Through the Looking Glass and What Immanuel Found There

Nick Huggett
University of Illinois Chicago

July 30, 2024

1 Kant and the Chiral Arguments for Substantivalism

When Kant (early in his career) considered the arguments for absolute space – those that we considered in Chapter One – he came to a rather bleak conclusion: 'Everybody knows how unsuccessful the philosophers have been in their efforts to place this point once and for all beyond dispute, by employing the most abstract judgements of metaphysics. Nor am I familiar with any attempt to attain this end so to speak *a posteriori* ... apart, that is, from the treatise of the illustrious *Euler* the Elder ....' (1768, 366) Certainly then, Kant viewed Newton and Clarke's efforts as metaphysics, rather than proceeding from scientific considerations (contrary to our reading), and Euler as offering the first useful argument, showing that Newton's laws of motion cannot be adequately formulated given a relational account of space (though again remember our narrower reading of Euler's ambitions). However, even Euler's accomplishment is partial, since it "does not, however, consider the no less serious difficulties which arise if, in applying [Newton's laws], one attempts to represent them *in concreto*, employing the concept of absolute space. The proof [that] I am seeking here, is intended to furnish, not engineers, as was *Euler's* purpose, but geometers themselves with a convincing argument which they could use to maintain, with the certainty to which they are accustomed, the actuality of their absolute space." (1768, 366) In other words it is not enough to show that mechanics rules out relationism (which of course we have argued it does not, when properly conceived) one must also show that geometry entails the same conclusion.

In this chapter and the next we will consider the prospects for the kind of 'geometric proof' of absolute space that Kant suggests here. His idea is that something in the geometry of mirror images can only be properly explicated on the assumption that space is substantival. In the following chapter we will consider something that Kant only touches on, the idea that the 'mechanical arguments', which he viewed as incomplete, occur not only for the general theory of motion, but also for mirror asymmetric laws of nature. Let me say from the outset that there are two important issues that we will not be discussing. First I will have nothing to say about the vexed issue of the place of the 1768 paper in the development of Kant's views on space (he later gave up his commitment to absolute space). Second I make no claim that the arguments we discuss are those Kant had in mind (another, inter-connected, vexed issue). Instead they are what I take to be the strongest considerations that can be applied to the kind of examples that he raises. I do however think that any of the arguments he intended can be answered by the relationist using straight-forward modifications of the discussion here.

Kant's paper then attempts to argue from the geometry of mirror images that space must be absolute. The idea is quite simple once you see it:

"It is apparent from the ordinary example of the two hands that the shape of the one body may be perfectly similar to the shape of the other, and the magnitudes of their extensions may be exactly equal, and yet there may remain an inner difference between the two, this difference consisting in the fact, namely, that the surface which encloses the one cannot possibly enclose the other."

Think about your hands. Ignoring scars, warts and other minor differences they are very alike. In particular they have the same volume, the same surface area, the same width palms and the lengths of the fingers and thumbs are the same. Specifically, they are geometrically similar and of the same size. But they are also crucially different – they will not fit in the same space. A left hand will fit in left handed glove but a right hand will not (at least not without stretching the glove into a new shape). They are incongruent. We of course recognize these facts by saying that hands are 'handed' or 'chiral', and learning that they come in two types or 'chiralities', 'left' and 'right'. (Contrast this with, say, basket balls, or bricks, or pyramids.) And so concerning any handed object (or process, as we shall discuss later) there are three kinds of fact: first the fact that it is handed; second that has a particular handedness, left or right; and third that there is (or can be) an object of the same shape and size but opposite handedness. Now surely these facts are spatial, and so for the relationist can only be facts about the relations between bodies. But, asks Kant, how can that be?[1] Especially, left and right hands (or left and right anything) have exactly the same relational descriptions: the distance from tip to bottom of the index finger is the same, (fingers spread) the distance from thumb knuckle to pinky knuckle is the same, and so on for any two points on each hand. To put things another way, if you were given a list of all the relations between the parts of a hand (or the various bones say) and were asked to build an object that instantiated those relations, you could build either a left or right hand, and nothing could decide which was 'correct'. Thus leftness and rightness just cannot be facts about those relations; the relations of the hands agree but the chiralities do not. Similarly, it is a spatial fact that one of my hands will fit in a given glove, but the other will not, and yet the hands instantiate exactly the same pattern of relations. The relations just do not determine which hand will fit in a given glove. If the relationist cannot explain how these chiral facts are facts about

[1] Kant's most explicit concern is with the second of these facts, though we shall pursue them all.

relations then his account of space is just inadequate: to reverse some infamous sentiments, 'if the glove doesn't fit, then you must convict'.

What I am going to show (in agreement with philosophical consensus[2]) is that given the assumption that space is Euclidean – which Kant surely held – these arguments are not at all effective against the relationist. However, as Nerlich (1976) has pointed out, when a modern, more general, understanding of the mathematics of space is employed, a new, far more worrying problem for the relationist appears – that of accounting for the 'shape' of space. In the next chapter we will turn to an original relationist solution to the problem.

2 Chirality

Let's start by laying out the theory of handedness in n-dimensional Euclidean space. Given any point p and any $(n-1)$-dimensional Euclidean subspace M – a '(hyper-)plane of reflection' – we can define the *'reflection'* p_M of p in M: there is a unique line through p normal to M, let q be its point of intersection with M. Then p_M is the point on pq such that $\bar{p}q = \bar{p_M}q$. In other words, p_M is just the projection of p an equal distance through the 'mirror', M. For instance, in 2-dimensions, M is any straight line, and pp_M is a line at right angles to it, with p and p_M equally spaced on either side, as the figure shows.

And once the reflection of a point is defined, the reflection of a figure of any number of dimensions follows immediately: any figure S is a set of points, $\{p, q, ...\}$, and so its reflection, S_M, is the set of reflected points, $\{p_M, q_M, ...\}$. For example, in two dimensions the letters are reflected as shown in the figure. And in just the same way a left hand is the reflection of a right hand in a plane in three dimensions (a bathroom mirror makes it *appear* as if reflected objects are behind it). Note that as we said earlier, a figure and its mirror image have just the same internal relations, since the distance between any two points is preseved by a reflection – mirror images are geometrically similar and equally sized.

It may be the case for a body S that for some plane of reflection $S = S_M$, so that every point of S is reflected onto another point of S, and a reflection in M leaves the figure unchanged. If so then we say that M is a 'plane of (mirror) symmetry' of S. For example the triangle shown has three planes of symmetry and the letter F none. Hands have no plane of symmetry, but human bodies do (roughly). And finally, any body of fewer than n dimensions in n-dimensional Euclidean space has a plane of symmetry, namely any plane that it lies in: for instance, the letter F reflected through the plane in which it lies is unchanged, and a hand in four dimensions would be unchanged by a reflection in the 3-dimensional space in which it lies. If a body has no plane of symmetry then it is an *'enantiomorph'*.[3] So the letter F in

[2]See van Cleve and Frederick for a comprehensive collection of articles on the subject

[3]Some authors (e.g., Nerlich??) define this word mean that a body is enantiomorphic just in case that it is incongruent to its mirror images. Working in Euclidean space the two definitions come to the same thing, but in other spaces they come apart. I have adopted my definition in order to have a convenient terminology to discuss just this point.

two dimensional space, or a hand in three dimensions are enantiomorphs, but the triangle, the F in three dimesions, and the hand in four dimensions are not.[4]

With this terminology in place we can understand a 'chirality theorem' that is the key to understanding the geometry of handedness. In Euclidean space of any dimension, a body is congruent to its mirror image just in case it is *not* an enantiomorph.[5] Kant termed mirror images 'counterparts', and so he distinguished 'congruent counterparts' like the letter A or a cricket ball and their reflections, from 'incongruent counterparts' like the letter F or a hand and their reflections. The consequence of this result is that objects are handed just in case they are enantiomorphs, for then they can be distinguished – by their incongruence – from their counterparts: despite being similar and equally sized – since they are related by a reflection their internal relations are identical – they are of opposite chiralities.[6]

3 The Fitting Theory of Handedness

Now that we understand what it is for a body to be handed, we can press the case against the relationist more carefully. Consider again the various challenges that he faces.

(i) What is it to be handed? What we said is that to be handed is to be an enantiomorph, since then our chirality theorem says that in Euclidean space any mirror images will be incongruent yet possess the same internal relations. We will see that the relationist account of handedness runs along these lines, but that it will take some work to justify the use of the theorem. For the theory of chirality presented so far assumes the existence of a metric space: especially to define planes of symmetry and the operation of reflection we assumed the existence of a n-dimensional continuum of points with a Euclidean metric. So we have an account developed in the first place in an abstract mathematical space, but we are interested in the nature of the handedness of real physical objects in real physical space. The substantivalist believes that physical space is a metric space and so can immediately apply the theory developed to physical chirality. But the relationist does not believe that space literally is a metric space but rather a system of relations and so needs to explain how the theory of handedness is to be understood in satisfying relational terms. In particular, what is an enantiomorph? What are incongruent counterparts? And can an appropriate version of the theorem linking the notions be proven in relational space?

[4] It won't serve our purposes here to discuss the geometry of many dimensional spaces at length here. Just be aware that as in the examples mentioned, the number of dimensions makes a difference to the effects of reflections. One can see quite intuitively the differences between two and three dimensions, and they are representative of the differences between any n- and $(n+1)$-dimensional spaces.

[5] Two figures are congruent iff they can be perfectly superposed by 'rigid' translations and rotations – those that do not affect the geometry of the figures (i.e., no squashing or stetching is allowed).

[6] To be a little more careful, we will say that two bodies are counterparts if they are related by a reflection and *any series of rigid translations and rotations* – my left hand is still the counterpart of my right wherever I move it.

If we assume that we have answers to the last two questions – answers that I will provide below – then the two questions on the table – what is it to be handed and what is it to be an enantiomorph – are again one. Perhaps it sounds as if the definition – the absence of a plane of symmetry – is only accessible to a substantivalist, for an object could have holes, and any putative plane of symmetry might not exist as a material entity: in this case, the definition makes reference to spacetime regions — planes — that aren't material. For instance, a sphere is not an enantiomorph, but since it is only a surface there are no flat material surfaces through its inside, and apparently the relationist is left calling it an enantiomorph. But there's really not much of a problem here. One way that the relationist could respond is by setting up a rigid reference frame as we discussed in earlier chapters on mechanics, for then one can define the reflection of a coordinate point in an arbitrary coordinate plane and can pose the question of whether any reflection of the coordinates of the points of a body is the same set of coordinates. If so, for some coordinate plane, then that is a plane of symmetry, and the body is not an enantiomorph. However, this approach would require some messy though elementary manipulation of variables, and hence I will consider a more intuitive and neater relationist response.

The key to this answer, and indeed most of the rest of this chapter is to realize that the relationist need have no beef with *abstract* metric spaces, just with the idea that *physical* space literally is a metric space. Further, the relationist accepts the reality of distance relations between bodies, and so he can perfectly well answer the question of whether some system of bodies is geometrically embeddable in some metric space. If there is a system of points of the space – considered as an abstract mathematical entity – which systematically bear the same set of distance relations to one another as the bodies do, then the system of bodies is embeddable in that space, at those points. And note that this idea is very amenable to the kind of analysis that the relationist gave all along. He thinks that geometric space is an abstraction of some kind from the actual relations between bodies; obviously the space that is abstracted must be one in which the bodies and their relations are embeddable. I will have considerably more to say about the nature of this abstraction in the next chapter.

Thus the simplest answer to the current problem is that an object is an enantiomorph iff when embedded in n-dimensional abstract Euclidean space, that space contains no planes of symmetry.[7] Given this definition, obviously the relationist and substantivalist will be in complete accord about whether a body is an enantiomorph or not (assuming as always so far that space is Euclidean).

(ii) What is it to be incongruent counterparts? And can one prove that a body's mirror image is an incongruent counterpart iff the body is an enantiomorph *even in relational space*? The first question can be broken into two parts, when are bodies (in)congruent and when are bodies counterparts? We could use rigid

[7] Implicit in this definition is our continuing assumption that physical space is n-dimensional and Euclidean, *even for the relationist.* We will discuss what such a claim might mean for the relationist immediately below, and then at length in the next chapter, so the reader will have to accept a promissory note that this assumption will be shown to be legitimate.

reference frames as I suggested above to determine whether two bodies are mirror images, but it is simpler to rely again on embedding properties. Two bodies are counterparts just in case they can be embedded in n-dimensional Euclidean space in such a way that the points at which they are embedded are related by a (single) reflection and series of rigid rotations and translations. Once again this definition will bring the relationist and substantivalist into agreement about which bodies are counterparts (in Euclidean space).

Two bodies are congruent if no geometrically possible, continuous, rigid, relative motion would bring them into exact superposition. A motion is rigid if it preserves the internal relations of the bodies and it is continuous if all the distances between bodies vary continuously. But what is it for a motion to be geometrically possible? We need a notion of geometric possibility because we can't have bodies turning out to be incongruent just because forces of some kind would prevent them from being in superposition: two globes of the same radius are congruent, but if matter were impenetrable no physically possible motion would perfectly align them. So a motion is geometrically possible if it would be possible ignoring any mechanical considerations. Given that space is n-dimensional and Euclidean it is obvious which motions these are. A relational motion is a series of configurations of bodies in varying distance relations. The motion is geometrically possible iff every configuration in the series is embeddable in n-dimensional Euclidean space.[8]

Given these definitions it is now possible to use the chirality theorem to show that relational enantiomorphs are handed. First note that the homogeneity of Euclidean space entails that any geometrically possible motion is embeddable in such a way that the sequence of points at which any body is embedded forms a continuous curve. That is, if a motion is geometrically possible in the relationist's sense then an embedding maps it onto a corresponding continuous motion in Euclidean space. So if two bodies are congruent according to the relationist then their images in Euclidean space can be superposed by continuous rigid motions and so are congruent in that space, and so by the theorem – which of course holds in Euclidean space – are enantiomorphs, and so are relational enantiomorphs. Going the other way we simply need to note that if some continuous motion is possible in Euclidean space then it is the embedding of a possible continuous relative motion, and then the argument just runs in the opposite direction. And hence an object is an enantiomorph iff its mirror images are incongruent, even if space is relational. And so being handed amounts to being an enantiomorph.

(iii) Now we turn to what Kant took to be the trickiest problem for the relationist: what is is to have a particular handedness rather than another? To be

[8]This definition – and in particular the recognition that modal facts are in play – follows closely that given by Brighouse (1999, 59). The principal differences in the definitions are that mine states explicitly what relative motions are possible in terms of an embedding space, while hers is not explicit about how they are to be specified, and that she makes being in an 'orientable' space and being an enantiomorph definitional conditions for incongruent counterparts. Of course it is true (for spaces of constant curvature) that only enantiomorphs in orientable spaces can have incongruent counterparts, this is normally thought of as being a result of the chirality theorem, not as a matter of definition – it is more informative to make definitions as logically weak as possible, and then prove additional relations between them in terms of the theory in question.

left rather than right say? The problem, remember, is that left- and right-handed incongruent counterparts – hands say – are identically arranged bodies: since reflections leave distance relations unchanged any counterparts are described by the same system of relations. Therefore the *internal* distance relations are not sufficient to determine the handedness of a hand. So what does? Let's approach this question by imagining a world in which space is Euclidean and in which the only bodies are similar and equally sized hands, some congruent and others incongruent. The relation of congruence partitions the hands into two equivalence classes, each class containing the all the hands of one handedness. I think it is clear that which class we call 'left' and which we call 'right' is entirely conventional (even if we thought that we could tell that the hands in one set are congruent in some sense to my left hand, we could still call them 'right' if we felt like it). So the problem of handedness in this world is the problem of determining for a given hand which equivalence class it belongs to — whether that class called the 'left' one is a matter of linguistic convention. But then there really is no problem, for the hand belongs to the class whose members are congruent with it, and both the substantivalist and relationist are quite happy invoking the notion of congruence.

Now suppose that there are some other enantiomorphs in this world. Some similar (to the existing hands) but smaller hands, some (mutually similar, equally sized) fists, some (similar, equally sized) gloves and some (similar, equally sized) screws. These things are all handed, and so congruence will also partition each collection of objects into two equivalence classes. So now the problem of handedness is to decide, given two kinds of object, which pairs of the four equivalence classes contain the objects of the same handedness – to decide, given two objects of any type, whether they have the same handedness. Starting with geometrically similar hands, two such hands have the same handedness iff they are congruent up to a scale transformation. Two hands whether fisted or not have the same handedness iff they are congruent up to scale transformations and motions within the normal human range of motion for hands (e.g., making or unmaking a fist). A hand and a glove have the same handedness iff the outer surface of the hand and the inner surface of the glove are congruent up to scale transformations, normal human motions and glove transformations that don't leave the glove (badly) stretched. A hand and a screw have the same handedness iff when fingers of the hand are curled into the palm and the screw turned in the sense that the fingers point, the threads make the screw turn in the direction of the thumb.[9]

This then is the relational account of handedness. For each new kind of enantiomorph we designate some new relation of 'same handedness' to existing enantiomorphs based on the spatial relations that bodies of the two kinds bear to each

[9]There's a general way to define 'same handedness' between two kinds of enantiomorph. If an n-dimensional object is an enantiomorph then there is a set of $n+1$ points on its surface – identifiable by their relations to the rest of the body – that cannot be brought into coincidence with the corresponding set of points on its counterpart. Take such a set for enantiomophs of some kind – say hands – of some chirality – say left – then the two sets belonging to any other kind of enantiomorph will bear incompatible relations to the given set. So define one of the sets to have the same handedness as the original, and you have your purely relational notion of 'same handedness'.

other: for example, are they relations that allow for congruence (up to some transformations)? In this way we continue to partion all the enantiomorphs into just two equivalence classes. I think that the relational account is the correct account of how as a matter of fact we learn and make judgements about the relative handeness of enantiomorphs. We actually do make judgements about whether the surfaces of a hand and a glove can be brought into coincidence by sliding the hand into the glove. And when we learn to distinguish the left-handed variety of some enantiomorph, we have to learn what relations it has to existing enantiomorphs, and as a matter of fact the relations in question often are relations to our hands. But I don't think that this shows anything about the tenability of substantivalism: there is no reason for the substantivalist to eschew the relational account of handedness. For this reason I don't like to call the doctrine 'the relational theory', but prefer to dub it the 'fitting account of handedness' since that reminds us that what the particular handedness of a body depends on is whether it 'fits' in a certain way to other handed bodies – the paradigm case is the way that a left hand fits a left glove and right hand does not.

The examples I gave also indicate a couple of interesting features of handedness. First, while the definitions of same handedness for the various hands and for the various gloves seem completely natural, when it comes to screws things seem a bit more arbitrary: why not say the screw is right-handed if it moves forwards when turned in the tip-to-base sense of the fingers on my right hand? Or come to that, suppose we discovered that the world contained a great many irregular, similar, equally sized, enantiomorphic rocks. We might well come to distinguish some as having the same handedness as our right hands (and the others as being left-handed), but their irregularity threatens that the choice of how to do so is entirely arbitrary. Whatever fitting relation we use has its own mirror image, and it is a free choice which we use to compare rocks to hands. What this arbitrariness shows is that there is no notion of 'same handedness' intelligible independently of reference to specific bodies, that the various definitions of same handedness that we gave are not instances of some prior definition of same handedness, but always extensions of that concept. Even when the way to extend the concept is obvious – of course similar hands have the same handedness if they are congruent up to scale transformations – there is nothing in the existing notion of smae handedness – mere congruence in this case – logically entails the obvious extension. And so the references to particular kinds of bodies in the definitions I offered are inelliminable.

Second, some of the definitions are vague. What exactly is and is not within the 'normal human range of motion'? And when is a glove 'badly' stretched? I don't see any problem here. It's just a fact about many of the real world objects that we divide into left and right, that there are borderline cases. For example, after a while a pair of cheap gloves might become so stretched that you find it hard to distinguish left from right, and might even make different judgements on different occasions. All this point shows is that in everyday judgements about handedness, we don't use geometrically precise notions. It certainly still perfectly possible in geometry to define precise relations of same handedness, as we did for similar hands.

Finally, the fitting theory answers a question that can seem quite puzzling. Why do mirrors reflect left-right but not up-down? Look in a mirror and clearly the left side of your mirror image faces your right side, but his or her head is level with your head, not your feet. But what is the source of this asymmetry? Not the way mirrors work or the geometric definition of reflections, since these make absolutely no reference to orientation of the plane of reflection – the effect is the same if you turn the mirror through 90^o! One important point to make is that there is an ambiguity in the meaning of 'left' and 'right'. In the first place we use the terms to describe enantiomorphs of opposite chiralities, but we also use them to denote an axis in space, opposed to up-down and back-front axes. in a way the puzzle trades on this ambiguity: thought of as axes there is a parity between left-right and up-down (and back-front) and so it would be mysterious if only one were affected by a reflection. But what a mirror does is exchange left and right as chiralities not axes, and 'up' and 'down' just don't name chiralities. My feet and head are just not mirror images, so there's no reason to expect to see my feet reflected opposite my head and my head reflected opposite my feet, but my hands are mirror images of each other so of course the reflection of my right hand is a left hand and *vice versa*. To expect the reflection of my head to be feet is to misunderstand what transformation a reflection produces.

But this response doesn't totally satisfy. It tells us that I should (of course) expect the reflection of my left hand to be a right hand and of my head to be a head, but it doesn't immediately help us understand why that right hand ends up on the reflection of my left side, while the head ends up on the reflection of my head end. Commentators have pointed out that this puzzle depends on how we judge which side of my reflection corresponds to which side of me, and that this judgement is ambiguous in a way. Think of yourself standing in front of a full length mirror. And imagine moving around the back of it to place yourself where your image is (supposing it was kind enough to stay there while you did so). Clearly this will place your right hand where the reflection of your left hand is – hence left becomes right – and your head where that of your image is located – hence up and down are not exchanged. And now we have an asymmetry that does not (obviously) trade on the ambiguity concerning 'left-right', since changes of handedness don't seem to be part of the story. But this asymmetry can be traced to the asymmetric way you moved your body around the mirror – effectively rotating about a *vertical* line in the mirror, so around the sides. But now repeat the experiment rotating your body about a *horizontal* line in the mirror, jumping right over the top, head first. Now you are upside down with your head at the feet of your reflection (and *vice versa*) and your left (right) side on the left (right) side of your reflection. Comparing sides this way means mirror reverse the up-down axis not the left-right axis (though of course the mirror image of your left hand is a right hand and that of your feet is not a head!). And so the left-right *versus* up-down asymmetry is an asymmetry, not of the mirror, but created by your choice of how to match the bodies, and this problem solved.

Except that isn't quite enough either. Now we want to know why we don't think of comparing bodies by going over the mirror just as naturally as we imagine going around, and indeed why it seems so wrong to compare the bodies that way, and so right to do it the way we do. Peierls has suggested that it is gravity and our modes of movement that makes us think this way, because we find it so much easier to walk around things than to leap over them head first. But this answer doesn't seem sufficient to me, because it overlooks the fact that there is a crucial geometric difference in going around *versus* going over – namely that going over does not bring the two bodies into coincidence, while going around does! The point is that we judge which sides of another body are up, down, left and right (and back and front) by imagining ourselves (approximately) coincident with that body and then transferring the names we give to our sides to its sides. And then, since our bodies have symmetry about a plane running back-front and up-down, but not about any of the other planes that bisect us, if we bring our bodies into coincidence with their reflections we will find left and right reversed, but not up and down, and not back and front. So Peierls is absolutely correct to point out that how we compare sides is relevant to the question of why mirrors swap left-right but not up-down, and he may even be right that ease of motion has some psychological effect on that judgement, but it seems to me that the most important factor is geometric. And this account explains how I can tell which of Homer Simpson's hands is his left, even though they are not enantiomorphs; like many cartoon characters they lack the geometric detail – knuckles, life lines etc – to distinguish palms from backs, and so have a plane of symmetry dividing palm from back. I can still tell which is Homer's left hand by imagining myself, roughly, in the space he occupies and seeing which of his would be coincident with my left hand.

Finally, note that the puzzle was resolved in purely relational terms, since the idea of moving a body into (approximate) coincidence with another can be cashed out just in terms of continuous changes in their relations.

4 Is the Fitting Theory Sufficient?

I said that a substantivalist could adopt the relational account of handedness as capturing our practical employment of the concept of handedness, but that is not to say that substantivalist could not also argue for some 'deeper truth' about handedness which would tell against relationism. Indeed, Kant himself held such a view. '... yet there may remain an inner difference between the two, this difference consisting in the fact, namely, that the surface that encloses the one cannot possibly enclose the other.... the difference must be one which rests upon an inner ground.... [which] cannot, however, depend on the difference of the manner in which the parts of the body are combined ....' (1768, 371) I take it that Kant is assuming here is that it is not satisfactory to take the incongruence of enantiomorphic counterparts as a basic fact, but as deriving from some 'inner ground'. This ground of course cannot be traced to the internal relations of the counterparts, since they are the same. That the ground is metaphysically primitive with respect to incongruence is brought out by a thought experiment: 'imagine that the first created thing was a human hand. [It] would have to be either a right hand or a left hand.' (1768, 371)

Even in a world consisting of a single hand, in which it is not incongruent with a counterpart because it has none, Kant believes a hand must be either left or right, must have a particular handedness, determined by the inner ground.

Of course in response the proponent of the fitting theory (as a complete account of handedness) will reply that in such a universe there is nothing for the hand to fit to, and so the question of its rightness and leftness does not come up (in terms of the fitting analysis, the hand has the same handedness as itself, and the opposite handedness to nothing – so it's not so much that handedness fails to apply in the world, than that it is utterly trivial). Indeed, the fitting account of handedness is not hospitable to the idea that 'same (or opposite) handedness' can be applied across possible worlds. Consider the world of many similar, same sized hands again, and another like it (with a different number of hands, or differently distributed hands perhaps). In each world we have a partition into two chirality classes by (in)congruence, but can we compare the handedness of hands across the worlds? Bodies in two different worlds don't bear spatial relations to one another, so (in)congruence between bodies, as construed either by substantivalist or relationist doesn't make sense. And neither does any fitting relation, since how objects fit is a matter of their relations. So according to the fitting account there is no fact of the matter about whether similar, equally sized hands in distinct worlds have the same handedness, and so no hands in any other world are either the same or opposite handed from my right hand. And so the single hand, alone in the universe is neither left nor right, with respect to anything in its world or with respect to anything in any other world.

Kant thought this situation absurd, since then 'the hand would fit equally well on either side of the human body; but that is impossible', (1768, 371) and concluded that the fitting account, and hence relationism, was incomplete. Let's think about this argument. Suppose that an idealized human body, one in which the plane that divides the two sides is a perfect plane of symmetry, is introduced into the single hand world. The single hand must have the same handedness as exactly one of its hands, but which? Clearly nothing in the description of the thought experiment is sufficient to determine the answer: we are told the shape and size of the hand and body, but without any information about how the body is to be introduced relative to the hand, it is impossible to tell which attached hand has the same handedness as the lone hand. Kant obviously thinks that the description is incomplete, and that it cannot be completed without stating how the body is to be related to the hand. Namely, a full geometric description of a body requires not only its shape and size, but its handedness, taken as a primitive property, not analysed in terms of mutual relations, and so not a relational property at all. Then there are two possible single hand worlds, one left-handed and one right-handed, and in the first the hand will have the same handedness as the body's left hand and in the second as its right hand, and so it is determinate which side of the body the lone hand would fit.

How should the fitting theorist analyse the situation? Of course he thinks that there is only one kind of world with a lone hand, not two corresponding to opposite chiralities. But what will he say about worlds in which the body has been introduced? Are there two of them, one in which the hand has the same chirality as the left hand and one in which it has the same chirality as the right? Let's think through the story. We start with a single hand, and so a trivial partition into classes of objects with the same handedness. Then we introduce a reflection symmetric body. However this is done – whatever relations the body bears to the hand – we are left in a situation in which the lone hand has the same handedness as one of the body's hands, and the opposite handedness to the other. So now we have a partition into two equivalence classes, one of which contains two hands, and one which contains just one. And we can call either class left or right, but that's just a linguistic convention, not reflecting any facts about the world. And so, if the thought experiment is described just in terms of the system of relations instantiated by the hand, body and hand-body system, there is only one outcome. But if there is only one possible outcome to the experiment there cannot be any of the indeterminateness of which Kant complained. All that is to be said on this reconstruction of the argument is that when the body is introduced the hand will have the same handedness as just one of the hands, and there are no other facts that could further distinguish other possible outcomes: the answer to 'which' hands have the same handedness is just the ones that do! Moreover, this reconstruction reveals just how Kant's argument begs the question. He assumes that that there are two outcomes because he thinks that it is not only a fact that the hands of the body are of opposite chirality, but also that there are facts about which one is the left hand and which one the right. But of course the fitting theorist denies these extra facts, and so does not accept that there can be any fact about whether the lone hand ends up as the same chirality as, say, the left rather than right hand of the body. Kant's assumption begs the question against the fitting account, but without it there is only one possible outcome of the thought experiment, and hence no indeterminateness.

It might seem however, rather strong to conclude that there is only one outcome to Kant's experiment. After all, in a world in which it has been completed, it might seem true to say that 'while the lone hand in fact has the same handedness as *this* hand it could have had the same handedness as *that* one'. Assuming the fitting theory, whether or not you think that this is true will depend on your views of transworld identity. If you think that matters of identity supervene on the relations between bodies – since these are *ex hypothesi* the only physical properties obtaining in the worlds in question – then you will deny the statement. In any other similar possible world 'this' hand will also be (or be represented by) the same handedness hand. If, however, you accept that matters of identity do not supervene on the relations – so that the pronouns rigidly designate – then you may think that indeed the identities of the hands could have been reversed, and the statement true. And so it seems that this kind of fitting theorist will accept the existence of two possible outcomes to Kant's experiment, and so will have to concede that the outcome is underdetermined.

Even if you do think that identities supervene on relations, a similar experiment would have two different outcomes. Suppose that an exact duplicate of my body is to be deposited in the lone hand universe. The result could either be that the hand has same handedness as the hand of mine that wears a wedding ring (my left), or that it has the same handedness as the hand that has the nearly healed graze (my right). Again there are two possible outcomes, this time distinguished by the resulting arrangement of bodies, and again the state of the lone hand does not determine which will occur. And I think in the first case we would natural call the lone hand 'left-handed' after the body is introduced (and in the second 'right-handed'), and so might think that the lone hand was left (or right) all along.

There is one more way to the conclusion that there are two possible outcomes. Suppose that the body is again perfectly mirror symmetric, so that its hands are exact mirror images, instantiating just the same relations. But suppose that they differ in some other property, say mass. Then again there are two possible outcomes, one in which the hand has the same chirality as the heavy hand and one in which has the same chirality as the light one. And again, the fitting theory underdetermines which outcome will result.

But the fitting theorist (if his faith is strong) is unlikely to be moved by these examples. He is likely to respond that the underdeterminism does not follow from any defect in the fitting account of handedness, but from the terms of the thought experiment, which underdescribe the situation. The descriptions of the hand and body not only underdetermine which two hands have the same handedness but also how far apart the hand and body will be, for instance. But we don't feel the need to introduce some new property – 'far apartness' – of the hand to settle the issue. So the dialectic of this kind of argument runs as follows. We end up in one of two worlds distinguished by which hand of the body has the same handedness as the lone hand. How this relation will be instantiated is not determined by the internal relations of the hand and body, so some new monadic properties must be introduced to do the job, namely the inner ground that manifests itself as the left-right distinction. Then if the lone hand is left (right) it will have the same handedness as the left (right) hand of the body.[10] But this argument relies on the implicit premise that the relation 'same handedness' must be determined by monadic properties of the objects in question. And why think that? The relationist doesn't think that distance relations are determined by any monadic properties, so why should he think that 'same chirality' is?[11] But without this assumption the underdetermination is completely harmless. How to put the two bodies together is simply not fully determined by they are constituted individually, but so what?

I can find nothing troubling for the fitting theorist, and hence relationism, in the kinds of argument inspired by Kant. But one should also consider turning the question around and asking what the substantivalist can say about these examples. In fact I was a little misleading in something that I said earlier. I said that Kant assumes that the inner ground of the distinction between left and right is a monadic property, not a relation between bodies. But what he in fact thinks is

[10] And of course once the left/right property is introduced, there are four possible outcomes. For example the lone hand and the heavy hand could be left or right.

[11] Of course Leibniz did think that all relations are determined by monadic properties, and it seems to me that his program for implementing this claim in the case of spatial relations might run into difficulties with this argument.

that the difference between left and right lies in the different relationships that can hold between hands and absolute space. To be left is to stand in one relationship to space, and to be right in another. Now supposing that Kant had successfully demonstrated the inadequacy of the fitting account, he would still only have an argument for absolute space if these relationships solved the problems raised. Various commentators (especially Earman) have suggested that they would not. Indeed Kant fails to give any geometric characterization of the relations at all, and in fact he says that such a relation to absolute space cannot be 'immediately perceived' (p369), only the resulting incongruence.

There are however two relationships to absolute space that the substantivalist might invoke to define handedness. First she might claim that points of space enjoy transworld identities so that regions of space can be rigidly designated. Then pick a hand shaped region of space and there are two possible ways to create a hand, with the same handedness or with the opposite handedness as the region. Sadly this proposal is no help with the indeterminacy argument. If there is any way to distinguish the two hands of the body, because one is left, or wartier or heavier than the other, then there are two ways to place it in the space, one which makes the lone hand of the same chirality as one hand, and one which makes it have the same chirality as the other. (And if there is no distinction then there's again only one outcome.)

Second, substantival space allows for the definition of absolute geometric structures, for example an affine structure. In Euclidean space it is also possible to define an 'orientation' – effectively a hand at every point, so that there are no discontinuous jumps in handedness. If space is oriented then there are again two ways to have a lone hand, either of the same or opposite handedness as the orientation. Does this help solve Kant's underdetermination worries? Only if we take it that if a hand is left, say, it necessarily has the same handedness as the orientation in any space in which it belongs. Otherwise we will once again find two ways to place a body with distinguished hands in a space. The story will then be that if the lone hand has the same handedness as the orientation then it is left handed, and given that the body must be placed to make a specific hand also oriented, it is determinate which side of the body the lone hand will fit. However, given that there is no problem with the fitting account that needs to be solved, and given that it is hard to imagine the ground for such necessitations, this account has little to recommend it.

5 Looking Left and Right

There is one more issue that I'd like to address since it motivated Kant, and that is the question of what it takes to be able to perceive the difference between left and right handed objects.[12] As we saw, he did not think that we could immediately

[12] I also want to address this question because one of the things that David Albert has convinced me by his discussions of quantum and statistical mechanics is that our views about the nature of the physical world have interesting consequences for our views about the place of our mental

perceive the relation to absolute space that makes an enantiomorph a left hand, say. Instead, '... there is only one way in which we can perceive that which, in the form of a body, exclusively involves reference to pure space, and that is by holding one body against other bodies.' (p371-2) That is, when we compare a pair of incongruent counterparts we are aware not of their relations to absolute space, but that they bear different relations to absolute space. To the question 'how can we tell left from right', Kant has the following answer in mind: 'by perceiving the differences in the relationships of enantiomorphs to space'. I've already argued that there are no plausible candidates for such relationships, so I'm unsympathetic to Kant's idea. That said, it is worthwhile pursuing the issue to be sure that the fitting account is compatible with our ability to tell left from right, in case a new anti-relationist argument is lurking around, but ultimately to show that Kant's proposal is not only unworkable but superfluous.

To start with we should break up the problem of perceiving chirality in the same way that we broke up the problem of the geometry of chirality. How do we perceive enantiomorphy? How do we perceive that bodies are incongruent counterparts? How do we perceive that a body has a particular handedness, left rather than right say? The first two questions don't seem problematic at all (though Kant's words make it look as if he is troubled by the second). Let's take it that we can perceive the distances between objects (fallibly of course), then it seems no great mental feat to imagine a body embedded in Euclidean space and inspect it for planes of symmetry (which is not to say we might not get it wrong). And similarly it seems perfectly possible to imagine a pair of counterparts embedded in Euclidean space with an observed set of mutual relations and all the possible relative motions between them, and to ask oneself whether any of these motions bring them into coincidence. Just look at a pair of hands and think about moving them around, and you've done it. I don't, however, want to imply that perceptions of enantiomorphy and congruence require a conscious act of imagination. Given the relations between a set of bodies it is a perfectly calculable matter whether they have a plane of symmetry in Euclidean space, or whether two parts are similar but unsuperposable. Such a determination is quite within the powers of the mind, without any additional perceptual input regarding relations to absolute space.

And, since the fitting account works by extending a notion of 'congruence' to pairs of bodies that are not similar or not the same size in terms of the relations of the bodies and operations on them in Euclidean space, it seems similarly that there is no problem in our mental ability to partition any enantiomorphs into two classes of same handedness. Given a definition of same handedness, it is perfectly within the powers of the mind to see whether it applies between two bodies to which it is applicable. However there is one further issue. When we were considering chirality purely geometrically, there was no issue about which of the same handedness classes we called left and which right; on the fitting view these words denote no geometric property, they're just arbitrary names for the two classes. But when we think of our perceptions of left and right we cannot stop there. When I perceive my left

lives within it.

hand I perceive its same handedness as my left glove and foot, and its opposite handedness to my right hand and normal screws, but I also perceive that it is my *left* hand, which seems to be something additional. And so the question is what extra is needed in order for the mind to perform this judgement, as always *given that my hands are relationally identical*. The simplest answer is to point out that they are not exact mirror images at all, and so in fact are distinguishable by their internal relations after all. I can tell which hand is my left because I have learnt that it is the one with the ring, for instance. I don't find this answer ultimately satisfying, in part because I can still tell which hand is left when I take off my ring, and I think I still could if any partiuclar relational distinction in my hands were removed, but mostly because I think I could still tell my left from my right if the surface of my body were perfectly mirror symmetric. That is, if I could see no relational difference at all between my hands I would know which is my left. So the question to ask is, given the fitting view of handedness, what does it take to posses this ability?

Given certain plausible assumptions, it takes a brain state that is not mirror symmetric. Let me explain why. Consider then the following thought experiment involving someone whose body is perfectly mirror symmetric. In this experiment we shine a light on one of his hands and ask him to think to himself whether it is the left or right hand. Suppose that the initial brain state of the subject is symmetric about the plane dividing the body, so that the spatial arrangement of the parts is symmetric and so that any non-spatial physical properties of those parts are distributed symmetrically. First we shine the light on the left hand (we certainly know left from right, and we imagine ourselves in the laboratory shining the light, so *we* have no problem telling which is the subject's left). The subject's brain state will evolve in a certain way depending on its initial state and on which hand is illuminated, and so will his mental state, until, let's suppose, he correctly thinks, 'aha, it's left'.

Now repeat the experiment, starting the subject in the same symmetric initial brain state – our question is whether *that* state could correspond to a state of knowledge of which hand is left and which right. Assume that the laws that determine the evolution of the brain in this case are mirror image symmetric and deterministic.[13] Now because the brain is symmetric, the conditions at the start of the second experiment are a perfect mirror image of the conditions at the start of the first. And so our assumption guarantees that the evolution of the brain will be a perfect mirror image of the evolution of the first (since mirror symmetric, deterministic laws preserve mirror symmetry). So suppose further that the mental state of the subject is determined by the physical state of the brain. And then suppose, as the relationist is likely to do, that the (relevant) physical state of the brain is determined by the relative spatial arrangement of its parts and of the non-spatial physical properties of those parts. Then, since the brain states

[13] In Chapter Six we will consider in more detail what this means, and discuss laws that are not mirror symmetric, including some that happen to be true. After that discussion I will reconsider the present argument, but for now note that the processes likely to be involved in the evolution of the subject's brain are mirror symmetric.

in the first and second experiments are mirror images of each other, they always correspond to the same relative arrangement of parts and of properties, and so always correspond to the same mental state. In particular, the brain states that occur when the subject thinks the handedness of the illuminated hand are mirror images, and so correspond to *the same mental state*, 'aha, it's left'. But in the second experiment this is the wrong answer! And so we see that the subject does not really know the handedness of his hands at all. Indeed, his initial mental state seems to correspond, *inter alia*, to the knowledge that 'left' means illuminated, when applied to his hands.

However, if our brain states are asymmetrical, then we can do the job. To see how asymmetry defeats these kinds of considerations we don't need a realistic account of how the brain is configured – any asymmetry will do. So imagine a new subject with a different initial brain state, one in which the right side of her brain contains a discernible R-shaped part and the left a discernible L-shaped part. Now the initial configuration when her left hand is illuminated is not the mirror image of the initial configuration of when her right hand is illuminated – the L and the R have different spatial relations to the illuminated hand in the two cases. And so the brain states will not be mirror images, and so they will not (necessarily) correspond to the same mental states. And so there seems no reason to think that the relationist cannot account for all the facts concerning our ability to know which hand is left and which is right: the brain state described will – in principle – do the trick, and so once again, whatever Kant thought, there is no task here that only absolute space can do.

One last remark on this topic however. Consider the mirror image – of all parts internal and external – of our subject in the asymmetric brain state. Now the L-shaped region is on the right and the R-shaped region on the left (and of course are reflected L- and R-shaped). If we now illuminate her right hand, her initial condition will be the exact mirror image of the initial condition of the original asymmetric subject when her *left* hand was illuminated. And so by the reasoning we used earlier, our new subject will end up thinking incorrectly 'aha, it's left'. And similarly if we illuminate her left hand she will end up thinking 'aha, it's right'. And so if Earth is ever invaded by our duplicates from a mirror image universe, we will be able to tell our friends from our foes by testing whether they can correctly identify their left hands.

To conclude the discussion of the last two sections, I think it has been conclusively shown that none of the specific problems concerning the geometry or perception of chirality in Euclidean space are insurmountable or even awkward for the relationist. Indeed, even if one is a substantivalist, I would recommend relationism about these aspects of handedness as the most sensible approach. That said, in the next section and following chapter we will consider a new set of problems that arise when the restriction to Euclidean space is relaxed, and then in the following chapter the difficulties that arise when chirality plays a role in the laws of nature. But even though I think that Kantian arguments completely fail against the relationist in Euclidean space, I do think that there is a core concern that Kant raised, which we will have to resolve in the next chapter. And that is the question of the 'ultimate ground' of same and opposite handedness. The relationist account rests on a definition of congruence that depends on a notion of 'geometrically possible motions', or embeddability of a relative motion in a space with a suitable geometry. If that space is Euclidean, the notion slips by almost unremarked. But I want to open up the range of possible spaces so that the relationist has an account for any geometry that the world might turn out to have. And so it seems (as of course occurs even in the Euclidean case) that the relationist is simply helping himself to a notion of geometric possibility – embeddability in a given space – without any thought about whether such a notion is justified to him. Of course it is justified to a substantivalist because only those spatial arrangements that are embeddable in actual physical space are physically possible, but what is the ground of geometric possibility if substantival space is denied? Of course this problem is especially acute if – as many are – the relationist is of an empiricist bent, with a traditional empiricists skepticism of modality. In the next chapter I will propose what I think is a novel solution to this problem.

First however, a more recent problem concerning chirality that will lead us eventually to face the issue of geometric possibility.

5.1 Chirality in Non-Euclidean Spaces

It is finally time to relax our assumption – in this chapter and those preceeding – that space is Euclidean. Here and in the next chapter we will consider the implications of the more general framework for geometry and chirality in particular, and in later chapters for arguments concerning dynamics. I think, however, that it's fair to say that the question of how relationism in the tradition of Leibniz is to be implemented in non-Euclidean spaces has been neglected, and taken to offer few substantially new lessons. One important exception involves handedness, but I will show in the next section that there is rather more to be said.

Consider the simplest case first, that of a space with constant curvature. In such a space the metric is a constant and so if a configuration is embeddable at some point and orientation in the space, it is embeddable at any point and at any orientation in the space. Such a space is not flat, but it is isotropic and homogeneous – for example a spherical space (or indeed a space that, like Euclidean space, is flat). In such a space the geometry of handedness is a straight forward extension of that in Euclidean space: one generalizes the notion of a plane and normal lines to generalize the notion of a mirror image and enantiomorph. Congruence is also a well-defined notion since in such a space a body can be moved rigidly anywhere – since a configuration is embeddable anywhere in the space it can be moved continuously without any changes in its internal distance relations. And – given a crucial assumption that we will introduce below – an equivalent chirality theorem holds: if (and only if) a body is an enantiomorph then its counterparts are incongruent.

So the substantivalist has an account of handedness, exactly parallel to that in Euclidean space, but what is the relationist's account of being handed, being incongruent counterparts and being left (*versus* right) handed? Previously we relied on embedding configurations in an abstract Euclidean space in order to define enantiomorphs and to define 'geometric possibility' – specifically a motion was geometrically possible if every instantaneous configuration that comprises the motion is embeddable in Euclidean space – to explicate (in)congruence. Obviously, if space is not Euclidean then this account won't do. But it's equally obvious how to amend the account. Suppose space is non-Euclidean but of constant curvature, and suppose S is an abstract metric space that is isometric to it. Then a body in the space is an enantiomorph just in case, when embedded in S it has a plane of symmetry (in the extended sense). And a motion is geometrically possible just in case it is continuously embeddable in S. And two bodies are incongruent counterparts just in case the configuration containing them is embeddable in S so that no series of rigid motions of one (in S) will bring them into coincidence, but a reflection and a series of continuous rigid motions will. Then, supposing that the chirality theorem holds in S, it will again follow that the relationist will conclude that enantiomorphs have incongruent counterparts – since rigid motions are geometrically possible iff they are possible in S. And finally all the rest of the relationist account of handedness will proceed exactly as before.

If space does not have constant curvature then there is no natural way to extend the geometry of chirality. Of course our world is like this and we certainly experience handed objects, but that is no problem. In regions in which curvature is approximately constant, objects smaller than the region may be approximate enantiomorphs and if they are their approximate counterparts will not even be approximately congruent. Such is our experience of chiral objects. In such regions the relationist will develop his account in terms of geometric possibility as embeddability in an abstract space isometric to the physical space supposed by the substantivalist. And even in regions where the approximation is bad – if for example we compare two bodies in isometric regions connected by a region with very different geometry – one still may be able to give an intuitively correct notion of 'same handedness' in terms of some set of non-rigid motions that don't amount to a reflection – for example motions which stretch the body and then return it to its original size. So even in such regions the substantivalist may offer an intuitively correct account of handedness. And the relationist will again piggy back his account on the substantivalist's, using a notion of embeddability in an abstract space isometric to the substantival space in order to give a notion of geometric possibility.

But maybe things aren't so simple for the relationist. I see two worries that one should have. First, when we look at non-Euclidean spaces it becomes clear that the relationist is putting a lot into the notion of geometric possibility: where the substantivalist sees the possible arrangements of matter governed by the metric of the physical space that contains matter, the substantivalist takes the possibilities as primitive. I think it is more than fair to ask at this point about the metaphysics of this alleged possibility – are we really going to end up in a better situation than the substantivalist? And note that one doesn't escape this issue in Euclidean space, since even then the relationist needs to assume the notion of geometric possibility; in that case familiarity with Euclidean geometry makes it is easy to overlook the assumption, though it should be viewed as equally problematic. Second, one should start to suspect from these examples that geometric possibility as embeddability in an abstract space is not enough. For the substantivalist's account of chirality depends not only on the geometry of space, but *where* in space the bodies are located. Now, a given arrangement may only be embeddable in some regions and not others, but symmetries may mean that there are a number of places that a system of bodies might be embedded. What guarantee does the relationist have that he will be able to pick out the location of a body in space sufficiently well to mimic the substantivalist's account? I won't pursue this concern any further now because it will reappear in a simple form that is clearly problematic for the relationist in the next chapter, but let it be noted.

I mentioned above that the chirality theorem in non-Euclidean spaces requires a crucial assumption; this assumption is that space is 'orientable'. The idea is intuitive enough. Some spaces have a 'twist' in them, that make certain continuous rigid motions – those around the twist – equivalent to reflections. The simplest example is that of the Möbius loop, a space that is like a cylinder – so flat – but with the edges twisted before they are reconnected. Take an enantiomorph, say the letter F, and its mirror image in a near-by line, the symbol backF. A continuous, rigid motion of backF across the twist brings it into coincidence with F. So in this space – as in any non-orientable space of constant curvature – enantiomorphic counterparts are not incongruent, and our chirality theorem fails.

In general, an n-dimensional differential manifold is orientable if it admits a nowhere vanishing, continuous 'n-ad field', which one can think of as an n-dimensional hand (or other enantiomorph) at every point of space so that there are no discontinuous jumps in handedness from point to point.[14] The connection between orientability and the existence of an n-ad field is clear. Suppose such a field – an 'orientation' – exists, and take an enantiomorph and its mirror image. We can define a notion of 'same handedness' between the bodies and an n-ad, according to which, of course, one of the counterparts – say the original – has the same handedness and the other the opposite. Now move the mirror image continuously and rigidly along any path in space, and it must maintain its opposite handedness to the orientation. In particular, it will have the opposite chirality along any path that brings it to its counterpart, which, since it has the same chirality as the field, means that the two counterparts have opposite chiralities when superposed, which means they are incongruent. And if any two counterparts are incongruent, we can use one and our notion of same handedness to define an orientation by moving the one to define an unambiguously (same) handed n-ad at any point, generating an n-ad field. So if a space is non-orientable, so that no continuous n-ad field exists, we must conclude that there are no incongruent counterparts. (You can see what

[14]More precisely, an n-ad is a collection of n linearly independent vectors – a set of n axes for the space at each point. And in fact continuity is not quite the correct condition, for certain orientable spaces – the n-sphere being a prominent example – do not admit any continuous vector fields – they are 'hairless' – and so don't admit of continuous n-ad fields. Instead, the correct condition is that an n-dimensional space is orientable if it can be divided into overlapping regions that admit nowhere vanishing, continuous n-ad fields, and that *agree on the handedness of the fields in the regions of overlap* in a technical sense (the Jacobian of the transformation between them has positive determinant).

happens in such a space: if you move an enantiomorph from point p to point q, you will find that its handedness at q depends on the path, so an n-ad cannot be unambiguously defined at q, and so there is no orientation.)

And since handedness is dependent on the orientability of space, we have to ask what orientability means to the substantivalist and relationist. The substantivalist, accepting the physical reality of the spatial manifold and its geometry, can simply adopt the n-ad characterization of orientability. The relationist however, assuming matter does not form a plenum, has insufficient material points to form a differential manifold, and cannot simply invoke the standard geometric account. Indeed, Nerlich (1976) has claimed that this problem is unsurmountable for the relationist, a charge to which, to my mind, no one has yet given a completely satisfactory response. Ultimately the answer I will propose is that once again there is an abstract metric space controlling geometric possibility and it is the topology – in particular the orientability or non-orientability – of that space that counts. Of course, if that were all there were to say the substantivalist might reasonably feel that a fast one had been pulled. As I've promised before, we will have to consider carefully the justification for such a move in the next chapter.

References

Brighouse, Carolyn (1999), "Incongruent counterparts and modal relationism", *International Studies in the Philosophy of Science* 13.1: 53-68.

Van Cleve, James and Robert E. Frederick (eds.) (1991), *The Philosophy of Left and Right*. Dordrecht: Kluwer Academic Publishers.

Kant, Immanuel (1768), *Concerning the First Grounds of the Distinction of Regions in Space*, translated by J. Handyside. Originally published as "Von dem ersten Grunde des Unterschiedes der Gegenden im Raume", Königsberger Fragund Anzeigungsnachrichten 6-8 (Königsberg). In van Cleve and Frederick (1991): 27–38.

Nerlich, Graham (1976), *The Shape of Space*. Cambridge, UK: Cambridge University Press. Chapter 2 reprinted in Van Cleve and Frederick (1991): 151–172.