The Controversy About Interference of Photons

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Abstract

In the 1960s, the demonstration of interference effects using two laser-beams raised the question: can two photons interfere? Its plausibility contested Dirac's dictum, "Interference between two different photons never occurs". Disagreements about this conflict led to a controversy. This paper will chart the controversy's contour and show that it evolved over two phases. Subsequently, I investigate the reasons for its perpetuation. The controversy was initiated and fuelled by several misinterpretations of the dictum. I also argue that Dirac's dictum is not applicable to two photon interference as they belong to different contexts of interference. Recognising this resolves the controversy.

1 Introduction

Thomas Young, at the beginning of the 19th century, provided a novel explanation of several optical effects based on the interaction of multiple waves. The first articulation of this is found in his 1801 paper, where he explains the colours observed in thick and thin plates as the effects arising from the combination of two waves (Young 1802a, 34). In the subsequent work, he refers to this proposed phenomenon as "the *interference* of two coincident undulations, which either cooperate, or destroy each other" (Young 1802b, 116-117, emphasis added). Young further formalised this theory in the 1807 work through the analysis of the bright and dark fringe-pattern observed in a double-slit experiment.¹

The pattern in the double-slit experiment got a different explanation a century later, when the corpuscular interpretation of light and quantum mechanics were gaining acceptance. One of the influential formulations of the quantum mechanical interpretation was provided by Paul Dirac in his 1930 book. According to this, the fringe-pattern is not formed due to the summing of waves' intensities. Instead, the screen's bright regions are where more photons have arrived compared to the other regions. Each photon's probabilities for arriving at various regions are determined by the superposition principle of quantum mechanics. Dirac put this succinctly: "Each photon...interferes only with itself. Interference between two different photons never occurs" (Dirac [1930] 1958, 9). The fringes are produced by an ensemble of photons; but, the basic phenomenon involves only single photons.

In this transition from classical optics to quantum mechanics, the phenomenon retained the name "interference" even when no two particles were interfering.² However, in the 1960s,

^{1.} For a detailed historical analysis of interference by Young and Fresnel, see Kipnis (1991).

^{2.} This does not mean that the name became a misnomer in the quantum mechanical context. The new interpretation of the phenomenon lost the above classical feature; but, it gained new meanings for "interfere", most of which are contextualised in the problem of wave-particle duality.

some physicists started to deliberate about the possibility of two photons interference. This atavism conflicted with the above quoted Dirac's statement (henceforth abbreviated as D), which had gradually gained the status of a dictum. The revival of two photons interference happened in an experimental context where the technique to produce the interfering beams is fundamentally different from that of traditional experiments.

Interference experiments are traditionally executed not with light beams coming from independent sources, but ones generated from a single source.³ Any two superposing beams will not produce a visible fringe-pattern as they will not be coherent. One of the practical ways to achieve coherence, right from the initial days, was to use beams from a common source as that way their phase-difference would be stable.⁴ The technical feature of using a single source again played a central role during the 20th century revival of the light-quantum hypothesis. To understand how light-quanta give rise to fringes, scientists carried out experiments with attenuated light source and discovered the development of the pattern, speckle by speckle.⁵ The observation was explained in terms of the light-quanta going through the setup, one at a time. This explanation was based on the premise that interference experiments have a single source; the presence of more sources could not have guaranteed a single light-quantum at a given time. It is in this experimental context that D gets formulated. The use of a single source, thus, played a foundational role in guiding both the classical and quantum mechanical analyses of interference.

The availability of highly coherent sources like lasers post 1950s made it possible to conduct interference experiments with each beam coming from an independent source. With two sources, there can be scenarios where there are two photons – one from each source – in an experimental run. When the initial experiments of this kind reported the observation of fringe-pattern, questions about how these novel observations correspond with D started emerging. For instance, one of the early papers on this topic states:

Since the experimental demonstration of interference effects produced by the superposition of two independent light beams, the question has been debated whether the effect is to be regarded as evidence for the interference of photons from one beam with photons of the other beam. On the face of it, the observations appeared to contradict a well-known remark of Dirac (Pfleegor and Mandel 1967a, 766)

What started as a seemingly minor confusion eventually grew into a complex controversy about reconciling D with the new kind of "two-photon" interference experiments. At present, the controversy manifests in the form of differing stances about the D's validity.

This controversy in the quantum optics community has not been noticed by historians and philosophers of science, except a few who have observed some minor aspects of the overall confusion. Bromberg, in her 2006 and 2016 papers, points to some experiments that explore D's validity, with no mention of the disagreements surrounding it. In a 2010 paper, she notes the possibility of physicists sharing different views about D, albeit in a different context. Here, while analysing the impact of D on Hanbury-Brown and Twiss'

^{3.} In fact, interference experiments are commonly categorised based on how the beams are produced from a common source: amplitude-division and wavefront-division techniques.

^{4.} The necessity to use a common source for producing interference patterns is one of the initial sets of conditions emphasised by both Young and Fresnel, independently. Young's early works indicate that he was not aware of this condition. Only in his 1807 book, this condition becomes explicit (Kipnis 1991, 130–33). Fresnel's initial papers in 1815 mention the condition of common source. A much more thorough exposition of the complete list of conditions of coherence was presented in his 1822 work (ibid., 193).

^{5.} See, for instance, the experiments of Taylor (1909) and Dempster and Batho (1927).

experiments, Bromberg poses the following question, with no further analysis: "It would be interesting to know whether it was particularly in Britain that Dirac's statement held sway... was such faithfulness to Dirac peculiar to people trained or working in Britain? Was it true for Edward Purcell or others of the group at Harvard? Was it true... in Berlin" (Bromberg 2010, 9–11). Hentschel (2018) also does not mention the controversy while summarising the development of interference experiments with lasers. This is not surprising given he is of the opinion that D has been "quite definitely falsified" by modern experiments (2018, 149–150).

These scholars have overlooked the fact that the physics community is currently divided about D's validity. For instance, Glauber (1995, 12) suggests that "It is time to put the famous dictum to rest... and to forgive [Dirac] for writing... a highly simplistic remark which has sowed confusion among physicists ever since". In contrast to this, Shih (2003, 1018) believes that "Dirac is correct... What he said in his book is still valid if we slightly modify his statement". Even though this debate has been going on since the 1960s, there has been no attempt from physicists, and scholars of history and philosophy of science to understand why a seemingly straightforward disagreement persists. Also, establishing this controversy and identifying its characteristics provides a critical case study to the existing literature on the philosophical analyses of scientific controversies.⁶

2 Plan of the Paper

The agenda of the current paper is twofold. Since the controversy surrounding D has not been recognised within science studies, the first task is to provide a historical account of the controversy. Charting its evolution and identifying the main features of the controversy enables the subsequent task of investigating the reasons for its persistence.

Two photons interference (henceforth abbreviated as 2pI) has been an active area of research since the 1960s. 2pI and the development of multi-photon interferometry have become essential in other research areas like the evaluation of Bell inequalities and related topics, ghost-imaging techniques, and quantum lithography. In the large corpus of research papers that spans these areas of research, for the historical survey of the controversy, I will consider the papers that have directly commented on D's validity in the context of 2pI experiments.

The examination of the relevant papers reveals that the controversy is constituted by the differing views of physicists about the relation between two seemingly conflicting points:

- (a) Dirac's dictum that two photons do not interfere with each other
- (b) the modern interference experiments with two sources or photons exhibit interference effects.

^{6.} For analyses of the structures and closures of scientific controversies, see Engelhardt Jr. and Caplan (1989) and Machamer et al. (2000).

^{7.} Because of language and pragmatic constraints, I consider only research papers in English. Also, there are papers where the authors' stance on D is either implicit or unclear. For instance, Bouwmeester (2004, 139) starts his article by mentioning how Dirac could "probably have been more cautious" while commenting on the interference of photons. Beyond this allusion to the tension surrounding D, there is no indication of the author's stance on the controversy. Similarly, Kaltenbaek et al. (2006, 240502–2) mention how novel experiments in the 1960s were "partly motivated by the often overinterpreted quotation from Dirac" without further clarification about this over-interpretation. Even though these papers highlight the confusion surrounding D, they are not helpful for tracing the contour of the controversy.

Over the last sixty years, the views about these points have evolved over two phases. In the first phase (henceforth *Phase-1*) of the controversy – from 1963 to the late 1970s – some physicists held the view that (a) and (b) do not conflict with one another. Post 1983, with (b) gradually becoming established, (a) was no longer endorsed. In this subsequent phase (henceforth *Phase-2*) of the controversy, diverse opinions about why (a) is wrong have emerged.

In the following sections 3 and 4, I will chart the broad contours of the controversy by identifying the differing views in each phase. With these aspects laid out, from section 5 onwards, I investigate the sources of disagreements and the reasons for the controversy's perpetuation. I argue that the controversy initiated and prevails due to the continued superficial association of (a) and (b). As I show in section 7, their relation is sustained through several misinterpretations of D. In section 8, I argue that (a) and (b) belong to different contexts. Even though D and 2pI experiments respond to the query "Can two photons undergo interference?", their responses are incommensurable as each uses different meanings of interference.

3 Phase-1 of the Controversy

3.1 Development of 2bI Experiments

As early as 1955, it was known that independent thermal light beams could exhibit temporal coherence and produce beats. Similar experiments were replicated with independent laser beams by 1962. Magyar and Mandel (1963) were the first to demonstrate spatial coherence between two independent laser beams through an interference experiment (henceforth abbreviated as 2bI experiment). Since the sources emitted the pulses sporadically, a transient fringe-pattern was observed. In the paper, the authors do not make any comment on the implications of the novel observations on D. However, at the beginning of the paper, they do point out that D is about interference experiments with coherent sources and 2bI experiments are better understood as incoherent scenarios, as the beams' phase difference keeps fluctuating.

The distinction between coherent and incoherent interference experiments is further developed in a subsequent paper by Mandel (1964). In this theoretical paper, he models two scenarios of 2bI experiments from the quantum mechanical perspective: when the beams are in pure quantum states and when they are in mixed states. The quantum mechanical analysis of the interference effect of the beams in pure states is similar to the classical analysis: the expectation value of the total intensity at a particular place r and time $t - \langle I(r,t) \rangle$ modulates periodically and this accounts for the observed effect. When the beams are in mixed states, $\langle I(r,t) \rangle$ no longer furnishes any evidence at all of interference effects (ibid., A12). As transient interference effects are observed in these cases, Mandel suggests an alternative approach to represent them: "the detection of a pattern implies the observation of intensity at several space-time points. We are therefore led to examine the correlation of intensities at two space-time points r_1, t_1 and r_2, t_2 " (A13 ibid., emphasis in the original). The intensity correlation $\langle I(r_1, t_1)I(r_2, t_2) \rangle$

^{8.} See Forrester et al. (1955) and Javan et al. (1962). For a brief review of these and other early experiments, see Paul (1986).

^{9.} In quantum mechanics, the radiation fields that are classically considered "coherent" are represented by pure quantum states. The "incoherent" fields, which are usually encountered in 2bI experiments, are represented by mixed states. See Glauber (1963) and Mandel (1964) for a detailed discussion on this.

exhibits periodic modulation and this, according to Mandel, explains the interference effects observed in the 1963 experiment.

Unlike the previous paper, Mandel (1964) explicitly discusses the relation between 2bI experiment and D. In the introduction section, after pointing that D explains the interference in coherent scenarios, he mentions:

While [D] does not refer to or deny the possibility of observing interference in the superposition of incoherent beams, it has nevertheless sometimes been interpreted in this sense (ibid., A10) ...(2)

This thread is picked up again in the concluding section, where Mandel argues that 2bI experiments do not contradict D as the effects observed here are not 2pI effects, but still single photon interfering with itself (ibid., A14-15).¹⁰

A more precise experiment to investigate the controversy was carried out by Pfleegor and Mandel (1967b). The paper's intention is clarified at the beginning itself:

The demonstration, a few years ago, that two light beams derived from two independent lasers can give rise to observable interference fringes led to a certain amount of debate. The discussion centered... on the question whether the experiment disproved [D]... Partly with a view to answering this [sic] questions we have performed a further experiment of the same kind" (ibid., 1084)(3)

The authors superpose two laser beams on an interference detector. The beams are attenuated so that there is a single photon in the setup at a given time. By statistically determining the formation of interference effects, they conclude that these effects cannot be associated with the interference between two photons, one from each beam. Instead, these are still single photon interference (henceforth abbreviated as 1pI) effects. They end the paper by saying "Surprising as it might seem, [D] appears to be as appropriate in the context of this experiment as under the more usual conditions of interferometry" (ibid., 1088).¹¹

3.2 Birth of the Controversy

Apart from the papers by Mandel and his associates, there do not seem to be other research papers that discuss the implications of 2bI experiments on D during the decades of 1960s and 1970s. The surveyed papers do mention the presence of opposing views, but these claims are not adequately substantiated. For instance, Mandel (1964, A10) says that D has "sometimes been interpreted" to deny the possibility of 2bI experiments, but does not provide any reference. The two papers by Pfleegor and Mandel (1967a, 1967b) point to the "debates" and cite some references for this claim, none of which comment on D's validity. The non-presence of research papers that disagree with one another

^{10.} Mandel and Wolf (1965) reiterate the analysis found in the 1964 paper.

^{11.} Pfleegor and Mandel (1968) conduct a slightly modified experiment to achieve "greater statistical accuracy".

^{12.} See quote (2).

^{13.} See quotes (1) and (3).

^{14.} In both papers, the authors cite four papers for their "debate" claim. Leaving aside a German paper, the other three are Mandel (1964) and two papers that respond to the theoretical analysis of the 1964 paper. There is no discussion on D's validity in the latter two papers.

need not imply the absence of disagreement within the physics community. There were already discussions by 1969 on the challenges of teaching quantum mechanics due to 2bI developments. Sciamanda (1969, 1128) reports, "Not a few students (and graduates) have expressed difficulty in reconciling [2bI] phenomenon with a famous comment by Dirac". It might also be the case that the phase-1 physicists anticipated plausible objections about the new observations and preemptively responded to them. The comments and the intentions expressed in the 1967 papers suggest this possibility.¹⁵

In the surveyed papers, the main point of contention is about the possibility of 2bI experiments exhibiting "interference between two different photons", something which D denies. However, none of these papers substantiate in what sense the experiments contradict D. Neither is there clarification about the preliminary questions for raising such a concern: why do 2bI experiments – unlike the traditional ones – undermine D? how to interpret D in the new experimental context so that it can be evaluated? Among the papers, Pfleegor and Mandel articulate the contention more precisely. They mention that the effects of 2bI experiments can be regarded as "evidence for the interference of photons from one beam with photons of the other beam" (1967a, 766) and go on to show that "the effect readily cannot be described in terms of one photon from one source interfering with one from the other" (1967b, 1088). But, there is no discussion about interpreting "two different photons" in D as photons from different sources.

All the surveyed papers provide a similar argument for why 2bI effect does not contradict D. For Mandel (1964), since the detected photon's source cannot be ascertained, "just as in conventional interferometry, each photon is to be considered as being partly in both beams, and 'interferes only with itself'" (1964, A15). ¹⁶ Pfleegor and Mandel also conclude that the effects are due to the uncertainty about the detected photon's source.

As can be gathered from these developments, the possibility of conducting interference experiments with two independent lasers initiated the confusion about the possibility of 2pI and its implication on D. The initial attempt to resolve this – as can be seen in the 1963 paper – was to distinguish conventional and 2bI experiments based on the grounds of coherent and incoherent scenarios respectively, and claim that D is only about the former scenarios. The subsequent papers revised this opinion and showed that D applies to 2bI experiments. With this, the physicists' strategy for denying 2pI was to argue that the observed effects are, in fact, due to the "conventional" 1pI, and thereby, 2bI experiments still abide by D.

By 1967, however, the intention to confirm the conclusion seems to have become the central interest rather than the investigation of the question at hand. The experiments of Pfleegor and Mandel only confirm whether single photons in the 2bI scenario can produce 1pI effects. These experiments cannot comment on the possibility of 2pI, which is one of the conflicting points in the controversy. This oversight seems to be an outcome of the problem I have already pointed out earlier – the physicists' non-engagement with the fundamental questions about the controversy.

Only in a 1969 paper by Sciamanda, discussions on some of the important questions can be seen. At the beginning of the paper itself, Sciamanda observes that "if this two-laser experiment is to be interpreted as producing interference between different photons, then

^{15.} For instance, the following phrases from quotes (1) and (3) support the above suggestion: "On the face of it, the observations appeared to contradict [D]" and "with a view to answering this questions we have performed a further experiment". Mandel and Wolf (1965, 282) also mention that "this statement appears to contradict the effect". (Emphases added in these quotations.)

^{16.} A similar analysis is found in the 1965 paper by Mandel and Wolf.

so also must many other [older] interference experiments" and therefore, the "present difficulty with Dirac's statement should, then, have been long ago discussed in connection with the traditional beam-splitting experiments" (1969, 1128–29). For him, the confusion about D in 2bI experiments stems "from an intuitive temptation to view spatially separated sources as necessarily producing physically different photons," and when the naive view of photons is rejected, it becomes clear that the observed effects are still due to 1pI. Despite his non-engagement with the earlier papers' analyses, when seen from his perspective, Mandel and his team seem to have erected a strawman and shown it to be wrong.

3.3 Deferred Start

The confusion about the relation between 2pI and D could have begun a few years before 1963 when the experimental result of Hanbury Brown and Twiss (henceforth abbreviated as HBT) had raised several concerns. In 1954, HBT proposed a new kind of interferometer design based on the premise that two light beams from a source can exhibit intensity correlations. Subsequently, they demonstrated that two photodetectors receiving light from a source show correlations and inferred that "the photons in two coherent beams of light are correlated" (Hanbury Brown and Twiss 1956, 29). Since their experimental observation and conclusion did not sit well with the contemporary understanding of quantum mechanics, these were not immediately accepted. One plausible explanation of HBT effect – a photon splits and illuminates the two detectors – contradicted the belief that photons are indivisible (Silva and Freire 2013, 471). On the other hand, it was difficult to "imagine photons hanging about waiting for each other in space" (Hanbury Brown 2019, 617). Also, given the prevalence of D, it was asked how "photons could arrive in pairs if they didn't interfere with each other" (ibid., 618)?

Although the concern about the HBT effect contradicting D was explicitly pointed out, this did not gain traction for several reasons. Given that the HBT effect – its very possibility and its explanation – was already at the center of several controversies during that time, it is not surprising that a plausible impact on D did not get much attention. Also, HBT, in a subsequent theoretical paper, clarified that the fluctuation observed in each of the photodetector is better understood as a 1pI effect – "a phenomenon caused by the uncertainty in the energies of the individual photons" (Hanbury Brown and Twiss 1957, 308) – and ended their analysis by arguing that the correlation is "essentially as an interference effect exemplifying the wave rather than the corpuscular aspect of light" (ibid., 308). Thus, HBT defended D and employed the wave-particle duality of light to steer away from the 2pI interpretation of their effect.

Apart from HBT's effort to dissolve the conflict between D and HBT effect, I think there is another crucial reason why the conflict was not taken seriously. Since none of the concerned papers categorised the HBT effect as an "interference" effect, its proposal – and even its subsequent establishment post 1957 – was not explicitly perceived as clashing with D. It is only in a 1961 paper, Fano suggested that HBT correlations "can be properly described as interference effects". Fano models the HBT experiment as a scenario where two sources illuminate two photodetectors and explains that the inability to identify the source that photoionized a specific detector results in the quantum superposition of two "alternative sets of events" (Fano 1961, 540). Even though this analysis resembles the phase-1 views, interestingly, none of the phase-1 physicists refer to Fano's paper or comment on the similarity between the 2bI and HBT effect. In fact, Mandel (1964, A10) claims that 2bI effects, despite the "formal similarity" to the HBT effect, are "experimentally very

different". A few decades later, however, several physicists categorise the HBT effect as 2pI effects.¹⁷

4 Phase-2 of the Controversy

4.1 Development of 2pI experiments

The scepticism about 2pI that emerged in phase-1 gradually declined in the 1980s. Mandel himself provides a theoretical analysis of it in a 1983 paper. In the paper's introduction, Mandel mentions: "So far most of the experiments made use of laser sources... However, with the development of experimental techniques... other quantum states of the field have become accessible for interference experiments, and this raises new possibilities" (1983, 929). Since the experiments in phase-1 used independent laser beams, creating a scenario involving two photons, one from each source, was not feasible. To do that, a crucial precondition was the availability of light sources that produce single photons at a time. With such a technique being invented in 1977 by Kimble et al., Mandel revised his initial view, which he held till 1976. 18

In the 1983 paper, Mandel provides the quantum mechanical analysis of 2pI as a special case of the incoherent scenario. Similar to his 1964 paper, discussed in section 3.1, Mandel shows that the photon detection probability – quantum mechanics' equivalent of classical intensity – does not have the interference term when the sources are incoherent. In this scenario, the joint probability of two-photon detection – the second-order correlation function of detecting two photons, each at a specific place and time – has a modulating term, and this acts as "an alternative procedure for establishing the existence of interference effects" (1983, 931). Also, in contrast to the 1964 paper's analysis of interference with independent lasers, the 1983 paper incorporates a new kind of light source – individual atoms – for the quantum mechanical analysis of the incoherent interference scenario. This enables Mandel to examine the 2pI scenario, where each source is an atom emitting a photon at a time. For this scenario, Mandel shows that the joint probability detection is zero when the two detectors are separated by an odd number of half fringes. "Two photons can therefore never be found at certain pairs of points. This prediction has no classical analog" (1983, 929).

Mandel's analysis provided a way to test 2pI. The first demonstration of this was carried out by Ghosh and Mandel (1987) with two photons generated through the parametric down conversion. Interference effects of two photons emitted from independent sources were subsequently demonstrated.¹⁹ These effects were also reproduced with independent thermal beams.²⁰ Eventually, 2pI effects were generalised in the form of n-order interference effects, leading to interference experiments of four photons.²¹

^{17.} See, for instance, Paul (1986, 210) and Shih (2021, 216). However, there is no consensus at present whether all correlation effects of incoherent ("thermal") light beams can be considered as 2pI phenomena. This ongoing controversy is situated in the context of the ghost-imaging technique. See Abouraddy et al. (2001), Gatti et al. (2004) and subsequent literature on the topic.

^{18.} Mandel iterates his 1960s view in a 1976 paper. See Mandel (1976).

^{19.} See Riedmatten et al. (2003) and Kaltenback et al. (2006).

^{20.} See Scarcelli et al. (2004) and Zhai et al. (2006).

^{21.} See Sackett et al. (2000) and Walther et al. (2004).

4.2 Change of Course

The development of 2pI did have an impact on the controversy, albeit it took some time for the discourse to change. In the 1980s, only a few papers talked about the relation between D and 2pI. Most of these, as expected, are similar to the phase-1 discussions. For instance, Milonni (1984, 49–50) refers to the confusion about 2pI, which arose in 2bI experiments, to illustrate the problems of interpreting interference in quantum mechanics from the particle perspective. Contrary to this, Kidd et al. (1989, 31) claim that the 2bI experiments brought forth the inadequacy of the particle model of photons used in D. The only paper to incorporate the 1983 development and comment on the controversy was a 1986 paper by Paul. In the following decades, however, numerous papers engaged with the controversy, bringing in a new set of perspectives.

The survey of research papers from 1983 onwards reveals that the establishment of 2pI – which is antithetical to the dominant view of phase-1 – surprisingly did not dissolve the controversy. Instead, the situation seems to have been inverted: in contrast to the initial phase's scepticism about 2pI, the confirmation of 2pI post 1983 has given rise to doubts about the dictum. A spectrum of views about D exists at present, as both 1pI and 2pI are possible. Some physicists consider that D is correct only in the context of the traditional interference experiments. Few others believe that D can be suitably reformulated to be valid in the modern interference experiments as well. In contrast to these, some physicists think that the modern experiments make evident D's inherent flaw as it uses the "photon" language to analyse the phenomenon. These differing views indicate that the controversy about the relation between D and 2pI still persists. It transited to a new phase, where the disagreements are about the validity of D.

4.2.1 D is Contextual

A view to emerge soon after the 1983 development was by Paul (1986), who argued about the non-applicability of D for the modern interference experiments. For Paul, interference experiments involve "the superposition of electromagnetic fields". He finds the conventional definition of interference, based on the observation of fringe-pattern, to be situated in the experimental context of its time. Prior to the availability of lasers and sensitive photodetectors, the only way to detect interference was by producing fringe-pattern using a single source. The observations in this kind of experiments are aptly explained by the first part of D, "each photon interferes only with itself". The modern 2bI experiments, on the other hand, demonstrate "interference between photons spontaneously emitted by different atoms" (ibid., 210), and the effects manifest in the form of intensity correlations. Therefore, in these scenarios, the subsequent assertion of D – "interference between two different photons never occurs" – "proves to be false" (ibid., 230).

Paul's stance to restrict D to its historical context seems to be an obvious outcome of the 2pI developments. Nonetheless, this stance marks an important shift in the controversy as it is the first time D was argued not to be a "general rule".²² A similar view was later proposed by Davis and Parigger (1994), who responded to Louradour et al. (1993). In their paper, Louradour et al. reported better observations of conventional fringe-patterns produced in a 2bI experiment. It is not their results, but what they said in the paper's introduction that initiated a fresh discussion around the controversy: "Interference arising from light emitted by two separate optical sources was observed for the first time in 1963

^{22.} As mentioned in section 1, Bromberg (2010, 10-11) contrasts Paul and other Berlin physicists' unsympathetic view of D with the "faithfulness" of British physicists, like Mandel.

... thereby demonstrating that two different photons can interfere, contrary to Dirac's dictum" (1993, 242). As discussed in section 3.1, Magyar and Mandel (1963) carried out the first 2bI experiment; but, contrary to what Louradour et al. state, they did not claim to have demonstrated 2pI.

Wallace (1994), in his response to the comment of Louradour et al., points out that they make a hasty conclusion about D by ignoring photons' indistinguishability. Being ignorant of the developments that have happened post 1983, Wallace emphasises that "there is no flaw in the argument of Dirac, which should not surprise anyone" (1994, 905).²³ Another response to Louradour et al. was by Davis and Parigger (1994), who correctly note that the 2bI experiment of Louradour et al. produces – similar to that of Magyar and Mandel (1963) – the conventional 1pI interference patterns and thus D still applies to Louradour et al. experiment, contrary to their own claim. Drawing from the recent developments about 2pI, Davis and Parigger clarify that "while Dirac's statement is correct for first order Young's interference effects, interference between two different photons does occur in higher order correlation experiments" (1994, 951). This view about D was also endorsed by a few other physicists like Brown and Pike (1995, 1402) and Bachor (1998, 49).

4.2.2 D can be Reformulated

Another view about D emerged from the series of experiments that explored the non-local effect involved in 2pI. These experiments brought out an essential aspect of 2pI phenomenon: two parametric-down-conversion photons produce 2pI effects because they are in a quantum superposition state together. The necessity of superposition for both single and two-photons interference phenomena led to the generalisation of D. The first instance of this proposal is seen in a paper by Greenberger et al., who suggest to "generalise Dirac's famous dictum . . . We prefer to think of the down-converted pair as a single entity, a 'two photon.' It is this two-photon . . . that is interfering with itself" (1993, 26).

A few years later, Shih codified this suggestion in the following manner: "Dirac is correct. Two-photon interference is not the interference between two individual photons. What he said in his book is still valid if we slightly modify his statement: '... biphoton ... only interferes with itself. Interference between two different biphotons never occurs'" (2003, 1018).²⁵ Here, it is paradoxical to see both Shih's insistence on D's correctness and his appeal to modify it. Nonetheless, Shih formulates a dictum for 2pI that structurally resembles D, which is for 1pI. A few other physicists, like Scarcelli et al. (2004, 624) and Keller (2014, xvii), have also discussed 2pI phenomenon in a similar manner.

4.2.3 *D* is Wrong

In contrast to the above two views, which either narrow or extend the scope of D, certain physicists have held an extreme position that D is problematic, irrespective of the context

^{23.} In another instance of reproduction of earlier 2bI experiments, Hariharan et al. (1993) perform a modified version of Pfleegor and Mandel (1967a) experiment to generate more accurate results. Similar to Wallace, the authors do not engage with the latest 2pI experiments. They passingly mention that D is "preserved" in their experiment.

^{24.} See Franson (1989), Kwiat et al. (1990) and Ou et al. (1990).

^{25.} Shih (2003, 1018) defines biphoton as "an entangled photon pair". Shih (2021, 256) credits David Klyshko for coining the name "biphoton". Klyshko ([1980] 1988, xvii) defines biphoton as "photon pairs that arise simultaneously".

in which it is used. A version of this can be found in Glauber's response to the series of comments pertaining to Louradour et al. (1993). In this brief response, Glauber points that the "things that interfere in quantum mechanics are not particles. They are probability amplitudes for certain events". Not paying attention to this and using D in 1pI does not create much of a problem. But using it to interpret 2bI effects "would generate intolerable confusion . . . as photons interfering with one another" (Glauber 1995, 12). For Glauber, D is a naive statement that should not be used.²⁶

According to this view, D is flawed not because it fails in the 2pI context, but for the fundamental reason that it misinterprets interference in terms of photons. This criticism of D, then, is not directly an outfall of 2pI development; although the possibility of 2pI further strengthened the criticism of D. Another paper where this line of reasoning can be seen is Sudarshan and Rothman (1991). The main argument of the paper is that the analysis of conventional interference experiments based on photons is incorrect; instead field-theoretic approach should be adopted. In this larger debate of field versus particle interpretations, Sudarshan and Rothman point to the confusion about D in 2pI experiments to illustrate the problems of analysing interference from the viewpoint of photons. They note that "many authors, under the spell of [D] have the impression that different sources cannot interfere . . . this is not true . . . if one regards the interference as taking place between coherent states, then the question about what constitutes a photon disappears" (1991, 594).

5 Analysis of the Controversy

The enquiry about the relation between D and the novel kind of interference experiments has gathered several responses over the last sixty years. When viewed from the present, the phase-1 physicists appear to have been conservative about D, and this might have influenced their closed-mindedness about 2pI. The establishment of 2pI brought in different views, most of which attempt to rehabilitate D in a new context where both 1pI and 2pI are possible. The presence of multiple views has not been acknowledged, and there has not been sufficient engagement across these camps. For instance, in his 1986 review paper – where the status of D is one of the central topics – Paul does not discuss how his view about D deviates from others in spite of considering their papers. A later review paper by Pan et al. (2012) on multiphoton interferometry is altogether silent about the confusion surrounding D. With no analysis that evaluates and synthesises these plural views, important questions about this controversy remain unanswered. Why has the seemingly straightforward question about the relation between D and 2pI been unresolved for this long? What are the reasons for the presence of the diverse views on the issue? In other words, what factors are giving rise to disagreements? Do any of the views of phase-2 satisfactorily resolve the controversy?

An obvious starting point to answer these questions is to go back to Dirac's book and understand what he meant by D. After carrying this out in section 6, I will examine the physicists' views in section 7. Interestingly, most of these views misinterpret D. In each case, the interpretation of D is influenced by the physicist's opinion about 2pI. Thus, the deviation from the original meaning of D initiated the controversy and its varied interpretation sustained the controversy. Another crucial aspect that aggravated

^{26.} In his textbook, Hecht (2017, 413) discusses Glauber's comments to illustrate the "simplistic" nature of D.

the situation is the non-recognition of the fact that D and 2pI belong to different contexts. As I will show in section 8, the concept of "interference" in 2pI experiments differs from that of 1pI experiments, where D is situated. When this becomes clear, invoking D in the context of 2pI seems inappropriate.

6 Dirac's Dictum

Dirac's statement D appears in his book's first chapter, where he introduces the fundamental aspects of quantum mechanics. He illustrates the principle of superposition of states through the quantum mechanics interpretation of interference (henceforth QM-I). According to this principle, the state of a photon entering an interferometer is a superposition of two states, each corresponding to the photon's travel across one of the two paths in the interferometer. The interference effect is due to the probabilistic distribution of the photon across these two possibilities. Dirac clarifies that the probability here should not be understood as "the probable number of photons" in a particular path of the interferometer. This interpretation that a certain number of photons from each path interfere with one another would yield problematic scenarios, like "two photons would have to annihilate one another and other times they would have to produce four photons" (Dirac [1930] 1958, 9). Instead, the probability should be associated with single photons, such that a photon is "partly" in both paths. Dirac summarises this clarification through the remark: "Each photon then interferes only with itself. Interference between two different photons never occurs" (ibid., 9).

D, then, is situated in a discussion that analyses different ways of understanding the notion of probability in the principle of superposition. Dirac assesses these meanings by evaluating the phenomenon of interference they entail. If probability represents specific numbers of photons in each path, then interference has to be interpreted as photons interacting with each other – similar to classical waves superposing – by adding up or annihilating one another. In contrast, understanding probability at a single photon's level implies an interference between the possibilities associated with each photon's superposition state. The conclusion of this analysis is succinctly captured by D. The first statement of the dictum (henceforth D_1) – "Each photon then interferes only with itself" – suggests the preference for the QM-I over the classical optics interpretation of interference (henceforth CO-I), which is captured by the second statement (henceforth D_2) – "Interference between two different photons never occurs".

7 Misinterpretations of D

The comparison of D's meaning in the original context and its interpretations across the two phases reveals that the physicists have deviated from what Dirac had meant. Consider the phase-1 interpretation of D_2 by Pfleegor and Mandel (henceforth D_2^m) as the denial of "one photon from one source interfering with one from the other" (1967b, 1088). Even though D_2^m seems similar to D_2 , when the meaning of "interfering" here is unpacked, the differences come to the fore. Pfleegor and Mandel are not emphasising the denial of CO-I in D_2^m . Given the well-establishment of the QM-I during their time, it would have been trivial to set up a new experiment to demonstrate that two photons from each source do not annihilate or add up. When D_2^m is understood in the context of the paper, the

authors are claiming that 2pI does not happen – two photons, one from each source, do not quantum mechanically interfere.

This analysis clarifies a few important aspects of the phase-1 view. The physicists failed to realise that D_2 is not specifically about "interference between two photons", but articulates the denial of CO-I in this manner. D_2 could have been phrased in other ways – like "interference between several photons" – without losing the central meaning. Not realising this, the physicists thought D_2 is literally about "two photons" and presumed its relevance in 2bI experiments. D_2^m also suggests that the physicists' interpretation of D_2 is influenced by their stance about 2pI. When the physicists claim that their 2bI experiments do not contradict D_2 , they are not upholding D. Instead, through D_2^m , they are actually re-affirming their stance that 2pI is not possible. Thus, the phase-1 physicists interpreted D_2 to suit their purpose, and this initiated the controversy.

The interpretations of D in phase-2 differ from the previous views in an important manner. Unlike the phase-1 views that focused only on D_2 , the phase-2 views²⁷ interpret both D_1 and D_2 . This is evident in Paul's analysis of D: "While the first part of [D] undoubtedly provides the correct quantum-mechanical interpretation of all conventional interference experiments, its second part cannot be held as a general rule" as modern experiments with two sources have exhibited interference between different photons (1986, 209). This interpretation of D by Paul (henceforth D^p) amounts to the following: D_1^p is about 1pI, which takes place in conventional experiments; D_2^p claims that two independent photons do not undergo QM-I; and D_2^p has been proven wrong by the modern experiments.

Paul not only misinterprets D, but also claims D_2^p to be wrong, opposed to Dirac's affirmation of D_2 . This indicates that D^p deviates from D in another manner. Since D_1 and D_2 pertain to single source experiments, these are logically related such that both have to be true or false. It would be contradictory if D_2 is false and D_1 is true.²⁸ Paul, on the other hand, upholds "Each photon interferes only with itself" and denies "Interference between two different photons never occurs" without leading to a contradiction. This is because D_1^p is about 1pI experiments and D_2^p pertains to 2pI experiments. Paul splits the dictum into two independent parts and thereby could deny one of them. The splitting of the dictum is based on Paul's premise that modern experiments with two sources are different from conventional single source experiments which exhibit 1pI. Thus, Paul's interpretation of D, similar to the phase-1 view, seems to be influenced by his stance about 2pI.

8 Different Kinds of Interference

A premise similar to Paul's seems to have motivated a few other physicists, as discussed in section 4.2.2, to reformulate D for the modern experiments. According to them, as 1pI and 2pI scenarios differ in the object of interference – photon and biphoton respectively – D can be extended to the 2pI scenario by accommodating this change. These physicists do not clarify their interpretation of the two statements in D or in the reformulated dictum (henceforth D^g). Nonetheless, unlike others in the controversy, these physicists are aware that they are modifying the original dictum. This need to modify, however,

^{27.} Here, the view discussed in section 4.2.3 is an exception, since it rejects D altogether. Also, Glauber, while discussing D, represents D_2 accurately: "[this] statement was meant to dismiss absurd images of different photons eating or reinforcing one another" (Glauber 1995, 12).

^{28.} This is the case because D_1 is an assertion and D_2 is a denial. Given this, it would not be contradictory if D_1 is false and D_2 is true.

should have cautioned them about the hurdles of transposing D to a new context. Without attending to the nuances, the physicists presumed that replacing "photon" with "biphoton" in D would make the dictum suitable for the 2pI context. When the resulting statements are evaluated for their explanatory role in the concerned experimental observation, their irrelevance – as opposed to D's relevance for the 1pI effect – becomes evident.

In Dirac's discussion, D_1 and D_2 are not generic claims about QM-I and CO-I. These statements describe plausible phenomena in the experimental setup of an interferometer to explain the 1pI effect, the modulation of resultant intensity observed as the fringe-pattern. D_2 , then, is not just denying CO-I. Instead, it claims CO-I between photons from each path does not happen as this cannot consistently explain the intensity modulation. In contrast, D_1 affirms the quantum superposition of each photon as this coherently explains the observation. Recognising that D_1 and D_2 are competing explanations for the same observation implies that these statements are connected through an exclusive disjunction (XOR). This accounts for their logical relation discussed in the previous section.

In modern experiments with two sources, the photon detection probability of a detector at a particular place does not show any modulation. This prompted Mandel (1983), as discussed in section 4.1, to suggest the use of two detectors as the second-order correlation function contains the modulating term. Even though Mandel had claimed that this modulation corresponds to the "random motion" of the transient interference pattern (1983, 936), subsequent works clarified that each detector's reading in a 2pI experiment does not modulate when the second-order correlation function exhibits the 2pI effect.²⁹ For instance, in experimental setups that can exhibit both 1pI and 2pI effects, physicists have shown "complementarity between one and two-particle fringes: the conditions of seeing one preclude the possibility of seeing the other" (Greenberger et al. 1993, 24). So, the two effects are associated with distinct physical quantities: 1pI effect pertains to the modulation of field's intensity; 2pI effect is the modulation of the second-order correlation of resultant field. Also, their mutual complementarity implies that the effects are not related. 2pI effects cannot be understood as some high-order effects reducible to 1pIeffects. Therefore, 1pI and 2pI are fundamentally different kinds of interference. Apart from the commonality of quantum superposition in the involved phenomena, these two have different kinds of experimental design, object of interference, and interference effect.

The phase 2 physicists did not recognise this distinction and considered 2pI to be another variant of 1pI. This presumption is reflected in the way they tweak D for the new context. The resulting D^g restates D in terms of biphotons: each biphoton undergoes QM-I and biphotons do not partake in CO-I. These two statements are not untrue, but they are not about competing plausible explanations for 2pI like their counterparts in D. Although D_1^g captures an essential aspect of 2pI effect, the phenomena denied in D_2^g albeit being true in quantum mechanics – has no explanatory role in the concerned context. Thus, the physicists' reformulation fails to preserve the structural integrity of D and thereby renders D^g irrelevant to the 2pI context.

^{29.} As Shih points, there are experiments where both 1pI and 2pI effects are observed. However, he considers these as "trivial second-order phenomena" and suggests avoiding "this type of second-order interference... by arranging the experimental condition in such a way under which no classic first-order interferences are observable" (2021, 277).

9 Conclusion

No one had probably anticipated that using two sources, instead of one, in interference experiments would result in such a long-drawn controversy. As I have shown in the last four sections, the controversy originated and perpetuated due to several misinterpretations. The phase-1 physicists interpreted "interference between two different photons" in D as 2pI and denied its possibility. The subsequent establishment of 2pI could have highlighted the mistake and resolved the controversy. This development did nudge the physicists to alter D for the new context. However, the physicists made a weak distinction between 1pI and 2pI interference experiments, and considered the essential aspects of D – as seen in D^p and D^g – still applicable to 2pI. The relation between D and 2pI never got severed, and the controversy continues. 1pI and 2pI are different kinds of interferences. Dirac is clarifying a few aspects about 1pI in D and this does not pertain to 2pI.

The survey of the papers also reveals a common reason behind the numerous interpretations of D: the physicists, before commenting on D, did not peruse what Dirac is discussing. This interpretative laxity might be due to the presumption that D is law-like: since it is true for all interference experiments, it can be understood acontextually. Contrary to this belief, as I have shown in the above sections, several implicit specificities about D_1 and D_2 become apparent when D is read in its original context. So, surprising as it might sound, the exaltation of D to the status of a dictum seems to have led to its multiple interpretations.

Since D tersely embodied the essential change brought by quantum mechanics to interference, it became a handy mantra to define QM-I. This way of talking about interference became so prevalent and intertwined that, as the above analyses illustrate, D's interpretation varied over time to reflect the evolution of the concept of interference. Thus, the history of D is also a history of interference in the last ninety years. However, the continued emphasis of D inhibited recognising the next revolution in interference brought by the discovery of high-order effects.

References

- Abouraddy, Ayman F., Bahaa E. A. Saleh, Alexander V. Sergienko, and Malvin C. Teich. 2001. "Role of Entanglement in Two-Photon Imaging." *Physical Review Letters* 87 (12): 123602. https://doi.org/10.1103/PhysRevLett.87.123602.
- Bachor, Hans-A. 1998. A Guide to Experiments in Quantum Optics. Wiley. ISBN: 978-3-527-29298-1.
- Bouwmeester, Dirk. 2004. "High NOON for Photons." *Nature* 429 (6988): 139–141. https://doi.org/10.1038/429139a.
- Bromberg, Joan Lisa. 2006. "Device Physics Vis-à-vis Fundamental Physics in Cold War America: The Case of Quantum Optics." *Isis* 97 (2): 237–259. https://doi.org/10. 1086/504733.
- ——. 2010. "Modelling the Hanbury Brown-Twiss Effect: The Mid-Twentieth Century Revolution in Optics." In *Third International Conference on the History of Quantum Physics*. Accessed November 12, 2023. http://quantum-history.mpiwg-berlin.mpg.de/news/workshops/hq3/hq3_talks/22_bromberg.pdf.

- Bromberg, Joan Lisa. 2016. "Explaining the Laser's Light: Classical versus Quantum Electrodynamics in the 1960s." *Archive for History of Exact Sciences* 70 (3): 243–266. https://doi.org/10.1007/s00407-015-0166-8.
- Brown, R G W, and E R Pike. 1995. "A History of Optical and Optoelectronic Physics in the Twentieth Century." In *Twentieth Century Physics*, edited by Laurie M Brown, Abraham Pais, and Brian Pippard, 3:1385–1504. Institute of Physics Publishing, and American Institute of Physics Press.
- Davis, Lloyd M., and C. Parigger. 1994. "Comment on "Interference Fringes between Two Separate Lasers," by F. Louradour, F. Reynaud, B. Colombeau, and C. Froehly." *American Journal of Physics* 62 (10): 951–952. https://doi.org/10.1119/1.17690.
- Dempster, A. J., and H. F. Batho. 1927. "Light Quanta and Interference." *Physical Review* 30 (5): 644–648. https://doi.org/10.1103/PhysRev.30.644.
- Dirac, P. A. M. (1930) 1958. The Principle of Quantum Mechanics. Fourth. Oxford: Clarendon Press.
- Engelhardt Jr., H. Tristram, and Arthur L. Caplan. 1989. Scientific Controversies: Case Studies in the Resolution and Closure of Disputes in Science and Technology. Cambridge University Press.
- Fano, U. 1961. "Quantum Theory of Interference Effects in the Mixing of Light from Phase-Independent Sources." *American Journal of Physics* 29 (8): 539–545. ISSN: 0002-9505. https://doi.org/10.1119/1.1937827.
- Forrester, A. Theodore, Richard A. Gudmundsen, and Philip O. Johnson. 1955. "Photoelectric Mixing of Incoherent Light." *Physical Review* 99 (6): 1691–1700. https://doi.org/10.1103/PhysRev.99.1691.
- Franson, J. D. 1989. "Bell Inequality for Position and Time." *Physical Review Letters* 62 (19): 2205–2208. https://doi.org/10.1103/PhysRevLett.62.2205.
- Gatti, A., E. Brambilla, M. Bache, and L. A. Lugiato. 2004. "Correlated Imaging, Quantum and Classical." *Physical Review A* 70 (1): 013802. https://doi.org/10.1103/PhysRevA. 70.013802.
- Ghosh, R., and L. Mandel. 1987. "Observation of Nonclassical Effects in the Interference of Two Photons." *Physical Review Letters* 59 (17): 1903–1905. https://doi.org/10.1103/PhysRevLett.59.1903.
- Glauber, Roy J. 1963. "Coherent and Incoherent States of the Radiation Field." *Physical Review* 131 (6): 2766–2788. https://doi.org/10.1103/PhysRev.131.2766.
- ——. 1995. "Dirac's Famous Dictum on Interference: One Photon or Two?" *American Journal of Physics* 63 (1): 12. https://doi.org/10.1119/1.17790.
- Greenberger, Daniel M, Mike A Horne, and Anton Zeilinger. 1993. "Multiparticle Interferometry and the Superposition Principle." *Physics Today* 46:22–29.
- Hanbury Brown, R. 2019. "Excerpt from the Book Boffin: A Personal Story of the Early Days of Radar, Radio Astronomy and Quantum Optics." In *Quantum Photonics:* Pioneering Advances and Emerging Applications, edited by Robert W. Boyd, Svetlana G Lukishova, and Victor N. Zadkov, 615–619. Springer.

- Hanbury Brown, R., and R. Q. Twiss. 1956. "Correlation between Photons in Two Coherent Beams of Light." *Nature* 177:27–29.
- ——. 1957. "Interferometry of the Intensity Fluctuations in Light. I. Basic Theory: The Correlation between Photons in Coherent Beams of Radiation." *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* 242 (1230): 300–324.
- Hariharan, P., N. Brown, and B.C. Sanders. 1993. "Interference of Independent Laser Beams at the Single-photon Level." *Journal of Modern Optics* 40 (1): 113–122. https://doi.org/10.1080/09500349314550131.
- Hecht, Eugene. 2017. Optics. 5th ed. Essex: Pearson Education Limited.
- Hentschel, Klaus. 2018. Photons: The History and Mental Models of Light Quanta. Springer.
- Javan, A., E. A. Ballik, and W. L. Bond. 1962. "Frequency Characteristics of a Continuous-Wave He–Ne Optical Maser." *Journal of the Optical Society of America* 52 (1): 96–98. https://doi.org/10.1364/JOSA.52.000096.
- Kaltenbaek, Rainer, Bibiane Blauensteiner, Marek Žukowski, Markus Aspelmeyer, and Anton Zeilinger. 2006. "Experimental Interference of Independent Photons." *Physical Review Letters* 96 (24): 240502. https://doi.org/10.1103/PhysRevLett.96.240502.
- Keller, Ole. 2014. Light: The Physics of the Photon. CRC Press.
- Kidd, Richard, James Ardini, and Anatol Anton. 1989. "Evolution of the Modern Photon." American Journal of Physics 57 (1): 27–35.
- Kimble, H. J., M. Dagenais, and L. Mandel. 1977. "Photon Antibunching in Resonance Fluorescence." *Physical Review Letters* 39 (11): 691–695. https://doi.org/10.1103/PhysRevLett.39.691.
- Kipnis, Nahum. 1991. History of the Principle of Interference of Light. Basel: Springer Basel AG.
- Klyshko, David N. (1980) 1988. *Photons Nonlinear Optics*. Gordon and Breach Science Publishers. ISBN: 2-88124-669-9.
- Kwiat, P. G., W. A. Vareka, C. K. Hong, H. Nathel, and R. Y. Chiao. 1990. "Correlated Two-Photon Interference in a Dual-Beam Michelson Interferencer." *Physical Review A* 41 (5): 2910–2913. https://doi.org/10.1103/PhysRevA.41.2910.
- Louradour, F., F. Reynaud, B. Colombeau, and C. Froehly. 1993. "Interference Fringes between Two Separate Lasers." *American Journal of Physics* 61 (3): 242–245. https://doi.org/10.1119/1.17298.
- Machamer, Peter, Marcello Pera, and Aristides Baltas, eds. 2000. Scientific Controversies: Philosophical and Historical Perspectives. New York: Oxford University Press.
- Magyar, G, and L. Mandel. 1963. "Interference Fringes Produced by Superposition of Two Independent Maser Light Beams." *Nature* 198 (4877): 255–256.

- Mandel, L. 1964. "Quantum Theory of Interference Effects Produced by Independent Light Beams." *Physical Review* 134 (1A): A10–A15. https://doi.org/10.1103/PhysRev.134. A10.
- ——. 1976. "The Case for and Against Semiclassical Radiation Theory." Edited by E. Wolf. *Progress in Optics* 13:27–68. https://doi.org/10.1016/S0079-6638(08)70018-0.
- ——. 1983. "Photon Interference and Correlation Effects Produced by Independent Quantum Sources." *Physical Review A* 28 (2): 929–943. https://doi.org/10.1103/PhysRevA.28.929.
- Mandel, L., and E. Wolf. 1965. "Coherence Properties of Optical Fields." Reviews of Modern Physics 37 (2): 231–287. https://doi.org/10.1103/RevModPhys.37.231.
- Milonni, P. W. 1984. "Wave-Particle Duality of Light: A Current Perspective." In *The Wave-Particle Dualism*, edited by S. Diner, D. Fargue, G. Lochak, and F. Selleri, 27–68. Dordrecht: D. Reidel Publishing Company.
- Ou, Z. Y., X. Y. Zou, L. J. Wang, and L. Mandel. 1990. "Observation of Nonlocal Interference in Separated Photon Channels." *Physical Review Letters* 65 (3): 321–324. https://doi.org/10.1103/PhysRevLett.65.321.
- Pan, Jian-Wei, Zeng-Bing Chen, Chao-Yang Lu, Harald Weinfurter, Anton Zeilinger, and Marek Żukowski. 2012. "Multiphoton Entanglement and Interferometry." *Reviews of Modern Physics* 84 (2): 777–838. https://doi.org/10.1103/RevModPhys.84.777.
- Paul, H. 1986. "Interference between Independent Photons." Reviews of Modern Physics 58 (1): 209–231.
- Pfleegor, R. L., and L. Mandel. 1967a. "Interference Effects at the Single Photon Level." *Physics Letters A* 24 (13): 766–767. https://doi.org/10.1016/0375-9601(67)90259-9.
- ——. 1967b. "Interference of Independent Photon Beams." *Physical Review* 159 (5): 1084–1088. https://doi.org/10.1103/PhysRev.159.1084.
- ——. 1968. "Further Experiments on Interference of Independent Photon Beams at Low Light Levels." *Journal of the Optical Society of America* 58 (7): 946–950. https://doi.org/10.1364/JOSA.58.000946.
- Riedmatten, H. de, I. Marcikic, W. Tittel, H. Zbinden, and N. Gisin. 2003. "Quantum Interference with Photon Pairs Created in Spatially Separated Sources." *Physical Review A* 67 (2): 022301. https://doi.org/10.1103/PhysRevA.67.022301.
- Sackett, C. A., D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, et al. 2000. "Experimental Entanglement of Four Particles." *Nature* 404 (6775): 256–259. https://doi.org/10.1038/35005011.
- Scarcelli, G., A. Valencia, and Y. Shih. 2004. "Two-Photon Interference with Thermal Light." *Europhysics Letters* 68 (5): 618. https://doi.org/10.1209/epl/i2004-10280-8.
- Sciamanda, Robert J. 1969. "Dirac and Photon Interference." American Journal of Physics 37 (11): 1128-1130. https://doi.org/10.1119/1.1975229.
- Shih, Yanhua. 2003. "Entangled Biphoton Source Property and Preparation." Reports on Progress in Physics 66 (6): 1009–1044. https://doi.org/10.1088/0034-4885/66/6/203.

- Shih, Yanhua. 2021. An Introduction to Quantum Optics: Photon and Biphoton Physics. 2nd ed. CRC Press. ISBN: 978-1-00-029279-4.
- Silva, Indianara, and Olival Freire. 2013. "The Concept of the Photon in Question: The Controversy Surrounding the HBT Effect circa 1956–1958." *Historical Studies in the Natural Sciences* 43 (4): 453–491.
- Sudarshan, E. C. G., and Tony Rothman. 1991. "The Two-slit Interferometer Reexamined." American Journal of Physics 59 (7): 592–595. https://doi.org/10.1119/1.16778.
- Taylor, G. I. 1909. "Interference Fringes with Feeble Light." *Proceedings of the Cambridge Philosophical Society* 15 (1): 114–115.
- Wallace, Philip R. 1994. "Comment on "Interference Fringes between Two Separate Lasers," by F. Louradour, F. Reynaud, B. Colombeau, and C. Froehly [Am. J. Phys. 61(3), 242–245 (1993)]." American Journal of Physics 62 (10): 950–950. https://doi.org/10.1119/1.17689.
- Walther, Philip, Jian-Wei Pan, Markus Aspelmeyer, Rupert Ursin, Sara Gasparoni, and Anton Zeilinger. 2004. "De Broglie Wavelength of a Non-Local Four-Photon State." *Nature* 429 (6988): 158–161. https://doi.org/10.1038/nature02552.
- Young, Thomas. 1802a. "II. The Bakerian Lecture. On the Theory of Light and Colours." *Philosophical Transactions of the Royal Society of London* 92:12–48. https://doi.org/10.1098/rstl.1802.0004.
- ——. 1802b. "XIV. An Account of Some Cases of the Production of Colours, Not Hitherto Described." *Philosophical Transactions of the Royal Society of London* 92:387–397.
- ———. 1807. A Course of Lectures on Natural Philosophy and the Mechanical Arts. Vol. 1. London: Taylor and Walton.
- Zhai, Yan-Hua, Xi-Hao Chen, and Ling-An Wu. 2006. "Two-Photon Interference with Two Independent Pseudothermal Sources." *Physical Review A* 74 (5): 053807. https://doi.org/10.1103/PhysRevA.74.053807.