A Field & Signal Analysis Approach to Quantum Measurement

Peter Morgan
Yale University
peter.w.morgan@yale.edu

May 17th, 2023

Lisbon Philosophy of Physics Seminar (by Zoom)

A recording is available on YouTube at https://www.youtube.com/watch?v=5Jx2WIa5eTs
Devices → *noisy* signals ↔ events → data

The violation of Bell inequalities — signal analysis & field theory

Classical mechanics — *add* noncommutativity & quantum noise and *discuss* analyticity

The measurement problem — joint probabilities

Generalized Probability — as a way to discuss intervention & causality

Quantum and QND fields — modulation & measurement

Interacting quantum fields — a signal analysis approach

An evolution of ideas in:

- “Classical states, quantum field measurement”, *Physica Scripta* 2019
- “An algebraic approach to Koopman classical mechanics”, *Annals of Physics* 2020
- “The collapse of a quantum state as a joint probability construction”, *Journal of Physics A* 2022
- “A source fragmentation approach to interacting quantum field theory”, arXiv:2109.04412

At first, quite operational, but, by the end, quite realist

Different theory frameworks: same experiment \(\rightarrow\) same data

but what we can imagine and design and choose to perform changes

A signal\&data approach is well-suited to machine learning

Let some preconceptions go and let more abstract algorithms do the heavy lifting instead

Supplementary Information

Write a paper \(\rightarrow\) journal editor

\(\rightarrow\) physicists \& engineers

“raw data” is never completely raw

Many devices

signal lines

data lines

Real-time Signal Analysis

Recorded Experimental Data

Compressed Digital Data

Bokulich\&Parker 2021

Morgan\&Morrison, "Models as Mediators"

Borges, "On Exactitude in Science"

Martínez-Ordaz 2023

Bokulich\&Parker 2021

Morgan\&Morrison, "Models as Mediators"

Borges, "On Exactitude in Science"
Bohr 1921: “in a certain respect we are entitled in the quantum theory to see an attempt of a natural generalisation of the classical theory of electromagnetism.”

Bohr 1949: “It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.”

Dirac 1949: “My own opinion is that we ought to search for a way of making fundamental changes not only in our present Quantum Mechanics, but actually in Classical Mechanics as well.”

Bell 1975: “‘Observables’ must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.”
Bohr 1921: “in a certain respect we are entitled in the quantum theory to see an attempt of a natural generalisation of the classical theory of electromagnetism.”

Bohr 1949: “It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.”

Dirac 1949: “My own opinion is that we ought to search for a way of making fundamental changes not only in our present Quantum Mechanics, but actually in Classical Mechanics as well.”

Bell 1975: “Observables’ must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.”

Use the Poisson bracket to make new ‘observables’ the opposite of adding hidden variables to quantum mechanics

Bell 1990, “Against ‘measurement’” : “experiments have results.”

We collate an Experimental Dataset into multiple Measurement Datasets, operationally, by device and by analysis, not as “measurements of particle properties”
Quantum Field Theory and Signal Analysis are both grounded in actually recorded measurement results, which are about the noisy signals on the signal lines out of devices, which indicate *something* about the devices’ surroundings, whatever that is.
taking quantum field theory to be about a field of measurements noisy signal analysis

Quantum Field Theory and Signal Analysis are both grounded in actually recorded measurement results, which are about the noisy signals on the signal lines out of devices, which indicate something about the devices’ surroundings, whatever that is.

We take quantum field theories to be our best theories but we still take particle properties to cause events

suggestion(1): hesitate before mentioning particles or systems
suggestion(2): hesitate before mentioning a field that is measured

A quantum field has a hat because it is a field of measurement operators.
Quantum Field Theory and Signal Analysis are both grounded in actually recorded measurement results, which are about the noisy signals on the signal lines out of devices, which indicate something about the devices’ surroundings, whatever that is.

We take quantum field theories to be our best theories but we still take particle properties to cause events

suggestion(1): hesitate before mentioning particles or systems

suggestion(2): hesitate before mentioning a field that is measured

A quantum field has a hat because it is a field of measurement operators

About noise: Quantum noise (cf “shot” noise) is different from Thermal noise (see #16)
If we have data about many millions of events, we have to write algorithms that decide how to assign each event to a particle.

If we add data about more events, the assignment of events to particles will sometimes be fragile.

Events-to-particles algorithms are global, after-the-events, and fragile.
If we have data about many millions of events, we have to write algorithms that decide how to assign each event to a particle. If we add data about more events, the assignment of events to particles will sometimes be fragile.

Events-to-particles algorithms are global, after-the-events, and fragile.

Signal-to-events algorithms, often implemented in hardware, are non-Markovian because events must be reported only once, but are less fragile and nonlocal than events-to-particles algorithms because adding data about more events doesn’t change the other events.
For QM, we have measurements $\hat{M}_1, ..., \hat{M}_n$, which is uninformative unless we have a list of metadata $\text{Description}_1, ..., \text{Description}_n$, which should be enough for another experimenter to reproduce the measurement results. We could write $\hat{M}_{\text{Description}_1}, ..., \hat{M}_{\text{Description}_n}$

For QFT, measurement operators are not point-like: we use $\hat{M}_f = \int \hat{M}(x)f(x)d^4x$

We have measurements $\hat{M}_{f_1}, \hat{M}_{f_2}, ..., \hat{M}_{f_n}$, where $f_1, f_2, ..., f_n$ are smearing functions, test functions, window functions, or ..., as descriptions of how a measurement is different from point-like
For QM, we have measurements $\hat{M}_1, \ldots, \hat{M}_n$, which is uninformative unless we have a list of metadata $\text{Description}_1, \ldots, \text{Description}_n$, which should be enough for another experimenter to reproduce the measurement results. We could write $\hat{M}_{\text{Description}_1}, \ldots, \hat{M}_{\text{Description}_n}$.

For QFT, measurement operators are not point-like: we use $\hat{M}_f = \int \hat{M}(x)f(x)d^4x$. We have measurements $\hat{M}_{f_1}, \hat{M}_{f_2}, \ldots, \hat{M}_{f_n}$, where $f_1, f_2, \ldots, f_n$ are smearing functions, test functions, window functions, or ..., as descriptions of how a measurement is different from point-like.

- QFT: $\hat{M}_f$ commutes with $\hat{M}_g$ if $f(x)$ and $g(x)$ are causally separated.
- QNDFT: $\hat{M}^{\text{QND}}_f$ always commutes with $\hat{M}^{\text{QND}}_g$.

Quantum Non-Demolition Field Theory (what I have called a random field theory)
For quantum optics $\sim$ QNDFT, see #27.

Alice and Bob both have two Avalanche PhotoDiodes, an Electro-Optic Modulator, a Random Bit Generator, and a clock; a central apparatus modulates the ground state.

The time when an APD’s signal rises to a higher level is recorded, and which APD it was, and what the EOM setting was: when and 2 bits. This compressed record does not let us analyze any other signal details.
the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD.

An APD is not made from ordinary clay. (An APD is not conscious, but it is complicated.)

An APD mostly burbles along while it interacts with its surroundings.

An APD sometimes gets cross at the world and takes it to a higher level.

That’s more interesting than using devices that do nothing.

An APD’s electronics calms it down so it can burble again.

An APD’s frequency scales:

- optical @ PHz
- electronic @ GHz
- thermodynamic @ MHz
- human @ Hz

An APD’s electronics might even know that an APD is very likely to get cross and stop it going there.

An APD sometimes gets cross with itself even when it’s dark and quiet.

An APD gets cross differently if we intervene to change its surroundings.

An APD tells a story even when it’s not cross. (which Zlatko listened to. Nature 2019)

The electronics might even know that an APD is very likely to get cross and stop it going there.

An APD’s electronics calms it down so it can burble again.
the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD

An APD is not made from ordinary clay (An APD is not conscious, but it is complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level

That’s more interesting than using devices that do nothing

An APD knows nothing about particles, but it does get cross

An APD’s electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it’s dark and quiet

An APD gets cross differently if we intervene to change its surroundings

An APD tells a story even when it’s not cross

The electronics might even know that an APD is very likely to get cross and stop it going there

An APD’s frequency scales:

- optical @ PHz
- electronic @ GHz
- thermodynamic @ MHz
- human @ Hz

(An APD is not conscious, but it is complicated)

An APD's frequency scales:

- optical @ PHz
- electronic @ GHz
- thermodynamic @ MHz
- human @ Hz

which Zlatko listened to, Nature 2019
the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD.

An APD is not made from ordinary clay (An APD is not conscious, but it is complicated).

An APD mostly burbles along while it interacts with its surroundings.

An APD sometimes gets cross at the world and takes it to a higher level.

An APD’s electronics calms it down so it can burble again.

An APD sometimes gets cross with itself even when it’s dark and quiet.

An APD gets cross differently if we intervene to change its surroundings.

An APD tells a story even when it’s not cross (which Zlatko listened to, Nature 2019).

The electronics might even know that an APD is very likely to get cross and stop it going there.

An APD’s frequency scales:

- optical@PHz
- electronic@GHz
- thermodynamic@MHz
- human@Hz

That’s more interesting than using devices that do nothing.

An APD knows nothing about particles, but it does get cross.
An engineer worked hard to create an APD

An APD is not made from ordinary clay (An APD is not conscious, but it is complicated)

An APD mostly burbles along while it interacts with its surroundings

An APD sometimes gets cross at the world and takes it to a higher level

An APD’s electronics calms it down so it can burble again

An APD sometimes gets cross with itself even when it’s dark and quiet

An APD gets cross differently if we intervene to change its surroundings

An APD tells a story even when it’s not cross (which Zlatko listened to, Nature 2019)

An APD’s electronics might even know that an APD is very likely to get cross and stop it going there

An APD’s frequency scales:

optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz
the signal analysis of Avalanche PhotoDiodes

An engineer worked hard to create an APD.

An APD is not made from ordinary clay. (An APD is not conscious, but it is complicated.)

An APD mostly burbles along while it interacts with its surroundings.

An APD sometimes gets cross at the world and takes it to a higher level.

That’s more interesting than using devices that do nothing.

An APD knows nothing about particles, but it does get cross.

An APD’s electronics calms it down so it can burble again.

An APD sometimes gets cross with itself even when it’s dark and quiet.

An APD gets cross differently if we intervene to change its surroundings.

An APD tells a story even when it’s not cross. (which Zlatko listened to, Nature 2019)

An APD’s frequency scales:

optical@PHz, electronic@GHz, thermodynamic@MHz, human@Hz
An engineer worked hard to create an APD.

An APD is not made from ordinary clay. (An APD is not conscious, but it is complicated.)

An APD mostly bumbles along while it interacts with its surroundings.

An APD sometimes gets cross at the world and takes it to a higher level.

An APD’s electronics calms it down so it can burble again.

That’s more interesting than using devices that do nothing.

That’s more interesting than using devices that do nothing.

An APD knows nothing about particles, but it does get cross.

An APD sometimes gets cross with itself even when it’s dark and quiet.

An APD gets cross differently if we intervene to change its surroundings.

An APD tells a story even when it’s not cross.

The electronics might even know that an APD is very likely to get cross and stop it going there.

An APD’s frequency scales:

- optical @ PHz
- electronic @ GHz
- thermodynamic @ MHz
- human @ Hz

Gregor gets measurement results (Alice sees almost 400,000 APD events in 10 seconds)

16 colors represent the 4 APD and EOM bits: □ ▧ + × (brightness represents Alice’s two bits, shapes represent Bob’s two bits, red is the diagonal, …)

For over 15,000 of Alice’s 400,000 events, Bob also records an event within 3 nanoseconds

When Alice and Bob both record an event within 3 nanoseconds, the majority are green or yellow
Gregor gets measurement results (Alice sees almost 400,000 APD events in 10 seconds)

16 colors represent the 4 APD and EOM bits:

-  □
-  ◆
-  +
-  ×

3.0ns
-3.0ns
longdist35-Alice+-3ns-0-10s

after coincident events have been identified,
collate events by relative timing information and by APD# and EOM setting, to give 16 histograms

and collate by APD# and EOM setting to give a 4×4 table
If we had transformed the recorded experimental data innocuously we could have used commutative algebras to model the algorithms.

In QM, we model Bell-violating statistics using noncommuting operators.

In CM as usual, we do not have noncommuting operators.
If we had transformed the recorded experimental data innocuously we could have used commutative algebras to model the algorithms.

In QM, we model Bell-violating statistics using noncommuting operators. In CM as usual, we do not have noncommuting operators. Without noncommutativity, CM is computationally incomplete.

How can we add noncommutativity to CM?
transformations and noncommutativity

If we had transformed the recorded experimental data innocuously we could have used commutative algebras to model the algorithms.

In QM, we model Bell-violating statistics using noncommuting operators. In CM as usual, we do not have noncommuting operators. Without noncommutativity, CM is computationally incomplete.

How can we add noncommutativity to CM?

About (non)locality:

Alice & Bob’s Electro-Optic Modulation could be $\ll \sim 1$MHz, nonetheless giving approximately the same $4 \times 4$ table of numbers. For elementary QM models, the EOM rate makes no difference at all, but a low EOM rate does not probe (non)locality.

For quantum fields, locality is closely associated with measurement incompatibility because microcausality only allows noncommutativity at time-like separation.
Gregor’s experiment in a modulated non-steady-state form

At a fine-grained scale, Gregor’s experiment is *not* at equilibrium
At a coarse-grained scale, Gregor’s experiment *is* at equilibrium

*About (non)locality:* thermodynamic equilibrium depends on boundary conditions
Gregor’s experiment in a modulated non-steady-state form

At a fine-grained scale, Gregor’s experiment is not at equilibrium

At a coarse-grained scale, Gregor’s experiment is at equilibrium

About (non)locality: thermodynamic equilibrium depends on boundary conditions

What happens when we first turn on the power?

The event rate increases in each of the four APDs

The coincident event rate increases

The violation of Bell inequalities increases at some rate

Are these rates the same, or how are they different, and how do these rates change at different distances?

These rates are technologically important as well as foundationally significant

Some experiments make more sense as signal analysis than as probes of particle properties
algebraic QM and CM

There are *abstract* measurements $\hat{M}_1, \hat{M}_2, \hat{M}_3, \ldots, \hat{M}_1 + \hat{M}_2, \ldots, \hat{M}_1 \hat{M}_2, \ldots$

linear operators $\equiv$ random variables, spectrum $\equiv$ sample space, noncommutative or commutative, associative, distributive, with unit

With no dynamics, the tradition is: QM $=$ noncommutative, CM $=$ commutative
There are *abstract* measurements $\hat{M}_1, \hat{M}_2, \hat{M}_3, ..., \hat{M}_1 + \hat{M}_2, ..., \hat{M}_1 \hat{M}_2, ...$

linear operators $\equiv$ random variables, spectrum $\equiv$ sample space, noncommutative or commutative, associative, distributive, with unit

With no dynamics, the tradition is: QM $=$ noncommutative, CM $=$ commutative

A (statistical) state $\rho$ maps measurement operators to *expected measurement results*

$$\rho(\hat{M}_1), \rho(\hat{M}_2), \rho(\hat{M}_3), ..., \rho(\hat{M}_1 + \hat{M}_2), ..., \rho(\hat{M}_1 \hat{M}_2), ..., \rho(\hat{M}_1^n), ..., \rho(\delta(\hat{M}_1 - u))$$

positive: $\rho(\hat{A}^\dagger \hat{A}) \geq 0$; normalized: $\rho(1) = 1$;

von Neumann linearity: $\rho(\lambda \hat{A} + \mu \hat{B}) = \lambda \rho(\hat{A}) + \mu \rho(\hat{B})$

compatible with the adjoint: $\rho(\hat{A}^\dagger) = \rho(\hat{A})^*$; where $(\hat{A} \hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger$
There are *abstract* measurements \( \hat{M}_1, \hat{M}_2, \hat{M}_3, \ldots, \hat{M}_1 + \hat{M}_2, \ldots, \hat{M}_1 \hat{M}_2, \ldots \)

linear operators \(\equiv\) random variables, spectrum \(\equiv\) sample space, noncommutative or commutative, associative, distributive, with unit

With no dynamics, the tradition is: \(\text{QM} = \text{noncommutative}, \text{CM} = \text{commutative}\)

A (statistical) *state* \(\rho\) maps measurement operators to *expected measurement results*

\[
\rho(\hat{M}_1), \rho(\hat{M}_2), \rho(\hat{M}_3), \ldots, \rho(\hat{M}_1 + \hat{M}_2), \ldots, \rho(\hat{M}_1 \hat{M}_2), \ldots, \rho(\hat{M}_1^n), \ldots, \rho(\delta(\hat{M}_1 - u)) \rho(e^{i \lambda \hat{M}_1})
\]

positive: \(\rho(\hat{A}^\dagger \hat{A}) \geq 0\); normalized: \(\rho(1) = 1\);

von Neumann linearity: \(\rho(\lambda \hat{A} + \mu \hat{B}) = \lambda \rho(\hat{A}) + \mu \rho(\hat{B})\)

compatible with the adjoint: \(\rho(\hat{A}^\dagger) = \rho(\hat{A})^*\), where \((\hat{A} \hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger\)

We can also use measurement operators to *modulate* the state \(\rho\) to give different expected measurement results, \(\rho_A(\hat{M}) = \frac{\rho(\hat{A}^\dagger \hat{M} \hat{A})}{\rho(\hat{A}^\dagger \hat{A})}\),

from which the GNS-construction gives us a Hilbert space

The GNS-construction lets us think of \(\rho_v(\hat{M})\) as \(\langle v | \hat{M} | v \rangle\) and of \(\rho_{Av}(\hat{M})\) as \(\frac{\langle v | \hat{A}^\dagger \hat{M} \hat{A} | v \rangle}{\langle v | \hat{A}^\dagger \hat{A} | v \rangle}\)
Take classical mechanics to be an algebra of functions on phase space that has **three** binary operations: addition, multiplication, *and* the Poisson bracket:

\[ u + v \\
\]

\[ u \cdot v \\
\]

\[ \{ u, v \} \]
Take classical mechanics to be an algebra of functions on phase space that has three binary operations: addition, multiplication, and the Poisson bracket

\[
\begin{align*}
\hat{u} + \hat{v} &= u + v \\
\hat{u} \cdot \hat{v} &= u \cdot v \\
\{\hat{u}, \hat{v}\} &= \{u, v\}
\end{align*}
\]

We can introduce “Multiply by \(w\)”, \(\hat{Y}_w(u) = w \cdot u\), and “Poisson by \(w\)”, \(\hat{Z}_w(u) = \{w, u\}\),

A familiar example: “Poisson by the Hamiltonian function” gives a generator of time evolution, \(\hat{Z}_H(u) = \{H, u\}\), the Liouvillian operator

We have \([\hat{Y}_v, \hat{Y}_w] = 0\), but \([\hat{Z}_v, \hat{Y}_w] = \hat{Y}_{\{v, w\}} \neq 0\) and \([\hat{Z}_v, \hat{Z}_w] = \hat{Z}_{\{v, w\}} \neq 0\), generating a noncommutative algebra with addition and composition
Take classical mechanics to be an algebra of functions on phase space that has \textit{three} binary operations: 
addition, multiplication, \textit{and} the Poisson bracket 
\[
\begin{align*}
\hat{Y}_w(u) &= w \cdot u, \\
\hat{Z}_w(u) &= \{w, u\}
\end{align*}
\]

We can introduce “Multiply by \(w\)” , \(\hat{Y}_w(u) = w \cdot u\), 
and “Poisson by \(w\)” , \(\hat{Z}_w(u) = \{w, u\}\) 

A familiar example: “Poisson by the Hamiltonian function” gives 
a generator of time evolution, \(\hat{Z}_H(u) = \{H, u\}\), the Liouvillian operator 

We have \([\hat{Y}_v, \hat{Y}_w] = 0\), but \([\hat{Z}_v, \hat{Y}_w] = \hat{Y}_{\{v, w\}} \neq 0\) and \([\hat{Z}_v, \hat{Z}_w] = \hat{Z}_{\{v, w\}} \neq 0\), 
generating a noncommutative algebra with addition and composition 

\textit{I suggest:}

We can use the \(\hat{Y}\)’s and \(\hat{Z}\)’s of a more powerful CM\(_+\) without restriction 

We do not \textit{have to} follow the way of quantization and the Correspondence Principle 
if what we want is noncommutativity and measurement incompatibility 
and an algebraic measurement theory shared with QM
The Poisson bracket: \( \{u, v\} = \frac{\partial u}{\partial p} \frac{\partial v}{\partial q} - \frac{\partial u}{\partial q} \frac{\partial v}{\partial p} \)

\[
\begin{align*}
\hat{Y}_q[u] &= q \cdot u, & \hat{Z}_p[u] &= \{p, u\} = \frac{\partial}{\partial q} u, & [\hat{Y}_q, \hat{Z}_p] &= -1 \\
\hat{Y}_p[u] &= p \cdot u, & \hat{Z}_q[u] &= \{q, u\} = -\frac{\partial}{\partial p} u, & [\hat{Y}_p, \hat{Z}_q] &= 1 \\
\hat{Y}_H[u] &= \frac{1}{2}(q^2 + p^2) \cdot u, & \hat{Z}_H[u] &= \{H, u\} = \left(p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p}\right) u
\end{align*}
\]

We work with the transformations generated by the Poisson bracket, not with the Poisson bracket directly. \( \{u, v\} \not\rightarrow [\hat{u}, \hat{v}] \)
The classical simple harmonic oscillator

The Poisson bracket: \( \{u, v\} = \frac{\partial u}{\partial p} \frac{\partial v}{\partial q} - \frac{\partial u}{\partial q} \frac{\partial v}{\partial p} \)

\[ \hat{Y}_q[u] = q \cdot u, \quad \hat{Z}_p[u] = \{p, u\} = \frac{\partial}{\partial q} u, \quad [\hat{Y}_q, \hat{Z}_p] = -1 \]

\[ \hat{Y}_p[u] = p \cdot u, \quad \hat{Z}_q[u] = \{q, u\} = -\frac{\partial}{\partial p} u, \quad [\hat{Y}_p, \hat{Z}_q] = 1 \]

\[ \hat{Y}_H[u] = \frac{1}{2}(q^2 + p^2) \cdot u, \quad \hat{Z}_H[u] = \{H, u\} = \left(p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p}\right) u \]

The Gibbs thermal state at temperature \( kT \) (in a generating function form, introducing \( j \)):

\[ \rho_{\text{Gibbs}} \left( e^{j \hat{Y}_q + j \mu \hat{Y}_p} \right) = e^{-kT(\lambda^2 + \mu^2)/2} \]
The classical simple harmonic oscillator

The Poisson bracket: \( \{ u, v \} = \frac{\partial u}{\partial p} \frac{\partial v}{\partial q} - \frac{\partial u}{\partial q} \frac{\partial v}{\partial p} \)

\[
\hat{Y}_q[u] = q \cdot u, \quad \hat{Z}_p[u] = \{ p, u \} = \frac{\partial}{\partial q} u, \quad [\hat{Y}_q, \hat{Z}_p] = -1
\]

\[
\hat{Y}_p[u] = p \cdot u, \quad \hat{Z}_q[u] = \{ q, u \} = -\frac{\partial}{\partial p} u, \quad [\hat{Y}_p, \hat{Z}_q] = 1
\]

\[
\hat{Y}_H[u] = \frac{1}{2}(q^2 + p^2) \cdot u, \quad \hat{Z}_H[u] = \{ H, u \} = (p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p}) u
\]

The Gibbs thermal state at temperature kT (in a generating function form, introducing j):

\[
\rho_{\text{Gibbs}} \left( e^{i\lambda \hat{Y}_q + i\mu \hat{Y}_p} \right) = e^{-kT(\lambda^2 + \mu^2)/2}, \quad \rho_{\text{Gibbs}} \left( e^{\alpha \hat{Z}_p + \beta \hat{Z}_q} \right) = e^{-(\alpha^2 + \beta^2)/8kT}
\]

set \( \hat{Y}_q = (a + a^\dagger)\sqrt{kT} \), \( \hat{Z}_p = \frac{(a - a^\dagger)}{2\sqrt{kT}} \), \( [a, a^\dagger] = 1 \), ensuring \( [\hat{Y}_q, \hat{Z}_p] = -1 \), and we set \( a |\alpha\rangle = 0 \) (and \( b |\alpha\rangle = 0 \) &c for \( \hat{Y}_p \) and \( \hat{Z}_q \))

We can construct modulated, non-equilibrium states, \( \langle \alpha | \hat{A}^\dagger \hat{M} \hat{A} | \alpha \rangle \), and hence a Hilbert space
The Poisson bracket: \( \{u, v\} = \frac{\partial u}{\partial p} \frac{\partial v}{\partial q} - \frac{\partial u}{\partial q} \frac{\partial v}{\partial p} \)

\[
\hat{Y}_q[u] = q \cdot u, \quad \hat{Z}_p[u] = \{p, u\} = \frac{\partial}{\partial q} u, \quad [\hat{Y}_q, \hat{Z}_p] = -1
\]

\[
\hat{Y}_p[u] = p \cdot u, \quad \hat{Z}_q[u] = \{q, u\} = -\frac{\partial}{\partial p} u, \quad [\hat{Y}_p, \hat{Z}_q] = 1
\]

\[
\hat{Y}_H[u] = \frac{1}{2} (q^2 + p^2) \cdot u, \quad \hat{Z}_H[u] = \{H, u\} = (p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p}) u
\]

The Gibbs thermal state at temperature \( kT \) (in a generating function form, introducing \( j \)):

\[
\rho_{\text{Gibbs}}(e^{i\lambda \hat{Y}_q + j\mu \hat{Y}_p}) = e^{-kT(\lambda^2 + \mu^2)/2}, \quad \rho_{\text{Gibbs}}(e^{\alpha \hat{Z}_p + \beta \hat{Z}_q}) = e^{-(\alpha^2 + \beta^2)/8kT}
\]

We work with the transformations generated by the Poisson bracket, not with the Poisson bracket directly

\( \{u, v\} \not\rightarrow [\hat{u}, \hat{v}] \)

We can construct modulated, non-equilibrium states, \( \langle \alpha | \hat{A}^\dagger \hat{M} \hat{A} | \alpha \rangle \), and hence a Hilbert space

Instead of trying to map \( (q, p) \not\rightarrow (\hat{q}, \hat{p}) \), as quantization tries to \( \text{(but fails)} \), we can map \( \text{CM}_+ \) to QM, \( (q, j \frac{\partial}{\partial q}) \mapsto (\hat{q}_1, \hat{p}_1), (p, j \frac{\partial}{\partial p}) \mapsto (\hat{q}_2, \hat{p}_2) \)

Crucially, \( kT \) is not \( \hbar \), but it is also about an \textit{irreducible} noise
What is the difference between quantum and thermal noise?

- $\hbar$ has units action, whereas $kT$ has units energy
- In QFT, the quantum vacuum is Poincaré invariant, thermal noise is not
  This difference of symmetry properties can be used in $\text{CM}_+$
- Adopting this for $\text{CM}_+$, $\hbar$ is an amplitude of Poincaré invariant noise
  $kT$ is an amplitude of thermal noise

This gives a new reason to think that we must work with field theories, because we can only define the Lorentz group in $1+n$-dimensions

$\hbar \rightarrow 0$ is a mean-field approximation, not a classical approximation
For the Gibbs state of the Simple Harmonic Oscillator, $\hat{Z}_H$ is anti-Hermitian, so we consider $j\hat{Z}_H$, which is Hermitian,

\[
j\hat{Z}_H = j \left( p \frac{\partial}{\partial q} - q \frac{\partial}{\partial p} \right) = j \left( \hat{Y}_p \hat{Z}_p + \hat{Y}_q \hat{Z}_q \right) = j (ba - b^\dagger a^\dagger) = \frac{1}{2} \left[ (a - jb^\dagger)^\dagger (a - jb^\dagger) - (a + jb^\dagger)^\dagger (a + jb^\dagger) \right] \not\geq 0
\]

so $\langle \alpha | j\hat{Z}_H | \alpha \rangle = 0$.
unboundedness of the Hermitian generators of time-like evolution

For the Gibbs state of the Simple Harmonic Oscillator, 
\( \hat{Z}_H \) is anti-Hermitian, so we consider \( j\hat{Z}_H \), which is Hermitian,

\[
\begin{align*}
\hat{Z}_H &= j \left( p \frac{\partial}{\partial q} - q \frac{\partial}{\partial p} \right) = j \left( \hat{Y}_p \hat{Z}_p + \hat{Y}_q \hat{Z}_q \right) \\
&= j (ba - b^\dagger a^\dagger) \\
&= \frac{1}{2} \left[ (a - jb^\dagger)(a - jb^\dagger) - (a + jb^\dagger)(a + jb^\dagger) \right] \neq 0
\end{align*}
\]

so \( \langle \alpha | j\hat{Z}_H | \alpha \rangle = 0 \)

The Hamiltonian operator in QM is bounded below \( \longrightarrow \) analytic properties; the corresponding operator in \( \text{CM}_+ \), \( j\hat{Z}_H \), is not (though \( \hat{Y}_H \) is)

\( \text{CM}_+ \) includes (1) noncommutativity and (2) quantum noise, however (3) analyticity is mathematically useful but is not included so we can say that QM is an analytic form of \( \text{CM}_+ \)

Accepting that analyticity is a difference instead of trying to fix it gives us a relationship that is usefully different from quantization, but (1) and (2) ensure that the measurement theory is the same
For a measurement $A$, with sample space $A = \{\alpha_m\}$, $\hat{A} = \sum_m \alpha_m \hat{P}_m$, and a measurement $B$, with sample space $B = \{\beta_n\}$, $\hat{B} = \sum_n \beta_n \hat{Q}_n$,

For solo measurements, with density operator $\hat{\rho}$,
we obtain the result $\alpha_m$ with probability $\rho(\hat{P}_m) = \text{Tr}[\hat{\rho} \hat{P}_m]$ and
we obtain the result $\beta_n$ with probability $\rho(\hat{Q}_n) = \text{Tr}[\hat{\rho} \hat{Q}_n]$. 

"The collapse of a quantum state as a joint probability construction", PM, JPhysA 2022
For a measurement $A$, with sample space $\mathcal{A} = \{\alpha_m\}$, $\hat{A} = \sum_m \alpha_m \hat{P}_m$, and a measurement $B$, with sample space $\mathcal{B} = \{\beta_n\}$, $\hat{B} = \sum_n \beta_n \hat{Q}_n$.

For solo measurements, with density operator $\hat{\rho}$,

- we obtain the result $\alpha_m$ with probability $\rho(\hat{P}_m) = \text{Tr}[\hat{\rho} \hat{P}_m]$ and
- we obtain the result $\beta_n$ with probability $\rho(\hat{Q}_n) = \text{Tr}[\hat{\rho} \hat{Q}_n]$.

For two measurements, of $A$ first, followed by $B$, we say that the result $\alpha_m$ “collapses” the state from $\hat{\rho}$ to the collapsed state $\hat{\rho}_m$,

\[
\hat{\rho}_m = \frac{\hat{P}_m \hat{\rho} \hat{P}_m}{\text{Tr}[\hat{P}_m \hat{\rho} \hat{P}_m]} = \frac{\hat{P}_m \hat{\rho} \hat{P}_m}{\text{Tr}[\hat{\rho} \hat{P}_m]},
\]
For a measurement $A$, with sample space $\mathcal{A} = \{\alpha_m\}$, $\hat{A} = \sum_m \alpha_m \hat{P}_m$, and a measurement $B$, with sample space $\mathcal{B} = \{\beta_n\}$, $\hat{B} = \sum_n \beta_n \hat{Q}_n$.

For solo measurements, with density operator $\hat{\rho}$,
we obtain the result $\alpha_m$ with probability $\rho(\hat{P}_m) = \text{Tr}[\hat{\rho} \hat{P}_m]$ and
we obtain the result $\beta_n$ with probability $\rho(\hat{Q}_n) = \text{Tr}[\hat{\rho} \hat{Q}_n]$.

For two measurements, of $A$ first, followed by $B$, we say that
the result $\alpha_m$ “collapses” the state from $\hat{\rho}$ to the collapsed state $\hat{\rho}_m$,
$$\hat{\rho}_m = \frac{\hat{P}_m \hat{\rho} \hat{P}_m}{\text{Tr}[\hat{P}_m \hat{\rho} \hat{P}_m]} = \frac{\hat{P}_m \hat{\rho} \hat{P}_m}{\text{Tr}[\hat{\rho} \hat{P}_m]},$$
then we measure $B$ in that state, so we obtain the result $\alpha_m$ followed by $\beta_n$ with conditional probability
$$p(\beta_n|\alpha_m) = \text{Tr}[\hat{\rho}_m \hat{Q}_n] = \frac{\text{Tr}[\hat{P}_m \hat{\rho} \hat{P}_m \hat{Q}_n]}{\text{Tr}[\hat{\rho} \hat{P}_m]},$$
so the joint probability is
$$p(\alpha_m \text{ and } \beta_n) = \text{Tr}[\hat{P}_m \hat{\rho} \hat{P}_m \cdot \hat{Q}_n] = \text{Tr}[\hat{\rho} \cdot \hat{P}_m \hat{Q}_n \hat{P}_m].$$
We have \( p(\alpha_m \text{ and } \beta_n) = \text{Tr}[\hat{P}_m \hat{\rho} \hat{P}_m \cdot \hat{Q}_n] = \text{Tr}[\hat{\rho} \cdot \hat{P}_m \hat{Q}_n \hat{P}_m] \), so the positive operators \( \hat{J}_{mn} = \hat{P}_m \hat{Q}_n \hat{P}_m \) generate the joint probabilities \( \text{Tr}[\hat{\rho} \hat{J}_{mn}] \).

Instead of collapse affecting a state, we can take collapse to affect the next measurement.

If \([\hat{A}, \hat{B}] = 0\), then \( \hat{P}_m \hat{Q}_n \hat{P}_m = \hat{P}_m \hat{Q}_n = \hat{Q}_n \hat{P}_m \hat{Q}_n \sim \text{no action} \)

We can use the positive operators \( \hat{J}_{mn} \) to construct a “collapse product”, a measurement \( A \bowtie B \), with sample space \( A \times B \), even if \([\hat{A}, \hat{B}] \neq 0\).
We have \( p(\alpha_m \text{ and } \beta_n) = \text{Tr}[\hat{P}_m \hat{\rho} \hat{P}_m \cdot \hat{Q}_n] = \text{Tr}[\hat{\rho} \cdot \hat{P}_m \hat{Q}_n \hat{P}_m] \), so the positive operators \( \hat{J}_{mn} = \hat{P}_m \hat{Q}_n \hat{P}_m \) generate the joint probabilities \( \text{Tr}[\hat{\rho} \hat{J}_{mn}] \).

Instead of collapse affecting a state, we can take collapse to affect the next measurement. If \([\hat{A}, \hat{B}] = 0\), then \( \hat{P}_m \hat{Q}_n \hat{P}_m = \hat{P}_m \hat{Q}_n = \hat{Q}_n \hat{P}_m \hat{Q}_n \sim \text{no action} \)

We can use the positive operators \( \hat{J}_{mn} \) to construct a “collapse product”, a measurement \( A \triangleright \triangleleft B \), with sample space \( A \times B \), even if \([\hat{A}, \hat{B}] \neq 0 \)

The existence of a joint probability is traditionally “classical”, so we can instead use commuting operators \( \hat{A}' \) and \( \hat{B}' \) and a different state \( \hat{\rho}' \) that give the same joint probability, \( \text{Tr}[\hat{\rho}' \cdot \hat{P}_m' \hat{Q}_n'] = \text{Tr}[\hat{\rho} \cdot \hat{P}_m \hat{Q}_n \hat{P}_m] \)

For the mathematically inclined, we can use the Neumark Dilation Theorem to construct a joint PVM \( \hat{A} \hat{B} \sim A \triangleright \triangleleft B \) (for a larger Hilbert space)
We can think of what we have just constructed as a “super-Heisenberg picture”, for which both unitary evolution and collapse are applied to measurements.

The Schrödinger picture applies both unitary evolution and collapse to the state.

The Heisenberg picture applies unitary evolution to measurements, but applies collapse to the state.
We can think of what we have just constructed as a “super-Heisenberg picture”, for which both unitary evolution and collapse are applied to measurements.

The Schrödinger picture applies both unitary evolution and collapse to the state.

The Heisenberg picture applies unitary evolution to measurements, but applies collapse to the state.

or as the “Bohr picture”, because it’s rather classical and, for Bohr, measurements affect other measurements†

or as the “QND picture” or as the “Consistent Histories picture”, because it’s commutative

or as the “Everett picture”, because it’s no-collaps

or as the “Einstein picture”, because it’s rather classical (but with a Poincaré invariant noise)

† Howard 2004
we can (and somehow must) extend this to many measurements

For a sequence of three or more measurements (*many* more for signal analysis),
we can use the *sequential product*, \( \hat{X} \circ \hat{Y} = \sqrt{\hat{X}} \cdot \hat{Y} \cdot \sqrt{\hat{X}} \),
or more elaborate *systematic* constructions of sums of positive operators

Collapse of the quantum state after measurement is ambiguous

\[
\rho \left( \sqrt{\hat{P}_i^{(A)} \hat{P}_j^{(B)} \hat{P}_k^{(C)}} \sqrt{\hat{P}_i^{(A)} \hat{P}_j^{(B)} \hat{P}_i^{(A)}} \right) \text{ or } \rho \left( \hat{P}_i^{(A)} \hat{P}_j^{(B)} \hat{P}_k^{(C)} \hat{P}_i^{(A)} \right) \?
\]

\((A \flow B) \flow C \neq A \flow (B \flow C)\)

We can use any ordering, but each makes a different assertion about dependencies
\(\flow\) is nonassociative, so, more complicated than the Heisenberg cut,
we have a *Heisenberg bracketing ambiguity*

We may not like the square root in \((A \flow B) \flow C\), but AB-preparation may be
a more natural pairing in the apparatus context than BC-measurement
For signal analysis, when we have many measurements at time-like separation, we can use $\hat{M}_1, \ldots, \hat{M}_{1000000}$, with many ambiguous collapses, or we can use $\hat{M}_1', \ldots, \hat{M}_{1000000}'$, which all commute, unambiguously, with no collapses.

We can think of this as Bohr’s ideal of a classical model for compatible measurements measurements at timelike separation can give joint probabilities.

Time reversal is easy for the QND construction, but with collapse (for just 3 measurements) we have

$$\rho\left(\hat{P}_i^{(A)} \hat{P}_j^{(B)} \hat{P}_k^{(C)} \hat{P}_j^{(B)} \hat{P}_i^{(A)}\right)$$

or

$$\rho_k\left(\hat{P}_k^{(C)} \hat{P}_j^{(B)} \hat{P}_i^{(A)} \hat{P}_j^{(B)} \hat{P}_k^{(C)}\right),$$

with the collapses running in reverse.
“Collapse” is not (only or necessarily) a dynamical process

We can (also) take it to be a JOINT PROBABILITY ALGORITHM

Belavkin 1994 Quantum Non-Demolition (QND) Measurements
Anastopoulos 2006 Sequential Measurements
Tsang&Caves 2012 Quantum-Mechanics–Free-Subsystems
“Collapse” is not (only or necessarily) a dynamical process

We can (also) take it to be a JOINT PROBABILITY ALGORITHM

A necessary tradeoff:
QM is effective for incompatible measurements, but less so for joint measurements
Collapse is QM’s way of constructing joint measurement probabilities

CM is effective for joint measurements, but less so for incompatible measurements
The Poisson bracket is CM’s way of constructing incompatible measurements
It’s a running joke that Correlation $\neq$ Causality, so, for example, *causal modeling* adds *Interventions*, with $p(Y = y \mid \text{do}(X = x))$ as a way to discuss counterfactuals, with that and other graphical and logical tools *on top of* classical modeling.

The Poisson bracket gives us a transformation algebra that can model interventions in an intrinsic and classically natural way,

$$
p(Y = y \mid \text{do}(X = x)) = \frac{\text{Tr}[\hat{P}_{Y=y} \cdot \hat{P}_{X=x} \hat{\rho} \cdot \hat{P}_{X=x}]}{\text{Tr}[\hat{P}_{X=x} \hat{\rho}]} = \frac{\text{Tr}[\hat{P}_{X=x} \hat{P}_{Y=y} \hat{P}_{X=x} \cdot \hat{\rho}]}{\text{Tr}[\hat{P}_{X=x} \hat{\rho}]}.
$$

Interventions are what people do, which is on the edge of classical modeling

Suggestion: “intervention” is a usefully different way
to think about “contextuality” or “measurement incompatibility”
We can say the vacuum state of a quantum or QND field is a broadband, noisy carrier “signal” for \textit{probabilistic} modulations of measurement results.

\[
\rho_v(\delta(\hat{M}_f - u)) = \frac{e^{-u^2/2(f,f)}}{\sqrt{2\pi(f,f)}}
\]

\[
\rho_v(e^{i\lambda \hat{M}_f}) = e^{-\lambda^2(f,f)/2}
\]

\[
(f,g) \text{ determines the geometric structure of a free quantum or QND field}
\]

We can also modulate joint measurements:

\[
\rho_v(e^{i\lambda_1 \hat{M}_{f_1}} e^{i\lambda_2 \hat{M}_{f_2}} \ldots)
\]

Call this a “super-characteristic function”
We can say the vacuum state of a quantum or QND field is a broadband, noisy carrier “signal” for probabilistic modulations of measurement results.

\[ \rho_v(\delta(\hat{M}_f - u)) = \frac{e^{-u^2/(2(f,f))}}{\sqrt{2\pi (f,f)}} \]

\[ \rho_v(e^{i\lambda \hat{M}_f}) = e^{-\lambda^2(f,f)/2} \]

\[ \rho_v\left(e^{i\lambda(\hat{M}_g - (f,g)}e^{i\lambda \hat{M}_f}e^{i\lambda \hat{M}_g}\right) \]

\[ \rho_v\left(\hat{M}_g^\dagger e^{i\lambda \hat{M}_f} \hat{M}_g\right) \]

\( (f,g) \) determines the geometric structure of a free quantum or QND field.

We can also modulate joint measurements: \( \rho_v(e^{i\lambda_1 \hat{M}_{f_1} e^{i\lambda_2 \hat{M}_{f_2} \ldots}}) \)

Call this a “super-characteristic function”.

Analysis of global properties of a model needs global tools, which may have a discrete spectrum.
For a Gaussian state, we can completely fix the algebraic structure with one equation:

$$\rho_v(e^{i\lambda_1 \hat{M}_1} e^{i\lambda_2 \hat{M}_2} \cdots) = \exp \left[ - \sum_{i,j} \lambda_i \lambda_j (f_i^*, f_j) / 2 - \sum_{i<j} \lambda_i \lambda_j [(f_i^*, f_j) - (f_j^*, f_i)] / 2 \right]$$

This equation consists of two parts:

- **Gaussian noise term**:
  $$- \sum_{i,j} \lambda_i \lambda_j (f_i^*, f_j) / 2$$

- **Noncommutativity term**:
  $$- \sum_{i<j} \lambda_i \lambda_j [(f_i^*, f_j) - (f_j^*, f_i)] / 2$$

\(\rho_v\) is a state if \((f_i, f_j)\) is a positive semi-definite matrix.
states for quantum and QND fields

For a Gaussian state, we can completely fix the algebraic structure with one equation:

$$\rho_v(e^{i\lambda_1 \hat{M}_1} e^{i\lambda_2 \hat{M}_2} \cdots) = \exp \left[ -\sum_{i,j} \lambda_i \lambda_j (f_i^*, f_j) / 2 - \sum_{i<j} \lambda_i \lambda_j [(f_i^*, f_j) - (f_j^*, f_i)] / 2 \right]$$

Gaussian noise term
noncommutativity term

$\rho_v$ is a state if $(f_i, f_j)$ is a positive semi-definite matrix

We can fix the geometric structure in multiple ways:

**Klein-Gordon:**

$$(f, g) = \hbar \int \tilde{f}^*(k) \tilde{g}(k) 2\pi \delta(k \cdot k - m^2) \theta(k_0) \frac{d^4 k}{(2\pi)^4}$$

**Quantum optics:**

$$(f, g) = -\hbar \int \tilde{f}^*_{\alpha \mu}(k) k^\mu k^\nu \tilde{g}^\alpha_{\nu}(k) 2\pi \delta(k \cdot k) \theta(k_0) \frac{d^4 k}{(2\pi)^4}$$

two space-like 4-vectors

which are manifestly Poincaré invariant
For a Gaussian state, we can completely fix the algebraic structure with one equation:

$$\rho_v(e^{i\lambda_1 \hat{M}_1} e^{i\lambda_2 \hat{M}_2} \cdots) = \exp \left[ -\sum_{i,j} \lambda_i \lambda_j (f_i^* f_j) / 2 - \sum_{i<j} \lambda_i \lambda_j [(f_i^* f_j) - (f_j^* f_i)] / 2 \right]$$

- Gaussian noise term
- Noncommutativity term

$$\rho_v$$ is a state if $$(f_i, f_j)$$ is a positive semi-definite matrix.

We can fix the geometric structure in multiple ways:

- **Klein-Gordon:** 
  \[ (f, g) = \hbar \int \tilde{f}^*(k) \tilde{g}(k) 2\pi \delta(k \cdot k - m^2) \theta(k_0) \frac{d^4 k}{(2\pi)^4} \]

- **Quantum optics:** 
  \[ (f, g) = -\hbar \int \tilde{f}_{\alpha \mu}(k) k^\mu k^\nu \tilde{g}_{\nu \alpha}(k) 2\pi \delta(k \cdot k) \theta(k_0) \frac{d^4 k}{(2\pi)^4} \] 

  two space-like 4-vectors

  which are *manifestly* Poincaré invariant

remove the “$$\theta(k_0)$$” for an everywhere commutative Gaussian QND field with a Planck-scale noise.
-operators and the relationship between QFT and QNDFT

An alternative way to construct the same Gaussian state,
\[ \hat{M}_f = a_{f^*} + a_f^\dagger, \quad [a_f, a_g^\dagger] = (f, g), \quad a_f |v\rangle = 0, \]
so that \[ [\hat{M}_f, \hat{M}_g] = (f^*, g) - (g^*, f) \]
An alternative way to construct the same Gaussian state,
\[ \hat{M}_f = a_{f^*} + a_f^{\dagger}, \quad [a_f, a_g^\dagger] = (f, g), \quad a_f |\nu\rangle = 0, \]
so that \([\hat{M}_f, \hat{M}_g] = (f^*, g) - (g^*, f)\)

For the complex Klein-Gordon field and for quantum optics,
we can find involutions \(f \mapsto f^\bullet, f^{\bullet\bullet} = f\),
for which \((f^\bullet, g^\bullet) - (g^{\bullet\bullet}, f^\bullet) = 0,\)
for all test functions \(f\) and \(g\)

For \(\hat{M}^{\text{QND}}_f = a_{f^{\bullet\bullet}} + a_{f^\bullet} \neq \hat{M}_f^\bullet,\)
\([\hat{M}^{\text{QND}}_f, \hat{M}^{\text{QND}}_g] = 0\)

The \(\hat{M}^{\text{QND}}_f\) generate a QND field: a commutative algebra of
quantum non-demolition measurements, and an isomorphic Hilbert space
-operators and the relationship between QFT and QNDFT

An alternative way to construct the same Gaussian state,
\[ \hat{M}_f = a_{f^*} + a_f^\dagger, \quad [a_f, a_g^\dagger] = (f, g), \quad a_f |v\rangle = 0, \]
so that \[ [\hat{M}_f, \hat{M}_g] = (f^*, g) - (g^*, f) \]

For the complex Klein-Gordon field and for quantum optics,
we can find involutions \( f \mapsto f^*, f^{\bullet\bullet} = f \),
for which \( (f^{\bullet\bullet}, g^*) - (g^{\bullet\bullet}, f^*) = 0 \),
for all test functions \( f \) and \( g \)

For \( \hat{M}_f^{\text{QND}} = a_{f^{\bullet\bullet}} + a_f^\dagger \neq \hat{M}_f^{\bullet\bullet} \), \[ [\hat{M}_f^{\text{QND}}, \hat{M}_g^{\text{QND}}] = 0 \]

The \( \hat{M}_f^{\text{QND}} \) generate a QND field: a commutative algebra of quantum non-demolition measurements, and an isomorphic Hilbert space

The algebra generated by the \( \hat{M}_f^{\text{QND}} \) is not isomorphic to that generated by the \( \hat{M}_f \)

For quantum optics: \( \tilde{f}^{\bullet}(k) = \frac{1}{2}(1+j\star)\tilde{f}(k) + \frac{1}{2}(1-j\star)\tilde{f}(-k) \)
\( f \mapsto f^\star \) is Lorentz invariant but not translation invariant or local, but both the Quantum and QND Field Theories are Poincaré invariant
If we allow the use of the vacuum projection operator \( \hat{V} = |v\rangle\langle v| \), then the algebra generated by \( \hat{V}, \hat{M}_f \) is isomorphic to the algebra generated by \( \hat{V}, \hat{M}_f^{\text{QND}} \).

Anything we can model with quantum optics+\( \hat{V} \), we can also model with QND optics+\( \hat{V} \) (classical, but with Poincaré invariant noise).

\( \hat{V} \) is nonlocal insofar as \([\hat{V}, \hat{M}_f] \neq 0 \) for any \( f \), but we implicitly use \( \hat{V} \) whenever we use a state transition probability.
the Wightman axioms (adapted from Haag’s Local Quantum Physics)

for which, despite how simple they look, there are no known well-defined interacting models in 3+1-dimensions, after 70 years

- A Hilbert space $\mathcal{H}$ supports a unitary representation of the Poincaré group; there is a unique lowest energy Poincaré invariant vacuum vector $|\nu\rangle$

- Quantum fields are operator–valued distributions, linear maps $\hat{M}: f \mapsto \hat{M}_f$
  from a space of modulation functions into a $\ast$–algebra $\mathcal{A}$

- Quantum fields can be a Lorentz scalar, vector, ...

- Microcausality: commutativity at space-like separation

- Completeness: the action of the quantum field on $|\nu\rangle$ generates $\mathcal{H}$

[Omitting spinors/fermions, which matter, but not today.]
Thinking about QNDFT and signal analysis suggests at least three ways in which the Wightman axioms are too strong:

- A Hilbert space $\mathcal{H}$ supports a unitary representation of the Poincaré group; there is a unique lowest energy Poincaré invariant vacuum vector $|\nu\rangle$
  
  **QNDFT:** Allow the vacuum to be *not* a lowest frequency state

- Quantum fields are operator–valued distributions, linear maps $\hat{M}: f \mapsto \hat{M}_f$ from a space of modulation functions into a $*$-algebra $\mathcal{A}$

  **Allow quantum fields to be nonlinear maps into $\mathcal{A}$**

- Quantum fields can be a Lorentz scalar, vector, ...

- Microcausality: commutativity at space-like separation
  
  **QNDFT:** Allow commutativity at *all* separations, $[\hat{M}_f, \hat{M}_g] = 0$

- Completeness: the action of the quantum field on $|\nu\rangle$ generates $\mathcal{H}$

  [Omitting spinors/fermions, which matter, but not today.]
There are two linearities implicit in the Wightman axioms:

von Neumann linearity of the state, \( \rho_v(\lambda \hat{A} + \mu \hat{B}) = \lambda \rho_v(\hat{A}) + \mu \rho_v(\hat{B}) \),

and linearity of the field, \( \hat{M}_\lambda f + \mu g = \lambda \hat{M}_f + \mu \hat{M}_g \).
There are two linearities implicit in the Wightman axioms: von Neumann linearity of the state, $\rho_v(\lambda \hat{A} + \mu \hat{B}) = \lambda \rho_v(\hat{A}) + \mu \rho_v(\hat{B})$, and linearity of the field, $\hat{M}_{\lambda f + \mu g} = \lambda \hat{M}_f + \mu \hat{M}_g$.

If we double the amplitude of a modulation, we do not expect that will double the effects of that modulation [with the certainty required for linearity to be axiomatic]

$\hat{M}_{\lambda f + \mu g} \neq \lambda \hat{M}_f + \mu \hat{M}_g$

We can also argue that renormalization can be formalized as a nonlinear dependence on window and modulation functions

Loosely: renormalization scale ← experimental details ← test functions

For real-space renormalization, nontrivial blocking algorithms are nonlinear

“A source fragmentation approach to interacting quantum field theory”, arXiv:2109.04412
The Reeh-Schlieder theorem for a Wightman field:

local operators acting on the vacuum vector $|\nu\rangle$ can approximate any vector

⇒ what path integrals can approximate
can be approximated by local operators $\hat{M}_{F_i[f_j]}$

This is an inverse problem: find local, nonlinear fragment functionals $F_i[\cdot]$ and free field QFTs that give the same results as our best path integrals

NOT the last word! I hope we can find something better

I suppose that tables and chairs can be modeled as something like a caustic
A concise list of the difficulties of QFT

1. Divergences  (non-dynamical nonlinear resonance)
2. No precise ontological picture  (signal analysis & incompatibility\(\iff\)intervention)
3. No particles  (nonlinearity & dispersion \(\longrightarrow\) caustics over time?)
4. Haag’s theorem  (subalgebra of an algebra generated by many free fields)
5. The measurement problem  (collapse of a quantum state as a joint probability construction)

from Oldofredi&Öttinger 2022
A final generalization

Given measurements $\hat{M}_{\text{Description}_1}, \ldots, \hat{M}_{\text{Description}_n}$, all we need so we can construct a Gaussian state over that collection of measurements is a positive semi-definite matrix $(\text{Description}_i, \text{Description}_j)$

The matrix does not have to be linear in $\text{Description}_i$ and $\text{Description}_j$

The domain and the manifest symmetries of the matrix fix the theory
A final generalization

Given measurements $\hat{M}_{Description_1}, \ldots, \hat{M}_{Description_n}$, all we need so we can construct a Gaussian state over that collection of measurements is a positive semi-definite matrix $(Description_i, Description_j)$

The matrix does not have to be linear in $Description_i$ and $Description_j$

The domain and the manifest symmetries of the matrix fix the theory

For interacting fields, introduce and combine many such matrices, while ensuring the properties required for $\rho_\nu(e^{i\lambda_1 M_{f_1}}e^{i\lambda_2 M_{f_2}}\ldots)$ to be a state are satisfied

For gravity, we have to describe how we measure the geometry of space-time

If $\hbar$ appears nontrivially in the construction of matrices, then it’s quantum
Quantum and Classical/QND are types of description, not types of system.

Instead of collapse of the state, we can use the QND picture, but noncommutativity is useful for modeling intervention & causality.

Signal analysis suggests the introduction of nonlinearity into the Wightman axioms.

Quantum and Classical have been converging, in numerous ways, for decades:

- Generalized Probability Theories, phase space methods, contextuality, non-demolition measurement,
- Koopman CM, time-frequency analysis, stochastic methods, semi-classical methods, superdeterminism,
- causal modeling, Cohen 1988 on characteristic functions, Abramsky 2020 on Boole’s “Conditions of Possible Experience”

“Classical states, quantum field measurement”, Physica Scripta 2019
“An algebraic approach to Koopman classical mechanics”, Annals of Physics 2020
“The collapse of a quantum state as a joint probability construction”, Journal of Physics A 2022
“A source fragmentation approach to interacting quantum field theory”, arXiv:2109.04412