Schrödinger's Cat: Qbit or Cbit?

W.M. Stuckey*

Abstract

In 1935, Schrödinger introduced what he considered to be a reductio against the Copenhagen interpretation of quantum mechanics. His argument was based on a "ridiculous case" that is widely used today to portray the counterintuitive nature of quantum superposition. Schrödinger imagined that a cat was placed out of sight in a box with a mechanism that would kill the cat within an hour with 50% probability. Since the deadly mechanism employed a quantum process for its trigger, he supposed the cat was in a quantum superposition of 50% Live Cat + 50% Dead Cat. In this paper, we point out that if Schrödinger's Cat actually represents a quantum superposition of 50% Live Cat + 50% Dead Cat, as is commonly asserted, then the cat-box system is the physical instantiation of a quantum bit of information (Qbit). This agrees with the Copenhagen interpretation, which says there is no fact of the matter as to whether the cat is dead or alive until a measurement is made. Accordingly, the state 50% Live Cat + 50% Dead Cat must be the outcome with 100% probability for some measurement complementary to the measurement 'open the box' with its two possible measurement outcomes of Live Cat or Dead Cat. If one cannot provide a physically meaningful complementary measurement to 'open the box' with a clear empirical consequence represented by the state 50% Live Cat + 50% Dead Cat as its (certain) measurement outcome, then the state 50% Live Cat + 50% Dead Cat only represents a distribution of outcomes for many trials of that single 'open the box' measurement. That is, the state 50% Live Cat + 50% Dead Cat is not a quantum superposition and Schrödinger's Cat is merely the physical instantiation of a classical bit of information (Cbit) in support of Schrödinger's reductio. The double-slit experiment is provided as an example of a Qbit to illustrate what is meant by complementary measurements (position x and momentum p for the double-slit experiment).

*Department of Physics, Elizabethtown College, Elizabethtown, PA 17022, USA

1 Introduction

In quantum mechanics, if the states $|\Psi 1\rangle$ and $|\Psi 2\rangle$ are solutions of Schrödinger's equation, then any linear combination of $|\Psi 1\rangle$ and $|\Psi 2\rangle$ is also a solution. This is called *quantum superposition* (or just superposition) and it led to an interesting debate with Einstein and Schrödinger on one side and Bohr and Heisenberg representing the Copenhagen interpretation on the other.

One point of contention had to do with what was true about reality independent of our measuring it. Einstein [1] and Schödinger [2] said that the act of measuring the physical system merely gave us information about Nature that existed whether we had decided to reveal it or not. In contrast, the Copenhagen interpretation said the outcome of the measurement didn't exist until the physical system was actually measured [3][4][5, p. 180].

In 1935, Schrödinger challenged that point using superposition "to drive the Copenhagen model *ad absurdum*" [6] (although he did not mention the Copenhagen interpretation per se). If the Copenhagen interpretation of superposition was correct [2], "One can even set up quite ridiculous cases." To make his point, he assumed a cat was closed out of sight in a box with a radioactive material that would decay with 50% probability within an hour. If a radioactive decay occurred, a deadly gas would be released in the box killing the cat. Since the decay was represented by a quantum wavefunction in a superposition of 50% "yes" and 50% "no" regarding the decay after one hour, the cat was also represented by a quantum wavefunction in a superposition of 50% "alive" and 50% "dead" [7][8][9] (Figure 1). Schrödinger wrote [2]:

> The ψ-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

Today this is known as Schrödinger's Cat [11]. Schrödinger's point was that clearly the cat is either dead or alive before we make our 'open the box' measurement. So, unless Bohr and Heisenberg were willing to admit they were wrong to say quantum reality didn't exist until it was measured, they would have to accept there were some things that were just 'too big' to be described quantum mechanically. Either way, quantum mechanics was incomplete. First, quantum mechanics didn't say anything about an underlying reality independent of measurement; worse, it said measurement outcomes establishing the exact values for some (complementary) properties simultaneously (like position and momentum) were impossible. Einstein and Schrödinger both used a pair of particles with entangled position and momentum to argue that each particle possessed definite values for both properties even though [2]:

> This cannot be fully verified in a single experiment, for that is never the case with quantum statements, but it is true nonetheless, because, whoever might have had doubts and decided to check anyway, could not be disappointed.

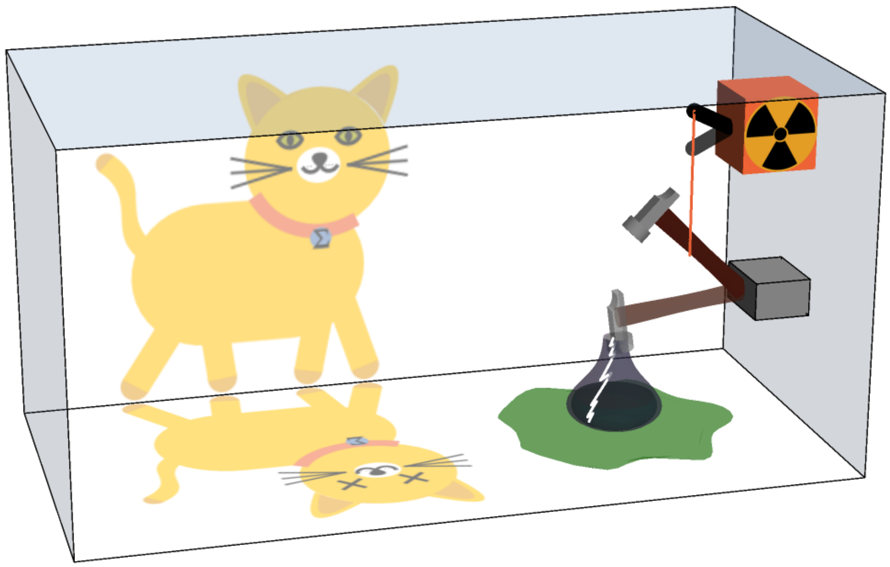

Figure 1: Schrödinger's Cat. Image from: Timothy McDevitt [10].

So, if Einstein and Schrödinger were right, quantum mechanics was incomplete in that sense. Second, quantum mechanics didn't say anything about a size limit to its applicability, so if some things really were 'too big' to be treated quantum mechanically, it was incomplete in that sense.

As to the second point, experimentalists have put larger and larger objects into superposition states, so there doesn't seem to be any reason to suspect a limit to the applicability of quantum mechanics based on size [12]. Sure, experimentalists are a long way from putting something as large as a cat into quantum superposition [13], but let's grant that possibility to explore the first point for those who want to use Schrödinger's Cat as an example of quantum superposition.

If the Copenhagen interpretation is correct, then Schrödinger's cat is literally in the state 50% Live Cat + 50% Dead Cat until the measurement is made, i.e., not Live Cat or Dead Cat just waiting for our measurement to reveal it. This is the mystery of quantum superposition and while many authors have used Schrödinger's Cat to convey this mystery, it is rare to see anyone mention complementary measurements in doing so [14].

That is, while these authors describe the cat as being both alive and dead when it is in this superposition state, the vast majority only ever talk about opening the box to find the cat alive or dead. They never propose a measurement whose outcome is the state 50% Live Cat + 50% Dead Cat with 100% probability. Indeed, they often say we never observe the superposition state. But, if the cat is *truly* in the superposition state of 50% Live Cat + 50% Dead Cat, then there is a measurement one can do on the cat-box system whose empirically observable and 100% certain outcome is 50% Live Cat + 50% Dead Cat. Remember, any linear combination of solutions $|\Psi 1\rangle$ and $|\Psi 2\rangle$ is also a solution *in itself*. The measurement with an outcome of 50% Live Cat + 50% Dead Cat is called the *complementary measurement* to the 'open the box' measurement with its two possible outcomes (pure states) of Live Cat or Dead Cat.

If there are no other measurements one can make on the cat-box system other than 'open the box', then the state 50% Live Cat + 50% Dead Cat represents a distribution over the two possible Live Cat or Dead Cat outcomes for that measurement. That is, the state 50% Live Cat + 50% Dead Cat is a *mixed state* and it is not actually an observable consequence (pure state outcome) of any measurement. Let us explore this difference in terms of quantum information theory.

2 Qbit Versus Cbit

If your query (measurement) of a physical system can produce one of only two possible answers (outcomes), then you are acquiring a *bit* of information about Nature. If that bit of information is quantum, then we are talking about a quantum bit or Qbit. If that bit of information is classical, then we are talking about a classical bit or Cbit. [We adopt Mermin's terminology of Qbit and Cbit here [15].]

The physical example of a Cbit we will use here is a ball and a box. Suppose we have a box that may or may not contain a ball. There is only one measurement we can make, i.e., we open the box, and the measurement outcome is a yes-no answer to the question, "Does this box contain a ball?" Since the ball either resides in the box or it doesn't, the probability that it resides in the box (p_1) plus the probability that it doesn't (p_2) must sum to 1, i.e., $p_1 + p_2 = 1$. We can depict that fact with a vector $\vec{p}$ in a vector space with axes $|1\rangle$ and $|2\rangle$ (Figure 2). $|1\rangle$ is the "yes" outcome of the measurement and $|2\rangle$ is the "no" outcome.

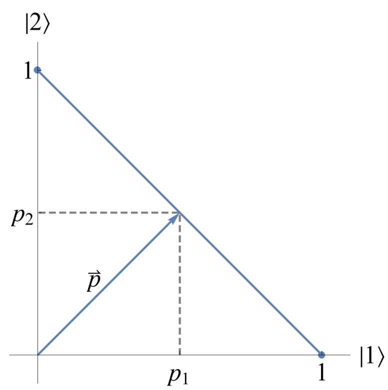

Figure 2: Probability state space for the classical bit. Image from: Timothy McDevitt [10].

The vectors $|1\rangle$ and $|2\rangle$ are called *pure states*, as they correspond to actual, single measurement outcomes. Since there is only one measurement possible on this classical bit, the state vector $\vec{p}$ pointing along the line in Figure 2 does not represent a possible single measurement outcome. It represents what is called a *mixed state*, i.e., a distribution of outcomes for that single measurement. Notice that the pure states for the Cbit are not connected via other pure states, they are only connected by mixed states. That is, there are only (two) discrete transformations between pure states for the Cbit, the identity (trivial) and the exchange ($|1\rangle \leftrightarrow |2\rangle$) [15].

Finally, there are some things that most people would consider to be true for our Cbit. First, whether or not the box contains a ball in each trial of the experiment has nothing to do with our opening the box. When we open the box, we are just finding out what is true (as Einstein and Schrödinger believed about quantum systems). Second, it makes no sense for the box to contain 50% ball and 50% no ball. Third, there is only one measurement we can make on the ball-box system, i.e., we can open the box.

In the next section, we'll look at how the Qbit differs from the Cbit. We'll give a Qbit example in detail, so you can appreciate what is needed to show that the combination of the two possible outcomes of a measurement constitutes a quantum superposition per the Qbit.

3 The Double-Slit Experiment: A Qbit

The physical example of a Qbit we will use here is the double-slit experiment with photons. In the classical double-slit experiment, a source sends monochromatic electromagnetic plane waves through a pair of slits to a detector (Figure 3a). If the detector is far enough away from the slits, an interference pattern is observed (Figure 3b).

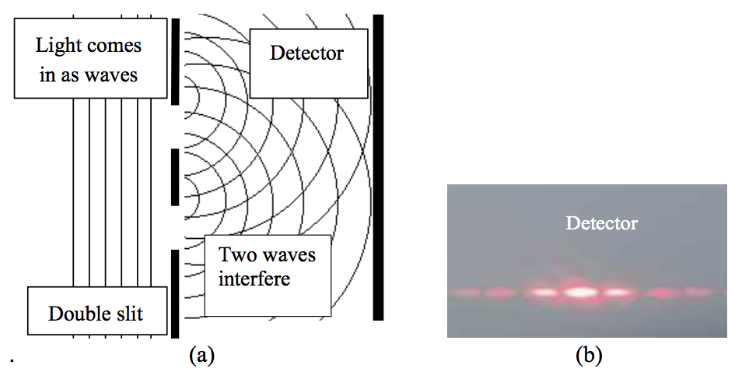

Figure 3: Double-Slit Experiment with Electromagnetic Plane Waves. Image from: Hui Peng [16].

In the quantum counterpart, we reduce the intensity of the source until it is emitting only one photon at a time. Each photon then impinges on the detector leaving a single dot located someplace along the detector. The pattern made on the detector by many such photons depends on where the detector is located relative to the slits. If the detector is located immediately behind the slits, the dots appear immediately behind the two slits forming a pattern of two vertical lines, i.e., an image of the slits (Figure 4). This is a *position measurement* x for the Qbit; the two possible outcomes are a dot in the image region of Slit 1 or a dot in the image region of Slit 2.

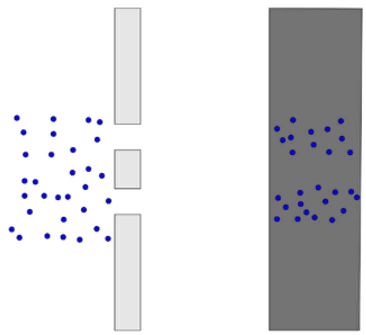

Figure 4: Detector Immediately Behind Slits. Image from: Physics in a minute: The double slit experiment. https://plus.maths.org/content/physics-minute-double-slit-experiment-0

If the detector is located far behind the slits (relative to the slit separation), the dots gradually clump into an interference pattern along the detector (Figure 5). Since the slits are illuminated equally and in phase, there is only one possible outcome for this measurement, i.e., all photons land in the image region of a constructive interference fringe (none land in the image region of a destructive interference fringe). This is a *momentum measurement* p for the Qbit, because the interference pattern allows you to compute the wavelength λ, which gives you the momentum $p = \frac{h}{\lambda}$ for a photon in the experiment (where h is Planck's constant). A lens can be inserted between the slits and detector so as to magnify the pattern. In that case, it is the location of the lens relative to the slits that determines the pattern made by many photons on the detector.

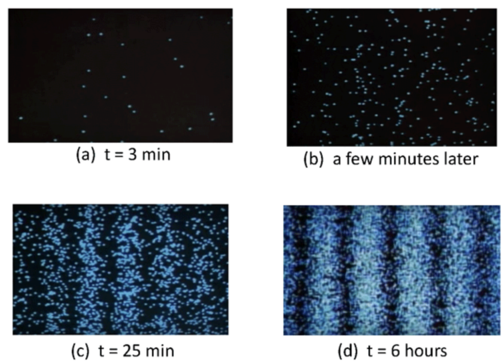

Figure 5: Detector Far Behind Slits. Image from: Ocean Optics Web Book, The Nature of Light. https://www.oceanopticsbook.info/view/light-and-radiometry/the-nature-of-light

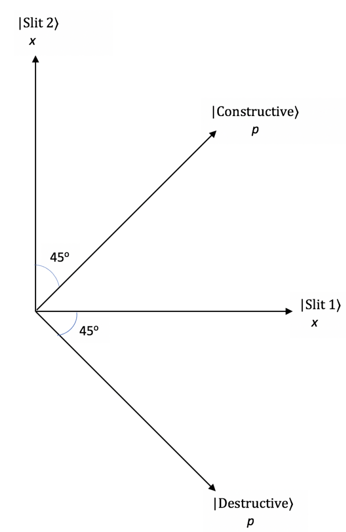

Figure 6: Hilbert Space for the Double-Slit Experiment.

The vector depiction in Hilbert space for the two possible measurement outcomes for each of the two measurements x and p is given by Figure 6. The pure state outcomes for a position measurement x are the vectors $|\text{Slit 1}\rangle$ and $|\text{Slit 2}\rangle$. The pure state outcomes for the complementary momentum measurement p are the vectors $|\text{Constructive}\rangle$ and $|\text{Destructive}\rangle$. From Figure 6 we see that

$$|\text{Constructive}\rangle = \frac{|\text{Slit 1}\rangle + |\text{Slit 2}\rangle}{\sqrt{2}} \tag{1}$$

and

$$|\text{Destructive}\rangle = \frac{|\text{Slit 1}\rangle - |\text{Slit 2}\rangle}{\sqrt{2}} \tag{2}$$

Notice that in the $|\text{Destructive}\rangle$ outcome, the phase of a photon associated with Slit 2 differs from that associated with Slit 1 by π, i.e., $\mathrm{e}^{i\pi} = -1$. So, if the slits are illuminated equally and in phase (as is true in our example), $|\text{Destructive}\rangle$ will be the regions where destructive interference fringes appear in the interference pattern (no photon detection events). In order to have photon dots appear in these regions, i.e., in order to have measurement outcomes for $|\text{Destructive}\rangle$, the initial state $|\Psi\rangle$ must have some non-zero projection onto $|\text{Destructive}\rangle$. Our initial state is $|\Psi\rangle = |\text{Constructive}\rangle$, which is orthogonal to $|\text{Destructive}\rangle$, so all photon dots (all measurement outcomes) for the p measurement on our initial state will be in the constructive interference fringes (Figure 5).

If you added a π phase plate to Slit 2, for example, then the initial state would be $|\text{Destructive}\rangle$ and all the photon dots would all appear in that region. That is, your initial state would be $|\Psi\rangle = |\text{Destructive}\rangle$, so the p measurement would produce $|\text{Destructive}\rangle$.

The probability that a photon in the constructive fringes is associated with Slit 1 is found by projecting the vector $|\text{Slit 1}\rangle$ onto the vector $|\text{Constructive}\rangle$ and squaring the result, i.e., $|\langle\text{Slit 1}|\text{Constructive}\rangle|^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 0.5$ (50%). Ditto for Slit 2 giving the same 50% result. This is what it means for p to be complementary to x in quantum information theory, i.e., when your measurement outcome gives you information allowing you to compute p precisely (perfect interference pattern allowing you to obtain exact λ) your information about x is totally random. Let's pause to see how this relates to Schrödinger's Cat.

Our initial state is $|\Psi\rangle = |\text{Constructive}\rangle$, i.e., 50% Slit 1 + 50% Slit 2 (the slits are equally illuminated by the source and have equal phase), in total analogy with our initial state of 50% Live Cat + 50% Dead Cat for Schrödinger's Cat. If we put our detector (or lens) very close to the slits, we are doing a position measurement x on our initial state, so we find that 50% of the photons strike immediately behind Slit 1 and 50% of the photons strike immediately behind Slit 2. [This is what we computed above.] That's analogous to the 'open the box' measurement of our initial state 50% Live Cat + 50% Dead Cat in the Schrödinger's Cat experiment. There we find a live cat 50% of the time we open the box and we find a dead cat 50% of the time we open the box.

For example, Peng [16][17] divided the region between the slits (in a diaphram) and detector (screen) into three Zones: Zone 1 is closest to the slits (position measurement in this Zone), Zone 3 is farthest from the slits (momentum measurement in this Zone), and Zone 2 is between Zones 1 and 3 ("transition region" between position and momentum measurements) (Figure 7). While Peng didn't do the experiment with single photons, the intensity of light on the detector screen serves to provide a probability for where any photon in the single-photon experiment would land. Peng's images also show clearly the binary outcome regions for the Qbit measurements in question. Remember, we are doing physics here, so theoretical measurements must correspond to actual configurations of physical equipment and theoretical measurement outcomes must correspond to empirical observations. In Peng's position measurement x, the lens was placed very close to the slits and the measurement outcome was just an image of the two slits (D-1 in Figure 8).

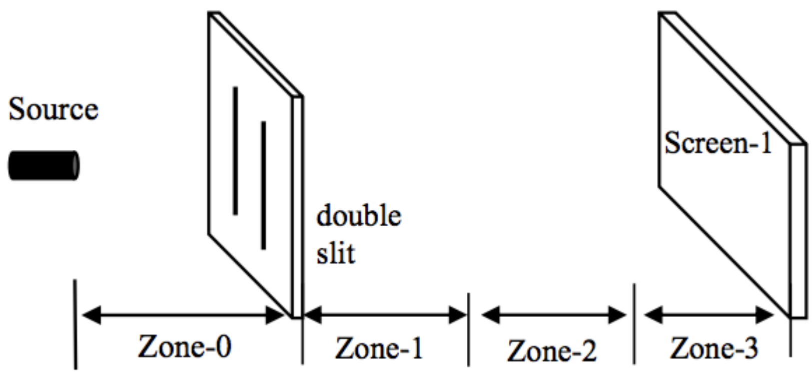

Figure 7: Peng's Zones for the Double-Slit Experiment. Image from: Hui Peng [16].

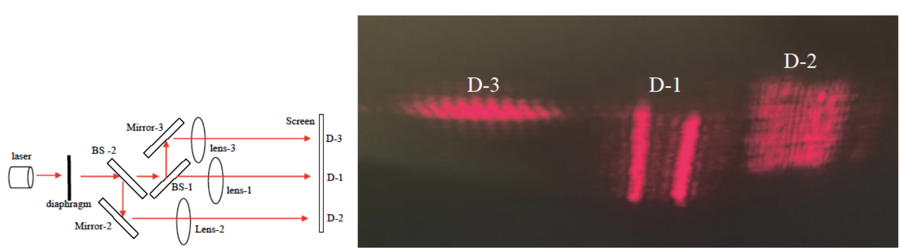

Figure 8: Peng's Zone Results for the Double-Slit Experiment. Image from: Hui Peng [17].

Since the double-slit experiment is a Qbit, there is a measurement whose outcome is the initial state itself with 100% probability, i.e., the initial state is a pure state (an eigenstate for some measurement operator). The state we are measuring is given by Eq. 1, so there must be some measurement we can perform on this Qbit that will give us $|\text{Constructive}\rangle$ with certainty, i.e., all outcomes (photon dots on the detector screen) are associated with $|\text{Constructive}\rangle$ (contribute to the region of constructive interference fringes when the slits are illuminated equally and in phase). Again, that measurement is the momentum measurement p, i.e., $|\text{Constructive}\rangle$ is an eigenstate of the momentum measurement operator. The corresponding empirical observation in Peng's experiment occurred when the detector was placed far from the slits, so the measurement outcome was an interference pattern (Figure 9).

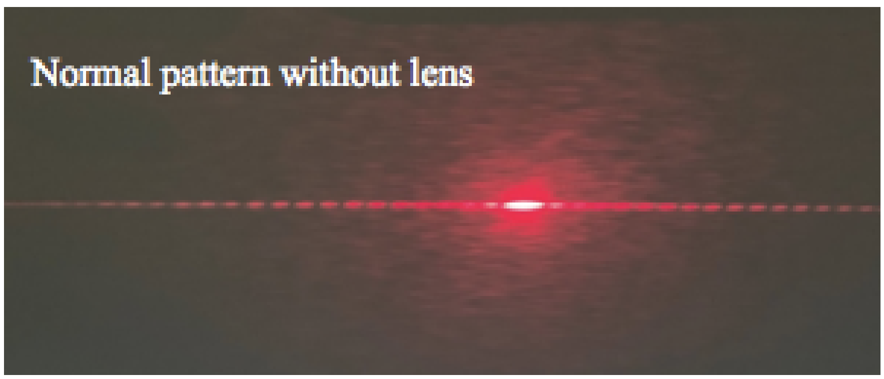

Figure 9: Peng's p measurement. There is no lens and the detector (screen) is 2500mm from the slits, which are separated by 1mm. Image from: Hui Peng [17].

Finally, since the double-slit experiment is a Qbit, there is a continuum of measurement configurations and outcomes between x and p. In Peng's experiment, those are his "transition" measurements and outcomes corresponding to the lens being translated in continuous fashion from near the slits to far from the slits (D-2 in Figure 8).

In summary, when making a 'which slit' or position measurement x on the state $|\Psi\rangle = |\text{Constructive}\rangle$, we obtain one of two outcomes $|\text{Slit 1}\rangle$ or $|\text{Slit 2}\rangle$ with equal probability. This measurement is done by putting the detector very close to the slits, so that a photon passing through a slit does not have any access to the detector except immediately behind that slit. So, if Slit 1 is left of center and Slit 2 is right of center, and we're doing an x measurement, a photon striking the detector left of center is associated with Slit 1 with 100% probability and a photon striking the screen right of center is associated with Slit 2 with 100% probability. As the detector is moved farther and farther from the slits, a photon dot left of center becomes more and more likely to be associated with Slit 2; and a photon dot right of center becomes more and more likely to be associated with Slit 1. When the detector is very far from the slits compared to the slit separation (2500mm compared to 1mm for Peng's experiment), a photon dot left of center is equally likely to be associated with Slit 2 as Slit 1; and a photon dot right of center is equally likely to be associated with Slit 1 as Slit 2. Again, this is the complementary measurement to the position measurement x, so the 'which slit' information is now completely randomized. All of this is to say, when making an 'interference' or momentum measurement p on the state $|\text{Constructive}\rangle$, we obtain $|\text{Constructive}\rangle$ with 100% probability.

This continuous progression of measurement configurations has outcomes represented in Figure 6 by a continuous set of basis vectors rotated from $|\text{Slit 1}\rangle$ and $|\text{Slit 2}\rangle$ to $|\text{Constructive}\rangle$ and $|\text{Destructive}\rangle$. This means pure states are connected continuously to pure states for the Qbit, i.e., pure states transform continuously to pure states in Hilbert space. This is quantum superposition and it alone suffices to distinguish classical probability theory from quantum probability theory [10]. For example, as Hardy notes in his first reconstruction of quantum mechanics based on information-theoretic principles [18], if you delete just one word ("continuous") from his Axiom 5, "then we obtain classical probability theory instead" of quantum probability theory. And concerning their information-theoretic reconstruction of quantum mechanics, Masanes & Müller write [19], "... if Requirement 4 is strengthened by imposing continuity of the reversible transformations, then [classical probability theory] is ruled out and [quantum theory] is the only theory satisfying the requirements."

4 Conclusion

We have shown that the initial state 50% Live Cat + 50% Dead Cat for the Schrödinger's Cat experiment is analogous to the initial state 50% Slit 1 + 50% Slit 2 for the double-slit experiment. And, the 'open the box' measurement on 50% Live Cat + 50% Dead Cat is analogous to a position measurement x on 50% Slit 1 + 50% Slit 2; 'open the box' produces outcomes of Live Cat or Dead Cat with equal frequency while an x measurement produces outcomes of Slit 1 or Slit 2 with equal frequency. Given that alone, we have a measurement on each physical system that produces a bit of information about Nature, but we don't know if either of these physical systems is a Qbit. Consequently, we don't know that either of these initial states provides an example of quantum superposition.

If the initial state is a quantum superposition, we can find the complementary measurement whose outcome is the initial state itself with 100% probability. And, we can describe the measurement configurations producing the continuum of measurement outcomes between the outcomes of the two complementary measurements. Otherwise, all we have is a Cbit whose initial state is a mixed state, i.e., not a quantum superposition.

We were able to find a measurement complementary to the position measurement x for the double-slit experiment, thus proving it is a Qbit and its initial (pure) state is a quantum superposition. The momentum measurement p complementary to the position measurement x was produced by moving the detector far from the slits where the measurement outcome was 50% Slit 1 + 50% Slit 2. That is, the outcome of the p measurement on the state 50% Slit 1 + 50% Slit 2 was that very same state, i.e., an interference pattern where all photon detection events were associated with $|\text{Constructive}\rangle$. The measurement configurations between x and p producing a continuum of measurement outcomes between the bases $|\text{Slit 1}\rangle$-$|\text{Slit 2}\rangle$ and $|\text{Constructive}\rangle$-$|\text{Destructive}\rangle$ in Hilbert space, were created by moving the detector from near the slits for x to far from the slits for p. Those measurements and outcomes are easy to instantiate physically and observe empirically.

In order to show that Schrödinger's Cat provides an example of quantum superposition, refuting Schrödinger's reductio and supporting the Copenhagen interpretation, one must produce physical measurements with empirical outcomes analogous to those for the double-slit experiment [14]. We admit that we cannot provide any such physical measurements, let alone their corresponding empirical outcomes, so we leave that as a challenging exercise for those who want to use Schrödinger's Cat as an example of quantum superposition.

References

[1] A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Physical Review **47**(10), 777–780 (1935). `https://link.aps.org/doi/10.1103/PhysRev.47.777`

[2] Ewrin Schrödinger, "The present situation in quantum mechanics," Naturwissenschaften **23**(48), 807-812 (1935). Translated by John Trimmer `https://www.unicamp.br/~chibeni/textosdidaticos/schrodinger-1935-cat.pdf`.

[3] Jan Faye, "Copenhagen Interpretation of Quantum Mechanics," The Stanford Encyclopedia of Philosophy (Summer 2024 Edition), ed. E.N. Zalta. `https://plato.stanford.edu/archives/sum2024/entries/qm-copenhagen/`.

[4] T.J. Axon, "Introducing Schrödinger's cat in the laboratory," American Journal of Physics **57**(4), 317–321 (1989).

[5] David Prutchi and Shanni Prutchi, *Exploring Quantum Physics Through Hands-On Projects* (John Wiley & Sons, Inc., Hoboken, New Jersey, 2012).

[6] Gerd Kortemeyer and Catherine Westfall, "History of physics: Outing the hidden curriculum?," American Journal of Physics **77**(10), 875-881 (2009).

[7] J.G. Loeser, "Three perspectives on Schrödinger's cat," American Journal of Physics **52**(12), 1089-1093 (1984).

[8] L.E. Ballentine, "Comment on 'Three perspectives on Schrödinger's cat [Am. J. Phys. 52, 1089 (1984)]'," American Journal of Physics **54**(10), 947-948 (1986).

[9] J.G. Loeser, "Reply to Ballentine's comment [Am. J. Phys. 54, 947 (1986)]," American Journal of Physics **54**(10), 948-949 (1986).

[10] W.M. Stuckey, Michael Silberstein, and Timothy McDevitt, *Einstein's Entanglement: Bell Inequalities, Relativity, and the Qubit* (Oxford University Press, Oxford, UK, 2024).

[11] Robert P Crease, *Ursula Le Guin: the pioneering author we should thank for popularizing Schrödinger's cat.* https://physicsworld.com. Quantum Opinion AND Reviews (27 May 2024).

[12] Christopher Gerry and P. L. Knight, "Quantum superpositions and Schrödinger cat states in quantum optics," American Journal of Physics **65**(10), 964-974 (1997). https://www.researchgate.net/publication/243491864_Quantum_superpositions_and_Schrodinger_cat_states_in_quantum_optics.

[13] N. G. van Kampen, "The Scandal of Quantum Mechanics," American Journal of Physics **76**(11), 989-990 (2008).

[14] Robert Griffiths, "EPR, Bell, and quantum locality," American Journal of Physics **79**(9), 954-965 (2011). https://arxiv.org/pdf/1007.4281.

[15] N. David Mermin, *Quantum Computer Science: An Introduction* (Cambridge University Press, Cambridge, UK, 2007).

[16] Hui Peng, "Photons distributing as waves — New mystery of double slit experiment," TechRxiv. March 27, 2023. https://www.techrxiv.org/doi/full/10.36227/techrxiv.22335739.v1.

[17] Hui Peng, "Photoelectric Effect to Photowaves Phenomena — New Phenomena Requiring Interpretation," TechRxiv. April 25, 2023. https://www.techrxiv.org/doi/full/10.36227/techrxiv.22659172.v1.

[18] L. Hardy, *Quantum Theory from Five Reasonable Axioms.* https://arxiv.org/abs/quant-ph/0101012 (2001).

[19] Ll. Masanes and M.P. Müller, "A derivation of quantum theory from physical requirements," New Journal of Physics **13**, 063001 (2011). `https://arxiv.org/abs/1004.1483`.