

OPEN SYSTEMS AND AUTONOMY

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ABSTRACT. This paper designs and defends a conceptual framework for the disambiguation of scientific language regarding open and closed systems. We argue that the open-closed distinction should always be precisified by specifying a characteristic quantity that is conserved if and only if the system is closed. Open systems are those for which conservation of the characteristic quantity fails. This precisification is in accord with much but not all existing practice. We show that an open system can have well-posed autonomous dynamics and need not be embeddable in any larger system. We distinguish two kinds of autonomy and show that they dissociate from the open-closed distinction. We argue that this framework clears the path towards a new approach to the modelling of autonomous open systems in quantum physics and cosmology.

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1. INTRODUCTION

The term ‘open system’ is widely used in science along with its contrary ‘closed system’. It is often taken to be of great importance whether a system is ‘closed’ or ‘open’, and this feature may even be thought to be definitive, or at least

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necessary, for a system to be studied by a particular science. Since Schrödinger puzzled about how living things can maintain their order and structure out of equilibrium, it has been observed that the Second Law of Thermodynamics refers to ‘isolated’ systems, while “organisms are open systems” (Olby 1971, p. 128). The relevant sense of open here involves the exchange of both energy and matter with the environment (Von Bertalanffy 1950). Order and self-organisation can also emerge in nonliving systems that are open only in the sense of being driven by some force or flow of energy without exchanging matter with the environment. This is the sense in which complex systems are always open (Ladyman and Wiesner 2020).

Much of physics deals with closed systems (in various senses of the term distinguished and precisified below), but there are many models of open systems in physics, and open systems are of fundamental importance. For example, consider the significance of Brownian motion for the development of current physics. There are many such physical systems, including importantly stars, whose distinctive physical behaviour is only possible because they are open (Phillips 2013). Of course, all real physical systems are open in the sense that they interact or exchange matter or energy with their environments to some, even if only to a negligible, extent. However, the salient feature of the Brownian motion of a pollen grain, or the damped oscillation of pendulum in the atmosphere of the Earth, is that interactions with the environment *non-negligibly* affect the system’s dynamics. The ‘scale-relativity’ of the open-closed distinction as applied to concrete systems in scientific practice is an important part of the analysis of this paper (and is the subject of Section 3). There are absolute distinctions between open and closed systems in the context of models and theories, however, these correspond to differences of degree in practice. Differences of degree add up to give the dynamics at different scales studied by physics. The difference of degree between a few particles and 10^{23} makes gases and their emergent modal structure, for example, the gas laws, cf. Ladyman and Lorenzetti (2023).

The notions of open and closed in physics are entwined with ideas of autonomy, conservativeness, embeddability in a larger system, and formal properties of time evolution such as being completely positive, Markovian, unitary or measure-conserving. While the concepts of open and closed systems are of widespread foundational significance, the exact meaning of the terms ‘open’ and ‘closed’ varies between different contexts even within physics. To complicate matters even more sometimes the term ‘closed’ is used to mean ‘isolated’, and this proliferation of concepts and language greatly complicates foundational and philosophical work. It is

important to be clear about the relevant concepts and terminology so that fallacious reasoning based on conflation of the meanings of terms does not arise. Much of this paper designs and defends a conceptual framework for the disambiguation of scientific language regarding open and closed systems. Despite remaining very close to scientific practice, we arrive at some rather surprising conclusions. Most surprising of all, that the universe, which is usually taken to be a closed system, may be open.

The next section analyses the open-closed distinction in more detail. We find that although the distinction is made in different ways in different domains, its conceptual core is the conservation or not of some characteristic quantity. Section 3 discusses the importance of scale-relativity for distinguishing whether concrete systems are modelled as open or closed. Section 4 introduces the key idea of *autonomy* defined in terms of the feature of a model of a system whereby the model contains well-posed equation of motion for the dynamical variables of the system alone. Autonomous models may nonetheless encode the dependence of system's evolution on the environment, as with models of Brownian motion and many other models of open systems in physics. Hence, we further distinguish 'parameterised' autonomy, which involves parameters that represent features of the environment, from 'absolute autonomy which does not involve such parameters. Section 4 also shows that autonomy of both types is independent of the open-closed distinctions of the next section. Autonomous open systems models are an important part of science at various scales. Section 5 considers the special case of open quantum systems and argues, following Cartwright (1983) and Cuffaro and Hartmann (2021), that there is no reason to take embeddability in a larger unitary system to be a fundamental feature of open quantum systems. Section 6 draws on the previous sections to argue that the argument that the universe as a whole must be subject to unitary time evolution conflates different senses of closed, and that it is in fact an open question whether we should model the universe as an open or closed quantum system.

2. OPEN AND CLOSED

Two other foundational distinctions are required for our analysis. The first distinction is motivated by the observation that the same concrete system, a particular pendulum in a laboratory for example, can be modelled as closed, if friction is ignored, or as open if it is not. Hence, it is important to distinguish between how the system is modelled and the system itself when discussing ideas of open and closed. In general, concepts and terms can be applied in the *material mode*

to *concrete systems*, and in the *formal mode* to *models*.¹ Our usage of these terms is as follows. To discuss a physical system X in the material mode is to make an ontological claim about the properties of X . To discuss X in the formal mode is to refer to the representations (linguistic or mathematical structures) of X .² In what follows we often clarify concepts and terms relating to open and closed systems by drawing attention to the formal mode/material mode distinction. Of course, any application of terms in material mode involves the conceptualisation of the system in some way. However, since the same material system may be modelled as open or a closed system the material/formal distinction is required to say unambiguously whether the system is open or closed. More generally we can obviously model real systems in many ways and consider many aspects of them while also idealising or abstracting, and sometimes the questions we ask are about the system as modelled.

The second distinction is that between a system and its environment. In material mode, unless the system is everything there is, the open-closed distinction always concerns the relationship between the system and its environment. However, in formal mode the environment may not be modelled at all and the system may be treated as if it is everything there is. For example, it may stipulated that the system conserves or does not conserve a characteristic quantity without the environment or its effects on the system being considered at all. In scientific practice (except in cosmology), the environment is always the local surroundings of the system not the whole universe. The immediate environment of the system is taken to be all that matters for the purposes of the modelling the evolution of the system. The distinction between a system and its environment, surroundings or other systems is usually predicated on there being some kind of boundary around the system so that whether the system is open or closed depends on what happens at the boundary. However, in some models there is no boundary just the distinction between the system and its environment. Again, in formal mode there may be no environment since it may not be modelled at all, but of course in material mode every system except the whole universe has an environment. The local environment may be assumed to screen off any effects that the rest of the universe has on the system because they have to go through it, but of course if there is action at a distance this is not so.

¹The idea of material and formal modes is used by [Ladyman and Ross \(2007\)](#) who draw upon a similar though non-identical distinction due to [Carnap \(1934\)](#).

²This distinction is implicit in Cartwright's discussion of open quantum systems when she says, "to demand a physical correlate of unitarity is to misunderstand what functions it serves in the quantum theory" ([Cartwright 1983](#), p. 203). In our terms she is saying that the quantum open-closed distinction, which just is the unitary-nonunitary distinction (as Section 5 discusses in more detail) is made in formal mode and may not correspond to anything in material mode.

The principal notions of open and closed systems applied in physical practice can then be distinguished as follows:

Definition 1. *Closed Systems in Material Mode.*

1. *In the most general and strongest sense a system is closed (and isolated) iff it is not interacting with or affected by anything else.*
2. *In chemistry (and often in engineering) a system is closed iff it is contained within a boundary that no matter crosses. Heat or light can cross the boundary of chemically closed systems.³*
3. *In classical mechanics a system is closed iff it has no net (non-conservative) forces acting upon it (while also not exchanging matter with its environment).*
4. *In thermodynamics a systems is closed iff it does not exchange heat or work with its environment (while not exchanging matter with its environment).⁴*
5. *In statistical mechanics a system is closed if the number of particles is constant (and the probability flow is ‘incompressible’).⁵*
6. *In quantum theory a system is closed iff its coherence does not change.*

Obviously a system that is chemically closed need not be closed mechanically or thermodynamically, and indeed most reactions are either endothermic or exothermic. It is not true that a mechanically closed system must be isolated since a mechanical system may be closed whilst being subject to conservative forces. It is true, however, that a mechanically open system, which is acted on by non-conservative forces, is thermally open in that there is net work done on the system for any closed state space loop. In general, systems being isolated and systems being closed must be distinguished. An isolated system is a system that is not interacting with the environment or affected by anything else, so there is no exchange of matter, energy, information, or anything else across the boundary of the system. This may or may not be connected to failure of conservation since, for example, a system can be conservative whilst having boundary flows when these flows are net zero. In practice neither being closed nor isolation are absolute, but both can be effectively true (as noted in Section 1 and discussed in the next section).

³Ensuring that experiments were performed in a closed system in this sense, together with the precise measurement of masses and volumes was critical to the Chemical Revolution, as was quantifying the heat flow [Ladyman et al. \(2024\)](#).

⁴This is described in different ways with some authors using ‘closed’, and others using ‘isolated’ to mean not exchanging heat or work with the environment.

⁵Probability is here understood as a kind of modal structure in material mode. If probability is purely epistemic then of course there is no correlate of the conservation of volume measure.

Clearly, the open-closed distinction is related to conservation such that in the context of particular physical theories there is typically a *characteristic quantity* such that the system is closed iff the quantity is conserved by its time-evolution, and otherwise open. A closed mechanical system is conservative with respect to energy, which is equivalent to there being no net non-conservative force since systems acted on by purely conservative forces are also energy conserving. A closed chemical system is conservative with respect to mass. A closed statistical mechanical system is conservative with respect to the number of particles and probability current. A closed quantum system is conservative with respect to quantum coherence. Note that simple-harmonic motion that is conservative with respect to energy, and so mechanically closed, may be driven by an external conservative potential, and so not isolated.

All the above definitions can be understood in material mode (but only effectively unless they are applied to the whole universe), and when the conservation of the characteristic quantity is encoded formally within a model, they all have formal mode correlates as follows:

Definition 2. *Closed Systems in Formal Mode.*

- 1.* *In the most general and strongest sense a model is closed (and isolated) iff it conserves everything that matters to the dynamics.*
- 2.* *In a chemically closed model mass is conserved, and so is the number of nuclei of each atomic species.*
- 3.* *In a model of a closed system in classical mechanics energy is conserved (the Hamiltonian is a constant of the motion).*
- 4.* *In a model of a closed system in thermodynamics heat and work are conserved.*
- 5.* *In statistical mechanics a model of a closed system has constant phase space dimension and volume measure.*
- 6.* *In quantum theory closed systems are those that undergo unitary time evolution.*

The formal mode and material mode descriptions may come apart. For example, a concrete engineering system may be modelled as conserving mass, and so as closed in the relevant formal mode sense, even though it leaks and so is not closed in the relevant material mode sense, and a concrete quantum system may be modelled as evolving unitarily, and so as closed in the relevant sense, even though it is subject to a small degree of decoherence and so is not closed in the relevant material

mode sense. Systems that are not conservative are often assumed to be contained within a larger system that is conservative, for example, if the particles that leak out of a chemical system are present in the environment they are conserved in the wider system. In the context of open quantum systems it is often assumed that they are embedded in larger closed quantum systems (as noted above and discussed in Section 6).

In foundational contexts the terms ‘open’ and ‘closed’ *must* always be precisely specified because discussion often involves consideration of different theoretical domains in which different ideas of open and closed are at issue, and hence different quantities are conserved. Hence, it is fallacious to argue that a system should be modelled as open in the formal mode sense of non-unitary time evolution, because it is interacting with another system, because that means only that it is not closed in the general material mode sense, unless some grounds are given to connect the different precisifications of the open-closed distinction. In fact, as discussed in detail in Section 6, it is not true that systems being affected by their environments cannot be modelled with unitary time evolution, so the general sense of closed in material mode does not entail the formal mode sense of closed in quantum physics.

In general, systems can be modelled by explicitly representing the environment in detail, or by only representing its effect on the system. Either way, the whole environment is never really modelled, but only the immediate surroundings of the system, so that together they can be thought of as a system that is effectively isolated from the rest of the universe. In this way, the dynamics of the system that depend on interactions with the environment can be modelled without modelling the whole universe, for example, by treating the environment as a heat bath, or as a source of random perturbations as in Brownian motion. The success of science in finding descriptions of effectively isolated systems, and open systems with local environments that are effectively isolated makes it seem like the only way to understand open systems is as part of bigger closed systems. Similarly, it might be assumed that any open system can always be modelled as part of a bigger closed system, and since there is nothing more than the universe it might seem obvious that it must be a closed system in every sense. This assumption and the conclusion drawn from it are challenged in Section 6.

Many models work by assuming that open systems can be embedded in bigger closed systems, but in every such case it is always possible to make physical measurements to estimate the variables or parameters that represent the effect of the

environment on the system, for example, the temperature of a heat bath, or viscosity of a liquid medium. In the case of the universe to embed it in anything bigger is speculative. The embeddability of open systems in larger closed and isolated systems is discussed further in Section 6. The next section considers how applying the open-closed distinction in any of its forms to concrete systems is scale-relative.

3. SCALE-RELATIVITY

We start with a simple observation regarding gravitational interactions and isolation that goes back at least to Russell (1903), cf. Barbour (2001). The point is that gravitational interactions cannot be screened in Newtonian gravitation (and in fact also general relativity). Thus, every massive body in the universe interacts with every other massive body. Hence, in material mode strictly speaking *there is no such thing as an isolated system*. Quantum field theory also implies that no system is ever really isolated since the vacuum is constant source of fluctuations. Furthermore, there is a fundamental ontological sense in which all systems studied in science are ‘metaphysically open’ in the sense that they are always interacting *to some degree* with other systems even non-gravitationally (unless the whole universe is modelled as per the discussion of Section 6). As Nielsen and Chuang (2010) put it ‘...in the real world there are no perfectly closed systems, except perhaps the universe as a whole.’ (p.353).

In the light of this observation it might seem that the material mode attribution of open in terms of non-isolation loses its conceptual purchase since it applies to everything. The reason it does not is because of the general feature of scientific theorising, modelling and practice called ‘scale-relativity’ by Ladyman and Ross (2007). The idea is that arguably all, and certainly most, of physical science is concerned with effective rather than fundamental ontology. Chemical bonds, fluids and quasi-particles are obviously not fundamental entities but they are part of a domain, regime or scale that is spatial, temporal and energetic. For example, when we say that a flask of liquid is in thermal equilibrium with its environment this is implicitly relative to timescales that are short compared to the half-life of the liquid (which is evaporating and will eventually be entirely gone). Clearly, as Russell pointed out, all systems are materially open in the non-isolation sense because they are interacting gravitationally and otherwise to some degree. However, in many situations these interactions have negligible effects on aspects of their state. In such contexts they may be understood as isolated in the material mode since we

can understand such statements in terms of *effective* isolation in accordance with scale-relativity.

It is difficult to overemphasise the significance of this observation for the interpretation of scientific language. In particular, when combined with the distinction between material and formal modes, the notion of scale-relativity renders transparent various aspects of scientific practice, that appear ambiguous or confusing otherwise. Most significantly, very often in science we find that *properties are represented as absolute in a model when applied to a concrete system in material mode they are scale-relative*. For example, the smoothness of a surface is broken at small length scales, and the regularity of the oscillations of Newton's cradle is broken at long time scales. Being closed qua isolated (1. above) fits this pattern because systems are represented as isolated in absolute terms in a model, since exogenous interactions are completely excluded, whereas in material mode the system is effectively isolated in the sense that its dynamics are relatively unaffected by anything else over relevant timescales. Newtonian mechanics allows us to consider the solar system in just these terms: although it is subject to gravitational forces from the rest of the universe we can assume that its acceleration can be modelled as approximately uniform and linear and thus, via Corollary VI to the Laws (? , p.423), treated as though it were not accelerating at all. This idealisation is justified precisely because effects arising from the non-uniformity of the gravitational field can be ignored at the scale in question. Furthermore, as noted in the previous section, even when systems are modelled as open, it is assumed that the system and its environment can be studied as if they form a larger system that is itself effectively isolated.

Scale relativity is also, of course, significant for the understanding of the other sense of open and closed discussed above – that is, the sense based upon conservation of a characteristic quality. When we talk of ‘closed systems’ in material mode we are referring to systems in which some relevant physical feature or quantity is conserved to some degree of approximation over the relevant timescale. For example, in practice the principle of conservation of mass is not exactly true of a chemical reaction, but in a large well-engineered chemical system any egress or ingress of matter is negligible compared to the size of the system and has no effect on the evolution of its overall properties such as, for example, the half-life of the reagents or the time when the reaction will effectively be over. Similarly, we talk about concrete mechanical systems being closed in the material mode we are referring to the approximate conservation of the total energy of the system over the

relevant timescale. A closed mechanical system has energy-conservative dynamical evolution, while an open mechanical system has dissipative dynamical evolution where in each case the conservation and non-conservation is indexed to a time-scale (or spatial scale). For example, the earth-moon-sun system is understood as a closed mechanical system on the basis that the loss of energy is negligible on the relevant scales.

Conservation of the characteristic quantity may imply that interactions with the rest of the universe are not important to the systems dynamics at the relevant scale, as in a closed mechanical system, but it may not as in the chemical case, where the exogenous application of heat to the system is relevant to its evolution. In general, interactions with their environment can be crucial to the dynamics of closed systems at the relevant scale, because there are situations where there is an exogenous driving force or a net zero flux that is dynamically relevant even though the characteristic quantity is conserved. However, it is also often the case that a system can be understood as isolated on a given scale just because it can be understood as conservative of some quantity at that scale. It is appropriate to talk of ‘open systems’ in physics in material mode when referring to concrete systems in which the relevant physical feature or quantity is not conserved over the relevant timescale.

As noted above, isolation and non-isolation, and conservation and non-conservation are typically represented as absolute in a model when in material mode they are scale-relative. That is, a system is represented as isolated or as conservative of the relevant quantity for the purpose of modelling, because at the scale picked out by the modelling context the evolution of the system is as if it was isolated, or the property is approximately conserved. Typically the scales in question are timescales, but spatial and numerical scales, and macro and coarse-grained states may also be relevant. For example, the individual particles in a gas are affected by the motions of atoms in other solar systems, but the evolution of the aggregate properties of the gas such as its pressure and temperature are not.

For mechanical systems the formal mode correlate of being closed qua conserving energy is typically found in terms of a Hamiltonian representation with both a privileged conserved function, the Hamiltonian, and a conserved phase space measure, the Liouville measure. For mechanical systems the formal mode correlate of being an open qua non-conservative system is typically found in terms of a contact Hamiltonian representation with non-conservation of the Hamiltonian, and a non-conserved phase space measure ([Bravetti and Tapias 2015](#); [Bravetti et al. 2017](#),

2020). A vivid analogy can be made between conservation of the phase space measure and the flow of an incompressible fluid and, in turn, between non-conservation of the phase space measure and compression of a phase space fluid, hence the term ‘measure compression’ is used to talk in formal mode about open systems.

In experimental contexts it is often the case that scientists select and prepare physical target systems such that they can be represented as isolated and conservative on the scale relevant for the experimental inferences we wish to draw. Thus, experimental scientists often select and prepare physical target systems with scale relative ontological features that make them closed or isolated in the relevant sense.⁶ The important point is that such a target-model relationship is only one part of scientific practice. There are also, however, examples where scientists model systems they select and prepare as open as in experiments on decoherence or cooling. For real systems, the criterion for being open or closed is always whether the quantities in question, are conserved or not conserved. Conservation is scale-dependent because in real systems it is always approximate, and because the system is studied at a particular scale or scales. Interactions may be ignored in an effective description because they have a negligible effect or because they are irrelevant to some aspects of the system and their dynamics.

Scale-relativity applies to other features of systems too. For example, in quantum information theory it is possible to have a model of the effect of noise in a channel at one scale that is Markovian, even though at another scale the evolution is not Markovian (Preskill 2018). The next section considers general ways the dynamics of both open and closed systems can be modelled with a focus on autonomy as a feature of the representation of dynamics within the model.

4. AUTONOMY

The concept of *autonomy* traces back to its original ancient Greek coinage in the play *Antigone* by Sophocles.⁷ The eponymous heroine is about to be led away to be buried alive for transgressing a law laid down by Kreon, King of Thebes, forbidding the burial of her brother. By way of a rather odd consolation, the chorus sing to her:

Is it not with fame and praise that you depart to the corpses’ depths?
You were not struck down by wasting sicknesses, nor did you pay

⁶This selection is closely related to Cartwright’s idea of a nomological machine (Cartwright 1999)

⁷Here we are following McNeill (2011).

the wages of the sword, but autonomous [*αὐτόνομος*] you alone of mortals go living into Hades.⁸

The implication is that Antigone is ‘autonomous’ in the sense that she, rather than the King or the gods, is the author of the laws of behaviour that she obeys in the act of burning her brother. Thus to be autonomous, in the Sophoclean sense, is to be the creator of your own laws: *auto-nomos*.

In science and the philosophy of science the word autonomy is used such that the laws in question are natural rather than civic or supernatural. That which is understood to be autonomous is then a system or level of description rather than an individual. Thus we have that an autonomous system or level of description is one which in some sense has its own laws.

Most prominently, we find discussions of the autonomy of higher level laws from lower-level laws in the context of the reduction emergence debate. For example, we might understand the autonomy of higher-level laws as criterion of genuine emergence and explicitly characterise autonomy in terms of the coarse-grained dynamics being independent of microdynamics (Franklin 2020; Robertson 2020; Palacios 2022). Paradigmatically, in continuum fluid mechanics we find autonomous laws for macro-scale variables independent of molecular scales dynamics (Darrigol 2013; Batterman 2018).

The sense of autonomy we are interested in here is related but more basic. We are interested in autonomy defined as a formal feature of a model that amounts to the models capacity to produce independent and suitably behaved solutions to the equations of motion. We will understand autonomy in formal mode and defined it as follows:

Definition 3. *Autonomy: a model of a physical system is autonomous iff it includes no explicit dynamical variables that encode degrees of freedom other than those of the system, and there are well-posed dynamical equations for these variables.*

Our definition of autonomy allows that a model of a system can be autonomous even when boundary or initial conditions must be specified, c.f. Sloan (2023). This seems sensible since boundary or initial conditions are always needed in practice. It also allows us to dismiss putative failures of autonomy that result purely from improperly specified boundary conditions.

In material mode a concrete system is autonomous to the extent that its dynamics can be modelled as *effectively* autonomous. We thus have that when we

⁸Quotation is from McNeill (2011) p. 412.

talk about a *system* being autonomous we are making scale-relative statements about that typically relate to how a system can be modelled to a given degree of accuracy for a given purpose.

There are three ways for a model not to be autonomous. First, and most obviously, the model may include dynamical variables that encode degrees of freedom other than those of the system. For example, a model of the earth-moon system that also includes the sun is not an autonomous model of that system both in a trivial sense that it includes a dynamical variables corresponding to the sun and in the non-trivial sense that there is no well-posed sub-equation within the model for the earth-moon variables on their own. The second way for a model to be non-autonomous is if there *are* only variables for the system's degrees of freedom, but the equations of motion are not well-posed due to the presence of underspecified functions leading to an underdetermined system of equations. For example, a model of a banknote being dropped off a building *can* be written in terms of variables for the banknote together with a conservative gravitational force and a non-conservative damping force. However, the damping force term will involve a underspecified function varying over space and time, and this means the equation of motion are underdetermined.⁹ The third way in which a model may fail to be autonomous is via the failure of the system of equations to be well-posed due to a formal breakdown of integrability such as a collision singularity in gravitational n-body dynamics (Saari 1973).

Lack of autonomy might sometimes be due to openness but in general openness is compatible with autonomy because there are entire frameworks for constructing autonomous models of open classical and quantum systems (Breuer and Petruccione 2002; Sloan 2018). Autonomy and being closed or isolated dissociate, and there is good reason to keep them separate in discussion of open systems (there are examples to illustrate this point below). It is thus not always the case that autonomy can be restored by simply enlarging the boundaries of the 'system' being represented. Autonomy is an analytic property of a system of equations relating to the generation of determinate evolution of the variables describing a system. Its failure may or may not be related to the model 'missing out' couplings to other

⁹This is the Neurath's banknote example famously discussed by Cartwright (1999). Here we take a more modest view of the power of the example as showing merely that a model can fail to be autonomous rather than fail to be law-like at all. Indeed, although the underspecified model fails to produce well-posed dynamical equations of motion it clearly does have nomic representational capacity that tracks features of its material mode counterpart. For example, time translation invariance.

systems. A model of a system with the dynamics including parameters that represent the effects on the system of an environment or bath is autonomous in the sense above. Hence, as mentioned above, autonomy is compatible with the system being open in various senses.

If autonomy is defined in this way there are two substantively different ways models can be autonomous:

Definition 4. *Two Types of Autonomy.*

Absolute Autonomy (A-Autonomy) : There are well-posed dynamical equations in the model that are completely independent of any further systems.

Parameterized Autonomy (P-Autonomy): There are well-posed dynamical equations for the system but the dynamics depends on parameters or non-dynamical variables that encode features of other systems.

If the trinary distinction between non-autonomy, parameterized autonomy, and absolute autonomy is combined with the binary distinction between open (qua non-conservative of some characteristic quantity) and closed (qua-conservative of some characteristic quantity), we get six possible categories:

	Non-Autonomous	P-Autonomy	A-Autonomy
Open	Open Non-Autonomous	Open P-Autonomy	Open A-Autonomy
Closed	Closed Non-Autonomous	Closed P-Autonomy	Closed A-Autonomy

Each category can be explicitly defined and illustrated with an example from classical mechanics as per below.

Definition 5. *Six Categories of Dynamical Model.*

Open, Non-Autonomous: The characteristic quantity is not conserved and there are not well-posed dynamical equations for the dynamical variables of the model. Classical Mechanics: banknote system modelled mechanically by a point particle in a Newtonian gravitational potential field with an under-specified damping function.

Closed, Non-Autonomous: The characteristic quantity is conserved and there are not well-posed dynamical equations for the dynamical variables of the model. Classical Mechanics: n-body Newtonian gravitational system with collision singularities.

Open, Parameterized Autonomy: The characteristic quantity is not conserved and there are well-posed dynamical equations for the dynamical variables of the model such that the dynamics depends on parameters or non-dynamical variables that encode the effect of other systems. Classical Mechanics: damped oscillator model in Newtonian mechanics with external potential (pendulum).

Closed, Parameterized Autonomy: The characteristic quantity is conserved and there are well-posed dynamical equations for the dynamical variables such that the dynamics depends on parameters or non-dynamical variables that encode the effect of other systems. Classical Mechanics: undamped oscillator model in Newtonian mechanics with external potential (pendulum).

Open, Absolute Autonomy: The characteristic quantity is not conserved and there are well-posed dynamical equations for the dynamical variables of the model that are completely independent of any further systems. Classical Mechanics: damped oscillator in Newtonian mechanics with internal potential (spring).

Closed, Absolute Autonomy: The characteristic quantity is conserved and there are well-posed dynamical equations for the dynamical variables of the model that are completely independent of any further systems. Classical Mechanics: undamped oscillator in Newtonian mechanics with internal potential (spring).

The six categories based on the open-closed distinction and the two types of autonomy should not distract from the simple main point of this section namely that open systems can be and often are modelled as autonomous. Hence, we must not conflate a system being closed with it being modelled as autonomous.

We might still, however, expect that autonomous open system models must always ultimately be reinterpretable in terms of parameterized autonomy. That is, while there may be open system models with have well-posed dynamical equations of their own and no explicit parameterized dependence on other systems, these models should always be understood as *implicitly* representing further systems via a parameter. Such a claim would amount to insisting that our example of a damped oscillator in Newtonian mechanics with internal potential is only *apparently* absolutely autonomous because we should really take the spring potential to have as its origin micro-physical degrees of freedom, whose inclusion would ‘close’ the model leading to energy conservation or else at least allow us to understand the model as

only autonomous in the parameterised sense. We return to the status of absolute autonomy in open system models in the context of cosmology in Section 6.

The next section considers the interrelation between openness and the different forms of autonomy in the context of open quantum systems, and considers properties of dynamical maps that further constrain the terms of our investigation.

5. OPEN QUANTUM SYSTEMS

In the context of quantum physics it is very important to distinguish quantum-open from other notions of open in Section 2. Many authors move back and forth between the formal mode notion of quantum-open, and material mode notions of closed systems having to do with being isolated or not interacting with the environment. For example, [Nielsen and Chuang \(2010\)](#) say that “the dynamics of a closed system are described by a unitary transform” (p. 357), but they frame this statement with reference to whether or not the system is interacting with the outside world. However, as noted above, there are many unitary quantum models of systems that are subject to effects from the outside world, as when particles are confined in a potential well, and there are many unitary models of systems that lose energy (and so have a time-dependent Hamiltonian) cf. ([Breuer and Petruccione 2002](#), p. 110). Furthermore, it is possible to coherently drive quantum systems so that they move over macroscopic distances ([Alberti et al. 2009](#)). In general, any application of quantum mechanics in which the effects of electromagnetism are encoded in a potential cannot describe a system that is isolated in the material mode since the potential comes from the environment of the system, and this is orthogonal to the question of whether or not the model of the system has a unitary dynamics. It is also possible to model isolated systems with non-unitary dynamics in contexts where the Hamiltonian is not essentially-self adjoint. Furthermore, there can be decoherence without dissipation ([Unruh 2012](#)). Hence, quantum-open is not necessary or sufficient for openness in the other senses of Section 2. Note as well that it is not the strength of interaction that determines whether it requires a quantum-open model.¹⁰

Let us consider some important formal properties of unitarity in relation probability, purity, entropy and well-posedness. First, most straightforwardly, unitarity is sufficient but not necessary for probability conservation since there are non-unitary dynamical equations that preserve probability. Unitary time evolution

¹⁰If collapse is taken to be a real physical process then it may violate energy conservation ([Carroll and Lodman 2021](#)). We consider quantum mechanics without collapse.

conserves the norm induced by the inner product. In particular, it can be shown that unitary time evolution is sufficient (although not necessary) for the preservation of the *purity* of a quantum state when represented as a density operator, ρ , which is a positive semi-definite operator of unit trace. Explicitly, the purity of a state is given by $\gamma = \text{Tr}(\rho^2)$. Pure states are such that $\gamma = 1$. The quantum operation $\Lambda(\rho) = U\rho U^\dagger$ preserves the purity of ρ if U is a unitary operator (Jaeger 2007). Purity preservation is equivalent to the conservation of linear entropy since $S_L = 1 - \gamma$. There is also a connection between unitarity and the conservation of the informational or von Neumann entropy $S = -\text{tr}(\rho \ln \rho)$. In particular, the von Neumann entropy is also invariant with respect to the quantum operation $\Lambda(\rho) = U\rho U^\dagger$ since we have that $S(U\rho U^\dagger) = S(\rho)$ (Breuer and Petruccione 2002, §2.3). Hence, unitary dynamics conserves both the linear and von Neumann entropies, and in so far as this is taken to be genuinely thermodynamic property, the connection between unitary dynamics and being closed in the thermodynamic sense is a natural one.

There are strong results connecting unitarity with well-posedness. In particular, even for time dependent Hamiltonians a general condition on the *boundedness* of the Hamiltonian implies the existence of a unitary propagator, which in turn implies that a quantum model will necessarily have well-posed equations of motion. The results are somewhat technical but in essence amount to restriction that the Hamiltonian is a strongly continuous map from the real numbers into bounded self-adjoint operators on a Hilbert space. This then allows the definition of a Dyson expansion which in turn allows one to prove the existence of a unitary propagator. The continuity properties of the propagator are then sufficient to prove the existence of well-posed equations of motion (Reed and Simon 1975, Theorem X.69). There is thus a strong formal connection allowing one to infer well-posed dynamics from unitarity. The implication does not run the other way round, however, and we can model non-unitary dynamics via master equations that lead to well-posed systems of dynamical equations.

To understand the contrast between open and closed quantum system models let us first consider the paradigmatic equation for a unitary quantum dynamics of density operators, the von Neumann equation:

$$(1) \quad \dot{\rho} = -i[H, \rho]$$

where the evolution of density operator ρ will be unitary provided the Hamiltonian H is self-adjoint. The paradigmatic equation for open quantum systems is the

Lindblad equation which can be written:

$$(2) \quad \dot{\rho} = -i[H, \rho] + \sum_{i,j} a_{ij} (F_i \rho F_j^\dagger - \frac{1}{2} \{F_j^\dagger F_i, \rho\})$$

$$(3) \quad = -i[H, \rho] + \mathcal{D}_\gamma(\rho)$$

where the F_i are bounded operators, $\{, \}$ is the anti-commutator, and the matrix a_{ij} is positive semi-definite (Breuer and Petruccione 2002, Eq. 3.63). The Lindblad equation is made up of a unitary part identical to the von Neumann equation together with a non-unitary part. For an initial pure state the unitary part reduces to the Schrödinger equation (hence, the Lindblad equation may be regarded as more general). In physical models the non-unitary part of the dynamics, encoded in the super-operator $\mathcal{D}_\gamma(\rho)$, corresponds to a dissipator term that encodes the parameterized effects of decoherence, thermal damping and noise.

The non-unitary dynamics of a system having the Lindblad form is a sufficient (but not necessary) condition for probability to be conserved.¹¹ Furthermore, there are general results suggesting Lindblad type equations lead to well-posed partial differential equations in physical contexts (Azouit et al. 2016). The connection between the form of the dynamical map and the modelling of autonomous systems and their environments runs even deeper than the conservation of probability and well-posed differential equations. To see this we need to consider a selection of important results from the theory of open quantum systems.

A Markov process is a generalization of a deterministic process for which the probabilistic state of a given system at some time is wholly determined by the dynamics together with the probabilistic state at some other time. Markovian systems are thus *memoryless* in a specific, non-time directed, sense. For any Markov process the dynamical maps uniquely map each state in its state space to another state in the same space and can be composed, that is:

$$(4) \quad \Lambda_{t+s}\rho = \Lambda_t\Lambda_s\rho$$

Recall from above that a density operator, ρ , is defined to be a positive semi-definite operator of unit trace. For Λ_t to map density operators to density operators it must be a *trace-preserving positive map*. Trace-preservation is a necessary and sufficient condition for a dynamical map to be probability preserving. A *completely positive trace-preserving map* is then such that $\Lambda_t \otimes I_n$ is also a positive map for all n , where I_n is the identity map on the state space of some arbitrary further system of dimension n .

¹¹See Cuffaro and Hartmann (2021) for discussion and a short proof.

The *Stinespring's dilation theorem* (Stinespring 1955) shows that if the dynamics of a system \mathcal{S} is given by a completely positive trace-preserving map, then corresponding to ρ , there is a unique pure state of a larger system $\mathcal{S} + \mathcal{E}$ with unitary dynamics from which we can derive the non-unitary dynamics of \mathcal{S} . Following Cuffaro and Hartmann (2021), we can interpret the theorem as implying that *if* the map Λ_t is Completely Positive (CP), then the *open system model* of S is *embeddable* into a larger *closed system model* of $\mathcal{S} + \mathcal{E}$. Thus, if an ‘embeddable system’ is defined to be an open-quantum system that can be embedded into a larger model of a closed quantum system, then the Stinespring’s dilation theorem implies that open-quantum systems with CP dynamics are embeddable.¹²

There is then a natural connection between open quantum systems and parameterized autonomy as follows. The semi-classical limit of an open quantum system model is a dissipative classical model as is indicated by the interpretation of the non-unitary part of the Lindblad equation as a dissipator. Most vividly, we can show that the Caldeira-Leggett master equation recovers the equations of motion for a damped Brownian particle when the Ehrenfest-type relations are used to derive the equations of motion for the first and second order moments (Breuer and Petruccione 2002, p. 175). As such, we typically think of an open quantum model as the quantum analogue of a open classical model.

We would therefore expect to be able to derive open quantum models from the quantization of open classical models. However, there is a severe formal challenge to understanding open quantum models as the quantization of classical open models. Even in the simplest case of an open classical model with a linear, velocity dependent friction term the quantization operation is not well defined since such systems are not symplectic. That is, such models do not fit with the usual canonical approach to quantization built upon the symplectic structure encoded in the Poisson bracket and Hamilton’s equations. The response to this challenge follows one of two approaches:

The first one employs new quantization schemes, while the second one (the so-called system-reservoir approach) treats the particle as part of a larger quantum system which is quantized according to the

¹²It is worth considering in this context the view expressed by Cartwright (1983) that it is “mistaken” to adopt an understanding of open quantum systems (such as that of Davies) that is based upon the assumption that open quantum systems can always be assumed to be part of a larger system that is subject to unitary time evolution (p. 205). Clearly, Cartwright’s judgement is consistent with the Stinespring theorem in so far as there are representations of open system dynamics that can fail to be CP. For more details see Cuffaro and Myrvold (2013) and Cuffaro and Hartmann (2021).

usual rules. Of course, only the second approach can be considered as fundamental. (Joos 2013, p. 80)

This final line is telling. In the system-reservoir approach one starts with a classical closed system model which is such that the spatially contiguous environment (the reservoir) is given a fine-grained representation and we have absolutely autonomous and closed dynamics. We then apply canonical quantization and derive a full quantum description for the system and reservoir. Through a combination of limits and tracing out, we derive a model for the system degrees of freedom that encodes the quantum effects of friction but does not represent the reservoir in the quantum dynamics. Thus we derive an open system (conservation fails) which has *parameterized* autonomy. The most famous example of such a procedure is the derivation of the Caldeira-Leggett model, mentioned above, where the system-reservoir approach is deployed with the environment explicitly modelled as a set of non-interacting harmonic oscillators which are linearly coupled to the system (Breuer and Petruccione 2002; Joos 2013).

This is to follow the logic of the Stinespring’s dilation theorem the other way round. Rather than starting with an open system quantum master equation that we show to embeddable in a larger closed system dynamics, we first establish the larger closed system dynamics based upon the classical model, and then construct the reduced open system quantum dynamics by reducing a closed system quantum model. Is there an alternative? Must all open systems – both classical and quantum – be understood as embeddable and, moreover, autonomous only in the parameterised sense? The final section considers these questions in the context of cosmology.

6. THE UNIVERSE AS AN OPEN QUANTUM SYSTEM

The different senses of openness discussed in Section 2 leave open the question of whether the universe is a closed system. That is, the way we answer the question depends on how ‘the universe’ and ‘closed’ are understood. If the universe is taken to be everything that there is, then it is not interacting with or affected by anything else. Thus, it is closed in the material sense of 1. above. However, that does not tell us anything about whether it is closed in any other sense, unless it is assumed that all that there is conserves every physical property (so that the universe is closed in every sense in the material mode, and should be modelled as closed in every sense in the formal mode 1.*).

A specific form of this argument can be made in the context of quantum cosmology, that is, the application of quantum theory to the universe. It is standardly

assumed that the universe must be modelled as evolving unitarily, because there is no other system for it to interact with, and isolated quantum systems are assumed to be subject to unitary time evolution. Thus, isolation is taken to imply closed in all senses and conservation of all the relevant characteristic quantities and unitary time evolution. The universe cannot be quantum-open. Case closed.

Against this argument we offer the following response. First, as the system of study in cosmology, the universe is not in fact all there is. Even in the material mode, the universe in the context of cosmology is not everything there is but only the large scale physical structure of the universe. Second, as discussed in the previous section, quantum mechanical systems that are interacting with an environment can also be modelled with unitary time evolution, thus the relationship between isolation and unitarity is complicated. As discussed in the previous section, in quantum mechanics ‘closed’ is a term of art that means ‘subject to unitary time-evolution’, and it does not mean what is meant by ‘closed’ when it is said that the universe as a whole is closed because it is all there is in the sense of being isolated. If what we mean when we ask if the universe is a closed quantum system is whether quantum cosmological models are required to implement unitary time evolution, then the question turns out to be open for two reasons. The universe might be quantum-open since we might doubt that the model should be describing an isolated system since we may wish to include endogenous interactions and these may (or may not) require non-unitary time evolution. Then, even neglecting the question of non-isolation and endogenous interactions, the universe might be quantum-open since there is no formal or physical necessity binding us to connect absence of interactions to unitary evolution. Indeed, models exist of classical cosmology in which measure compression, a close cousin of non-unitarity, obtains (Sloan 2018, 2023; Bravetti et al. 2023). Hence, whether the universe is quantum-closed is an open question.

The discussion of the previous section allows us to frame two more precise arguments that the universe must be modelled as quantum-closed. First, if open quantum systems are always embedded in larger closed quantum systems then the universe must be a closed quantum system, since there is nothing to embed it within. Secondly, if the derivation of a open quantum model is, fundamentally speaking, always via the system-reservoir approach then such models cannot describe the universe as the system in question, since there is no exogenous system to act as a reservoir. We conclude by rebutting each of these arguments.

We start with the embeddability argument. The most straightforward response is just to note that embeddability does not imply embeddedness. That is,

although the fact that the dynamical map is completely positive may imply that formally we can embed an open quantum system model in a larger closed quantum system model, it may still be the case that no such larger system exists. A more further response is that it remains to be seen whether a quantum model of the universe will correspond to a master equation with completely positive dynamics. There exist physically important master equations like the Caldera-Leggett equation that are not completely positive.¹³ Furthermore, the standard argument that we should treat complete positivity as a physically motivated necessary condition on consistent master equations comes precisely from the assumption that quantum dynamics must always be understood as fundamentally unitary. In general terms, there are reasons to doubt such an assumption coming from what [Cuffaro and Hartmann \(2021\)](#) call the open systems view. Moreover, in the cosmological context, whether the universe is an open or closed system was precisely the question we started with – thus complete positivity and embeddability does not give us an independent reason to expect the universe to be a closed system but rather a more sophisticated way of encoding what such a closed systems perspective amounts to.

Let us then turn to the quantization argument. The most obvious response is to simply doubt the fundamental assumption. Since there are alternative quantization schemes for classical open systems that do not proceed via the system-reservoir approach ([Dekker 1981](#)), it may just prove the case that they are more fundamental, in the cosmological context. Moreover, as argued by [Bondar et al. \(2012, 2016\)](#) there is also a methodological justification for quantizations that proceed via the more direct route. In particular, the idea is to derive master equations based upon observed data recast in the form of Ehrenfest relations together with a specified mathematical structure of the equations of motion. The virtue of this approach, according to [Bondar et al. \(2016\)](#), is that it guarantees that the resulting equations of motion have the desired physical structure to reproduce the supplied dynamical observations (p. 1633). By contrast, the system-reservoir approach to the derivation of a master equation need not lead to a master equation that can reproduce the observations nor that the equations have a desired mathematical structure (p. 1633).

The second response to the quantization argument is that the universe need not be taken to be all that there is. We might understand the ‘system’ described by a cosmological model as being given by coarse grained large scale degrees of freedom

¹³We can see this straightforwardly by manipulating the equation as far as we can towards the Lindblad form (2) and finding that the matrix a_{ij} cannot be made positive semi-definite without the addition of extra terms. See ([Breuer and Petruccione 2002](#), pp. 172-3) and [Ferialdi \(2017\)](#).

and thus leave open the possibility for endogenous environmental interactions that lead to dissipative effects. Thus we might be able to apply the system-reservoir to derive an open quantum cosmology model, whether or not there is a more fundamental fine-scale dynamics that is unitary.

Finally, and most excitingly, we can respond to the quantization argument by considering the possibility, noted above, of classical open systems cosmology models. In particular, classical cosmological models can be provided in the framework of contact geometry (Sloan 2018, 2023; Bravetti et al. 2023). Such models feature compression of the Liouville measure and they are thus open systems in the relevant classical sense. Moreover, they are absolutely autonomous in our terminology. The quantization of such models, should it prove possible, should be expected to lead to models in which the universe is described as an open quantum system. Such models can also be expected to be absolutely autonomous. As such, the universe may turn out to be an open quantum system after all.

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