# A $_{\text{Signal Analysis}}^{\text{Field \&}}$ Approach to Quantum Measurement

Peter Morgan

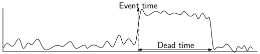
Yale University
*peter.w.morgan@yale.edu*

May 17th, 2023

Lisbon Philosophy of Physics Seminar (by Zoom)

A recording is available on YouTube at https://www.youtube.com/watch?v=5Jx2WIa5eTs

# outline

- Devices $\longrightarrow$ *noisy* signals $<$ events $\longrightarrow$ data / *noisy* surroundings 

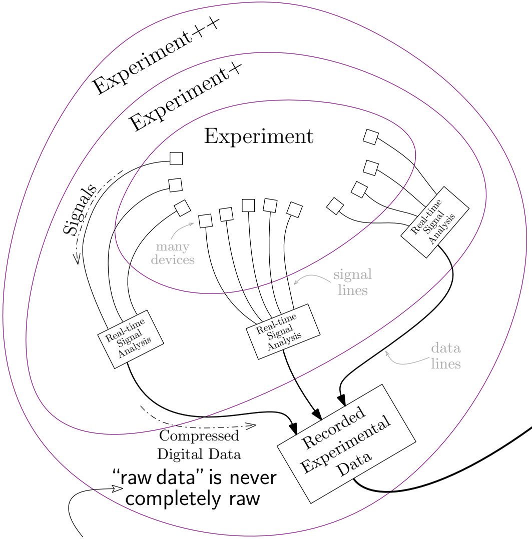
- The violation of Bell inequalities — signal analysis & field theory
- Classical mechanics — *add* noncommutativity & quantum noise and discuss analyticity
- The measurement problem — joint probabilities
- Generalized Probability — as a way to discuss intervention & causality
- Quantum and QND fields — modulation & measurement
- Interacting quantum fields — a signal analysis approach

An evolution of ideas in:

"Classical states, quantum field measurement", Physica Scripta 2019
"An algebraic approach to Koopman classical mechanics", Annals of Physics 2020
"The collapse of a quantum state as a joint probability construction", Journal of Physics A 2022
"A source fragmentation approach to interacting quantum field theory", arXiv:2109.04412

and, ancient history, "Bell inequalities for random fields", Journal of Physics A 2006

# signal&data analysis




At first, quite operational, but, by the end, quite realist

Different theory frameworks: same experiment ⟶ same data

*but* what we can imagine and design and choose to perform changes

A signal&data approach is well-suited to machine learning

Let *some* preconceptions go and let more abstract algorithms do the heavy lifting instead

Post-analysis

Supplementary Information

Write a paper
⟶journal editor
⟶physicists & engineers

Bokulich&Parker 2021 Morgan&Morrison, "Models as Mediators" Borges, "On Exactitude in Science" Martínez-Ordaz 2023

Bohr 1921: "in a certain respect we are entitled in the quantum theory to see an attempt of a natural generalisation of the classical theory of electromagnetism."
[Peter&Alisa Bokulich 2005]

Bohr 1949: "It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms."
[Zinkernagel 2015]

Dirac 1949: "My own opinion is that we ought to search for a way of making fundamental changes not only in our present Quantum Mechanics, but actually in Classical Mechanics as well."
[Alisa Bokulich 2004]

Bohr 1921: "in a certain respect we are entitled in the quantum theory to see an attempt of a natural generalisation of the classical theory of electromagnetism."
Peter&Alisa Bokulich 2005

Bohr 1949: "It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms."
Zinkernagel 2015

Dirac 1949: "My own opinion is that we ought to search for a way of making fundamental changes not only in our present Quantum Mechanics, but actually in Classical Mechanics as well."
Alisa Bokulich 2004

Bell 1975: "'Observables' must be *made*, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables."

Use the Poisson bracket to *make* new 'observables'

the *opposite* of adding hidden variables to quantum mechanics

---

Bell 1990, "Against 'measurement'": "experiments have results."

We *collate* an Experimental Dataset into multiple Measurement Datasets, operationally, by device and by analysis, *not* as "measurements of particle properties"

Quantum Field Theory and Signal Analysis are both grounded in actually recorded measurement results,
which are about the noisy signals on the signal lines out of devices,
which indicate *something* about the devices' surroundings, *whatever that is*

Quantum Field Theory and Signal Analysis are both grounded in actually recorded measurement results,
which are about the noisy signals on the signal lines out of devices,
which indicate *something* about the devices' surroundings, *whatever that is*

We take quantum field theories to be our best theories
but we *still* take particle properties to cause events
suggestion(1): *hesitate* before mentioning particles or systems
suggestion(2): *hesitate* before mentioning a field *that is measured*
A quantum field has a $\widehat{\text{hat}}$ because it is a field *of measurement operators*

Quantum Field Theory and Signal Analysis are both grounded in actually recorded measurement results,
which are about the noisy signals on the signal lines out of devices,
which indicate *something* about the devices' surroundings, *whatever that is*

We take quantum field theories to be our best theories
but we *still* take particle properties to cause events
suggestion(1): *hesitate* before mentioning particles or systems
suggestion(2): *hesitate* before mentioning a field *that is measured*
A quantum field has a $\widehat{\text{hat}}$ because it is a field *of measurement operators*

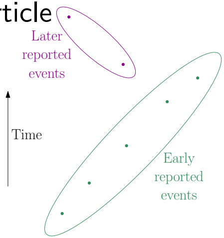
About noise: Quantum noise (cf "shot" noise) is different from Thermal noise (see #16)

# algorithms for signals, events, and particles

devices ⟶ *noisy* signals < events ⟶ data / *noisy* surroundings } ⟶ *perhaps* particles

If we have data about many millions of events, we have to write algorithms that decide how to assign each event to a particle

If we add data about more events, the assignment of events to particles will sometimes be fragile

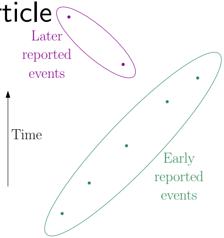



Events-to-particles algorithms are global, after-the-events, and fragile

# algorithms for signals, events, and particles

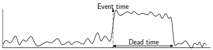
devices ⟶ *noisy* signals ⋜ { events ⟶ data ; *noisy* surroundings } ⟶ *perhaps* particles

If we have data about many millions of events, we have to write
algorithms that decide how to assign each event to a particle
If we add data about more events, the assignment
of events to particles will sometimes be fragile




Events-to-particles algorithms are
global, after-the-events, and fragile

Signal-to-events algorithms, often implemented in hardware,
are non-Markovian because events must be reported only once
but are less fragile and nonlocal than events-to-particles algorithms because
adding data about more events doesn't change the other events



For QM, we have measurements $\hat{M}_1, \ldots, \hat{M}_n$, which is uninformative unless
we have a list of metadata $\mathit{Description}_1, \ldots, \mathit{Description}_n$, which should be
enough for another experimenter to reproduce the measurement results
We could write $\hat{M}_{\mathit{Description}_1}, \ldots, \hat{M}_{\mathit{Description}_n}$

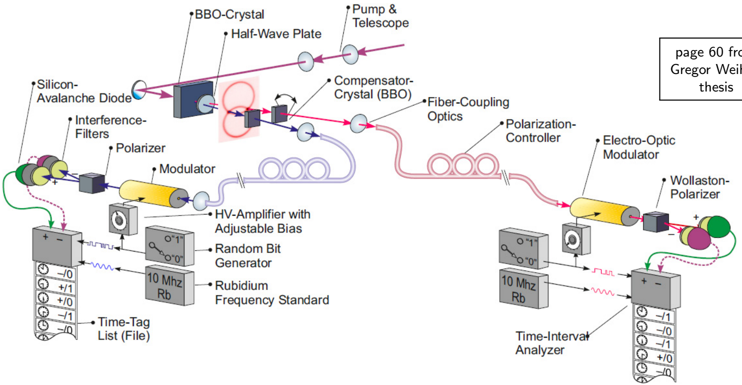
For QFT, measurement operators are not point-like: we use $\hat{M}_f = \int \hat{M}(x) f(x) \mathrm{d}^4 x$
We have measurements $\hat{M}_{f_1}, \hat{M}_{f_2}, \ldots, \hat{M}_{f_n}$, where $f_1, f_2, \ldots, f_n$
are *smearing functions*, *test functions*, *window functions*, or ...,
as descriptions of how a measurement is different from point-like

For QM, we have measurements $\hat{M}_1, \ldots, \hat{M}_n$, which is uninformative unless we have a list of metadata $\mathcal{D}escription_1, \ldots, \mathcal{D}escription_n$, which should be enough for another experimenter to reproduce the measurement results
We could write $\hat{M}_{\mathcal{D}escription_1}, \ldots, \hat{M}_{\mathcal{D}escription_n}$

For QFT, measurement operators are not point-like: we use $\hat{M}_f = \int \hat{M}(x) f(x) \mathrm{d}^4 x$
We have measurements $\hat{M}_{f_1}, \hat{M}_{f_2}, \ldots, \hat{M}_{f_n}$, where $f_1, f_2, \ldots, f_n$ are *smearing functions*, *test functions*, *window functions*, or ..., as descriptions of how a measurement is different from point-like

- QFT: $\hat{M}_f$ commutes with $\hat{M}_g$ if $f(x)$ and $g(x)$ are $\genfrac{}{}{0pt}{}{\text{causally}}{\text{space-like}}$ separated
- QNDFT: $\hat{\mathtt{M}}_f^{\text{QND}}$ *always* commutes with $\hat{\mathtt{M}}_g^{\text{QND}}$
  *Quantum Non-Demolition Field Theory* (what I have called a *random field theory*)
  For quantum optics∼QNDFT, see #27

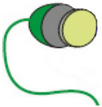
Alice and Bob both have two Avalanche PhotoDiodes,
an Electro-Optic Modulator, a Random Bit Generator, and a clock;
a central apparatus modulates the ground state




page 60 from Gregor Weihs's thesis

The time when an APD's signal rises to a higher level is recorded, and
which APD it was, and what the EOM setting was: when and 2 bits
This compressed record does not let us analyze any other signal details

# the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD 

An APD is not made from ordinary clay (An APD is not *conscious*, but it *is* complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level




That's more interesting than using devices that do nothing
An APD knows nothing about particles, but it does get cross

An APD's electronics calms it down so it can burble again

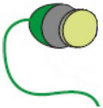
An APD sometimes gets cross with itself even when it's dark and quiet

An APD gets cross differently if we intervene to change its surroundings

An APD tells a story even when it's not cross (which Zlatko listened to, Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there

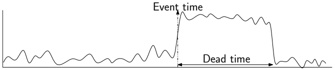
An APD's frequency scales:
optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz

An engineer worked hard to create an APD 

An APD is not made from ordinary clay (An APD is not *conscious*, but it *is* complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level




That's more interesting than using devices that do nothing
An APD knows nothing about particles, but it does get cross

An APD's electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it's dark and quiet

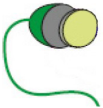
An APD gets cross differently if we intervene to change its surroundings

An APD tells a story even when it's not cross (which Zlatko listened to, Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there

An APD's frequency scales:

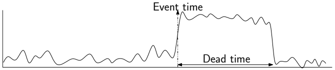
optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz

An engineer worked hard to create an APD 

An APD is not made from ordinary clay (An APD is not *conscious*, but it *is* complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level




That's more interesting than using devices that do nothing

An APD knows nothing about particles, but it does get cross

An APD's electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it's dark and quiet

An APD gets cross differently if we intervene to change its surroundings

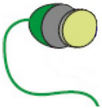
An APD tells a story even when it's not cross (which Zlatko listened to, Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there

An APD's frequency scales:

optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz

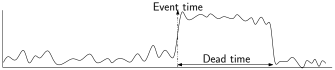
# the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD 

An APD is not made from ordinary clay (An APD is not *conscious*, but it *is* complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level




That's more interesting than using devices that do nothing
An APD knows nothing about particles, but it does get cross

An APD's electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it's dark and quiet

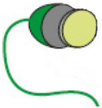
An APD gets cross differently if we intervene to change its surroundings

An APD tells a story even when it's not cross (which Zlatko listened to, Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there

An APD's frequency scales:
optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz

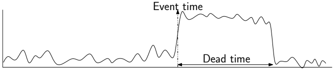
# the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD 

An APD is not made from ordinary clay (An APD is not *conscious*, but it *is* complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level




That's more interesting than using devices that do nothing
An APD knows nothing about particles, but it does get cross

An APD's electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it's dark and quiet

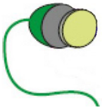
An APD gets cross differently if we intervene to change its surroundings

An APD tells a story even when it's not cross (which Zlatko listened to, Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there

An APD's frequency scales:
optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz

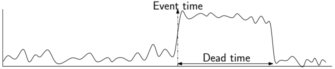
# the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD 

An APD is not made from ordinary clay (An APD is not *conscious*, but it *is* complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level




That's more interesting than using devices that do nothing
An APD knows nothing about particles, but it does get cross

An APD's electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it's dark and quiet

An APD gets cross differently if we intervene to change its surroundings

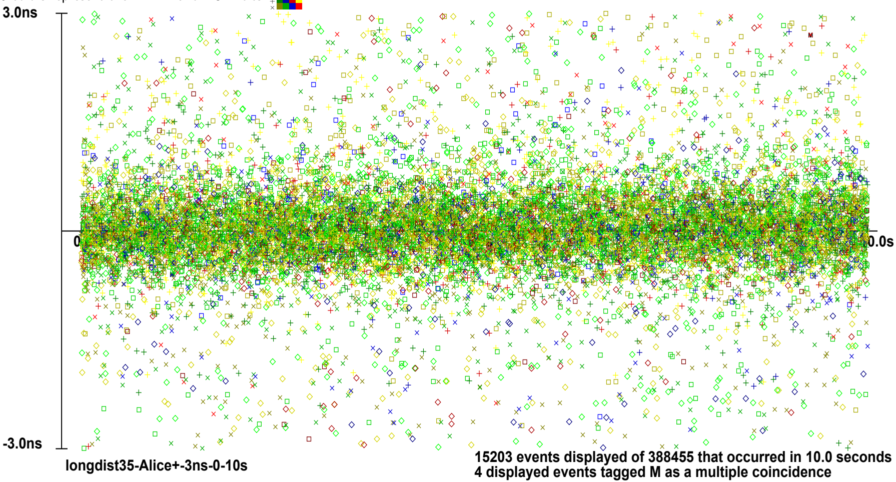
An APD tells a story even when it's not cross (which Zlatko listened to, Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there

An APD's frequency scales:
optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz

# Gregor gets measurement results (Alice sees almost 400,000 APD events in 10 seconds)

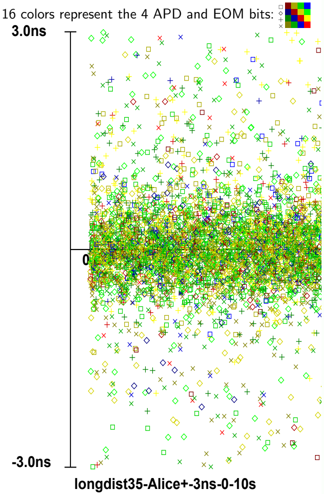
16 colors represent the 4 APD and EOM bits: (brightness represents Alice's two bits, shapes represent Bob's two bits, red is the diagonal, ...)

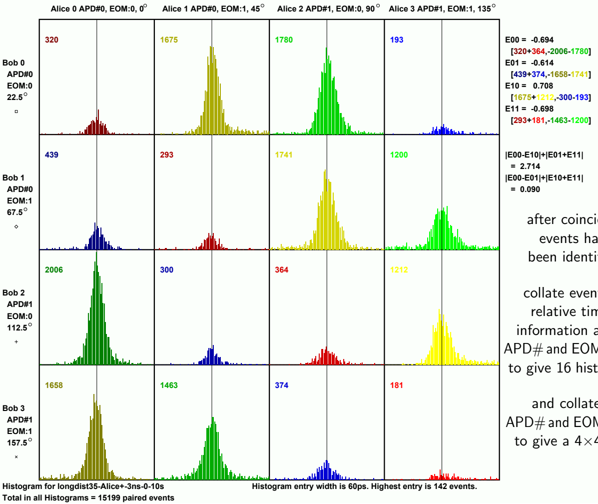



For over 15,000 of Alice's 400,000 events, Bob also records an event within 3 nanoseconds

When Alice and Bob both record an event within 3 nanoseconds, the majority are green or yellow

# Gregor gets measurement results (Alice sees almost 400,000 APD events in 10 seconds)




after coincident events have been identified,

collate events by relative timing information and by APD# and EOM setting, to give 16 histograms

and collate by APD# and EOM setting to give a 4×4 table

# transformations and noncommutativity

If we had transformed the recorded experimental data innocuously
we could have used commutative algebras to model the algorithms

In QM, we model Bell-violating statistics using noncommuting operators
Fine 1982 Landau 1987

In CM as usual, we do not have noncommuting operators

If we had transformed the recorded experimental data innocuously
we could have used commutative algebras to model the algorithms

In QM, we model Bell-violating statistics using noncommuting operators
Fine 1982 Landau 1987

In CM as usual, we do not have noncommuting operators
Without noncommutativity, CM is computationally incomplete

How can we add noncommutativity to CM?

# transformations and noncommutativity

If we had transformed the recorded experimental data innocuously
we could have used commutative algebras to model the algorithms

In QM, we model Bell-violating statistics using noncommuting operators
Fine 1982 Landau 1987

In CM as usual, we do not have noncommuting operators
Without noncommutativity, CM is computationally incomplete

How can we add noncommutativity to CM?

*About (non)locality:*

Alice&Bob's Electro-Optic Modulation could be $\lll\sim$1MHz,
nonetheless giving approximately the same $4\times4$ table of numbers
For elementary QM models, the EOM rate makes no difference at all,
but a low EOM rate does not probe (non)locality

For quantum fields, locality is *closely* associated with measurement incompatibility
because *microcausality* only allows noncommutativity at time-like separation

# Gregor's experiment in a modulated non-steady-state form

At a fine-grained scale, Gregor's experiment is *not* at equilibrium

At a coarse-grained scale, Gregor's experiment *is* at equilibrium

*About (non)locality:* thermodynamic equilibrium depends on boundary conditions

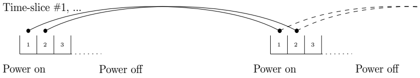
# Gregor's experiment in a modulated non-steady-state form



At a fine-grained scale, Gregor's experiment is *not* at equilibrium

At a coarse-grained scale, Gregor's experiment *is* at equilibrium

*About (non)locality:* thermodynamic equilibrium depends on boundary conditions




What happens when we first turn on the power?

The event rate increases in each of the four APDs

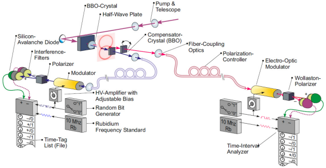
The coincident event rate increases

The violation of Bell inequalities increases at some rate

Collate by absolute timing information after power on

Are these rates the same, or how are they different, and how do these rates change at different distances?

These rates are technologically important as well as foundationally significant




Some experiments make more sense as signal analysis than as probes of particle properties

# algebraic QM and CM

There are *abstract* measurements $\hat{M}_1, \hat{M}_2, \hat{M}_3, \ldots, \hat{M}_1 + \hat{M}_2, \ldots, \hat{M}_1 \hat{M}_2, \ldots$

linear operators≡random variables, spectrum≡sample space, noncommutative or commutative, associative, distributive, with unit

With no dynamics, the tradition is: QM=noncommutative, CM=commutative

# algebraic QM and CM

There are *abstract* measurements $\hat{M}_1, \hat{M}_2, \hat{M}_3, \ldots, \hat{M}_1+\hat{M}_2, \ldots, \hat{M}_1\hat{M}_2, \ldots$

linear operators≡random variables, spectrum≡sample space, noncommutative or commutative, associative, distributive, with unit

With no dynamics, the tradition is: QM=noncommutative, CM=commutative

A (statistical) *state* $\rho$ maps measurement operators to *expected measurement results*

$$\rho(\hat{M}_1), \rho(\hat{M}_2), \rho(\hat{M}_3), \ldots, \rho(\hat{M}_1+\hat{M}_2), \ldots, \rho(\hat{M}_1\hat{M}_2), \ldots, \quad \rho(\hat{M}_1^n), \ldots, \begin{matrix}\rho(\delta(\hat{M}_1-u))\\ \rho(\mathrm{e}^{\mathrm{j}\lambda\hat{M}_1})\end{matrix}$$

positive: $\rho(\hat{A}^\dagger\hat{A})\ge 0$; normalized: $\rho(1)=1$;
von Neumann linearity: $\rho(\lambda\hat{A}+\mu\hat{B})=\lambda\rho(\hat{A})+\mu\rho(\hat{B})$
compatible with the adjoint: $\rho(\hat{A}^\dagger)=\rho(\hat{A})^*$; where $(\hat{A}\hat{B})^\dagger=\hat{B}^\dagger\hat{A}^\dagger$

# algebraic QM and CM

There are *abstract* measurements $\hat{M}_1, \hat{M}_2, \hat{M}_3, \ldots, \hat{M}_1+\hat{M}_2, \ldots, \hat{M}_1\hat{M}_2, \ldots$

linear operators≡random variables, spectrum≡sample space, noncommutative or commutative, associative, distributive, with unit

With no dynamics, the tradition is: QM=noncommutative, CM=commutative

A (statistical) *state* $\rho$ maps measurement operators to *expected measurement results*

$$\rho(\hat{M}_1), \rho(\hat{M}_2), \rho(\hat{M}_3), \ldots, \rho(\hat{M}_1+\hat{M}_2), \ldots, \rho(\hat{M}_1\hat{M}_2), \ldots, \quad \rho(\hat{M}_1^n), \ldots, \begin{matrix}\rho(\delta(\hat{M}_1-u))\\ \rho(\mathrm{e}^{\mathrm{j}\lambda\hat{M}_1})\end{matrix}$$

positive: $\rho(\hat{A}^\dagger\hat{A})\geq 0$; normalized: $\rho(1)=1$;
von Neumann linearity: $\rho(\lambda\hat{A}+\mu\hat{B})=\lambda\rho(\hat{A})+\mu\rho(\hat{B})$
compatible with the adjoint: $\rho(\hat{A}^\dagger)=\rho(\hat{A})^*$; where $(\hat{A}\hat{B})^\dagger=\hat{B}^\dagger\hat{A}^\dagger$

We can also use measurement operators to *modulate* the state $\rho$ to give different expected measurement results, $\rho_A(\hat{M})=\frac{\rho(\hat{A}^\dagger\,\hat{M}\,\hat{A})}{\rho(\hat{A}^\dagger\hat{A})}$,
from which the GNS-construction gives us a Hilbert space

The GNS-construction lets us think of $\rho_v(\hat{M})$ as $\langle v|\hat{M}|v\rangle$ and of $\rho_{Av}(\hat{M})$ as $\frac{\langle v|\hat{A}^\dagger\,\hat{M}\,\hat{A}|v\rangle}{\langle v|\hat{A}^\dagger\hat{A}|v\rangle}$

Take classical mechanics to be an algebra of functions on phase space
that has *three* binary operations:
addition, multiplication, *and* the Poisson bracket

$$u+v$$
$$u\cdot v$$
$$\{u,v\}$$

Take classical mechanics to be an algebra of functions on phase space
that has *three* binary operations:
addition, multiplication, *and* the Poisson bracket $u+v$, $u\cdot v$, $\{u,v\}$

We can introduce "Multiply by $w$", $\hat{Y}_w(u)=w\cdot u$,
and "Poisson by $w$", $\hat{Z}_w(u)=\{w,u\}$

A familiar example: "Poisson by the *Hamiltonian function*" gives
a generator of time evolution, $\hat{Z}_H(u)=\{H,u\}$, the *Liouvillian* operator

We have $[\hat{Y}_v,\hat{Y}_w]=0$, but $[\hat{Z}_v,\hat{Y}_w]=\hat{Y}_{\{v,w\}}\neq 0$ and $[\hat{Z}_v,\hat{Z}_w]=\hat{Z}_{\{v,w\}}\neq 0$,
generating a noncommutative algebra with addition and composition

Take classical mechanics to be an algebra of functions on phase space
that has *three* binary operations:
addition, multiplication, *and* the Poisson bracket

$$u+v \quad u\cdot v \quad \{u,v\}$$

We can introduce "Multiply by $w$", $\hat{Y}_w(u)=w\cdot u$,
and "Poisson by $w$", $\hat{Z}_w(u)=\{w,u\}$
A familiar example: "Poisson by the *Hamiltonian function*" gives
a generator of time evolution, $\hat{Z}_H(u)=\{H,u\}$, the *Liouvillian* operator

We have $[\hat{Y}_v,\hat{Y}_w]=0$, but $[\hat{Z}_v,\hat{Y}_w]=\hat{Y}_{\{v,w\}}\neq 0$ and $[\hat{Z}_v,\hat{Z}_w]=\hat{Z}_{\{v,w\}}\neq 0$,
generating a noncommutative algebra with addition and composition

*I suggest:*
We can use the $\hat{Y}$'s *and* $\hat{Z}$*'s* of a more powerful $CM_+$ without restriction
We do not *have to* follow the way of quantization and the Correspondence Principle
if what we want is noncommutativity and measurement incompatibility
and an algebraic measurement theory shared with QM

The Poisson bracket: $\{u,v\} = \frac{\partial u}{\partial p}\frac{\partial v}{\partial q} - \frac{\partial u}{\partial q}\frac{\partial v}{\partial p}$

We work with the transformations generated by the Poisson bracket, not with the Poisson bracket directly
$\{u,v\} \not\leftrightsquigarrow [\hat{u},\hat{v}]$

$$\hat{Y}_q[u] = q\cdot u,\quad \hat{Z}_p[u] = \{p,u\} = \ \frac{\partial}{\partial q}u,\quad [\hat{Y}_q,\hat{Z}_p] = -1$$

$$\hat{Y}_p[u] = p\cdot u,\quad \hat{Z}_q[u] = \{q,u\} = -\frac{\partial}{\partial p}u,\quad [\hat{Y}_p,\hat{Z}_q] = \ 1$$

$$\hat{Y}_H[u] = \tfrac{1}{2}(q^2+p^2)\cdot u,\quad \hat{Z}_H[u] = \{H,u\} = \left(p\cdot\frac{\partial}{\partial q} - q\cdot\frac{\partial}{\partial p}\right)u$$

The Poisson bracket: $\{u,v\} = \frac{\partial u}{\partial p}\frac{\partial v}{\partial q} - \frac{\partial u}{\partial q}\frac{\partial v}{\partial p}$

We work with the transformations generated by the Poisson bracket, not with the Poisson bracket directly
$\{u,v\} \not\leftrightarrow [\hat{u},\hat{v}]$

$\hat{Y}_q[u] = q\cdot u,\quad \hat{Z}_p[u] = \{p,u\} = \frac{\partial}{\partial q}u,\quad [\hat{Y}_q,\hat{Z}_p] = -1$

$\hat{Y}_p[u] = p\cdot u,\quad \hat{Z}_q[u] = \{q,u\} = -\frac{\partial}{\partial p}u,\quad [\hat{Y}_p,\hat{Z}_q] = 1$

$\hat{Y}_H[u] = \frac{1}{2}(q^2+p^2)\cdot u,\quad \hat{Z}_H[u] = \{H,u\} = \left(p\cdot\frac{\partial}{\partial q} - q\cdot\frac{\partial}{\partial p}\right)u$

The Gibbs thermal state at temperature kT (in a generating function form, introducing j):

$$\rho_{\mathrm{Gibbs}}\left(\mathrm{e}^{\mathrm{j}\lambda\hat{Y}_q+\mathrm{j}\mu\hat{Y}_p}\right) = \mathrm{e}^{-\mathsf{kT}(\lambda^2+\mu^2)/2}$$

We work with the transformations generated by the Poisson bracket, not with the Poisson bracket directly
$\{u,v\} \not\leftrightsquigarrow [\hat{u},\hat{v}]$

The Poisson bracket: $\{u,v\}=\frac{\partial u}{\partial p}\frac{\partial v}{\partial q}-\frac{\partial u}{\partial q}\frac{\partial v}{\partial p}$

$$\hat{Y}_q[u]=q\cdot u,\quad \hat{Z}_p[u]=\{p,u\}=\ \frac{\partial}{\partial q}u,\quad [\hat{Y}_q,\hat{Z}_p]=-1$$

$$\hat{Y}_p[u]=p\cdot u,\quad \hat{Z}_q[u]=\{q,u\}=-\frac{\partial}{\partial p}u,\quad [\hat{Y}_p,\hat{Z}_q]=\ 1$$

$$\hat{Y}_H[u]=\tfrac{1}{2}(q^2+p^2)\cdot u,\quad \hat{Z}_H[u]=\{H,u\}=\left(p\cdot\frac{\partial}{\partial q}-q\cdot\frac{\partial}{\partial p}\right)u$$

The Gibbs thermal state at temperature kT (in a generating function form, introducing j):

$$\rho_{\mathrm{Gibbs}}\left(\mathrm{e}^{\mathrm{j}\lambda\hat{Y}_q+\mathrm{j}\mu\hat{Y}_p}\right)=\mathrm{e}^{-\mathrm{kT}(\lambda^2+\mu^2)/2},\quad \rho_{\mathrm{Gibbs}}\left(\mathrm{e}^{\alpha\hat{Z}_p+\beta\hat{Z}_q}\right)=\mathrm{e}^{-(\alpha^2+\beta^2)/8\mathrm{kT}}$$

set $\hat{Y}_q=(a+a^\dagger)\sqrt{\mathrm{kT}}$, $\hat{Z}_p=\dfrac{(a-a^\dagger)}{2\sqrt{\mathrm{kT}}}$, $[a,a^\dagger]=1$, ensuring $[\hat{Y}_q,\hat{Z}_p]=-1$, and we set $a|_{\mathrm{kT}}\rangle=0$ (and $b|_{\mathrm{kT}}\rangle=0$ &c for $\hat{Y}_p$ and $\hat{Z}_q$)

We can construct modulated, non-equilibrium states, $\dfrac{\langle_{\mathrm{kT}}|\hat{A}^\dagger\,\hat{M}\,\hat{A}|_{\mathrm{kT}}\rangle}{\langle_{\mathrm{kT}}|\hat{A}^\dagger\hat{A}|_{\mathrm{kT}}\rangle}$, and hence a Hilbert space

We work with the transformations generated by the Poisson bracket, not with the Poisson bracket directly
$\{u, v\} \not\leftrightarrow [\hat{u}, \hat{v}]$

The Poisson bracket: $\{u, v\} = \frac{\partial u}{\partial p}\frac{\partial v}{\partial q} - \frac{\partial u}{\partial q}\frac{\partial v}{\partial p}$

$$\hat{Y}_q[u] = q \cdot u, \quad \hat{Z}_p[u] = \{p, u\} = \frac{\partial}{\partial q}u, \quad [\hat{Y}_q, \hat{Z}_p] = -1$$

$$\hat{Y}_p[u] = p \cdot u, \quad \hat{Z}_q[u] = \{q, u\} = -\frac{\partial}{\partial p}u, \quad [\hat{Y}_p, \hat{Z}_q] = 1$$

$$\hat{Y}_H[u] = \tfrac{1}{2}(q^2 + p^2) \cdot u, \quad \hat{Z}_H[u] = \{H, u\} = \left(p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p}\right) u$$

The Gibbs thermal state at temperature kT (in a generating function form, introducing j):

$$\rho_{\mathrm{Gibbs}}\left(\mathrm{e}^{\mathrm{j}\lambda\hat{Y}_q + \mathrm{j}\mu\hat{Y}_p}\right) = \mathrm{e}^{-\mathrm{kT}(\lambda^2+\mu^2)/2}, \quad \rho_{\mathrm{Gibbs}}\left(\mathrm{e}^{\alpha\hat{Z}_p + \beta\hat{Z}_q}\right) = \mathrm{e}^{-(\alpha^2+\beta^2)/8\mathrm{kT}}$$

set $\hat{Y}_q = (a + a^\dagger)\sqrt{\mathrm{kT}}$, $\hat{Z}_p = \frac{(a - a^\dagger)}{2\sqrt{\mathrm{kT}}}$, $[a, a^\dagger] = 1$, ensuring $[\hat{Y}_q, \hat{Z}_p] = -1$, and we set $a|_{\mathrm{kT}}\rangle = 0$
(and $b|_{\mathrm{kT}}\rangle = 0$ &c for $\hat{Y}_p$ and $\hat{Z}_q$)

We can construct modulated, non-equilibrium states, $\frac{\langle_{\mathrm{kT}}|\hat{A}^\dagger\,\hat{M}\,\hat{A}|_{\mathrm{kT}}\rangle}{\langle_{\mathrm{kT}}|\hat{A}^\dagger\hat{A}|_{\mathrm{kT}}\rangle}$, and hence a Hilbert space

Instead of trying to map $(q, p) \not\leftrightarrow (\hat{q}, \hat{p})$, as quantization tries to *(but fails)*,
we *can* map $\mathrm{CM}_+$ to QM, $(q, \mathrm{j}\frac{\partial}{\partial q}) \mapsto (\hat{q}_1, \hat{p}_1)$, $(p, \mathrm{j}\frac{\partial}{\partial p}) \mapsto (\hat{q}_2, \hat{p}_2)$
Crucially, kT is not $\hbar$, but it is also about an *irreducible* noise

# quantum and thermal noise



What is the difference between quantum and thermal noise?

- $\hbar$ has units action, whereas kT has units energy
- In QFT, the quantum vacuum is Poincaré invariant, thermal noise is not
  This difference of symmetry properties *can* be used in $\mathrm{CM}_+$
- Adopting this for $\mathrm{CM}_+$, $\hbar$ is an amplitude of Poincaré invariant noise
  kT is an amplitude of thermal noise

This gives a new reason to think that we must work with field theories,
because we can only define the Lorentz group in $1{+}n$-dimensions

$\hbar \to 0$ is a mean-field approximation, *not* a classical approximation

For the Gibbs state of the Simple Harmonic Oscillator,
$\hat{Z}_H$ is anti-Hermitian, so we consider $\mathrm{j}\hat{Z}_H$, which is Hermitian,

$$\begin{aligned}
\mathrm{j}\hat{Z}_H &= \mathrm{j}\left(p\cdot\frac{\partial}{\partial q} - q\cdot\frac{\partial}{\partial p}\right) = \mathrm{j}\left(\hat{Y}_p\hat{Z}_p + \hat{Y}_q\hat{Z}_q\right) \\
&= \mathrm{j}\,(ba - b^\dagger a^\dagger) \\
&= \tfrac{1}{2}\left[(a-\mathrm{j}b^\dagger)^\dagger(a-\mathrm{j}b^\dagger) - (a+\mathrm{j}b^\dagger)^\dagger(a+\mathrm{j}b^\dagger)\right] \not\geq 0
\end{aligned}$$

so $\langle_{kT}|\mathrm{j}\hat{Z}_H|_{kT}\rangle = 0$

For the Gibbs state of the Simple Harmonic Oscillator,
$\hat{Z}_H$ is anti-Hermitian, so we consider $\mathrm{j}\hat{Z}_H$, which is Hermitian,

$$\begin{aligned}\mathrm{j}\hat{Z}_H &= \mathrm{j}\left(p\cdot\frac{\partial}{\partial q} - q\cdot\frac{\partial}{\partial p}\right) = \mathrm{j}\left(\hat{Y}_p\hat{Z}_p + \hat{Y}_q\hat{Z}_q\right)\\ &= \mathrm{j}\,(ba - b^\dagger a^\dagger)\\ &= \tfrac{1}{2}\left[(a-\mathrm{j}b^\dagger)^\dagger(a-\mathrm{j}b^\dagger) - (a+\mathrm{j}b^\dagger)^\dagger(a+\mathrm{j}b^\dagger)\right] \ngeq 0\end{aligned}$$

so $\langle_{kT}|\mathrm{j}\hat{Z}_H|_{kT}\rangle = 0$

The Hamiltonian operator in QM is bounded below $\longrightarrow$ analytic properties;
the corresponding operator in $\mathrm{CM}_+$, $\mathrm{j}\hat{Z}_H$, is not (though $\hat{Y}_H$ is)

$\mathrm{CM}_+$ includes (1) noncommutativity and (2) quantum noise, however
(3) analyticity is mathematically useful but is *not* included
so we can say that QM is an analytic form of $\mathrm{CM}_+$

Accepting that analyticity is a difference instead of trying to fix it
gives us a relationship that is usefully different from quantization,
*but* (1) and (2) ensure that the measurement theory is the same

For a measurement A, with sample space $\mathcal{A} = \{\alpha_m\}$, $\hat{A} = \sum_m \alpha_m \hat{P}_m$, and
a measurement B, with sample space $\mathcal{B} = \{\beta_n\}$, $\hat{B} = \sum_n \beta_n \hat{Q}_n$,

For solo measurements, with density operator $\hat{\rho}$,
we obtain the result $\alpha_m$ with probability $\rho(\hat{P}_m) = \mathrm{Tr}[\hat{\rho}\hat{P}_m]$ and
we obtain the result $\beta_n$ with probability $\rho(\hat{Q}_n) = \mathrm{Tr}[\hat{\rho}\hat{Q}_n]$.

For a measurement A, with sample space $\mathcal{A}=\{\alpha_m\}$, $\hat{A}=\sum_m \alpha_m \hat{P}_m$, and
a measurement B, with sample space $\mathcal{B}=\{\beta_n\}$, $\hat{B}=\sum_n \beta_n \hat{Q}_n$,

For solo measurements, with density operator $\hat{\rho}$,
we obtain the result $\alpha_m$ with probability $\rho(\hat{P}_m)=\mathrm{Tr}[\hat{\rho}\hat{P}_m]$ and
we obtain the result $\beta_n$ with probability $\rho(\hat{Q}_n)=\mathrm{Tr}[\hat{\rho}\hat{Q}_n]$.

For two measurements, of A first, followed by B, we say that
the result $\alpha_m$ “collapses” the state from $\hat{\rho}$ to the collapsed state $\hat{\rho}_m$,

$$\hat{\rho}_m=\frac{\hat{P}_m\hat{\rho}\hat{P}_m}{\mathrm{Tr}[\hat{P}_m\hat{\rho}\hat{P}_m]}=\frac{\hat{P}_m\hat{\rho}\hat{P}_m}{\mathrm{Tr}[\hat{\rho}\hat{P}_m]},$$

## "The collapse of a quantum state as a joint probability construction", PM, JPhysA 2022





For a measurement A, with sample space $\mathcal{A} = \{\alpha_m\}$, $\hat{A} = \sum_m \alpha_m \hat{P}_m$, and
a measurement B, with sample space $\mathcal{B} = \{\beta_n\}$, $\hat{B} = \sum_n \beta_n \hat{Q}_n$,

For solo measurements, with density operator $\hat{\rho}$,
we obtain the result $\alpha_m$ with probability $\rho(\hat{P}_m) = \mathsf{Tr}[\hat{\rho}\hat{P}_m]$ and
we obtain the result $\beta_n$ with probability $\rho(\hat{Q}_n) = \mathsf{Tr}[\hat{\rho}\hat{Q}_n]$.

For two measurements, of A first, followed by B, we say that
the result $\alpha_m$ "collapses" the state from $\hat{\rho}$ to the collapsed state $\hat{\rho}_m$,

$$\hat{\rho}_m = \frac{\hat{P}_m\hat{\rho}\hat{P}_m}{\mathsf{Tr}[\hat{P}_m\hat{\rho}\hat{P}_m]} = \frac{\hat{P}_m\hat{\rho}\hat{P}_m}{\mathsf{Tr}[\hat{\rho}\hat{P}_m]},$$

then we measure B in that state, so we obtain the result $\alpha_m$ followed by $\beta_n$
with *conditional* probability

$$p(\beta_n|\alpha_m) = \mathsf{Tr}[\hat{\rho}_m\hat{Q}_n] = \frac{\mathsf{Tr}[\hat{P}_m\hat{\rho}\hat{P}_m\hat{Q}_n]}{\mathsf{Tr}[\hat{\rho}\hat{P}_m]},$$

so the *joint* probability is

$$p(\alpha_m \text{ and } \beta_n) = \mathsf{Tr}[\hat{P}_m\hat{\rho}\hat{P}_m \cdot \hat{Q}_n \quad ] = \mathsf{Tr}[\hat{\rho} \cdot \hat{P}_m\hat{Q}_n\hat{P}_m].$$

We have $p(\alpha_m \text{ and } \beta_n) = \mathsf{Tr}[\hat{P}_m\hat{\rho}\hat{P}_m \cdot \hat{Q}_n] = \mathsf{Tr}[\hat{\rho} \cdot \hat{P}_m\hat{Q}_n\hat{P}_m]$,
so the positive operators $\hat{J}_{mn} = \hat{P}_m\hat{Q}_n\hat{P}_m$ generate
the joint probabilities $\mathsf{Tr}[\hat{\rho}\hat{J}_{mn}]$.

Instead of collapse affecting a state,
*we can take collapse to affect the next measurement*

If $[\hat{A}, \hat{B}] = 0$, then $\hat{P}_m\hat{Q}_n\hat{P}_m = \hat{P}_m\hat{Q}_n = \hat{Q}_n\hat{P}_m\hat{Q}_n \sim$ no action

We can use the positive operators $\hat{J}_{mn}$ to construct a "collapse product",
a measurement A⊳B, with sample space $\mathcal{A}\times\mathcal{B}$, even if $[\hat{A}, \hat{B}] \neq 0$

We have $p(\alpha_m \text{ and } \beta_n) = \mathrm{Tr}[\hat{P}_m\hat{\rho}\hat{P}_m \cdot \hat{Q}_n] = \mathrm{Tr}[\hat{\rho} \cdot \hat{P}_m\hat{Q}_n\hat{P}_m]$,
so the positive operators $\hat{J}_{mn} = \hat{P}_m\hat{Q}_n\hat{P}_m$ generate
the joint probabilities $\mathrm{Tr}[\hat{\rho}\hat{J}_{mn}]$.

Instead of collapse affecting a state,
*we can take collapse to affect the next measurement*

If $[\hat{A}, \hat{B}] = 0$, then $\hat{P}_m\hat{Q}_n\hat{P}_m = \hat{P}_m\hat{Q}_n = \hat{Q}_n\hat{P}_m\hat{Q}_n \sim$ no action

We can use the positive operators $\hat{J}_{mn}$ to construct a "collapse product",
a measurement A⧐B, with sample space $\mathcal{A}\times\mathcal{B}$, even if $[\hat{A}, \hat{B}] \neq 0$

The existence of a joint probability is traditionally "classical", so we can instead
use commuting operators $\hat{A}'$ and $\hat{B}'$ and a different state $\hat{\rho}'$ that give
the same joint probability, $\mathrm{Tr}[\hat{\rho}' \cdot \hat{P}'_m\hat{Q}'_n] = \mathrm{Tr}[\hat{\rho} \cdot \hat{P}_m\hat{Q}_n\hat{P}_m]$

For the mathematically inclined, we can use the Neumark Dilation Theorem
to construct a joint PVM $\widehat{AB} \sim$ A⧐B (for a larger Hilbert space)

We can think of what we have just constructed as a *"super-Heisenberg picture"*,
for which *both* unitary evolution
*and* collapse are applied to measurements

The Schrödinger picture applies *both* unitary evolution
*and* collapse to the state

The Heisenberg picture applies unitary evolution to measurements,
*but* applies collapse to the state

We can think of what we have just constructed as a *"super-Heisenberg picture"*,
for which *both* unitary evolution
*and* collapse are applied to measurements

The Schrödinger picture applies *both* unitary evolution
*and* collapse to the state

The Heisenberg picture applies unitary evolution to measurements,
*but* applies collapse to the state

or as the *"Bohr picture"*, because it's rather classical and, for Bohr, measurements affect other measurements[†]
or as the *"QND picture"* or as the *"Consistent Histories picture"*, because it's commutative
or as the *"Everett picture"*, because it's no–collapse
or as the *"Einstein picture"*, because it's rather classical (but with a Poincaré invariant noise)

[†] Howard 2004

For a sequence of three or more measurements (*many* more for signal analysis),
we can use the *sequential product*, $\hat{X}\circ\hat{Y}=\sqrt{\hat{X}}\cdot\hat{Y}\cdot\sqrt{\hat{X}}$,
or more elaborate *systematic* constructions of sums of positive operators

Collapse of the quantum state after measurement is ambiguous

$$\rho\Big(\sqrt{\hat{P}_i^{(A)}\hat{P}_j^{(B)}\hat{P}_i^{(A)}}\,\hat{P}_k^{(C)}\sqrt{\hat{P}_i^{(A)}\hat{P}_j^{(B)}\hat{P}_i^{(A)}}\Big)\text{ or }\rho\big(\hat{P}_i^{(A)}\hat{P}_j^{(B)}\hat{P}_k^{(C)}\hat{P}_j^{(B)}\hat{P}_i^{(A)}\big)?$$

(A⧟B)⧟C $\neq$ A⧟(B⧟C)

We can use any ordering, but each makes a different assertion about dependencies

⧟ is nonassociative, so, more complicated than the Heisenberg cut,
we have a *Heisenberg bracketing ambiguity*

We may not like the square root in (A⧟B)⧟C, but AB-preparation may be
a more natural pairing in the apparatus context than BC-measurement

For signal analysis, when we have *many* measurements at time-like separation, we can use $\hat{M}_1, \ldots, \hat{M}_{100\ldots000}$, with many ambiguous collapses,
or we can use $\hat{M}'_1, \ldots, \hat{M}'_{100\ldots000}$, which all commute, unambiguously, with no collapses

We can think of this as Bohr's ideal of a classical model for compatible measurements

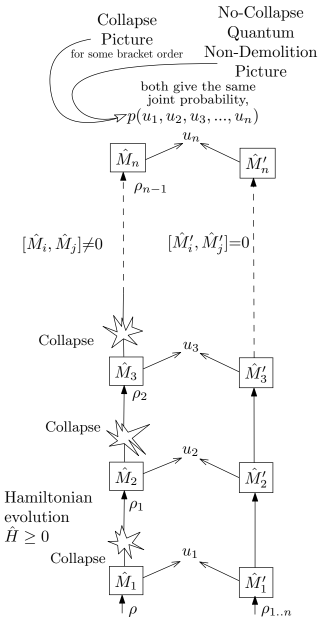
measurements at timelike separation *can* give joint probabilities

Time reversal is easy for the QND construction, but with collapse (for just 3 measurements) we have

$$\rho\big(\hat{P}_i^{(A)}\hat{P}_j^{(B)}\hat{P}_k^{(C)}\hat{P}_j^{(B)}\hat{P}_i^{(A)}\big)$$

or $\rho_R\big(\hat{P}_k^{(C)}\hat{P}_j^{(B)}\hat{P}_i^{(A)}\hat{P}_j^{(B)}\hat{P}_k^{(C)}\big)$,

with the collapses running in reverse



# “Collapse” is **not**

(only or necessarily)

# a dynamical process

We can (also) take it to be a

# JOINT PROBABILITY ALGORITHM

Belavkin 1994 Quantum Non-Demolition (QND) Measurements
Anastopoulos 2006 Sequential Measurements
Tsang&Caves 2012 Quantum-Mechanics–Free-Subsystems

# “Collapse” is **not**
(only or necessarily)
# a dynamical process

We can (also) take it to be a
# JOINT PROBABILITY ALGORITHM

Belavkin 1994 Quantum Non-Demolition (QND) Measurements
Anastopoulos 2006 Sequential Measurements
Tsang&Caves 2012 Quantum-Mechanics–Free-Subsystems

A necessary tradeoff:

QM is effective for incompatible measurements, but less so for joint measurements

Collapse is QM's way of constructing joint measurement probabilities

CM is effective for joint measurements, but less so for incompatible measurements

The Poisson bracket is $\mathrm{CM}_+$'s way of constructing incompatible measurements

# probability, intervention & causality

It's a running joke that Correlation $\neq$ Causality, so, for example,
*causal modeling* adds *Interventions*, with $p(Y = y \mid \mathrm{do}(X = x))$
as a way to discuss counterfactuals, with that and other
graphical and logical tools *on top of* classical modeling

The Poisson bracket gives us a transformation algebra that can model
interventions in an intrinsic and classically natural way,

$$p(Y = y \mid \mathrm{do}(X = x)) = \frac{\mathrm{Tr}[\hat{P}_{Y=y} \cdot \hat{P}_{X=x}\hat{\rho}\hat{P}_{X=x}]}{\mathrm{Tr}[\hat{P}_{X=x}\hat{\rho}]} = \frac{\mathrm{Tr}[\hat{P}_{X=x}\hat{P}_{Y=y}\hat{P}_{X=x} \cdot \hat{\rho}]}{\mathrm{Tr}[\hat{P}_{X=x}\hat{\rho}]}$$

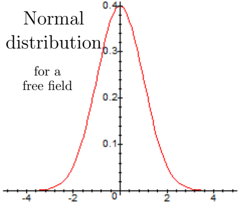
Interventions are what people do, which is on the edge of classical modeling
Suggestion: "intervention" is a usefully different way
to think about "contextuality" or "measurement incompatibility"

# quantum and QND fields — modulation & measurement

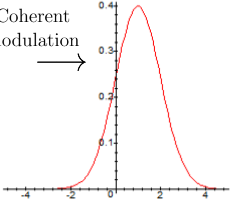
We can say the vacuum state of a quantum or QND field is a broadband, noisy carrier "signal" for *probabilistic* modulations of measurement results

Normal distribution

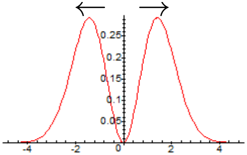
for a free field



Coherent modulation →



Once-raised modulation ← →



$$\rho_v(\delta(\hat{M}_f - u)) = \frac{e^{-u^2/2(f,f)}}{\sqrt{2\pi(f,f)}}$$

$$\frac{e^{-(u-((g,f)-(f,g))/j)^2/2(f,f)}}{\sqrt{2\pi(f,f)}}$$

(omitted)

probability distributions

$$\rho_v(e^{j\lambda\hat{M}_f}) = e^{-\lambda^2(f,f)/2}$$

$$\rho_v(e^{-j\hat{M}_g} e^{j\lambda\hat{M}_f} e^{j\hat{M}_g})$$

$$\frac{\rho_v\left(\hat{M}_g^\dagger e^{j\lambda\hat{M}_f}\hat{M}_g\right)}{\rho_v\left(\hat{M}_g^\dagger\hat{M}_g\right)}$$

characteristic functions

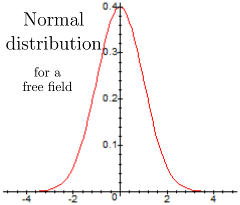
$(f, g)$ determines the geometric structure of a free quantum or QND field

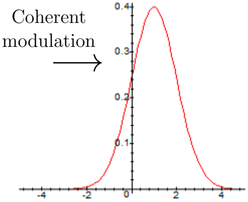
We can also modulate joint measurements: $\rho_v(e^{j\lambda_1\hat{M}_{f_1}} e^{j\lambda_2\hat{M}_{f_2}}\cdots)$

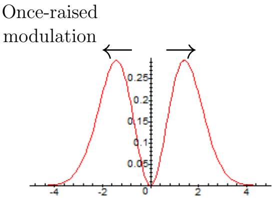
Call this a "super-characteristic function"

# quantum and QND fields — modulation & measurement

We can say the vacuum state of a quantum or QND field is a broadband, noisy carrier "signal" for *probabilistic* modulations of measurement results




$$\rho_v(\delta(\hat{M}_f - u)) = \frac{e^{-u^2/2(f,f)}}{\sqrt{2\pi(f,f)}} \qquad \frac{e^{-(u-((g,f)-(f,g))/j)^2/2(f,f)}}{\sqrt{2\pi(f,f)}} \qquad \text{(omitted)}$$

probability distributions

$$\rho_v(e^{j\lambda\hat{M}_f}) = e^{-\lambda^2(f,f)/2} \qquad \rho_v(e^{-j\hat{M}_g} e^{j\lambda\hat{M}_f} e^{j\hat{M}_g}) \qquad \frac{\rho_v\left(\hat{M}_g^\dagger e^{j\lambda\hat{M}_f}\hat{M}_g\right)}{\rho_v\left(\hat{M}_g^\dagger\hat{M}_g\right)}$$

characteristic functions

$(f,g)$ determines the geometric structure of a free quantum or QND field

We can also modulate joint measurements: $\rho_v(e^{j\lambda_1\hat{M}_{f_1}} e^{j\lambda_2\hat{M}_{f_2}}\cdots)$

Call this a "super-characteristic function"

Analysis of global properties of a model needs global tools, which may have a discrete spectrum

# states for quantum and QND fields

For a Gaussian state, we can completely fix the algebraic structure with one equation:

$$\rho_v(\mathrm{e}^{\mathrm{j}\lambda_1\hat{M}_{f_1}}\mathrm{e}^{\mathrm{j}\lambda_2\hat{M}_{f_2}}\cdots)=\exp\Big[\underbrace{-\sum_{i,j}\lambda_i\lambda_j(f_i^*,f_j)/2}_{\text{Gaussian noise term}}\underbrace{-\sum_{i<j}\lambda_i\lambda_j[(f_i^*,f_j)-(f_j^*,f_i)]/2}_{\text{noncommutativity term}}\Big]$$

$\rho_v$ is a state if $(f_i,f_j)$ is a positive semi-definite matrix

# states for quantum and QND fields

For a Gaussian state, we can completely fix the algebraic structure with one equation:

$$\rho_v(\mathrm{e}^{\mathrm{j}\lambda_1\hat{M}_{f_1}}\mathrm{e}^{\mathrm{j}\lambda_2\hat{M}_{f_2}}\cdots)=\exp\Big[\underbrace{-\sum_{i,j}\lambda_i\lambda_j(f_i^*,f_j)/2}_{\text{Gaussian noise term}}\underbrace{-\sum_{i<j}\lambda_i\lambda_j[(f_i^*,f_j)-(f_j^*,f_i)]/2}_{\text{noncommutativity term}}\Big]$$

$\rho_v$ is a state if $(f_i, f_j)$ is a positive semi-definite matrix

We can fix the geometric structure in multiple ways:

Klein-Gordon: $(f,g)=\hbar\int\tilde{f}^*(k)\tilde{g}(k)2\pi\delta(k\cdot k-m^2)\theta(k_0)\frac{\mathrm{d}^4k}{(2\pi)^4}$

Quantum optics: $(f,g)=-\hbar\int\underbrace{\tilde{f}^*_{\alpha\mu}(k)k^\mu}\,\underbrace{k^\nu\tilde{g}^\alpha{}_\nu(k)}2\pi\delta(k\cdot k)\theta(k_0)\frac{\mathrm{d}^4k}{(2\pi)^4}$

two space-like 4-vectors

which are *manifestly* Poincaré invariant

# states for quantum and QND fields

For a Gaussian state, we can completely fix the algebraic structure with one equation:

$$\rho_v(\mathrm{e}^{\mathrm{j}\lambda_1\hat{M}_{f_1}}\mathrm{e}^{\mathrm{j}\lambda_2\hat{M}_{f_2}}\cdots) = \exp\Big[-\underbrace{\sum_{i,j}\lambda_i\lambda_j(f_i^*,f_j)/2}_{\text{Gaussian noise term}}-\underbrace{\sum_{i<j}\lambda_i\lambda_j[(f_i^*,f_j)-(f_j^*,f_i)]/2}_{\text{noncommutativity term}}\Big]$$

$\rho_v$ is a state if $(f_i, f_j)$ is a positive semi-definite matrix

We can fix the geometric structure in multiple ways:

Klein-Gordon: $(f,g)=\hbar\int\tilde{f}^*(k)\tilde{g}(k)2\pi\delta(k\cdot k-m^2)\,\cancel{\theta(k_0)}\,\frac{\mathrm{d}^4k}{(2\pi)^4}$

Quantum optics: $(f,g)=-\hbar\int\underbrace{\tilde{f}^*_{\alpha\mu}(k)k^\mu\,k^\nu\tilde{g}^{\alpha}_{\ \nu}(k)}_{\text{two space-like 4-vectors}}2\pi\delta(k\cdot k)\,\cancel{\theta(k_0)}\,\frac{\mathrm{d}^4k}{(2\pi)^4}$

which are *manifestly* Poincaré invariant

remove the "$\theta(k_0)$" for an everywhere commutative Gaussian QND field with a Planck-scale noise

# ⇅-operators and the relationship between QFT and QNDFT

An alternative way to construct the same Gaussian state,

$$\hat{M}_f = a_{f^*} + a_f^\dagger, \qquad [a_f, a_g^\dagger] = (f,g), \qquad a_f|v\rangle = 0, \qquad \text{using the same } (f,g)$$

so that $[\hat{M}_f, \hat{M}_g] = (f^*,g) - (g^*,f)$

# $\updownarrow$-operators and the relationship between QFT and QNDFT

An alternative way to construct the same Gaussian state,

$$\hat{M}_f = a_{f^*} + a_f^\dagger, \qquad [a_f, a_g^\dagger] = (f,g), \qquad a_f|v\rangle = 0, \qquad \text{using the same } (f,g)$$

so that $[\hat{M}_f, \hat{M}_g] = (f^*,g) - (g^*,f)$

For the *complex* Klein-Gordon field and for quantum optics,

we can find involutions $f \mapsto f^\bullet$, $f^{\bullet\bullet} = f$,

for which $(f^{*\bullet},g^\bullet) - (g^{*\bullet},f^\bullet) = 0$, for all test functions $f$ and $g$

For $\hat{\mathtt{M}}_f^{\text{QND}} = a_{f^{*\bullet}} + a_{f^\bullet}^\dagger \neq \hat{M}_{f^\bullet}$, $\qquad [\hat{\mathtt{M}}_f^{\text{QND}}, \hat{\mathtt{M}}_g^{\text{QND}}] = 0$

The $\hat{\mathtt{M}}_f^{\text{QND}}$ generate a QND field: a commutative algebra of quantum non-demolition measurements, and an isomorphic Hilbert space

An alternative way to construct the same Gaussian state,

$$\hat{M}_f = a_{f^*} + a^\dagger_f, \qquad [a_f, a^\dagger_g] = (f,g), \qquad a_f|v\rangle = 0,$$ using the same $(f,g)$

so that $[\hat{M}_f, \hat{M}_g] = (f^*,g) - (g^*,f)$

For the *complex* Klein-Gordon field and for quantum optics,

we can find involutions $f \mapsto f^\bullet$, $f^{\bullet\bullet} = f$,

for which $(f^{*\bullet},g^\bullet) - (g^{*\bullet},f^\bullet) = 0$, for all test functions $f$ and $g$

For $\hat{\mathtt{M}}^{\mathrm{QND}}_f = a_{f^{*\bullet}} + a^\dagger_{f^\bullet} \neq \hat{M}_{f^\bullet}$, $\qquad [\hat{\mathtt{M}}^{\mathrm{QND}}_f, \hat{\mathtt{M}}^{\mathrm{QND}}_g] = 0$

The $\hat{\mathtt{M}}^{\mathrm{QND}}_f$ generate a QND field: a commutative algebra of quantum non-demolition measurements, and an isomorphic Hilbert space

The algebra generated by the $\hat{\mathtt{M}}^{\mathrm{QND}}_f$ is *not* isomorphic to that generated by the $\hat{M}_f$

For quantum optics: $\widetilde{f^\bullet}(k) = \frac{1}{2}(1+\mathrm{j}\star)\tilde{f}(k) + \frac{1}{2}(1-\mathrm{j}\star)\tilde{f}(-k)$

$f \mapsto f^\bullet$ is Lorentz invariant but not translation invariant or local, *but*

both the Quantum and QND Field Theories are Poincaré invariant

*If* we allow the use of the vacuum projection operator $\hat{V} = |v\rangle\langle v|$,
then the algebra generated by $\hat{V}, \hat{M}_f$ is isomorphic to
the algebra generated by $\hat{V}, \hat{\mathbb{M}}_f^{\mathrm{QND}}$

Anything we can model with quantum optics$+\hat{V}$,
we can also model with QND optics$+\hat{V}$ (classical, but with Poincaré invariant noise)

$\hat{V}$ is nonlocal insofar as $[\hat{V}, \hat{M}_f] \neq 0$ for any $f$, but
we implicitly use $\hat{V}$ whenever we use a state transition probability

# the Wightman axioms (adapted from Haag's *Local Quantum Physics*)

for which, despite how simple they look, there are no known well-defined interacting models in 3+1-dimensions, after 70 years

- A Hilbert space $\mathcal{H}$ supports a unitary representation of the Poincaré group; there is a unique lowest energy Poincaré invariant vacuum vector $|v\rangle$
- Quantum fields are operator–valued distributions, linear maps $\hat{M} : f \mapsto \hat{M}_f$ from a space of modulation functions into a $*$–algebra $\mathcal{A}$
- Quantum fields can be a Lorentz scalar, vector, ...
- Microcausality: commutativity at space-like separation
- Completeness: the action of the quantum field on $|v\rangle$ generates $\mathcal{H}$

[Omitting spinors/fermions, which matter, but not today.]

# the Wightman axioms (adapted from Haag's *Local Quantum Physics*)

Thinking about QNDFT and signal analysis suggests at least three ways in which the Wightman axioms are too strong

- A Hilbert space $\mathcal{H}$ supports a unitary representation of the Poincaré group; there is a unique lowest energy Poincaré invariant vacuum vector $|v\rangle$
  **QNDFT: Allow the vacuum to be *not* a lowest frequency state**
- Quantum fields are operator–valued distributions, linear maps $\hat{M} : f \mapsto \hat{M}_f$ from a space of modulation functions into a $*$–algebra $\mathcal{A}$
  **Allow quantum fields to be *nonlinear* maps into $\mathcal{A}$**
- Quantum fields can be a Lorentz scalar, vector, ...
- Microcausality: commutativity at space-like separation
  **QNDFT: Allow commutativity at *all* separations,** $[\hat{M}_f, \hat{M}_g] = 0$
- Completeness: the action of the quantum field on $|v\rangle$ generates $\mathcal{H}$

[Omitting spinors/fermions, which matter, but not today.]

There are *two* linearities implicit in the Wightman axioms:

von Neumann linearity of the state, $\rho_v(\lambda\hat{A}+\mu\hat{B})=\lambda\rho_v(\hat{A})+\mu\rho_v(\hat{B})$,

*and* linearity of the field, $\hat{M}_{\lambda f+\mu g}=\lambda\hat{M}_f+\mu\hat{M}_g$

There are *two* linearities implicit in the Wightman axioms:

von Neumann linearity of the state, $\rho_v(\lambda\hat{A}+\mu\hat{B})=\lambda\rho_v(\hat{A})+\mu\rho_v(\hat{B})$,

*and* linearity of the field, $\hat{M}_{\lambda f+\mu g}=\lambda\hat{M}_f+\mu\hat{M}_g$

If we double the amplitude of a modulation, we do *not* expect

that will double the effects of that modulation

[with the certainty required for linearity to be axiomatic]

$$\hat{M}_{\lambda f+\mu g} \neq \lambda\hat{M}_f+\mu\hat{M}_g$$

We can also argue that renormalization can be formalized as

a nonlinear dependence on window and modulation functions

Loosely: renormalization scale$\longleftarrow$experimental *details*$\longleftarrow$test functions

For real-space renormalization, nontrivial blocking algorithms are nonlinear

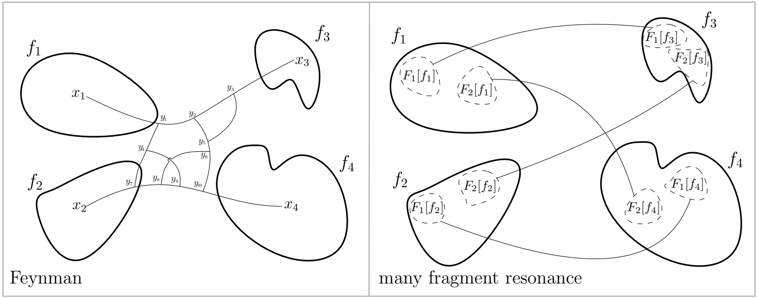
"A source fragmentation approach to interacting quantum field theory", arXiv:2109.04412

The Reeh-Schlieder theorem for a Wightman field:

*local operators acting on the vacuum vector* $|v\rangle$ *can approximate any vector*

$\Rightarrow$ what path integrals can approximate

can be approximated by local operators $\hat{M}_{F_i[f_j]}$




This is an inverse problem: find local, nonlinear *fragment functionals* $F_i[\cdot]$

and free field QFTs that give the same results as our best path integrals

NOT the last word! I hope we can find something better

I suppose that tables and chairs can be modeled as something like a caustic

# A concise list of the difficulties of QFT



1. Divergences (non-dynamical nonlinear resonance)
2. No precise ontological picture (signal analysis & incompatibility ⇝ intervention)
3. No particles (nonlinearity & dispersion ⟶ caustics over time?)
4. Haag's theorem (subalgebra of an algebra generated by many free fields)
5. The measurement problem (collapse of a quantum state as a joint probability construction)

from Oldofredi&Öttinger 2022

# A final generalization

Given measurements $\hat{M}_{Description_1}, \ldots, \hat{M}_{Description_n}$, all we need so we can construct a Gaussian state over that collection *of measurements* is a positive semi-definite matrix $\left(Description_i, Description_j\right)$

The matrix does *not* have to be linear in $Description_i$ and $Description_j$

The domain and the manifest symmetries of the matrix fix the theory

# A final generalization





Given measurements $\hat{M}_{Description_1}, \ldots, \hat{M}_{Description_n}$, all we need so we can construct a Gaussian state over that collection *of measurements* is a positive semi-definite matrix $\left(Description_i, Description_j\right)$

The matrix does *not* have to be linear in $Description_i$ and $Description_j$

The domain and the manifest symmetries of the matrix fix the theory

For interacting fields, introduce and combine many such matrices, while ensuring the properties required for $\rho_v(\mathrm{e}^{\mathrm{j}\lambda_1\hat{M}_{f_1}}\mathrm{e}^{\mathrm{j}\lambda_2\hat{M}_{f_2}}\cdots)$ to be a state are satisfied

For gravity, we have to $describe$ how we measure the geometry of space-time
If $\hbar$ appears nontrivially in the construction of matrices, then it's quantum

Quantum and Classical$_{+}$/$_{\text{QND}}$ are types of description, not types of system

Instead of collapse of the state, we can use the QND picture,
but noncommutativity is *useful* for modeling intervention&causality

Signal analysis suggests the introduction of nonlinearity into the Wightman axioms

Quantum and Classical have been
converging, in numerous ways, for decades

Generalized Probability Theories, phase space methods, contextuality, non-demolition measurement, Koopman CM, time-frequency analysis, stochastic methods, semi-classical methods, superdeterminism, causal modeling, Cohen 1988 on characteristic functions, Abramsky 2020 on Boole's "Conditions of Possible Experience"

"Classical states, quantum field measurement", Physica Scripta 2019
"An algebraic approach to Koopman classical mechanics", Annals of Physics 2020
"The collapse of a quantum state as a joint probability construction", Journal of Physics A 2022
"A source fragmentation approach to interacting quantum field theory", arXiv:2109.04412