On the reality of the quantum state once again: A no-go theorem for ψ-ontic models?

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Abstract

In a recent paper (Found Phys 54:14, 2024), Carcassi, Oldofredi and Aidala concluded that the ψ-ontic models defined by Harrigan and Spekkens cannot be consistent with quantum mechanics, since the information entropy of a mixture of non-orthogonal states are different in these two theories according to their information theoretic analysis. In this paper, I argue that this no-go theorem for ψ-ontic models is false by explaining the physical origin of the von Neumann entropy in quantum mechanics.

1 Introduction

The reality of the wave function has been a hot topic of debate since the early days of quantum mechanics (QM). Is the wave function real, directly representing the ontic state of a physical system, or epistemic, merely representing a state of incomplete knowledge about the underlying ontic state? In recent years, a rigorous approach called ontological models framework has been proposed by Harrigan and Spekkens to distinguish the ψ-ontic and ψ-epistemic views [1]. Moreover, several important ψ-ontology theorems that establish the reality of the wave function have been proved in the framework, and the strongest one is the Pusey-Barrett-Rudolph (PBR) theorem [2]. In this background, Carcassi, Oldofredi and Aidala's (COA) recent no-go theorem for ψ-ontic models [3] is unexpected. If it is correct, it will be a very important new result. In this paper, I will examine the COA theorem and argue that it is false.[1]

2 The ontological models framework

Before presenting my critical analysis of the COA theorem, I will briefly introduce the ontological models framework (OMF) in which the theorem is proved. The framework has two fundamental assumptions [1, 2]. The first assumption is about the existence of the underlying state of reality. It says that if a quantum system is prepared such that QM assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. This assumption is necessary for the analysis of the ontological status of the wave function, since if there are no any underlying ontic states, it will be meaningless to ask whether or not the wave functions describe them.

Here a strict ψ-ontic/epistemic distinction can be made. In a ψ-ontic ontological model, the ontic state of a physical system determines its wave function uniquely, and the wave function represents the ontic state of the system. While in a ψ-epistemic ontological model, the ontic state of a physical system can be compatible with different wave functions, and the wave function represents a state of incomplete knowledge – an epistemic state – about the actual ontic state of the system. Concretely speaking, a wave function or a pure state corresponds to a probability distribution $p(\lambda|P)$ over all possible ontic states when the preparation is known to be P, and the probability distributions corresponding to two different wave functions may overlap or not, depending on whether the ψ-epistemic view or the ψ-ontic view is true. Besides, a mixture of pure states $|\psi_i\rangle$ with probabilities p_i is represented by $\sum_i p_i p(\lambda|P_{\psi_i})$.

In order to investigate whether an ontological model is consistent with the empirical predictions of QM, we also need a rule of connecting the underlying ontic states with the results of measurements. This is the second assumption of OMF, which says that when a measurement is performed, the behaviour of the measuring device is only determined by the ontic state of the system, along with the physical properties of the measuring device. More specifically, the framework assumes that for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the predictions of QM then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|P) = p(k|M, P)$, where $p(k|M, P)$ is the Born probability of k given M and P.

[1] A referee of my recent paper [4] asked me to evaluate COA's paper in his/her report. This paper can be regarded as my fulfillment of this task.

3 The COA theorem

COA's proof of their theorem based on OMF is simple and clear and thus it can be readliy examined. COA analyzed the information entropy of a mixed state in both ψ-ontic models and QM, and argued that since they are different, ψ-ontic models are not consistent with QM according to their information theoretic analysis. The key then is to examine if the information entropy of a mixed state in these two theories are really different.

COA considered the mixed state $\rho = \frac{1}{2}(|\psi\rangle\langle\psi| + |\phi\rangle\langle\phi|)$, where $|\psi\rangle$ and $|\phi\rangle$ are two pure states. Its quantum information entropy is given by the von Neumann entropy, namely

$$H_{QM}(\rho) = -\frac{1+p}{2}ln\frac{1+p}{2} - \frac{1-p}{2}ln\frac{1-p}{2}, \quad (1)$$

where $p = |\langle\psi|\phi\rangle|$ is the absolute value of the inner product of the two states.

In ψ-ontic models, the two pure states $|\psi\rangle$ and $|\phi\rangle$ correspond to two δ probability distributions of ψ-related ontic states or two ψ-related ontic states λ_ψ and λ_ϕ. That the von Neumann entropy of a pure state is zero in QM requires that a pure state corresponds to a δ probability distribution of ψ-related ontic states or a unique ψ-related ontic state in ψ-ontic models;[2] otherwise the Shannon entropy for a pure state in ψ-ontic models already disagrees with the von Neumann entropy of the pure state in QM. Then, according to COA, the information entropy of a uniform mixture of these two states, λ_ψ and λ_ϕ, in ψ-ontic models is given by the Shannon entropy:[3]

$$H_{OM}(\rho) = -\frac{1}{2}ln\frac{1}{2} - \frac{1}{2}ln\frac{1}{2} = 1. \quad (2)$$

Now it can be seen that the information entropy of a mixed state in ψ-ontic models and in QM can be the same only when the pure states in the mixture are orthogonal, namely the inner product of the two pure states such as p in Eq. (1) is zero. Since the information entropy of a general mixed state (in which the pure states in the mixture are non-orthogonal) in ψ-ontic models and in QM are different, COA concluded that the ψ-ontic models cannot be consistent with QM.

[2]Even if there are hidden variables besides the wave function (in which case a pure state will correspond to a general probability distribution of ontic states that include both the unique ψ-related ontic state and other hidden variables), they cannot be measured in principle and thus no information about them can be obtained. This means that the existence of hidden variables does not change the information entropy of a pure state as given by the Shannon entropy for the unique ψ-related ontic state.

[3]Note that COA gave a different formulation of the Shannon entropy in their paper [3]. But their result is the same as the one given here. In my view, the formulation given here is simpler and clearer.

4 My analysis

In order to find whether and where the above proof goes wrong, we need to understand why the quantum information entropy of a mixed state is given by the von Neumann entropy, which, unlike the classical information entropy, relates to the inner product of the pure states in the mixture. First, for a mixture of orthogonal states $\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$, where $|\psi_i\rangle$ are certain orthogonal bases, the von Neumann entropy is just the Shannon entropy for a classical mixture. That is, the von Neumann entropy is $H(\rho) = -\sum_i p_i ln p_i$. Next, for a mixture of non-orthogonal states whose density matrix is equal to $\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$, its von Neumann entropy is equal to the Shannon entropy of the mixture of orthogonal states. That is to say, the von Neumann entropy of the mixture of non-orthogonal states is $H(\rho) = -\sum_i p_i ln p_i$, where p_i are determined by the inner product of the non-orthogonal states.[4]

Then, why? The reason is as follows. First, in QM orthogonal states can be distinguished by experiments with certainty, while non-orthogonal states cannot be distinguished by experiments with certainty. Thus, for a mixture of orthogonal states, the von Neumann entropy is the same as the Shannon entropy, since the orthogonal states in QM can be distinguished with certainty, just like the classical states in a classical mixture. Next, since the non-orthogonal states, unlike the classical states, cannot be distinguished with certainty, the von Neumann entropy for a mixture of non-orthogonal states is different from the Shannon entropy for a classical mixture.[5] In other words, the Shannon entropy requires that the states in the mixture should be distinguishable by experiments with certainty. Finally, since two mixed states in QM which have the same density matrix cannot be distinguished by experiments (which means that the information we can obtain from them must be the same), the von Neumann entropy of a mixture of non-orthogonal states, whose density matrix is equal to the density matrix of a mixture of orthogonal states, is equal to the von Neumann entropy of the mixture of orthogonal states, which is also the same as the Shannon entropy of the mixture of orthogonal states.

Based on the above analysis of the origin of the von Neumann entropy

[4] The appendix of COA's paper [3] gives a clear illustration of this result.

[5] Note that non-orthogonal states can be distinguished with probability less than one. It can be seen that the information entropy of a mixture of pure states depends on the probability of distinguishing these pure states. If the probability is one (i.e. the pure states in the mixture are orthogonal states), the information entropy assumes the maximum value, and if the probability is zero (i.e. the pure states in the mixture are all the same), the information entropy assumes the minimum value zero. If the probability is neither one nor zero (i.e. the pure states in the mixture are non-orthogonal states), the information entropy will assume a value between zero and the maximum value. This is consistent with the meaning of information entropy being a measure of the amount of randomness or uncertainty in the value of a random variable or the outcome of a random process such as the quantum measurement process.

in QM, we can see where COA's proof of their no-go theorem for ψ-ontic models goes wrong. The issue lies in that COA implicitly assumed that in ψ-ontic models, the two ontic states λ_ψ and λ_ϕ, which are represented by two non-orthogonal states $|\psi\rangle$ and $|\phi\rangle$, are classical states that can be distinguished by experiments with certainty. Only by this assumption, can the information entropy of a mixture of these two states be directly given by the Shannon entropy. It is this result that contradicts QM, in which the information entropy of a mixture of two non-orthogonal states is given by the von Neumann entropy that is different from the Shannon entropy. However, this assumption is not a part of OMF,[6] and as argued above, it is not consistent with QM either. In order that an ψ-ontic model is consistent with QM, it should additionally assume that the ontic states represented by non-orthogonal states cannot be distinguished with certainty (which is an assumption about the dynamics), and thus the information entropy of a mixture of these states should be given not by the Shannon entropy, but by the von Neumann entropy (when further considering the fact that two mixed states that have the same density matrix cannot be distinguished by experiments).

Therefore, COA did not successfully prove that the ψ-ontic models based on OMF are inconsistent with QM. What they proved is only that these models with an additional wrong assumption of the distinguishability of non-orthogonal states are inconsistent with QM. When replacing this wrong assumption with the correct assumption of the indistinguishability of non-orthogonal states, the ψ-ontic models based on OMF can agree with QM in giving the same information entropy of a mixed state.

Finally, it is worth noting that since OMF with its two fundamental assumptions does not include the dynamics for the ontic states, it is not a complete theory. This is well within expectations, and one should not complain about this. For example, one cannot calculate the information entropy directly on the epistemic state in OMF without a dynamics, since the information entropy depends on whether the ontic states in the epistemic state are distinguishable, which is determined by the dynamics.[7] Once we know the dynamics for the ontic states such as the Schrödinger dynamics for the state vectors in the Hilbert space (which represent the ontic states), then we can calculate the information entropy as given by the von Neumann entropy, which is just what we do in QM.

[6]OMF with its two fundamental assumptions does not state whether the ontic states represented by non-orthogonal states can be distinguished with certainty.

[7]Since distinguishing states requires an interaction between the measured system and the measuring device, which is determined by the dynamics, the distinguishability of states will depend on the dynamics in general. For example, if the dynamics is linear for the wave function such as in QM, then two non-orthogonal states cannot be distinguished with certainty, while if the dynamics is (deterministically) nonlinear for the wave function such as in nonlinear QM [5], then two non-orthogonal states may be distinguished with certainty and superluminal signaling may also be realized.

5 Conclusions

Carcassi, Oldofredi and Aidala (COA) recently argued that the ψ-ontic models defined by Harrigan and Spekkens cannot be consistent with QM. In this paper, I present a critical analysis of this no-go result. It is argued that in order to derive their result that the information entropy of a general mixed state in ψ-ontic models and in QM are different (and thus these models cannot reproduce all results of QM), COA implicitly assumed that in ψ-ontic models the ontic states represented by two non-orthogonal states are classical states that can be distinguished by experiments with certainty. However, this assumption is not a part of the ψ-ontic models defined by Harrigan and Spekkens, and it is not consistent with QM either. When replacing this assumption with the assumption of the indistinguishability of non-orthogonal states, the ψ-ontic models can agree with QM in giving the same information entropy of a general mixed state. Although COA's recent no-go result is not true, the reality of the quantum state and the ψ-ontic models still need to be further studied.

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