

APPLICATION OF RELATIONAL QUANTUM MECHANICS TO SIMPLE PHYSICAL SITUATIONS

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Abstract. I would interpret the principal ontological postulates of relational quantum mechanics in terms of what medieval philosophers called “relational properties”. Relational properties are exactly like all other properties, but they can be ascribed to a substance only in reference to another substance. If this interpretation is correct, a quantum event is a very complex situation. To individuate a quantum event, two substances and two properties are necessary, each one pertaining to one of the substances. Moreover, also a form of ontological replacement is needed. After elaborating on a simple symbolism based on these postulates, we investigate quantum situations, such as Wigner’s friend paradox, the strange result of a sequence of Stern and Gerlach measurements, and the probability flux of wave function.

Introduction

According to Mugnai (2012), one of the most standard medieval interpretations of relations is Walter Burley’s (1275-1344). He is a realist about relations. Even if Burley

is a realist, he does not arrive to make relations completely autonomous with respect to properties. People begin to speak of relations as autonomous entities only with the modern first-order predicate calculus and Bertrand Russell's *Principles of Mathematics* (1903, §§ 27ff.).

In Burley's approach, a relation is something real but needs four other ingredients: two subjects and two foundations (properties). For instance, "a is higher than b" means that there are both a and b and the high^{ness}ness of a and b.

In contemporary metaphysics of science, in consideration of the advent and high confirmation of quantum mechanics, philosophers introduced both relations not supervening on properties of relata (Calosi, Fano, Tarozzi, 2011) and even relations preceding relata (Muller, 2015). It is clear that Burley, even if he is a realist about relations, does not intend relations to precede relata. It is more difficult to establish whether he means that they completely determine the relation when he speaks of these four ingredients. An anachronistic historical question is: according to Burley's realism on relations, supervene they or not on properties? In what follows, we assume that in this four-ingredients approach, these four ingredients determine the relation. In other terms, a relation *is* these four ingredients. Mugnai (2012) defines this entity a "relational properties," and we will use that term.

In one of the most recent presentations of his interpretation of quantum mechanics, Rovelli (2021) says:

RQM interprets quantum mechanics as a theory about physical events, or facts. The theory provides transition amplitudes of the form $W(b, a)$ that determine the probability $P(b, a) = |W(b, a)|^2$ for a fact (or a collection of facts) b to occur, given that a fact (or a collection of facts) a has occurred.

Rovelli makes a similar statement in many other papers. I guess that RQM gives centrality in quantum ontology to what we have called “relational properties.” This bold statement needs, however, a couple of caveats.

I agree with those scholars who are wary of metaphysics, even if it is elaborated a posteriori with respect to empirical sciences. Anyway, I believe it is possible to formulate justified and fallible ontological statements based on our best-confirmed scientific theories (Corti, Fano, 2020). Quantum mechanics is an incredibly well-confirmed theory. Think, for instance, of the very risky prediction of the violation of Bell’s inequality, which has been largely confirmed.

Moreover, the problem with quantum mechanics is that there are many quantum ontologies. Many of these ontologies, such as wave function realism, many-worlds

interpretations, and Bohmian mechanics, go beyond the theory. On the contrary, relational interpretation attempts to be as adherent as possible to the theory. Relational is not entirely new with respect to Copenhagen's interpretation, and it is earning more and more consensus¹. Indeed, working with Jammer (1974), it would be interesting to outline the prehistory of the relational interpretation.

In the next section, I will say something about RQM. In the subsequent, I will introduce a simple symbolism. Then, we will apply this symbolism to investigate three typical quantum situations: viz. Wigner's Friend, a sequence of Stern-Gerlach measurements and the probability flux in a wave function.

Outline of RQM

RQM interprets quantum mechanics in an altogether naturalistic framework. Here, naturalism means that we can accept in our ontology only those entities whose existence is justified by our best scientific theories. In RQM, terms such as "observer" and "information" are used. To avoid misunderstandings, better to speak

¹ See, for instance, the recent issue of *Foundations of physics* edited by C. Calosi and devoted to RQM.

respectively of “physical systems” and “probabilistic dependence”. These two terms need a bit of clarification. A physical system is something that could have independent existence (a substance) and is identified by our physical theory; in our case, the theory is nonrelativistic quantum mechanics. We will indicate physical systems with small letters, such as f , w , and s . Probabilistic dependencies concern observables. As it is well known, the quantum term “observable” is epistemologically misleading. Many observables, such as Hamiltonian, are not directly measurable. Anyway, I will use the term in its ordinary quantum meaning. We will indicate observables with capital letters, such as A , B , and C , and their eigenvalues with small letters, a , b and c . Before measurement, an observable corresponds to a certain probability distribution on its eigenstates. Concerning probabilistic dependence, I propose these definitions:

One says that A 's eigenvalue a depends on B 's eigenvalue b when $p(a/b) \neq p(a)$.

A depends on B when each eigenvalue of A depends on at least one eigenvalue of B .

Why is this notion very similar to that of information? We know that 1 bit of information means, for instance, that a certain system s at time t_1 has probability $\frac{1}{2}$ that the observable S , for instance, has value \uparrow and $\frac{1}{2}$ of being \downarrow ,

whereas the observable O of system f is in state Ready. Then, at time t_2 , for instance, both S and O have value \downarrow . Let us repeat this process many times and find that at time t_2 $p(O = \uparrow/S = \uparrow) \approx 1$, whereas $p(O = \uparrow) = 1/2$. Then the eigenvalue \uparrow of O depends on the eigenvalue \uparrow of S . Moreover, something similar holds for the eigenvalue \downarrow . Therefore, we can say that the system f 's observable O depends on the system s 's observable S .

In other words, a measurement process is an interaction between two systems in which information passes from the observed system to the measurement apparatus. This passage of information is a probabilistic dependence.

These interactions are very common in the microphysical world. And they manifest themselves as probabilistic dependencies. It is clear that the term “interaction” is a genus ontological term referring to many different situations occurring at the micro-level. How often happens in our best scientific theories, it is very difficult to say much more of what happens physically beyond our equations. “Interaction” does not mean a lot, and we detect interactions through probabilistic dependence. *Measurements* are particular interactions that can be recorded through our peculiar sensible apparatuses.

In RQM, all ontological weight is on observables. The wave function is only a mathematical instrument. Observables are represented in the theory as operators. In operators, something is essentially indeterminate since operators act on states, and, in general, the reality of the different eigenstates is governed by a probability distribution. Operators refer to properties, but these properties cannot be ascribed to a physical system without indicating another physical system and one of its properties. Moreover, this ascription presupposes an interaction between the two systems. From an epistemological point of view, we have access to this ascription only when the interaction is a measurement.

In RQM, one can have a deterministic evolution of the state when there is no interaction. This determinism is very strange, as observed by Earman (1986, 203) since it is not a deterministic trajectory in spacetime but in Hilbert space. Moreover, most interesting processes are not of this kind but are interactions. Interactions are, for us, interesting when they are amplified enough; that is when they are measurements. All interactions are essentially indeterministic since they are described by the operators that are essentially incomplete; that is, they represent a multitude of possible properties. Note that a representation of an operator in a certain basis is a matrix, which if

diagonalized is a table of possible values of a certain observable.

The process of amplification of micro-interactions is only partially understood through decoherence models. In measurements, amplified interactions allow us to detect aspects of micro-reality. Zeh's and Zurek's models present a residue of many worlds without splitting of worlds². Decoherence models explain, in many cases, why, in the interaction between the microsystem and the macro measurement apparatus, most part of the interference contributions disappear at the macro level through the intervention of the environment. The process is essentially indeterministic. It is possible to ascribe properties to physical systems at the macro level, but this could happen only because one can neglect quantum effects. The ontology at different scales could differ (see Fano, 2023).

Quantum events

In fact, RQM proposes an ontology based on what I have called “relational properties”. Nonetheless, those entities in RQM literature are called “quantum events”. We will use these two terms interchangeably.

We have seen that a quantum event presupposes two physical systems and two properties (observables), one for

² On the topic of decoherence see, for instance, Bacciagaluppi, 2020.

each of the two systems. As already said, I represent systems with small letters, such as s , w , and f . Properties are eigenvalues of a given observable. Therefore, in general, one can use the following rule:

$(A = a, s, O = o)_f$ means “ascribe the eigenvalue a of the observable A to the system s in relation to the ascription of the eigenvalue o of the observable O to the system f ”.

Let us imagine a strongly idealized situation in which one produces a beam of silver atoms, which stay in the spin state:

$$|\varphi_z\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

Then, the beam crosses a Stern-Gerlach apparatus for measuring spin in the z -direction. We follow a single atom: let us call it s . Let us indicate this apparatus with the letter f . After having crossed the magnetic field, s will impact on a screen in the position corresponding, for instance, to \uparrow . Let us call the possible measurement results, that is the observable positions on the screen corresponding to \uparrow and \downarrow , respectively Up and Down. After having observed s that impact, for instance, in the position Up, one can write $(S_z = \uparrow, s, O = \text{Up})_f$. This is a simple quantum event.

I am aware that this symbolism is quite cumbersome and not useful from a physical point of view. Nevertheless,

representing quantum events in this explicit way helps to better understand RQM.

A quantum event occurs and, therefore, is a real relational property in Walter Burley's sense. Moreover, it is characterized by four ingredients, exactly as intended by the great medieval scientist and philosopher: two physical systems and two properties.

Note that RQM faces an intersubjectivity problem. The basic ontological entities are relational properties. This means that a system measured by two different apparatuses could result in different states. Nonetheless, two similar apparatuses measuring the same system should agree. This should hold not only because we love objectivity but also because it really happens in our experimental setting. For this reason, it is necessary to add adequate postulates to guarantee intersubjectivity (Adlam, Rovelli, 2022).

Before concluding this brief section, we should briefly dwell on the problem of time. As it is well known, in non-relativistic quantum mechanics, time is a parameter. Hilgevoord (2005) emphasized that time in quantum mechanics, even if it is not an operator, has a role quite similar to that of position. This becomes clear in quantum field theory, where, to guarantee Lorentz invariance, we should declass space to a role of parameter similar to that of time. Moreover, as it is well known, only string theory

promotes time to the level of operator in the same sense of position. Note finally that in loop quantum gravity, time does not become an observable, but it loses its role as independent variable. Anyway, in nonrelativistic quantum theory, time is a parameter playing an important role. To measure different instants of time could be meaningful, as in a sequence of Stern-Gerlach experiments. This means that sometimes it will be necessary to add another term to our clumsy symbolism: time t . Therefore, the preceding rule becomes:

$(A = a, s, O = o, t)_f$ means “ascribe *at time t* the eigenvalue a of the observable A to the system s in relation to the ascription of the eigenvalue o of the observable O to the system f ”.

Wigner's friend

Undoubtedly, Wigner's Friend paradox expresses in a very clear-cut manner one fundamental aspect of the measurement problem in quantum mechanics. Rovelli 2021 thinks that this paradox is the essence of the measurement problem. Indeed, Wigner's argument has been enormously influential. It persuaded its author for a while that there is a sort of psychophysical interaction (Wigner, 1967). Nonetheless, Wigner, with the appearance of the new decoherence models, changed his mind (Esfeld, 1999). Moreover, Everett proposed his interpretation of

quantum measurement reflecting on this paradox (Barrett, 2023). Finally, a recent reformulation of the argument caused a great quantity of very interesting discussion (see Corti, Fano, Tarozzi, 2023).

The formulation is quite simple. A Friend of Wigner f measure on a system s a superposition state like the preceding one:

$$|\varphi_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

For instance, she finds \uparrow . The Friend f is inside a certain closed laboratory. After having measured, she called Wigner, saying to him that she measured, without saying the result of her measurement. Wigner, w, applies quantum mechanics to the composed system of the system s and his friend f. Wigner finds that it is not possible to ascribe any result to s because of the entanglement between f and s. With obvious notation, we could say that the situation for f is:

“either ($S_z = \uparrow$, s, O = Up) or ($S_z = \downarrow$, s, O = Down)”

whereas for Wigner, the situation is:

“neither ($S_z = \uparrow$, s, O = Up) nor ($S_z = \downarrow$, s, O = Down)”.

It is evident that these two sentences are in contradiction.

Nevertheless, we have not used our complete identification of the quantum events. Indeed, as in the other cases, we should add the second subject of relational property, which we have neglected.

As we have already emphasized, Wigner and the Friend are nothing more than physical systems, and they should appear as second terms in the representation of quantum events. Therefore, the correct description of the two situations is:

“either $(S_z = \uparrow, s, O = \text{Up})_f$ or $(S_z = \downarrow, s, O = \text{Down})_f$ ”

“neither $(S_z = \uparrow, s, O = \text{Up})_w$ nor $(S_z = \downarrow, s, O = \text{Down})_w$ ”.

It is evident that in this complete relational formulation, the contradiction disappears.

Note that this interpretation is similar to the QBist approach since for f and w the state is different, but the quantum events $(S_z = \uparrow, s, O = \text{Up})_f$ and $(S_z = \downarrow, s, O = \text{Down})_f$ are ontologically different from $(S_z = \uparrow, s, O = \text{Up})_w$ and $(S_z = \downarrow, s, O = \text{Down})_w$. In another term, RQM also provides a clear ontology, whereas QBism is only epistemic.

The sequence of Stern-Gerlach experiments

Countless physicists and many philosophers have had their first impact on the awkwardness of quantum mechanics through the famous example of the sequence of Stern-Gerlach experiments. This happened because two much-used handbooks, such as Hughes (1989) – for philosophers – and Sakurai, Napolitano (2020) – for physicists – begin introducing this strange quantum behavior. In the past, from Bohr to Feynman, the awkwardness of quantum theory was often presented through the double-slit experiment, which is also very striking. Nevertheless, the double-slit experiment has two disadvantages: first, it is not an easy experiment to realize; second, continuum spectrum observables, such as position, involved in a double-slit experiment are much more problematic from the formal point of view. Obviously, there is also an RQM treatment for the double-slit experiment³.

My aim in this section and in the following is to show how the ontology of relational properties or quantum events, outlined in the preceding sections, helps to clarify two strange physical situations.

³ Here is an outline of this topic. The paradox is that on the second screen, the particle is detected as a particle, but during the passing through the first screen with two slits, it behaves as a wave; that is, it interferes. The relational point is that one can ascribe the position to the particle only in reference to the second screen, therefore, only after having impacted on it. This does not contradict the wave behavior of the particle when it passes the first screen. Note that this analysis is different from the verificationist consideration that one can ascribe properties only after measurement. The result is the same, but it is motivated by ontology from a relational perspective, whereas verificationism is motivated by epistemology. This argument deserves a deeper investigation.

Let us consider a beam of silver atoms whose z-spin is uniformly distributed in all directions. The physical situation is different from that presented in the preceding section. There, there was a pure state; here, there is a mixture. At time t_1 we measure the spin in the z direction. Approximately 50% of the silver atoms will have spin Up and 50% spin Down. At time t_2 , we take the Up beam. If we measure the spin in the z direction again, we will find almost all atoms in the Up state. Until now, it has been quite strange that the angular momentum in the z-direction could have only two values, whereas, in a classical macroscopic context, a continuous interval of values is allowed. Moreover, the behavior of a single atom is undetermined before measurement; again, this is something not classical, but awkwardness has not yet arrived at its climax. Let us measure spin in the x direction. We will find 50% of the atoms in spin Up and 50% in spin Down. Now, at time t_3 , let us take the beam that, according to our classical intuition, should be in a state Up with respect to observable S_z , and measure again in the z-direction. Instead, we will find 50% up and 50% down. Indeed, this behavior is the most strange in quantum mechanics.

Before investigating this paradox through RQM, note that this is a clear exemplification of Rovelli's two postulates

on information, introduced in his seminal paper of 1996 (pp. 1657-58):

Postulate 1 (Limited information). A maximum amount of relevant information can be extracted from a system.

Postulate 2 (Unlimited questions). It is always possible to acquire new information about a system.

It is better to reformulate these principles without the ambiguous term “information”:

Postulate 1* (Limited probability dependencies). A system could have a limited number of eigenvalues/observables probabilistic dependences with eigenvalues/observables of other systems.

Postulate 2* (Unlimited questions). It is always possible to put the eigenvalues/observables of a system in probabilistic dependence with new eigenvalues/observables of other systems.

The consequence of these two very deep ontological principles, largely confirmed by quantum experiments, is the following:

Ontological Replacement Corollary: if new relational properties are established, then other existing ones are lost.

The sequence of Stern-Gerlach experiments presented is a clear example of how the ontological replacement

corollary works. Indeed, at time t_2 we established a new relational property that is x spin direction Up, but at time t_3 we find that we have lost the relational property of the z spin direction Up.

Using our formalism with obvious notation, after the first z measurement, we will have 50% of $(S_z = \uparrow, s, O = \text{Up}, t_1)_f$ and 50% of $(S_z = \downarrow, s, O = \text{Down}, t_1)_f$. This means that for each one of the silver atoms, we established a relational property. Then, after the measurement in the x direction, we will have 50% of $(S_x = \uparrow, s, O = \text{Up}, t_2)_f$ and 50% of $(S_x = \downarrow, s, O = \text{Down}, t_2)_f$. This means that for each silver atom of the Up result in the z direction, we established a new relational property. Nonetheless, if we measure again the x Up beam in z direction we will find 50% of $(S_z = \uparrow, s, O = \text{Up}, t_3)_f$ and 50% of $(S_z = \downarrow, s, O = \text{Down}, t_3)_f$. This result confirms the Ontological Replacement Corollary. Moreover, there is no contradiction because the two quantum events refer to different times.

This analysis seems, however, incomplete. Indeed, what happened at time t_2 to cause the strange result one detects at time t_3 ? This question deserves further investigation.

In this example, time is something more than a label. The beam entering in the second z-direction apparatus is no longer the beam outgoing from the first z apparatus, but a

new quantum event characterized by $(S_x = \uparrow, s, O = Up, t_2)_f$, that is by spin \uparrow in x direction with respect to f.

The conclusion of this section seems, therefore, that the ontology of relational properties could explain adequately the paradigmatic and paradoxical sequence of Stern-Gerlach experiments only if we take into consideration the two postulates 1* and 2* and their consequence, i.e., the Ontological replacement Corollary.

Probability flux

Now we take into consideration another example proposed by Sakurai, Napolitano, 2020, in a section of his book meaningfully entitled “Interpretations of the wave function”, pp. 100-1. Let us consider a wave function expressed in the eigenvectors of position: $|\psi(\mathbf{x}, t)\rangle$; then let us define:

$$\rho = |\psi(\mathbf{x}, t)|^2$$

As a sort of probability density. Let us write the wave function as:

$$\psi(\mathbf{x}, t) = \sqrt{\rho} \exp \left[\frac{iS}{\hbar} \right]$$


Where S is the phase. This is always possible. Even if S does not influence the probability of finding the particle, it

is physically relevant. Indeed, it is possible to define a probability flux:

$$\mathbf{j} = \frac{\rho \nabla S}{m}$$

Where m is the mass. The last equation is extremely interesting from the ontological point of view since it shows that the wave function determines not only the probability of finding the particle in a given position but also, through its phase, it establishes the “velocity” with which this probability changes spatially.

This aspect of wave function radically challenges the interpretation of the wave function as indeterminacy. The wave function expressed in the basis of position establishes not only the probability with which to find the particle in a certain place, but also the flux of probability through different positions.


The metaphysics of indeterminacy has stalled because of the problem of the basis. If a certain state is interpreted as indeterminacy with respect to a certain basis, with respect to other bases, it can behave differently and incompatibly (Corti, 2021). 

Maybe this indeterminacy could be interpreted relationally. That is, without giving reality to the wave function. It is better to read ρ and \mathbf{j} as a probability of attributing a certain

property (in this case position) with respect to another substance. In this relational interpretation, the measuring apparatus establishes the preferred basis. In this way, it makes sense that there is a physical process with some “speed” in the appearance of a property. In other terms, the wave function is not real, but it is an instrument to evaluate not only the probability of a certain process of determination but also its flux. Moreover, this kind of ontology makes sense only in a relational context, in which also the choice of the measurement apparatus is determined.

Conclusion

In the present brief paper, I interpreted the RQM notion of quantum events on the basis of the medieval concept of relational property. This seems a good formulation of the ontology proposed in RQM. The RQM interpretation of the Wigner’s friend is well known, but I introduced a symbolism that clarifies how it works better. Then, I applied the same symbolism and ontological interpretation to the most incredible quantum phenomenon, that is, a certain sequence of Stern-Gerlach measurements, which gives very awkward results. To my knowledge, the RQM analysis of this experiment has not yet been realized. My result is that here, the two postulates proposed in the

original Rovelli's (1996) paper on RQM play a fundamental ontological role. Finally, I suggested why an indeterminacy ontology does not work in quantum mechanics. Only an ontology referring to the measurement apparatus could explain how the wave function represents our knowledge. 

It would be interesting and fruitful to investigate other situation with this approach, as the double-slits experiment, the violation of Bell's inequality and contextuality⁴.

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⁴ Part of the work has already been realized; see, for instance, Laudisa, 2001 and Smerlak, Rovelli, 2007.

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