Hidden Variables and Bell's Theorem: Local or Not?

Valia Allori

Dipartimento di Lettere, Filosofia e Comunicazione, Università di Bergamo
Via Pignolo 123, 24121 Bergamo, BG, Italy
Email: valia.allori@unibg.it
ORCR-ID: 0000-0002-8144-8549

Short Bio

Valia Allori has a Ph.D. in Physics from the University of Genova, and A Ph. D. in Philosophy from Rutgers University. She is currently Associate Professor at the University of Bergamo. She is interested in philosophy of physics, philosophy of science, and metaphysics.

Short summary

I compare and contrast two hidden variable strategies: the pilot-wave theory and superdeterminism. I show that both are contextual. Nonetheless, in contrast with the pilot-wave theory, superdeterminist contextuality makes it impossible to test the theory, and renders the theory uninformative. Thus, it is questionable whether a theory with these features is worth its costs.

Abstract

Bell's inequality is an empirical constrain on theories with hidden variables, which EPR argued are needed to explain observed perfect correlations if keeping locality. One way to deal with the empirical violation of Bell's inequality is by openly embracing nonlocality, in a theory like the pilot-wave theory. Nonetheless, recent proposals have revived the possibility that one can avoid nonlocality by resorting to superdeterministic theories. These are local hidden variables theories which violate statistical independence which is one assumption of Bell's inequality. In this paper I compare and contrast these two hidden variable strategies: the pilot-wave theory and superdeterminism. I show that even if the former is nonlocal and the other is not, both are contextual. Nonetheless, in contrast with the pilot-wave theory, superdeterminist contextuality makes it impossible to test the theory (which therefore becomes unfalsifiable and unconfirmable) and renders the theory uninformative (measurement results tell us nothing about the system). It is questionable therefore whether a theory with these features is worth its costs.

Keywords: pilot-wave theory; nonlocality; superdeterminism; contextuality

1. Introduction

It is commonly accepted that the violation of Bell's inequality shows that reality is nonlocal. That is, it is possible for the mutual influence between arbitrarily distant systems to be instantaneous. The pilot-wave theory is a deterministic theory of particles which reproduces all the predictions of quantum theory, including the violation of the above-mentioned inequality, and which is explicitly nonlocal. Many have resisted this nonlocality conclusion because they think that nonlocality is in tension with the special theory of relativity, according to which everything, including influences, travels at most at the velocity of light. This is the reason why some are willing to deny an assumption used to derive Bell's inequality, namely statistical independence, which was taken so far to be undeniable, in order to save locality. Hidden variable theories in which statistical independence is false are called superdeterministic. In this paper I wish to compare and contrast these two strategies. On the one hand we have the pilot-wave theory, which is nonlocal and in which statistical independence is true, and on the other hand we have superdeterministic theories which are local but violate statistical independence.

I start in section 2 to discuss how the nonlocality conclusion is reached. First, EPR wanted to justify observed perfect (anti)correlations assuming locality and this led them to a non-contextual hidden variable theory. That is, this theory contains variables, in addition to the wavefunction, representing genuine properties whose values do not change depending on how they are measured. Bell found a constrain such type of theory needs to obey which was experimentally found to be falsified, leaving with the only conclusion that the world is nonlocal. Then, in section 3 I show how the pilot-wave theory fits in this schema: its nonlocality accounts for the perfect (anti)correlations, while its hidden variables are contextual, and in virtue of that Bell's inequality is violated (that is, the predictions of the pilot-wave theory are the same as the ones of quantum theory). Then, in section 4 I move to discuss the hypothesis of statistical independence. I show that assuming statistical independence is tantamount as assuming that we can perform statistical inductive generalizations, which explains why this hypothesis was never questioned before. Among other consequences, I show how this makes superdeterministic theories impossible to falsify and to confirm. I also argue that the hidden variables of such superdeterministic theories are contextual. This allows for a clear comparison with the pilot-wave theory, which is spelled out in section 5. I argue that while the contextuality of the pilot-wave theory is not problematical and its origin is clear, no such thing is the case for superdeterministic theories. Their contextuality is unexplainable, and it ultimately makes this type of theories uninformative, as in these theories measurements never reveal any information about the system. Finally, if locality was considered a *desideratum* because it was supposed to help with relativity, I show that even if superdeterministic theories are local, their locality does not make their potential relativistic generalization any easier. In section 6 I summarize my conclusion that superdeterministic theories do not constitute a viable alternative to the pilot-wave theory, reinforcing the conclusion that nonlocality is a feature of nature.

2. From EPR to Bell, to Nonlocality

As it now seems to be commonly accepted, Bell has shown that no theory reproducing the predictions of quantum theory can be local: in all quantum theories there are instantaneous influences among arbitrarily distant systems. This conclusion was initially proven starting from the Einstein-Podolsky-Rosen (EPR) argument, and then by deriving the so-called Bell's inequality which was later tested and observed to be violated. In this section I reproduce this derivation and more generally discuss the assumptions as well as the conclusions. However, let me start by clarifying the meaning and the significance of locality.

2.1 Locality

Einstein first observed the nonlocality of quantum mechanics and took this to be a reason to reject the theory as originally proposed.[1] Locality, or local causality, is the idea that influences travel at finite velocity. Einstein, like everyone else at his time and before, thought that nature had to be local for arguably two reasons. First, an interaction being instantaneous seems to require a kind of explanation of the phenomena alternative to the one based on causality, as we think that causes precede effects in time rather than being simultaneous with them. Moreover, Einstein thought that one needs locality in order to be able to treat systems as isolated, as we constantly do in physics: we assume that what happens to a given object is substantially influenced only by objects which are close by, so that we can ignore the rest. In other words, when investigating a phenomenon, we think we can find its cause nearby. If instead the interaction propagates instantaneously, this cannot be guaranteed: the cause of something happening here may be arbitrarily far away. If so, the worry is that we might be unable to make progress in physics, because we would never be able to identify the cause of a given phenomenon. Notice that Newtonian mechanics violates local causality: the forces act instantaneously on matter. This was an objection to Newton, who unsuccessfully searched all his life for a local explanation. In any case, the concept of field as carrier of the interaction was later introduced to take care of the first problem, because it is mathematically represented by a function with a given value in every point in space. Also, in classical mechanics we do not have the second problem connected with nonlocality: since the gravitational force decreases like the inverse of the distance squared, the pull due to distant objects can be neglected, so we can safely treat systems as effectively isolated. While this is true also for classical electrodynamics, it is not the case for quantum mechanics, whose nonlocality is due to the collapse of the wavefunction. In fact, the strength of the collapse is unaffected by distance, and this prevents us from assuming systems as isolated. On top of this, the theory of special relativity imposes another constrain on locality, namely that the velocity of the propagation of the interaction cannot exceed the speed of light. So, even if we could treat quantum systems as isolated (as we can, as a matter of fact, due to decoherence), we still have the problem of the instantaneousness of the interaction.

2.2 From Locality to Incompleteness: EPR

Therefore, Einstein concluded, since reality needs to be local, quantum theory has to be incomplete: there have to be something, not specified in the standard quantum theory, which would locally account for the wavefunction collapse. Interestingly, Heisenberg (1949) conceded to Einstein that quantum theory is nonlocal. However, since he thought of relativity instrumentally as a theory of signals, he concluded that there is no tension with nonlocality,

[1] Einstein's remarks from Solvay 1927, translated in Bacciagallupi and Valentini (2009), pp. 485–487.

since it can be proven that such nonlocality cannot be used to send information.[2] Various versions of this type of incompleteness argument which rely on the assumption of local causality were proposed by Einstein[3] and culminated in the famous EPR paper (1935). Here is the argument in a nutshell, as reconstructed by Bricmont *et al.* (2022). Consider two particles in a spin singlet state. According to quantum theory, an individual particle in such a pair does not possess a definite spin property in any direction until this property is measured. So, assume the two particles fly in opposite directions. In this situation, if quantum theory were complete, then a spin measurement along some direction on one particle of the pair would instantaneously determine the spin measurement outcome for the other particle, regardless of their mutual distance, as being perfectly (anti)correlated with the result for the first particle. That is, if particle 1 is 'up' along some direction, then particle 2 is 'down' along that direction. However, this violates local causality, namely the idea that influences travel continuously at finite velocity. Thus, EPR concluded, the only other way of accounting for these observed perfect (anti)correlations is to grant that there are some 'pre-existing' values of spin properties along some direction which spin measurements actually reveal. In other words, (anti)correlations in the results are explained by (anti)correlations at the source.

2.3 Non-Contextuality

Let me add this very important remark. As just stated, according to Bricmont *et al.* (2022), for EPR the purpose of local hidden variable theories is to explain the observed perfect (anti)correlations. In order to fulfil such a purpose, the hidden variables must represent properties which can be faithfully measured and revealed in experiments; let's call them *genuine properties*. It is only because we can say that there are genuine properties which are revealed by the experiments that we can explain the observed perfect (anti)correlations. So, these properties have to have a value which should be the same independently of how we decide to find out what it is. That is, the context of the measurement should be irrelevant. That is obvious: the color of your eyes should not depend on whether I am measuring it while you are on a scale, so that I also measure your weight, or while I am taking your blood pressure. Your eye color is what it is, and it should remain the same, independently of how I decide to find out what it is. If your eye color depended on my choice on the conditions under which I am going to measure it, then it would not be a genuine property at all. This feature, namely that the value of a given property should not change depending on how it is measured, is called in the literature 'non-contextuality.' It amounts to asserting that the properties we are considering are genuine properties whose value can be faithfully revealed as the result of a suitable measurement. Notice that this means that local contextual hidden variables (HV) will not be an option to fix quantum theory for EPR. In fact they are not able to explain the observed perfect

---

[2] Notice that he did not address the problem that nonlocality may make impossible to treat systems as isolated. However, one could argue that he did not need to, as Heisenberg envisaged quantum theory to be empirically adequate, rather than a way of understanding the phenomena in terms of causes.

[3] See Allori (2023) for a critical review.

(anti)correlations because they are not genuine properties: experimental values need to be the values of the pre-existing properties (i.e. properties need to be non-contextual) otherwise the correlations remain a mystery. Schematically:

(1) [EPR]: perfect (anti)correlations & locality → non-contextual HV.

2.4 Bell's Inequality and Its Falsification

The main problem with the EPR argument is that, as Bell has shown, we cannot really reproduce the perfect (anti)correlations with local hidden variables, and because of this, the locality assumption has to be false. This is why. Bell (1964) started from the EPR argument and accordingly constructed a non-contextual hidden variable theory. He then observed that there is a constrain on such a theory, now known as Bell's inequality, which instead does not hold for quantum theory. That is:

(2) [Bell's inequality]: (local) non-contextual HV → inequality; [4]

from which, together with (1), it follows that:

(3) (1)&(2): [EPR] & [Bell's inequality]: perfect (anti)correlations & locality → inequality.

In this way, one can set up a crucial test, namely a test in which a local non-contextual hidden variable theory would predict something different from quantum theory. This test was later performed, and its results were compatible with quantum theory,[5] thereby falsifying the non-contextual hidden variable theory which EPR needed to explain the perfect (anti)correlations locally. From this, Bell concluded that any theory which matches the quantum predictions had to be nonlocal. That is:

(4) [Aspect]: crucial test result → inequality is false;

given that the observed perfect (anti)correlations in (3) and the result of the crucial test in (4) cannot be false (they are empirical findings), the only assumption to be questioned is locality. Namely, it has to follow that:

(5) (3)&(4): [EPR] & [Bell's inequality] & [Aspect]: perfect (anti)correlations & crucial test results → nonlocality.

The last line is called *Bell's theorem,* and its conclusion is that *the quantum world is nonlocal*.

---

[4] Equation (2) might misleadingly suggest that Bell's reasoning only applies to hidden variable theories. This is not the case, as discussed later: Bell has shown that all quantum theories, not just hidden variable ones, must be nonlocal. I have chosen to spell out Bell's reasoning in this way because that is how it was originally proposed in connection with eh EPR argument (in section 4.1 a more general line of argument is discussed). In addition, as I argue at the end of this section, in this way one can see Bell's inequality as a constraint on hidden variable theories and use it to rule out (local) non-contextual hidden variables.

[5] Aspect (1982).

I think it is also important to notice that Bell incidentally also proved that what EPR arguably wanted, namely to explain the observed perfect (anti)correlations using non-contextual hidden variables (and thus avoiding nonlocality), is impossible. In fact, Bell's inequality as written in (2) can be seen as a constraint that all hidden variable theories constructed in order to explain the empirical perfect (anti)correlations (that is, local non-contextual hidden variable theories) need to satisfy. Since this inequality was falsified as (4) shows, Bell has actually proven that local non-contextual hidden variables are not an option. That is, putting (2) and (4) together, one has:

(6) (2)&(4): [Bell's inequality] & [Aspect]: (local) non-contextual hidden variables & crucial test results → contradiction.

Considering that the results of the crucial test, being an empirical finding, cannot be false, this means that:

(7) non-contextual hidden variables (as needed by EPR, namely genuine properties to explain the perfect anti-correlations) are logically impossible.

So, given that hidden variables without locality are not a viable way of explaining the perfect (anti)correlations, the only empirically adequate option is to have nonlocality.[6]

3. The Pilot-Wave Theory

To summarize the result of the previous section, Bell's theorem shows that, assuming locality, the perfect (anti)correlations can only be explained by non-contextual hidden variables; however, non-contextual hidden variable theories have been empirically falsified by the violation of Bell's inequality, when seen as a constrain that such theories need to obey to. Therefore, they only other option to explain the perfect (anti)correlations is to assume that there are nonlocal interactions.

Nonetheless, there is a hidden variable theory which is nonlocal and whose predictions are the same as quantum theory. This theory is the pilot-wave theory.

3.1 Its Ontology and Its Evolution

This theory was not introduced in connection with Bell's inequality (the other way around was actually the case: Bell started from this theory to derive its inequality). Rather, the pilot-wave theory has been around since 1923, at least in some versions of it. It was proposed initially by de Broglie to explain quantum features without using notions such as the wave-particle duality

[6] Notice that, as suggested by Bricmont *et al*. (2022), the so-called no-go theorems (Gleason 1957, Kochen Specker 1967, etc.) prove the same thing stated in "(6): [Bell's inequality] & [Aspect]", namely that (local) non-contextual hidden variables are impossible. All these theorems provide examples of measurements for which the (local) non-contextual hidden variable theories need to obey a given constrain which cannot be logically satisfied. Also, von Neumann (1955) proposed the first of these impossibility theorems, but his poof was invalid, as his example involved a set of non-commuting operators, which cannot be measured at the same time, so there are no measurements associated with them.

and taken up independently by Bohm in 1952 with the purpose of explaining the quantum phenomena without mysteries and paradoxes. In fact, quantum mechanics as seen in physics books is notoriously difficult to take seriously from a scientific realist perspective, as it does not provide a clear picture of the world beyond the phenomena. Several alternative, more realist friendly, quantum theories have been proposed, among which the pilot-wave theory, the spontaneous localization theory, and the many-worlds theory. Among these, the most traditional, namely the closest to our pre-quantum understanding, is the pilot-wave theory.[7] It is a theory of microscopic point-particles which can be understood as compositing macroscopic objects, just like in the classical case. However, contrary to classical mechanics, the 'quantum' particles evolve in time according to an equation which constrains their velocities, rather than their acceleration. That is, in classical mechanics the evolution of a particle of position x is given by Newton's equation: $\frac{d^2}{dt^2}x(t) = -\frac{1}{m}\nabla V(x,t)$, where, with obvious notations, t represents time, m the particle's mass, and V the potential, describing the interaction of the particles with one another. Instead, in the pilot-wave theory the particle motion is given by the following equation: $\frac{d}{dt}x(t) = \frac{\hbar}{m}\nabla S(x,t)$, where $\hbar$ is the Planck constant divided by 2π, and S is a suitable function which describes the interaction. In virtue of obeying a different equation, the trajectories of the particles in the pilot-wave theory are highly non-classical.[8] Accordingly, this equation specifies what matter does, just like Newton's equation did classically. Newton's equation used the potential to describe the interaction, such that the gradient of the potential is the force. Instead in this case the situation is less straightforward. The function S is the phase of another function $\psi(x,t) = R(x,t)e^{iS(x,t)}$. This ψ function in turns evolves in time according to an equation which involves the potentials: $\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^1}{2m}\frac{d^1}{dx^2}\psi(x,t) + V\psi(x,t)$. This function is the wavefunction of standard quantum theory, which evolves according to the Schrödinger equation. The wavefunction may be seen as a wave oscillating in space, like electromagnetic waves, for single-particle systems. However, for a system of N particles, this is no longer possible because it is a function of all the particles positions: $\psi = \psi(x_1, x_2, \dots, x_N, t)$. That is, the wavefunction is a field in the space of the particles configurations. This is the sense in which the wavefunction has no classical analog. Otherwise, it has been argued by the so-called primitive ontologists that the wavefunction in the pilot-wave theory has essentially the same role the potential had in classical mechanics (see, e.g., Allori 2013).

If so, the equation of motion for the particles is to be regarded as the fundamental equation of the pilot-wave theory: particles are what matter is made of in this theory. The Schrödinger equation and the wavefunction instead are needed to complete the particles dynamics, but they are not to be understood as describing the behavior of the wavefunction as part of the ontology of matter of the theory. Instead, it is very common to present the pilot-wave theory as a theory of particles and waves, with two fundamental equations of motion. However, this is highly

[7] For a short review of this theory, see Allori and Zanghi (2004).
[8] Interesting examples of particle trajectories in the pilot-wave theory can be found in Norsen (2016).

misleading from the point of view of the primitive ontology approach, as this suggests that the wavefunction is a physical object in addition to the particles.[9] In contrast, in the primitive ontology approach the wavefunction and its evolution equation need to be specified to properly define the theory, just like the law of the forces are needed to properly define classical mechanics. But as classically potentials and forces are supposed to represent the interaction between material object rather than the objects themselves, in this understanding the wavefunction should also be thought as having to do with the interaction rather than representing matter.

3.2 Hidden Variables

The pilot-wave theory is a hidden variable theory: the complete description of a given physical system must include the specification of the configuration of the particles composing the system, in addition to the wavefunction. That is, the particles positions are the hidden variables of the pilot-wave theory, in the only sense that they are variables which need to be specified to complete the description, and their specification is hidden, in the sense that quantum theory alone does not specify them. Nonetheless, the name is inappropriate because it suggests that these variable are additional, somewhat secondary, to the wavefunction. However, as the discussion above is supposed to have clarified, they are the ontology of matter of the theory, so of course they need to be specified. And in this view, quantum theory which keeps them hidden is clearly incomplete.

Here is a very important remark. Even if, as we just saw, one can see the particles configurations as hidden variables, they are not the ones which EPR were thinking of. In the reading of Bricmont *et al*. (2023), EPR needed spin-along-some-direction values to be hidden variables, because the correlations were between these spin properties. EPR thought that the eigenvalues of the spin operator representing the experiment being were faithfully representing spin properties. In the pilot-wave theory the spin values can be thought as hidden variables, but neither them nor the particles configuration are what explains the perfect (anti)correlations, as explained below.

3.3 Nonlocality

The pilot-wave theory is explicitly nonlocal because the wavefunction, which represents the interaction, is a function of all the particles at the same time. Therefore, given a wavefunction entangling two particles at arbitrary distance, a change in one particle will give rise to a change in the other particles. That is, it is the pilot-wave (PW) theory's nonlocality which explains why the experimental results for the two particles are perfectly (anti)correlated:

(8) [PW]: perfect (anti)correlations → nonlocality.

---

[9] Presumably this is what people initially thought, but as we just saw the wavefunction is defined in configuration space, hence making this interpretation not straightforward to maintain, as it has been extensively argued elsewhere (see e.g. Allori, 2013).

EPR concluded that the only way of explaining these results is to assume the existence of hidden variables revealed by the experiment and corresponding to pre-existing properties of the measured system (non-contextual hidden variables) because they thought that nonlocality, which was the only alternative explanation, was absurd. Instead, the pilot-wave theory does what EPR thought was unthinkable, but that Bell made necessary, namely it explains the perfect (anti)correlation *via* nonlocal interaction: the particle on one side instantly communicates with the particle on the other side, even if arbitrarily distant.

3.4 Contextuality

Because of its nonlocality, the pilot-wave theory is compatible with the violation of Bell's inequality. Here is why. Since, as we have just seen, in the pilot-wave theory nonlocality explains the perfect (anti)correlations, the hidden variables that EPR were considering (such as spin values) do not have to be non-contextual in the pilot-wave theory. Indeed, Bell's inequality holds for non-contextual hidden variables, and the violation of Bell's inequality shows that such theories are impossible. So, a theory which wishes to explain the perfect (anti)correlations, and which violates Bell's inequality is a theory whose hidden variables have to be contextual. This is indeed a property of the pilot-wave theory: in this theory all the hidden variables that EPR wanted to add to explain the perfect (anti)correlations, such as spin properties, are contextual. In other words, as we have seen earlier that:

> (7) non-contextual hidden variables (as needed by EPR, namely genuine properties to explain the perfect anti-correlations) are logically impossible.

This, in turn, means:

> (9) Hidden variables (as needed by EPR) are contextual.

3.5 Naïve Realism about Operators

Having contextual properties may sound terrible. Nonetheless, upon reflection this is not necessarily the case. In fact, the contextuality of these properties only means that an experiment does not faithfully reveal the value of the property of the system before it is measured. That is, experiments are not really measurements. Classically we think of measurements as physical processes in which there is minimal change in the system. That is, the apparatus does not interact with the system to change it very much. For instance, when I measure my body temperature with a thermometer, the temperature I read is not the temperature of my body before I used the thermometer. Rather, it is the weighed mean between the temperature of the thermometer and my own temperature before the thermometer was used. Nonetheless, the material the thermometer is made of has been suitably chosen so that the mean temperature is close enough to my own temperature, ensuring that the instrument is able to faithfully reveal what it was before I put the thermometer in.

Instead, in the pilot-wave theory, what we usually call measurements are (almost) never such. All 'measurements' are more generally experiments, namely physical processes affecting the system, modifying it in such a way that the values we obtain from them do not necessarily reveal pre-existing properties. Spin experiments, namely experiments described by the spin operator, whose possible values are the spin eigenvalues, do not measure spin. Spin is simply what captures the way in which the system reacts in that particular experimental situation. It describes the interaction, just like the wavefunction does. In a spin measurement, since theory is deterministic, given the initial condition the result will be determined too. Nonetheless, one can see that the result is contextual, as there can be two spin measurements which yield to different results. In fact, consider a spin ½ particle with a wavefunction given by the superposition of two spins along direction z. Let it go through a Stern-Gerlach magnet with a magnetic field along z. Assuming that the spatial part of the wavefunction is symmetric, as a property of the dynamics, particles cannot cross the middle line $z = 0$. Assuming then that the particle starts above $z = 0$, it will go up, providing the result which we would label z-spin up. However, still assuming particles start from above $z = 0$, but now flipping the orientation of the magnetic field one would obtain opposite results: the particle would go down, and we would label such results z-spin down. Nonetheless, we do not conclude that the same particle has both z -spin up and down. Raher, we conclude that spin is not something we are measuring in this experiment. In fact, the very same particle will behave differently, and thus will provide different results, because these results are the product of the system-apparatus interaction, rather than representing something about the system in itself.[10] So, in this theory operators are not observables; they do not measure any pre-existing property. Rather, they only effectively systematize the experimental statistics.[11]

3.6 Position Measurements

There is only one property which one can faithfully measure in the pilot-wave theory: where particles are located. That is, particles positions are the only hidden variables which are not contextual. This is because they describe the ontology of matter of the theory, and since the pilot-wave theory is deterministic, the result of any experiment is determined beforehand by the configuration (and the wavefunction) of the system.

Notice that even if it true that in the example above we were not measuring spin, we were still measuring the position of the particle at the beginning of the experiment. Assume we have a positive-directed magnetic field along z, then if we observe a click above $z = 0$, then we will know the particle was originally located above $z = 0$. If it goes down, it was below $z = 0$. That is, one can measure the particle's initial position faithfully: it is the only non-contextual hidden variable of the pilot-wave theory. As anticipated, this is not problematical, as these hidden variables are not used to explain the perfect (anti)correlations (as instead EPR wanted), as

---

[10] Albert (1992).

[11] Daumner *et al.* (1996), Dürr, Goldstein and Zanghì (2004).

nonlocality does it for us. Indeed, the contextuality of these variables is needed to make this theory compatible with the violation of Bell's inequality, as already observed.

3.7 Testability

At this point, one may think that the pilot-wave theory may make empirical discoveries impossible. In fact, one may entertain the following reasoning. We perform experiments because we want to discover properties of the system. When we make measurements, we assume that we measure a property as it was before the experiment. Now in the pilot-wave theory this is no longer the case: every time we act on the system, this action cannot be neglected, and the result will likely not reflect something about the system as it was before we acted. So how can we possibly learn something about the world if in the moment we try to measure something we are unsuccessful because we massively change the system by simply doing experiments on it?

Luckily the situation is not as bleak. In fact, in the pilot-wave theory all measurements are position measurements: operators effectively describe the statistics of experimental data, which are ultimately position detections. This was clear from the spin example: we are not measuring spin, but we learn about the initial position of the system. That is, one can explain all experimental results in terms of where the particles are located. So, ultimately, we test the pilot-wave theory by testing the results of position measurements.

3.8 The Role of Positions

As we just seen, particle configurations thought as hidden variables are of no use for explaining the perfect (anti)correlations in EPR-style experiments, even if they are non-contextual: nonlocality explains them for us. Nonetheless, *one needs at least one non-contextual hidden variable to make the theory testable*: being able to know the position of particles can give us information about the system. In this regard, it has been argued that one would need at least one spatiotemporal variable to have a theory in which one could use techniques of constructive explanation (see Allori, 2013). If that is the case, for the pilot-wave theory particles positions would play this role.

4. A Local Alternative: Superdeterminism

Some have argued that Bell's nonlocality result is unacceptable and have tried to get around it. One possibility which has recently received attention is to reject a hidden assumption called statistical independence. Hidden variable theories which assume that statistical independence is false can be compatible with the violation of Bell's inequality without being nonlocal.

4.1 Bell's Nonlocality Conclusion without EPR

I am going to start this section by presenting a different way of arriving to the same nonlocality conclusion which does not explicitly rely on the EPR argument, and in which another assumption, namely statistical independence, is more obviously stated (Bell 1967). Consider a hidden variable theory in which the description provided by the wavefunction is supplemented by some hidden variable λ. Consider a repeated set of experiments on pairs of subsystems which have previously interacted, and which are now space-like separated. Assume measurements α_1 and α_2, with outcomes A_1 and A_2, are chosen, with observed probability distribution $P_{\alpha_1,\alpha_2}(A_1, A_2)$. Also, assume local causality. That is, assume that whatever may affect one subsystem after it gets separated by the other one does not have anything to do with what might affect the second subsystem. That means that the connection between the two subsystems entirely depends on λ which is available to both subsystems, so that whatever generates the outcome A_1 out of α_1 and λ is independent on whatever generates the outcome A_2 out of α_2 and λ. In formulas, the probability distributions of the outcomes, given the experimental setups and the hidden variables, factorizes: $P_{\alpha_1,\alpha_2}(A_1, A_2|\lambda) = P_{\alpha_1}(A_1|\lambda)P_{\alpha_2}(A_2|\lambda)$. The observed probability distribution is then obtained by integrating over λ: $P_{\alpha_1,\alpha_2}(A_1, A_2) = \int P_{\alpha_1,\alpha_2}(A_1, A_2|\lambda)\, dP(\lambda)$, where $dP(\lambda)$, the distribution of the possible hidden variables, is assumed not to depend on the choice of the experimental setups α_1 and α_2. This is the assumption called *statistical independence*. It states that what determines the experimental outcome, namely the hidden variable λ, does not depend on what experiments will be chosen to be performed. From these assumptions, namely locality and statistical independence, the so-called CHBH-Bell inequality follows,[12] which can be experimentally tested, and proves the same conclusion of nonlocality.[13]

4.2 Statistical Independence and Superdeterminism

As mentioned above, one may get around the nonlocality conclusion of Bell's theorem by rejecting statistical independence. Accordingly, some authors consider a special type of hidden variable theories in which not only there are properties revealed by experiments, but also these properties are 'superdetermined' as to preserve locality. That is, these theories reproduce the observations assuming there are hidden variables, namely properties not specified by quantum theory, and they are also, in contrast with other hidden variables theories, empirically equivalent to quantum theory. The nonlocality conclusion was established by pointing out how assuming locality and hidden variables, the theory had to have predictions which differed from the ones of quantum theory. Instead in this case, Bell's inequality does not hold because statistical independence is false, so that one does not have to reject locality. Theories using this strategy are difficult to construct (for the reasons explained below) but models have been proposed.[14]

4.3 Free Will?

What does it mean to violate statistical independence, does it help, and is it worth it? As anticipated, the hypothesis of statistical independence is the assumption that the experimental values are independent of the choice of what one measures. Sometimes this assumption is

---

[12] The name comes from John F. Clauser, Michael A. Horne, Abner Shimony, and Richard A. Holt (1969).

[13] For more on Bell's theorem, see Goldstein *et al.* (2011).

[14] See, most notably, t'Hooft (2016, 2020), Hossenfelder and Palmer (2020), Ciepielewski *et al.* (2020).

referred to as the 'free will' assumption, because one could express it by saying that the experimenter is free to choose what to measure. This connection is essentially due to the way Bell himself talked about superdeterminsm, which however contributed to much confusion: "There is a way to escape the inference of superluminal speeds and spooky action at a distance. But it involves absolute determinism in the universe, the complete absence of free will. Suppose the world is superdeterministic, with not just inanimate nature running on behind-the-scenes clockwork, but with our behavior, including our belief that we are free to choose to do one experiment rather than another, absolutely predetermined, including the 'decision' by the experimenter to carry out one set of measurements rather than another, the difficulty disappears."[15]

As emphasized by Bass and Le Bihan (2021), since there are different ways of making free will and determinism compatible, then there seems little reason to believe that such strategies would not work in superdeterminism. More generally, one can see that the issue of free will is of no importance in these matters by observing that to define what statistical independence is one needs no human beings. In fact, while it is the case that usually humans select the experimental settings, these could well be chosen by some sort of automatic random generator. If that is the case, then no genuine choice is involved or needed to define the hypothesis of statistical independence.

4.4 Cosmic Conspiracy and Fine Tuning

Instead, statistical independence has a lot more to do with random sampling and induction, as well as with conspiracies and fine tuning, than with free will, as we are going to see next. Notice that superdeterministic theories require two things. First, that statistical independence is violated: there has to be a dependency between the values of the experimental results and which experiment has been chosen to be performed. However, one also needs statistical independence to be violated in a very specific way, namely in such a way as to violate Bell's inequality, even if as a hidden variable theory, it should not. To give an example, suppose I am doing experiments on Fuji apples, of which I can measure color and shape. Then, if statistical independence were false, the results for a sample of Fuji apples on which I have measured their color would not be determined by the color the apples actually have. Rather, they would be determined by my decision to measure color rather than shape. That means that a hidden variable theory violating statistical independence is such that experiments on the samples produces some data (compatible with Aspect's results) not because these data tell us something about the property of the system, but rather because the chosen sample is so extremely special as to reproduce the data without any nonlocality.

The fact that the sample is very special seems to require some sort of cosmic conspiracy. In fact, by definition of hidden variable theory, a given system of a given type has a given set of

[15] Bell in a BBC interview, reproduced by Davis and Brown (1993). See also e.g. Conway and Kochen (2006).

properties, e.g., a pair of particles with spin have spin properties along any direction. Nonetheless, when I perform measurements on a sample of these pairs to investigate the theory, the results will display another distribution, which 'magically' makes the theory agree with the data.

Similarly, superdeterminism has been criticized on the basis that it requires fine-tuning, given that one could interpret superdeterminism as requiring experimental samples that seem designed to reproduce the data without being representative of the system under examination.[16] There is also a formal result establishing that statistical independence holds for typical initial conditions, so that superdeterminism can succeed only resorting to fine-tuned initial conditions.[17]

4.5 Unlawfulness

Notice that, if what we have said in the previous subsection is correct, superdeterminism has to 'fine-tune' the samples for all experiments, all the time, always. This is what makes a superdeterministic theory very hard to construct: every piece of the gigantic puzzle – arguably, the whole universe in space-time - needs to perfectly fit. This is of course the reason for the name 'superdeterminism.' Because of this, it has been argued that such a complete proposal would have to also be extremely complex, in such a way that they would not look lawlike.[18]

Relatedly, one could argue that superdeteminism does not support counterfactuals: 'had I done a different experiment, the results would have been different' is false. That is, if statistical independence were false, then testing a sample of Fuji apples and finding them to be red would be due to my choice of measuring their color, rather than their shape. Had I chosen to measure their shape I would not have gotten red for the very same apples. Because of this, one could therefore argue that superdeterminsim does not support laws of nature as we understand them.

4.6 No Random Sampling, No Statistical Generalizations and Uninformativeness

Be that as it may, I think that the most devastating objection against superdeterminism is that it is ultimately unscientific, if one defines as scientific a theory which is testable, confirmable or falsifiable.

Going back to the discussion in the previous sections, as discussed by Chen (2021), to say that statistical independence is violated is to say that *the sample being tested is not representative*. If the sample of Fuji apples were representative of Fuji apples in general, then my finding the apples in the sample to be red would tell me about the color of Fuji apples. But to say that a sample is representative means that there are some properties the Fuji apples in general have, say, being red, which I am revealing through measurements on the sample. Instead, if statistical

---

[16] Baas and Le Bihan (2021).
[17] Dürr and Teufel (2009).
[18] As emphasized by Chen (2021) the theory proposed by Ciepieswky *et al.* (2020), while not having this exact problem, still has a related problem, as it requires a radical ontology.

independence is false, then my finding red apples in the sample depends on my choice of measuring color, as opposed to some other property, and not on the apples actually being red. Thus, properties of the sample do not faithfully represent the properties of Fuji apples in general. But if the sample is not representative, by definition one could not infer anything about Fuji apples from the observation of the sample. This in turn means that *inductive generalization*, which is used all the time in science to gather information about unobserved systems though observed ones, with this type of theories *cannot be used*.

To put it in another way, statistical independence is equivalent to the hypothesis of random sampling, which is a necessary condition for a successful scientific investigation. When we perform repeated experiments on identically prepared systems to get information about that type of system, we assume that the measured random sample is representative of that type of system, measured or not. Suppose, for instance, that I wish to test the hypothesis that 75% of Fuji apples is completely red. I do not test individually all the Fuji apples. Rather, I randomly select a sample of Fuji apples and I test them. If the sample is suitably large and varied, I can use the principle of induction and infer that what I have found testing the sample is true also for all the Fuji apples, tested or not. If 75% of the tested Fuji apples is completely red, then I infer that this is true also for all Fuji apples, not only tested ones. In other words, I assume that I can choose a random sample of the type of system under investigation, so that what is found about that sample is also true for every system of that type. This assumption, namely that a random sample is a representative sample, is equivalent to the hypothesis of statistical independence, as it is the assumption that one can choose the test sample independently of many things, including what is chosen to be measured. An assumption of this sort is needed for scientific theorizing, because we need to be able to make statistical indictive inferences from the observed to the unobserved. So, arguably, if statistical independence were false, our samples would not be random, and as a consequence we would not be able to infer much about the unobserved systems from them: much of empirical science would therefore collapse.[19]

To sum up, therefore, in a superdeterministic world testing is completely uninformative, since one is never warranted to use induction to gather information from the observed about the unobserved. It rejects induction, we cannot know anything about systems of a given type by measuring a sample of systems of that type. So, we have no possibility of learning anything, and, as emphasized by Chen (2021), further justifying statistical independence seems just tantamount as trying to find a solution to the problem of induction.

4.8 Empirical Incoherence and Unfalisifiablity

Relatedly, as emphasized by Baas and Le Bihan (2021), if statistical independence were false, the corresponding theory would suffer from the problem of empirical incoherence (Barrett 1996). A theory is said to be empirically incoherent if its truth undermines our reasons to believe in it. A result confirming our theory will arguably give us more reasons to believe in it, while a

[19] Shimony *et al.* (1976), Goldstein *et al.* (2011), Maudlin (2019), Chen (2021).

falsification will give us reason to reject it. Thus, a theory which cannot be confirmed or falsified will be empirically incoherent, and as such it would undermine itself.

To confirm or to falsify a theory, one checks whether what the theory predicts is actually observed, and this is never possible for superdeterministic theories. In fact, for instance, the theory predicts that there are spin properties associated to any direction. A confirmation or a falsification of the theory would require that I can make a measurement to detect these properties to check whether they correspond or not with what the theory has predicted for them. Nonetheless, when I perform these experiments, their results do not reveal the pre-existing properties of the system (in which case confirmation or falsification would be possible). Rather, the values are what is needed to prevent locality to be violated.[20] We have already seen that superdeterministic theories are completely uninformative: they do not allow us to learn anything about the unobserved systems from observing some of them. Consequently, they are unfalsifiable and unconfirmable. Because of that, they seem hardly science to me.

4.9 Contextuality

The previous sections have discussed, perhaps with a different emphasis, material which has already been presented in the literature. Nonetheless, I think another feature of superdeterministic theories which I think deserves further analysis, namely the fact that *assuming statistical independence to be false makes sense only if experiments reveal no genuine properties*. In fact, as we have seen previously, one generally assumes that the sample being measured is representative of systems of that type. That means that there are properties of the samples which are shared also by all systems of that type, but also that such properties are revealed by the measurement. As we have said, measuring red apples assumes that redness is a property of Fuji apples which my measurement on the sample faithfully reveals. However, if statistical independence is violated, as we have repeated many times, my finding a red apple in the sample does not depend on the apple actually being red but on my choice of measuring color rather than, say, shape. So, the measurement result does not reveal a property of the apple, but rather something about the type of measurement which was performed. In other words, that means that any theory violating statistical independence, in particular superdeterministic theories, are contextual: experimental results depend on the type of experiment being performed rather than on the actual pre-existing value of the property being measured.

[20] Hossenfelder and Palmer (2020) have replied that the examples above are connected with macroscopic objects, while superdeterministic theories require a violation of statistical independence at the microscopic level, implicitly suggesting that the effects of such a violation would be macroscopic 'washed away' by phenomena such as decoherence, so that we could use statistical independence for all practical purposes at the macroscopic level, even if it does not hold microscopically. Nonetheless, this reply does not address the issue of being able to make empirical sense of a microscopic theory that violates statistical independence (Chen 2021), and no reason is provided to support that decoherence actually acts as needed.

Another way of seeing this is the following. In general, assuming the violation of statistical independence is unwarranted: there is no reason why the choice of the experiment to be performed should influence the values that I observe as experimental result. One can explain the violation of statistical independence is to assume the existence of a correlation between the experimental outcomes and the type of experiment being chosen to be performed, when the experiments which are performed are not genuinely measuring anything about the system. That is, one can explain the violation of statistical independence by assuming that experimental results are contextual. For instance, consider a set of identically prepared unmoving particles, and imagine I wish to measure their position. To measure where they are I need to see them, so I hit them with photons. This operation might change where they are if the photon which I use have enough momentum to move them. That is, my acting on them in order to measure their position will give me a result which does not reflect their position before the experiment. In other words, my measurement was not faithful: the observed value does not represent the pre-existing value of the property of the system. Rather, it tells a story about how the system (the particles) has interacted with the apparatus (the photons). Instead, if one thinks that experimental results faithfully reveal the values of pre-existing properties, a violation of statistical independence is absurd. In fact, assuming that there is a set of these properties and that they can be faithfully measured (i.e., measurements reveal the pre-existing values of these properties), violating statistical independence would mean that the choice to measure property 1, rather than 2, determines what measurement results one is going to get as values for property 1. That is, assume I am about to make measurements on Fuji apples, and assume that I can measure their color or their shape. Then, if measurements reveal these properties and I chose to measure color, then the violation of statistical independence says that the color I discover depends on the fact that I have decided to measure color, rather than shape. But this contradicts the assumption that these properties can be faithfully measured: the results for property 1 should depend only on the actual values the system has for property 1, rather than anything else, including what I happened to choose to measure. In the apple example, the 'red' result should only depend on the redness of the apple, not on the fact that I have decided to investigate what color it is instead of what shape it has. This is because I take redness to be a property of the apple that experiments can reveal. So, *one cannot violate statistical independence if there are genuine properties whose values are faithfully revealed by the measurement*. That is, theories which violate statistical independence, and in particular superdeterministic theories, have to be contextual, in the sense that experiments never have the possibility of revealing anything about the system.[21] This, as a side remark, makes the role of experiments in superdeterministic

[21] A reviewer of this paper has claimed that the Ciepielewski *et al.* (2020) model is a counterexample of my argument that all superdeterministic models are contextual because its matter density ontology has the same role of the position in the pilot-wave theory and thus it is non-contextual. This does not seem to be the case because the matter density field is not measurable, in virtue of being a function of the wavefunction, as proven in Dürr *et al.* (2003). If so, the matter density field cannot represent a genuine property. In any case, I agree that the Ciepielewski *et al*. model is an interesting case study and should be further explored by those interested in superdeterministic theories.

theories rather mysterious: if they reveal nothing about the system, why one would perform them to start with?

5. Comparison

The last remark above, namely that superdeterministic hidden variable theories have to be contextual, allows us to draw a close comparison with the pilot-wave theory: both the pilot-wave theory and superdeterministic hidden variable theories are contextual; but while in the pilot-wave theory what explains the perfect correlations is nonlocality, in superdeterministic theories it is the falsity of statistical independence.

5.1 Contextuality

Even if they have both contextual, the contextuality in each theory is very different. In superdeterministic theories, unlike in the case of the pilot-wave theory, there is no explanation of the contextuality of the hidden variables. In the pilot-wave theory the contextuality of the properties is understood by saying that experiments have an active role: they change the system in a substantive way. That is, one has contextual properties because experiments are not measurements but active physical processes. By analyzing what happens physically in the experiment we can predict what this result could be, and in turn explain this contextuality. In superdeterminism contextuality does not come as a result of understating experiments like we just sketched in the pilot-wave framework. There is no analysis of what happens in these experiments at all. The only thing that we should expect is that the results will be contextual because this is what superdeterminism is designed to do. Contextuality comes from the desire to reproduce the data, not from an analysis of what an experimental apparatus does to the system. Namely, the system-apparatus interaction will explain the observed report. Therefore, superdeterministic contextuality is *ad hoc*.

5.2 Testability

As we have seen previously, the pilot-wave theory can give us information about unmeasured systems and can be tested because among all the contextual variables, there is one, namely position, which is not contextual. This non-contextual variable is the one which allows us to understand what experiments are telling us: they do not measure properties like spin, as EPR thought, but they measure where the particle was. Given that, we can test the theory by checking its prediction about where the particle is supposed to evolve to, with what it was experimentally found. In contrast, superdeterministic theories are neither informative nor testable because of their universal contextuality: there is no single non-contextual variable which can allow us to go from the information about the observed to the one about the unobserved. Thus, there is nothing in the measurement results that tells us anything about the system, so we cannot learn anything from experiments, and we cannot confirm or falsify the theory. This should be not very surprising, as superdeterministic are designed to violate statistical independence and hence random sampling, as discussed in section 4.

5.3 Locality for What?

The bottom line therefore is that the choice between the pilot-wave theory and a superdeterministic theory is respectively between a theory which violates locality and one which violates statistical independence. The costs of nonlocality are the following: it is a mystery how the interaction can be instantaneous, and it is unclear whether systems can be thought as isolated if nonlocality is true (even if arguably decoherence can give an answer to that). The costs of superdeterminism are instead that it leads to conspiratorial, fine-tuned theories in which we cannot use induction, making the theory uninformative and untestable. Superdeterminists want to argue that these costs are worth keeping locality because we need locality to make quantum theory and relativity compatible. Even so, it is unclear how a local theory such as a superdeterminist theory can help in making any theory compatible with any other, given that what superdeterminism requires is the rejection of inductive reasoning. So, if superdeterministic theories have a problem of making sense of any scientific theory, it is very dubious that they can provide any insight into making quantum theory compatible with relativity.

To conclude, I think that superdeterminsm looks like an instance of the Quine Duhem thesis: you can always reject something to save what you want. So, if you want to save locality, there is always a way. All theories are underdetermined in this way. Nonetheless, I argue, one can break the underdetermination using sensible criteria, with a clear victory of the pilot-wave theory. In fact, the pilot-wave theory is a theory which is similar to past theories: as in classical mechanics, macroscopic objects are made of microscopic particles and their interaction is nonlocal; the only main difference is that such nonlocality is not attenuated by distance. Therefore, it requires a minimal change from previous theories, it is coherent with previous ways of understanding and explaining the phenomena, reductively and compositionally, it is simple enough, it is not convoluted, it is lawful, as well as testable. Instead, I have argued here that superdeterminism fails all these criteria: it requires a large departure from the way we understood explanation, confirmation, falsification and theory testing, it is complex, unlawful, fine-tuned.

6. Conclusions

Let's grant that Bell's theorem has proven that reality is nonlocal. One theory which respects this theorem is the pilot-wave theory, a hidden variable theory which is explicitly nonlocal. In this theory position is the only genuine property and in general experiments do not measure something other than the system-apparatus interaction. That is, operators represent contextual properties, which are not genuine natural properties. Some have tried to resist the nonlocality conclusion by rejecting the hypothesis of statistical independence: all experimental results are determined by the type of experiment we wanted to make. These superdeterministic hidden variable theories would then be local, but they would be such that making experiments on a sample will not give us information about the type of system under investigation. I have shown that this makes superdeterministic theories uninformative, unfalsifiable, and unconfirmable. Moreover, rejecting statistical independence makes sense only assuming that all hidden variables are contextual: they are not genuine properties which can be revealed by measurements. Nonetheless, even if the pilot-wave theory and superdeterminist theories are both contextual, their similarities end there. And while there is a reason for the contextuality of the pilot-wave theory, no such reason exists in superdeterministic theories, whose contextuality is therefore mysterious and *ad hoc*. It has been argued that retaining locality would be a desideratum for making quantum mechanics and relativity compatible. However, since locality has to come together with superdeterminism, it is not going to help with much at all. Therefore, I believe that there are no valid reasons to endorse superdeterminism.

Acknowledgements

I am thankful to Jean Bricmont and Travis Norsen for their insightful discussions about these topics, as well as to three anonymous reviewers whose sharp comments have helped me write a better paper.

References

- Albert, D. Z. (1992). *Quantum mechanics and experience*. Harvard University Press.
- Allori, V. (2013). Primitive Ontology and the Structure of Fundamental Physical Theories. In D. Z. Albert & A. Ney (Eds.). *The wave function: essays in the metaphysics of quantum mechanics* (pp. 58-75). Oxford University Press
- Allori, V. & Zanghì, N. (2004). What is Bohmian Mechanics. *International Journal of Theoretical Physics* 43, 1743-1755.
- Aspect, A., Dalibard, J., & Roger, G. (1982). Experimental test of Bell's inequalities using time-varying analyzers. *Physical Review Letters* 49(25), 1804.
- Baas, A. & LeBihan, B. (2021). What does the world look like according to superdeterminism? *The British Journal for the Philosophy of Science*. https://doi.org/10.1086/714815
- Bacciagaluppi, G. & Valentini, A. (2009). *Quantum theory at the crossroads: reconsidering the 1927 Solvay conference*. Cambridge: Cambridge University Press.
- Barrett, J. A. (1996). Empirical adequacy and the availability of reliable records in quantum mechanics. *Philosophy of Science* 63(1), 49-64.
- Bell, J. S. (1964). On the Einstein-Rosen-Podolsky paradox." *Physics* 1(3): 195–200.
- Bell, J. S. (1966). On the problem of hidden variables in quantum mechanics. *Review of Modern Physics* 38, 447–452.
- Bricmont, J., Goldstein, S., & Hemmick, D. (2022). From EPR-Schrödinger paradox to nonlocality based on perfect correlations. *Foundations of Physics* 52(3), 53.
- Bohm, D. (1952). A suggested interpretation of the quantum theory in terms of 'hidden' variables. *Physical Review* 85, 166-193.
- Chen, E. K. (2021). Bell's Theorem, Quantum Probabilities, and Superdeterminism. In E. Knox & A. Wilson (Eds.). *The Routledge companion to the philosophy of physics*. Routledge.
- Ciepielewski, G. S., Okon, E., & Sudarsky, D. (2021). On superdeterministic rejections of settings independence. *The British Journal for the Philosophy of Science*. https://doi.org/10.1086/714819
- Clauser, J. F., Horne, M. A., Shimony, A. & Holt, R. A. (1969). Proposed experiment to test local hidden-variable theories. *Physical Review Letters* 23(15), 880.
- Conway, J., & Kochen, S. (2006). The Free Will Theorem. *Foundations of Physics* 36(10): 1441.
- Davies, P.C.W., & Brown, J.R. (1993). *The ghost in the atom: a discussion of the mysteries of quantum physics*. Cambridge: Cambridge University Press.
- Daumer, M., Dürr, D., Goldstein, S. & Zanghì, N. (1996). Naïve realism about operators. *Erkenntnis* 45, 379–397.
- Dürr, D., Goldstein, S., & Zanghì, N. (2004). Quantum equilibrium and the role of operators as observables in quantum theory. *Journal of Statistical Physics* 116, 959-1055.
- Dürr, D., & Teufel, S. (2009). *Bohmian mechanics: the physics and mathematics of quantum theory*. Berlin: Springer.
- Einstein, Albert, Boris Podolsky, and Nathan Rosen. 1935. "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" *Physical Review* 47(10):777.

- Gleason, A. M. (1957). Measures on the closed subspaces of a Hilbert space. *Journal of Mathematics and Mechanics* 6, 885–893.
- Goldstein, S., Norsen, T., Tausk, D. V., & Zanghì, N. (2011). Bell's Theorem. *Scholarpedia* 6(10), 8378.
- Heisenberg, W. (1949). *The physical principles of the quantum theory*. New York: Dover.
- Hossenfelder, S., & Palmer. T. (2020). Rethinking superdeterminism. *Frontiers in Physics* 8,139.
- Kochen, S., & Specker, E.P. (1967). The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics* 17, 59–87.
- Maudlin, T. (2019). *Bell's other assumption(s)*. Retrieved from: https://www.youtube.com/watch?v=9VFrDcHnk2c
- Shimony, A., Horne, M.A., & Clauser, J.F. (1976). Comment on 'The Theory of Local Beables'. *Epistemological Letters* 13(1).
- t'Hooft, G. (2016). *The cellular automaton interpretation of quantum mechanics.* Spinger.
- t'Hooft, G. (2022). Explicit construction of local hidden variables for any quantum theory up to any desired accuracy. In V. Allori (Ed.). *Quantum mechanics and fundamentality* (pp.175-191). Spinger.
- Von Neumann, J. (1955). *Mathematical foundations of quantum mechanics*. Princeton: Princeton University Press.