour, etc., to calculate their evolution and so predict the weather in Geneva in a month time. On the other hand, one can wait exactly one month, let the process develop, and finally observe the weather in Geneva. In-between, one could spend one week to collect more data – or merely waste a week – and only then start computing the evolution of all air molecules. Let us concentrate on the two first mentioned extremal procedures. The question that we ask here is: Are these two procedures talking about the same unique concept of time, or are there multiple ones that play a role in physics? We contend that there are (at least) two different concepts of time, as exemplified by the two procedures above.

Before elaborating on that, we ought to distinguish be-

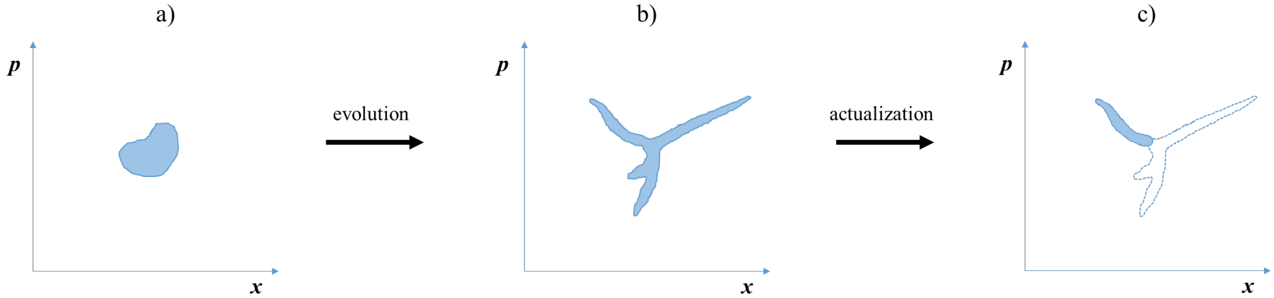

FIG. 1. Typical evolution of a classical chaotic system in phase space. The initial state (a) has some ontic indeterminacy, i.e., the position and momentum are in this case not determined further than the finite region depicted in figure. When evolved through the deterministic equations of motions (b), the indeterminacy spreads (while preserving the same volume), leading to multiple potential future evolutions, i.e. to indeterminism. However, only one of these potential evolutions will actualize (c), while ruling out the other previously potential futures (dashed); it is this process of actualization that makes creative time process.

tween different fundamental interpretations of physics, which become especially relevant in dealing with complex systems like the one determining the weather in our example. The standard narrative (i) has it that classical physics is deterministic, i.e. that each state is mapped one-to-one to all other possible states at all times (with time being the parameter that appears in the equations of motion). Although usually not stressed enough, this is based on two independent ingredients: the first is that since the equations of motion are ordinary differential equations, their solutions generally have a unique continuation in the parameter time,[1] given appropriate initial conditions; the second is that the initial conditions exist fully determinate with infinite precision (i.e., a state is represented by a dimensionless mathematical point in phase space, in turn described by an n-tuple of real numbers). Only taking these two assumptions together justifies the purported fundamental determinism of classical physics. Another possibility, (ii) is to keep the idea that the system is fundamentally deterministic, but realistically it is not possible to know the initial conditions with infinite precision. This becomes most relevant in chaotic systems (like the weather in our example) in which the future behavior of the considered system highly depends on the infinitesimal digits of the initial conditions. In this case one can only statistically predict what are the possible future states compatible with the partial knowledge of the initial conditions. Finally, (iii) one can instead uphold the more radical assumption that the initial conditions (alternatively, the state of a system) simply *do not exist* determined with infinite precision, but there is an *ontic indeterminacy* in nature (for a collection of works on this see, [3, 4, 8–11]; see also [12] and references therein for recent developments on metaphysical indeterminacy). In this case, although the equations of motion may remain unchanged, the system does not undergo a necessary evolution, but multiple future states are possible (perhaps with a determined objective tendency or propensity; see [4]), i.e., it is the ontic indeterminacy that leads to fundamental indeterminism (see Fig. 1). In what follows, when referring to indeterminacy we will have this kind of ontic property in mind and its consequent indeterminism.

Let us now go back to analyze the concepts of time that are at work in the example of the weather in Geneva in a month. The first procedure, under the ideal assumption of determinism (i), merely makes use of time as a parameter that simply indicates the coordinate of a certain evolution (e.g., sunny weather in one month from the start of the computation). Together with the three spatial coordinates (i.e., in Geneva in our example), this kind of time forms a manifold that contains all the events: a four-dimensional block universe picture where past, present and future are all on the same metaphysical footing (this is also known as *Eternalism* in the philosophy of time; see Sect. V). This can thus be called *geometric time*, the time that most physicists have come to think of as fundamental, both from classical physics and even more from relativity theory where time becomes “just” another dimension like space. In this sense, time is a relational label that allows one to position an event (with respect to another point of reference), but there is no real “becoming”.

On the other hand, let us consider the case of ontic indeterminacy in the physical state (iii). There, the actual development of a non-necessary physical process – such as the evolution of a highly chaotic system like the one that determines the weather, or of a quantum system – involves, in our opinion, a time that really “processes”. That is, a concept of time which sets an actual separation

[1] In exceptional cases, classical equations of motion of even simple systems do not have a unique (deterministic) solution, such as in the example of Norton’s dome [6]. However, in highly complex systems – such as Lagrangian fluid particles in high Reynolds-number turbulence – the fact that deterministic trajectories are not unique is rather common (see [7] and references therein).

of the future from the past (with the present at their interface). And it does so when new information that was not existing before gets created – i.e., even having access to the whole existing information of the initial conditions (even of the whole state of the universe) *before*, would not determine all the future physical properties of a system *afterwards*. Again, we stress that we are not talking here about epistemic uncertainty, but rather we explicitly take a physicalist stance on information: it quantifies something objectively encoded in the degrees of freedom of the universe. In this sense information gets *created* when non-necessary physical events come about, independently of any observer. Note that, – and this is what happens also in quantum physics – although the state of the system is fully determined and it contains the maximal information about the systems (i.e., it is pure), this does not fully determine the future evolution (see also Ref. [13]). The concept of time that stems from this can thus be called *creative time*.[2] On a similar note, G. Ellis remarked that "The most important property of time is that it unfolds. [...] The time that is the present at this instant will be the past at the next instant. [...] It is this fundamental feature of time that is not encapsulated by today's theoretical physics" [18] (see also [19]).

Let us emphasize that the concept of creative time has nothing to do with language, nor with subjective or psychological processes; it ticks due to objective processes that happen by themselves in nature. Accordingly, there are two types of events: necessary events (e.g., when two geometric worldlines cross each other) and non-necessary events when potentialities actualize. Those instances of actualization determine the objective present. In other words, this fundamental "becoming" comes from the realization of one single outcome among the possible ones (with the contextual exclusion of the other previously valid futures) due to ontic indeterminacy.

In the literature one often finds that time "unfolds" or that time "flows" to indicate that time is associated with something that really changes (so with creative time). Here we prefer to use the terms time "develops" or "processes", because also a film already shot does unfold, but nothing new is created there; whereas to flow is a somewhat misleading term for it reminds of the concept of speed which has time already in it. So, henceforth, we will write that geometric time passes while creative time processes or happens.

Coming back to the example of the weather in Geneva in a month, to carry out either of the envisioned procedures to determine the outcome takes some time. In the first procedure, i.e. when geometric time is at work, one maps the time that it takes to perform the computation $\Delta t'$ to the time appearing in the equations of motion. That is, to the interval $\Delta t = t_f - t_i$, where t_i is the parametric time associated to the initial state and t_f is the parametric time of the considered final state (i.e., the moment when we want to know the weather in Geneva). The (deterministic) computation lasts $\Delta t' < \Delta t = 1$ month, and returns as an outcome the weather in Geneva at t_f, say "sunny".[3]

One can also think of more subtle questions about the weather, such as "will there be (anytime in history) 700 consecutive days of rain in Geneva?". If the complex system that forms the weather is fully deterministic, answering this question seems only a matter of geometric time, although the answer may remain undecided for any finite value of geometric time. One can run the computation of the dynamics of the system and at some point find that the answer is "yes", if one finds the 700 consecutive days of rain, otherwise the algorithm will keep searching without ever halting (however, this question might be more complex, see Sect. III).

Two remarks are here in order. Firstly, note that $\Delta t'$ can be speed up by a more efficient computation, namely, this depends on how fast one "runs" through the parameter time as it appears in the equations of motion (this depends on the used algorithm and, physically, one can really think of the speed of the head of a Turing machine in reading the tape). Moreover, time dilation in relativity can be seen as a physical process to modify the interval of geometric time (see Subsect. VI A). So the map $\Delta t \to \Delta t'$ is a one-to-many function. Secondly, once one reaches the final state with the associated outcome "sunny", one gains full information about the initial state at t_i and its consequences, but not necessarily the full information about the final state if there is some fundamental indeterminacy, i.e., the actual weather that will be observed may vary.

On the contrary, the procedure involving creative time, namely, of waiting a month and carrying out an observation of the weather gives full information about the outcome, but provides in general little information about the initial state. This is because new information has been created along the way, through a series of indeterministic processes. So, the initial and the final states are not deterministically mapped to each other. The exactly same initial state under exactly the same physical conditions leads in general to different outcomes, so one can only infer a set of initial states that are compatible with the observed outcome. Creative time is thus associated to the possibility that physical properties are in general ontologically indeterminate (e.g., it is neither true that "the particle is within region r" nor that "the particle is not within r", where r is a region of space, see also Sect. IV). Creative time is required when an actualization event of the values of physical variables that were previously indeterminate occurs. It is the process of change from in-

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[2] The distinction between geometric and creative time was sketched in Refs. [3, 14–16] and previously also in Ref. [17] where the creative time was called "historical time".

[3] In principle the time for the "prediction" could take longer than the time interval to observe the outcome. In such a case, one would anyways ask for consistency of the calculation afterwards.

determinate to determinate – with an associated creation of novel information – that brings about this “happening” of time: creative time ticks when new information is created.

III. TWO TIMES IN MATHEMATICS

Although it is not common to discuss time in mathematics at all, the problem of the two different kinds of time introduced above has almost a perfect counterpart in mathematics. Let us now consider a standard mathematical object, e.g. a real number. Typical real numbers – in fact, almost all of them in a mathematical sense – are uncomputable, i.e. for each of these numbers there exists no algorithm – which is by definition a *finite* list of instructions – that outputs all their digits (they have infinite Kolmogorov complexity) [10]. On the contrary, all the (irrational) numbers that we usually consider as prototypical examples of reals, such as $\sqrt{2}$ or π, are all computable (the ratio between the diagonal of a square to its edge is a simple algorithm that outputs all the digits of $\sqrt{2}$ and similarly the ratio of the circumference to the diameter of a circle for π). Obviously, all the rational numbers are also computable: they can be directly written down or compressed (in the case of repeating decimals) in a finite string.

The standard way of regarding mathematics, the so-called classical-mathematics, is a form of Platonism which posits that, among the other mathematical entities, real numbers exist with their infinite series of digits, although there is in general no way to even label and thus grasp them (since there is no algorithm that generates them [20, 21]. Hence, classical Platonistic mathematics is a timeless language, which per se is not a bug, but it becomes problematic when mathematics is elevated to the language of science, i.e., it is used to the describe the physical world.

However, there are alternative approaches known as constructive mathematics, of which the most prominent is intuitionism. Therein, mathematical entities are not given at once, but rather are processes in development (digits get *created* continuously, one after the other). This provides mathematics with a concept of passage of time (and it would be only at infinite time – at the end of time, so to say – that the mathematical entities, such as real numbers, are completed into the ones defined by classical mathematics). The initiator of intuitionism, L.E.J. Brouwer, envisioned a “creative subject”, i.e., an idealized mind or mathematician, who is responsible for this progressive process of creation in time of, for instance, the digits of an uncomputable real numbers. Several authors, however, have distanced themselves from this controversial concept of a creative *subject*. In particular, one of us (N.G.) has put forward the idea that the digits of a typical real number are generated by a *true random number generator*,[4] i.e., a natural process that is able to create a piece of new information by changing a fundamentally indeterminate bit into a determined one [10].[5] This version of intuitionism could thus be labeled “objective” or “naturalistic” intuitionism. In this way, typical real numbers become a graspable concept directly linked to creative time, that is, to the change from the indeterminate to the determinate. Hence, naturalistic intuitionism, is a tensed mathematical language.

To think about computable real numbers, one just needs geometric time. In fact, it is possible to think that the full information about those numbers is contained in the (finite) algorithm that defines them, i.e., in their initial conditions. For instance, one can ask what is the 43800th digit of π and the answer is given by running an algorithm that outputs the digits of π and picking its 43800th digit. Note that it is not necessary to go through all the previous digits of π to compute 43800th one. There are so-called *digit-extraction algorithms* that allow one to directly compute the nth digit of π [23].[6] This can be seen as a further evidence that computable (irrational) numbers are already fully determined and do not require creative time. The point is that since this is found through a deterministic outcome, the answer to this kind of questions is fully contained in the algorithm, which can be run more or less fast, giving the ability to manipulate this “mathematical geometric time”. This exactly resembles the weather example in Sect. II for the deterministic evolution of the physical system that forms the weather.

In contrast, if one considers instead a typical, i.e., uncomputable, real number, things are different. There are no algorithms compressing the information of that number. The only existing “algorithm” is the number itself. Each next digit is generated by a genuinely natural random process, therefore, asking which is the 43800th digit of such a number requires to wait for all 43800 instances of creating of the “next digit” (see footnote 4), i.e., it requires “mathematical creative time”.

Finally, one can also ask a question of the kind “are there 700 consecutive sevens in the digits of a number.[7] If that number is uncomputable, then one has no choice but to wait and see. However, even for computable numbers, like e.g. π, the question is interesting. The situation is similar to the example of the weather – i.e., 700 consecutive days of rain if one assumes determinism – but the

[4] More precisely, it is not necessarily new digits that come into existence, but the new information reduces the indeterminacy of intuitionistic real numbers [21].

[5] For more works that relate constructive mathematics to physics, see [20] and [22].

[6] The most well-known of such algorithms is perhaps the Bailey-Borwein-Plouffe formula [24], that allows to directly compute the hexadecimal digits of π. Digit-extraction algorithms, also in base 10, are known for several other irrational computable real numbers, such as for e.

[7] The example of 700 consecutive sevens is borrowed from C. Posy [25].

axiomatic character of mathematics makes the solution to this question clearer. One can, in fact, think of programming two softwares: The first, S_1 is exactly the one we used in the physical example of the weather, i.e., the systematic search of a 700 consecutive sevens in π; on the other hand, the second software, S_2 outputs all the theorems that can be derived within Peano arithmetic and it halts should it find the negation of the conjecture. If the statement is true, S_1 will find this sequence of 700 sevens and therefore halt after a finite (geometric) time. If the statement is provable (within Peano's arithmetic), S_2 will halt after a finite (though presumably enormous) geometric time. Interestingly, if one assumes that neither halts, then the statement is false (if not $S1$ would halt), but not provable within Peano's arithmetic (if not S_2 would halt).[8] This would be an example of Gödel's celebrated theorem. Note that mathematicians don't use softwares like S_1 and S_2, but use their creativity to find shortcuts to analyse such statements, i.e., de facto they use creative time. In the case of the weather in Geneva, if one assumes that the system is indeterministic, like uncomputable numbers, then there is no choice but wait and see. However, if one trusts the equations describing the weather evolution and the fully determined initial conditions, then one could run analogues of S_1 and S_2, as for computable numbers. Note however, that this assumes one can compress the initial conditions into a finite algorithm (hence into a computer which is necessarily finite).

Let us acknowledge that this distinction between geometric and creative time in mathematics distances our view from the most accepted position in intuitionism, such as that of C. Posy [25] or M. van Atten [26, 27], who consider all numbers, whether computable or not, as generated by new information that comes about, so that they all require creative time in our parlance.

IV. LOGIC

We have seen that creative time emerges, both in physics and in mathematics, from indeterminism, i.e., from the possibility of having new information that comes into existence, thus reducing the indeterminacy (see Fig. 1). Another way to put it is to look at propositions about physical properties and study their logical structure. So-called classical logic upholds two main principles: the law of non-contradiction and the law of the excluded middle. Non-contradiction states that for any proposition p, it is impossible for both p and $\sim p$ to be true, whereas the excluded middle states that either p or $\sim p$ must be true. Let us start by defining how to attribute truth values to propositions about physical properties through an example: The proposition "a particle is inside region r" is true iff a (dichotomic) measurement[9] of that particle's position would for sure produce the outcome r, where r is an arbitrary chosen region of space. On the contrary, the proposition "a particle is *not* inside r" is true iff a measurement would never yield the result r.

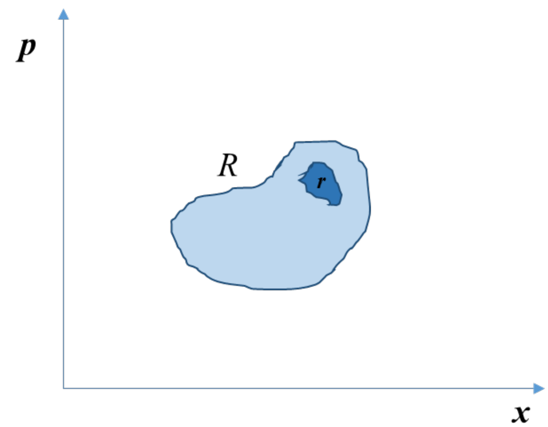

FIG. 2. Representation of ontic indeterminacy in phase space.

Now, the principle of non-contradiction imposes already that propositions about physics must be referred to a time. Indeed, if one states *simpliciter* that "the system is inside r" and "the system is not inside r", this is a violation of the principle of non-contradiction, since we do observe in nature the same system being sometimes inside r and sometimes not. One has then to refer to a time to qualify the propositions. If one adds "at time t" to both propositions, then the principle of non-contradiction holds because –at any time t – only one of these propositions would be true and the other not. On the contrary, if one labels one proposition with "at time t_1" and the other with "at time t_2", with $t_1 \neq t_2$, it is possible that they are both true. Hence, the principle of non-contradiction applies only to propositions at the same time. Note that this time is not necessarily creative, but it could just be the label appearing in the equations of motion, i.e., geometric time.

Coming instead to a scenario where there is ontic indeterminacy, we now show that the law of the excluded middle fails. Consider for instance a model of physics (classical, quantum, or even post-quantum) in which a (pure) state is represented by a finite volume, R, in phase-space (as opposed to a dimensionless mathematical point). That is, there is not merely a lack of knowledge about the system (as it is the case in, e.g., statistical physics), but this state contains the complete information existing at present about all the properties of the

[8] For the sake of completeness, there is also the fourth logical possibility that both S_1 and S_2 halt, a case which would prove Peano's arithmetic inconsistent.

[9] Actually, this should be an ideal – noise free, reproducible – measurement. By dichotomic we mean that this is not a position measurement but it can only tell whether the particle is inside or outside of r.

system (see also [3, 4]). Since, as we have already argued, this ontic indeterminacy generally leads to indeterminism in the future evolution of a system,[10] statements about future properties are in general neither true nor false because they are yet to be determined. And it is this process of determination that requires creative time. In general, in the case of ontic indeterminacy, one needs to introduce a multivalued tensed logic according to which a proposition is either true or false only at a time $\tau \geq \tau_0$ when it has become determined, but its truth value was "indeterminate" (as a third logical value) before τ_0 (for multivalued logic see [28, 29]). As an example, consider a true random number generator that outputs a (single determined) bit of information, say $b = 0$. That event defines the (creative) time instant τ_0. After such an event that brings into existence this new bit of information, the truth value of the proposition "b=0" is true and the proposition "b=1" is false, but before τ_0 both propositions had a truth value "indeterminate". This represents a breakdown of the principle of the excluded middle. Note that such a failure of the excluded middle is found in all constructive mathematics – e.g. intuitionism [25]. However, despite these many similarities, we emphasize that our approach differs substantially from "orthodox" intuitionism. First of all, as already remarked in Section III, we reject the Brouwerian notion of "creative subject" in intuitionistic logic, advocating for a naturalistic version of intuitionism in which new information is created in the universe by natural processes. Moreover, contrarily to what we have advocated, we stress that orthodox intuitionistic logic is not three-valued (nor multivalued for that matter) – as proven by Gödel [30]– and that logical propositions are typically timeless.

Finally, consider a system whose present state is represented by a finite volume in phase space R (see Fig. 2). This means that although the state is fully determined (but not as a collection of real numbers [9]), it does not fully determine the future. In this case, a statement like "the system *is* located in phase space within $r \subset R$" may lead to think that the principle of the excluded middle fails also for statements about the present. However, such a statement is not indeterminate, but actually false, for it is simply not the case that the system is located inside r in phase space. The only presently true statement about the state of the system is "the system is located at R in phase space" and nothing more. On the contrary, the statement "the system *will be* (at some future time) located in phase space at r" is currently indeterminate, because it is not impossible that the system will become more localized (in phase space). Once the evolution of the system develops, the statement will become either true or false. This process is what requires creative time. Hence, the failure of the excluded middle concerns only future statements, where the future is identified by a change of creative time.

[10] Except for integrable classical dynamical systems.

V. RELATION TO THE PHILOSOPHY OF TIME

Before discussing in detail how our proposal of creative and geometric times relates to some positions in the philosophy of time, we would like to stress that, as most physicists, we are not taking a Humean position. On the contrary, we believe that laws of nature are real causal connections (they have an ontological status) and that they guide the evolution of things of the world at different times. Note that these laws may not connects states at different times by necessity (determinism), but only state a propensity [4]. In either case, this puts an emphasis on the role of time in physical laws.

In the vast literature devoted to the philosophy of time (see, e.g., [31, 32] and references therein), one finds a main separation into two camps, following the seminal works of J. M. E. McTaggart [33]. The two camps are customarily referred to as A-theory and B-theory of time. While we acknowledge that the distinction in A- and B-theories presents a variety of modification and it may not even encapsulate the most striking features of time in physics (e.g., directionality, openness of the future, objective becoming, etc.), we cannot avoid drawing some parallel with our distinction between creative and geometric time, respectively.

In fact, starting in reversed order, B-theorists regard positioning in time in dyadic relational terms: given two events E_1 and E_2, it is either the case that E_1 is before E_2, or simultaneous to E_2, or after E_2 [34]. Clearly, these relations are somehow "static", i.e., they are not tensed and hold unchanged since ever and forever. For example, according to the B-theory, the extinction of the dinosaurs has always been, is and will always remain *before* the French revolution. Note however, that this relation seems to us to necessarily presume determinism, otherwise, in an indeterministic world, it would have not been true at a time earlier than the dinosaurs' extinction itself actualized, because that extinction was not necessary in the first place (nor was the French revolution). At the remote past time in which both events "dinosaurs' extinction" and "French revolution" were only potential events, their relation of before-after was also indeterminate. After the actualization of dinosaurs' extinction but without an actualization of the French revolution, one rules out the possibility that the French revolution is before (or simultaneous to) the dinosaurs' extinction, but it remains in general indeterminate whether the French revolution will ever actualize (although this may have a determined propensity [4]). Hence, B-theorists – who take the before-after relation as fixed – don't derive determinism but assume it from the outset.

Regarding the metaphysical status of time, the B-theory naturally fits to the spacetime of (deterministic) classical theory and the theory of relativity, where events are organised on a 4-dimensional manifold. Moreover, B-theorists uphold Eternalism – i.e., the doctrine that all events are temporally on an equal footing and their relation of being in the past, present or future of each other – merely reflects their placement on the manifold [35]. Clearly, the concept of geometric time as expressed above falls into this B-theory camp.

On the contrary, A-theorists uphold the view that characterizes the positioning in time with a monadic attribution: given an event E, it is either the case that E is present, or past, or future [34]. For A-theorists change really happens: "there is a way reality is (now, presently) which is complete [i.e., maximal given what exists (a pure state as physicist say)], but *was* different in the past and also *will be* different in the future"[35]. It is clearly this camp that to be more friendly to the concept of creative time explained above.

It seems that all A-theorists accept the theses of *temporalism*, i.e., that propositions change their truth-value over time, and of *temporal disparity*, i.e., the view that there is a metaphysical distinction between past, present and future [36]. The biggest disagreement is about which specific kind of temporal disparity one envisions. There are three main proposals: (i) *Presentism* [37, 38], which maintains that "no objects exist in time without being present" [35]; (ii) the *Moving spotlight* theory [39], an eternalist A-theory which takes events to be set on a fix manifold, but the present is singled out as if it was lit by a light moving along the manifold; and (iii) the *Growing-block* theory [40] (sometimes referred as the *Growing-universe*, or *Growing-past*), according to which events come into existence at present but, unlike in presentism, they persist in the past.

The moving spot-light, as an eternalist theory, does not seem compatible with creative time, showing that there is not a one-to-one correspondence between A-theories and the concept of creative time. However, temporalism is an essential ingredient for creative time to process and certain A-theories are the natural metaphysical framework for describing the processing of creative time. Let us focus on the the Growing-block view. More precisely, the latter states that the present is metaphysically privileged because it is then that events become determinate (or "real" in the standard parlance of philosophy of time, which we do not endorse). But as new events become present, the past ones remain equally "real", resulting in a growing block of reality [35]. In particular, we find parallels with a particular version of Growing-block theory, namely, E. Barnes and R. Cameron's *growing cloud of determinacy theory* [41], which upholds that the openness of the future has nothing to do with its alleged non-existence (a common conclusion in philosphy of time). In the words of the authors, "the future is as yet unsettled. [T]hink of this 'unsettledness' with respect to future states of the world as a type indeterminacy. For all times t_1 and t_2 such that t_2 is later than t_1, it is indeterminate at t_1 what the state of the world is at t_2" [41].

This is the theory of the passage of time which seems most aligned with our concept of creative time. The future exists as a collection of real potentialities and the actualization of these potentialities is the act of creation that defines creative time. The present is therefore, for us, the edge of the events that turn from potential (the open future) to actual (the settled past).[11] So, while past, present and future are for us all in a sense real, the past is the (growing) collection of physical properties that got already actualized, the future is the collection of the potential properties,[12] and the present is the transition from potential to actual (see also [4]).[13] Similar ideas were already put forward by H. Reichenbach who wrote: "The present, which separates the future from the past, is the moment when that which was undetermined becomes determined, and 'becoming' means the same as 'becoming determined'." [42]. Despite this, in Reichenbach, the present is still relative to someone or something, i.e. any moment can serve as the division between the determined past and the indeterminate future relative to who is simultaneous to that event (see [43] and reference therein for a discussion). Our proposal also has several similarities with S. McCall's "objective time flow" [43]. Therein, the author considers that the universe has, at any point in time, a dynamical tree structure, with a single determined past and multiple possible futures. The concept of becoming is brought about by the update of the tree structure, when only one branch becomes determined. However, McCall's proposal remains at an abstract level, whereas we present a naturalistic account of the indeterminate-actualization transition. Moreover, McCall does not conceive the possibility of objective (possibly biased) tendencies or propensities towards one possible future or another.

To conclude this section, we find it worth positioning our contribution within the metaphysical debate on time. First of all, although we advocate some form of Growing-block theory, we notice that most existing accounts thereof in the philosophy of time have been developed with an a priori metaphysical approach. The novelty of our approach is to provide a naturalistic characterization of the passage of time and of the present, as an objective transition from the past to the future. Moreover, one of the main criticisms against the standard Growing-block theory (the so-called *skeptic objection*), which rendered this approach quite unpopular among philosophers, is that since both past and present have the same ontological status (i.e., they exist, as opposed to the future), how can we know that we live in the present and not in the past? It seems that our ideas expressed above overcome this skeptical objection exactly due to its naturalistic character. In fact, although both past and present are determinate (and therefore different from the future), they are also different from each other. The

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[11] Although we have discussed the possibility that the remote past could again become indeterminate in Ref. [16].

[12] Note that some future properties have propensity one, hence are already actual [4].

[13] Note that the present is not a global, universal property, but there are as many local presents as actualization events.

present is when creative time processes, namely, when the determination (or actualization) happens and new information gets created. The present has a dynamical nature, whereas the past remains fixed.

VI. MORE INSIGHTS ON TIME(S) FROM PHYSICAL THEORIES

A. Relativity theory

As already recalled, geometric time is the conception of time derived from the regularities of some phenomena described by classical physics, like the motion of planets and satellites in the solar system which in turn led to a dominant deterministic world-view in physics. As remarked by G. E. M. Anscombe, "the high success of Newton's astronomy was in one way an intellectual disaster: it produced an illusion... for this gave the impression that we had here an ideal of scientific explanation; whereas the truth was, it was mere obligingness on the part of the solar system, by having had so peaceful a history in recorded time, to provide such a model." [44].

We agree with H. Reichenback that "the properties of time which the theory of relativity has discovered have nothing to do with its treatment as fourth dimension. This procedure was already possible in classical physics, where it was frequently used. However, according to the theory of relativity the four-dimensional manifold is of a new type; it obeys laws different from those of classical theory" [45]. And indeed it was the advent of the theory of (both special and general) relativity that popularized further the idea of geometric time. Therein, not only is time a parameter, but it seems to lose any special role with respect to space.

Geometric time intervals are not relativistically invariant. Indeed, the (geometric) time of a moving observer at speed v (with respect to a given reference frame) undergoes a dilation of a factor $\sqrt{1-\frac{v^2}{c^2}}$, according to the Lorentz transformations. Geometric time can be manipulated by using the scenario of the twin paradox. This means to prepare two identical systems (K and L), send one of them (say L) away, and finally bring L back together with K. Since L's local clock passes at a slower rate than the one of K, the geometric time elapsed for the system L will be shorter than the one elapsed for K (the higher v, the larger the dilation). Let us consider again the example of the weather in Geneva in a month time, and let us assume that the same computation of the outcome is carried out with a computer K and with a moving computer L (assuming that they are using the same algorithm and identical computers). If at the event in which they meet again, K's clock has registered a time $\Delta t'$ (which is the time that it takes for her computer to return an outcome), L's program would still be running due to time dilation. This shows again that geometric time can be manipulated. On the contrary, it is unclear whether there is any way to let creative time "run faster".

The question then naturally arises whether creative time, too, transforms relativistically. Looking at concrete experiments from particle physics, one would initially think that it necessarily does. Indeed, subatomic quantum particles like muons, undergo spontaneous decay which supposedly involves a genuine indeterministic process, and hence creative time. Muons have a mean lifetime of a only 2.2 µs, so on average they should be able to travel a half-survival distance of only about 456 meters without decaying [46]. The fact that we are able to detect cosmic rays muons (i.e., produced in the upper atmosphere) at sea level, however, means that they exist undecayed for a period (as seen from the reference frame of Earth) that is 25 times longer than their lifetime. Such an effect is explained by the fact that due to the muon's velocity close to the speed of light, the muon lifetime undergoes relativistic time dilation. This could be considered a strong argument to show that also creative time transforms under the Lorentz transformations.

Yet, one can also think that the muon evolves deterministically while propagating through the atmosphere (thereby involving only geometric time) into a state of quantum superposition of decayed-undecayed and the process that actualizes the outcome "decayed" (which involves creative time) only happens inside the detector, hence at rest with Earth. In this way, creative time would not necessarily follow relativistic transformations. While this is admittedly a wild conjecture, it seems not to contradict the experimental evidence. This calls for further investigations.

One might think that geometric time is more friendly to relativity than creative time (for a discussion on this see also [15]). Indeed, if there were only geometric time, then everything would be given all at once (determinism) and there would be no difference between time and space. But, actually, creative time might also be quite friendly to (special) relativity. Consider a true random number generator that produces (creates) a bit and let us fix the coordinate system such that this bit is produced at the origin. Along a time-like line passing through the origin, there is a time when the bit's value is indeterminate and a time when it is determinate. Next, consider a space-like line in the future. Assuming that the bit has a determined value only inside the future light cone (see [15]), one can "move" along that space-like line and notice, similarly to moving along the time-like line, a segment thereof where the bit's value is indeterminate and a segment where it is determinate. Note that along the time-like line there are only 2 regions –one of indeterminacy and one of determinacy–, while along the space-like line there are 3 regions, i.e., one can "move" from indeterminate to determinate and (without "returning") again to indeterminate.

Finally, the idea that creative time fundamentally separates the past from the future may *prima facie* seem add odds with relativity. However, the events of actualization that characterize the development of creative time

Time	Physics	Mathematics	Logic	Philosophy
Geometric	Determinism	Classical	Non-tensed	B-theory
Creative	Indeterminism	Intuitionistic	Tensed	A-theory

TABLE I. Summary of the relation of concepts of creative and geometric time to differnt disciplines.

are to be considered local therefore there is no (global) simultaneity of the present, in accordance with relativity. The event of actualization from indeterminate to determinate exists only within the forward light-cone, while remaining indeterminate in any other region of space-time (see [15] for a thorough discussion). Therefore distant regions have different ticking of creative time. However, one can conjecture that due to quantum nonlocality (of the propensities that characterize the indeterminacy), different "local presents" may be synchronised by the actualization at a distance of a nonlocal propensity (see also [4]): yet another wild conjecture.

B. Quantum theory

Quantum mechanics is the first physical theory in which indeterminism has been widely considered at a fundamental level. Indeed, Heisenberg uncertainty principle (which would be better named "indeterminacy"' principle) and later on violation of Bell's inequalities are strong indications that properties of quantum systems do not exist predetermined [47]. In this, quantum physics has inspired us to develop the concept of creative time, which happens at the moment when a quantum event, e.g. a measurement, determines a physical property (i.e. a single real outcome) that was previously only potential (as encapsulated by the probability amplitudes in the quantum state) [4, 48].

This can, however, also be applied in hindsight to classical theory that can consistently be interpreted as a fundamentally indeterministic theory [3, 10].

Creative time thus becomes associated with how, and under what circumstances the potentialities become actual, i.e. to the notorious measurement problem (also at the classical level, see [3, 4]). In fact, the concept of creative time is compatible only with these classes of interpretations that conceive a passage from the potential to the determinate [49]. In quantum theory these are the "spontaneous collapse theories" or interpretations , à la Copenhagen, where the collapse is induced by (some features of) the measurement. Other interpretations, such as the many-world, cannot accommodate the concept of creative time.

C. Thermodynamics

A long standing problem that would be impossible not to mention is why do we ubiquitously observe a direction of time while the microscopic laws of physics are time-reversal invariant. This is commonly explained through statistical consideration, i.e., the fact that the thermodynamic quantity entropy cannot decrease in a closed system (second law of thermodynamics) [50]. This tension between the observed asymmetry in time and the time symmetry of the underlying microphysics are exemplified by the Loschmidt Paradox and by the Zermelo objection that Boltzmann's H-Theorem is at odds with Poincaré recurrence theorem.

However, this is not necessarily the case even at the classical level, as discussed in detail in the works of B. Drossel and G. Ellis [8, 51–54].

In a similar fashion, our distinction between creative and geometrical time can help clarify this matter. If there are fundamentally indeterminate events that require creative time, then there is also a fundamental asymmetry between past and future, as encapsulated by the second law. So, while admittedly this still requires to address the measurement problem (classical or quantum), paradoxes like the ones of Loschmidt or Zermelo are solved by merely noticing that Poincaré recurrence theorem simply fails [3].

VII. CONCLUSIONS

We join T. Maudlin when he states that "it is a fundamental, irreducible fact about the spatio-temporal structure of the world that time passes" [55]. However, the formalization of physical theories has to a large extent expelled the real passage of time from the description of natural phenomena (see also [1]). We have shown that to understand physics we should rethink our concept of time and distinguish between a geometric time (that appears in the deterministic equations of motion) and a creative time (that happens concurrently with events of actualization of potentialities).

In physics, indeed, we have related these two conceptions of time to, respectively, determinism and indeterminism. We then showed a parallel in classical and intuitionistic mathematics. The former takes a Platonistic approach by assuming that mathematical entities (such as real numbers) are given all at once, therefore avoiding the necessity of time. On the other hand, we showed that a naturalistic interpretation of intuitionistic mathematics requires a concept of creative time. Consequently, we have advocated a tensed multivalued logic (which can be true, false or indeterminate) for scientific propositions. Finally, we have related our discussion to known philosophical theories of time, showing in particular that geometric time fits best with the worldview of B-theory, whereas creative time with A-theory. Creative time has also led us to conceive a variation to the Growing-block theory, which however overcomes fundamental criticisms by providing a naturalistic account of the difference between past, present and future. These parallels in different disciplines are summarized in Table I.

We conclude by stating that, in our view, physics is not only about sophisticated theories and fascinating technologies, it should also allow one to tell stories about "how nature does it". But all stories require time [19]. Hence, we would like to rephrase a famous aphorism by French writer F. Rabelais as: "science without time is but ruin of intelligibility".[14]

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