Phenomenology and independence II

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Introduction. In the previous paper[1] it was suggested that *Gödel's phenomenological program* has been usually replaced by its merely mathematical component in the recent years. Here, we intend to clarify a possible path for this program by means of the analysis of the *context principle.* In few words, Gödel seems to provide some kind of modal flavour for certain statements of set theory, namely, those that are 'forced upon us', say the axioms sanctioned by the so-called intrinsic criteria. In general, we claim that *modalizing* notions (not exclusively propositions) leads to conceptions such as those of intuitionists or phenomenologists. Therefore, the connection of Gödel's phenomenological program with these ideas seems natural, to say the least. Nevertheless, Gödel can be read as defending *modal collapse* to some degree. Therefore, the interesting appeal of modalizing seems lost. It is here where we defend that the context principle, read psychologically (or, to be more precise, in an intentional manner) seems to be of some help: the possibility of natural extensions of ZFC –as a psychological fact– is granted by how we work with the formal system and how certain notions appear in our framework.

1. What is modalizing? In the previous article, the intentional analysis of the primitive notions of set theory was vindicated as essential to Gödel purposes of naturally extending ZFC. Our purposes there were completely *negative*, that is, we tried to show how Gödel's phenomenological program turns out to be completely trivial and useless if certain *ad hoc* assumptions are granted. Here we will try to provide a *positive* companion of this initial approach.

We wish to defend here that *modal logic is the logic of intentionality.* This statement is not *that* crazy: after all, the logics of the intentional acts of believing, knowing, dreaming, etc. take usually the form of modal propositional calculi. Of course, not all intentional acts are of equal interest here; we usually concede more relevance to *intellectual* acts. A notorious example of *modalizing*, well-known to Gödel, is that given by intuitionism. Roughly, the intentional act here is 'being able to construct in intuition', whatever that means exactly[2]. Of course, if one collapses the modalities of intuitionistic logic, one obtains classical logic: indeed, classical mathematics does not distinguish between constructive and non-constructive truths or, equivalently, it does not recognize more modalities apart from mere truth.

Nevertheless, we must stress that we will be using the term 'modalization' in a loose way: we do not want to restrict ourselves to the transformation of a formal logical calculus into a modal one by extending the language and adding new rules[3]. Instead, we wish to include the corresponding conceptual modifications that arise from such introduction of modalities in the broad sense, the paradigmatic example being the intuitionistic treatment of species and spreads as a direct consequence of the intuitive modalization usually defended by its proponents[4].

2. Gödel modalizes... The second point that we want to make here is that Gödel provides a modalization corresponding to the intentional act of 'accessing through intuition', although this faculty of intuition differs from the intuitionistic one in power of representation. One could, in fact, read Gödel as extending the intuitionistic notion of intuition and thus

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[1] See [15].

[2] See [11].

[3] This means that, in particular, we are not interested in structural or formal aspects such as the translation between S4 and IPC. Also, we are not thinking in a modalization like that presented by [9].

[4] Again, see [11].

as extending the corresponding theory of truth. This happens, for example, when dealing with the possibility of axioms that 'force upon ourselves' certain properties of sets. To put it differently: (late) Gödel's platonistic realism, closely tied with phenomenology, intends to provide a *positive* account of the accessibility relation, and this is done by surpassing the negative restrictions of intuitionism. Such account is directly connected with a clarification of the notion of 'naturalness' that we have treated in the preceding paper[5].

3. ...right? An immediate objection to our proposal is the following: is it not extending the intuitionistic notion of intuition the *same* thing as collapsing the associated modalities?[6] Well, this is the case if we take the Gödelian faculty of intuition to be *exhaustive*, that is, to exhaust the set of all truths[7]. But it is *this* assumption the one encoding the modal collapse, *not* the extension of the faculty of intuition that we are suggesting here. In other words, what it is interesting in this positive account is precisely what we understand by Gödel's phenomenological program, namely, a *nontrivial* intentional analysis of the accessibility to the abstract realm. The nontriviality requirement implies that Gödelian intuition is not, without further explanation, exhaustive. Gödel's phenomenological program is then a clarification of this possibility of extending the power of intuition (in a more restricted sense).

4. Gödel collapses. Now, *in fact*, Gödel can be read as embracing modal collapse or, at least, nowhere he seems to defend modalities in the relevant sense[8]. The axioms 'forced upon us' are simply true, the presented modalization only applies provisionally for the candidates to natural axioms which, later, simply bear a truth-value, regardless of any intuitive content[9]. Therefore, we have required two steps in order to arrive to what Gödel dismissed in a single one. Why are we then interested in this seemingly redundant exercise of thought?

5. A justification. What we wish to argue is that intentionality –that is, a modalization based on a mathematical notion of intuition– should not be abandoned as uninteresting for Gödel's perspective (again: equating this faculty with the mere acquainting of truth is taking it to be exhaustive in our sense and, in virtue of this, Gödel's phenomenological program would lack of any interest whatsoever). In the previous article, the task of providing intentional analyses was left open or at least very poorly sketched. Here, we wish to show that the context principle may be of some aid, rephrased as a positive prescription of conscious experience. Additionally, this may help elucidating the 'psychological fact' defended by Gödel of the intrinsic possibility of non-arbitrarily extending ZFC with new and evident axioms[10]. As a matter of fact, we believe that what Gödel had in mind was the more specific term 'intentional'[11].

Modal collapse undermines the relevance of Gödel's phenomenological program: an intriguing extension of intuition (itself extending certain intellectual faculty *from the inside*) is taken down by the suppression of modality. This is why, instead of following this path, it seems reasonable to study how Gödel extends the idea of intuition by making clear the way in which we mentally access (alternatively: we have a conscious experience of) certain notions. This is where the context principle comes in handy.

6. A thought experiment. As a digression, consider the following thought experiment. Suppose a teacher wishes to illustrate a general definition by mean of examples. Of course, the order of presentation *affects* how the student may grasp the definition, as well as the temporal separation between the elements (in particular, it should be clear *what* examples exemplify, since otherwise they would stand as mere isolated statements). Moreover, the following considerations may

[5] Again, for more details, see [15].

[6] The problem of modal collapse belongs to contemporary philosophy of religion. We deliberately follow this terminology here.

[7] This interpretation is the generally accepted one. See [14], for example. Proponents of this position follow Gödel in defending that abstract intuition works in a similar way to the physical one, but leave the precise details of such analogy without further explanation.

[8] I am aware that this line may seem completely *ad hoc*. Here, I can only wait for the response of an expert better qualified in these topics.

[9] See [5].

[10] See [7].

[11] See [10] for details on Gödel's terminology.

seem problematic form certain points of view regarding how we access abstract objects. Suppose that what we grasp is the definition. Then, the examples do not illustrate anything, since they are merely instances of the already learned general case. But if we learn from examples, then it is not clear that what we have grasped is the content of the definition (the limit case: grasp the rule of a sequence by checking some initial segment)[12]. In other words: *both* the definition and the examples play a role in our understanding of the intended content: their joint co-influence provides a general *context* for the learner (such role is, in fact, what we have called *grammatical* elsewhere[13]).

7. The context principle. Frege's context principle, as a methodological one, tells us that one should not try to analyze a specific term alone, but always try to do it in the context of the complete statement in which it appears[14]. The classic example is that of natural numbers (not: 'what is 2?' but: 'where does 2 appear with sense?'). The context principle can be also read as a psychological regulation of our acquisition of meaning[15]. According to some, this principle is a landmark of mathematical constructivism[16].

8. The *positive* context principle. But this is not what we tried to illustrate in the preceding example above. Following our previous considerations, it seems natural to take this statement in a *positive* manner, namely, that we are able to grasp a new concept through the work in a determined context, that new concepts may arise with representational force from a context in which we work with others. In the previous thought experiment, the student first starts acquiring some *flavour*, in a completely unspecified manner and, later, the definition comes along as something 'natural'; the corresponding context influences and enables the way in which such definition is to be learned. Now, this is a good candidate in order to expand a broad and flexible notion of Gödelian intuition, rather than be assured of its infallibility and absoluteness[17].

9. The 'psychological fact'. Note that this 'obtaining a new representation' through an initial collection of representations, or this 'transition from one state of consciousness to another' fits quite smoothly with Gödelian accounts on the purpose of the phenomenological method regarding the search of new axioms[18]. In fact, Gödel concedes that this psychological fact is mediated by the work with the formal system itself, it is not immediate and pure, as if it were a revelation of the abstract realm:

> [...] there *do* exist unexplored series of axioms which are analytic in the sense that they only explicate the content of the concepts occurring in them, e.g., the axioms of infinity in set theory, which assert the existence of sets of greater and greater cardinality or of higher and higher transfinite types and which only explicate the content of the general concept of set. These principles show that ever more (and ever more complicated) axioms appear during the development of mathematics. For, *in order only to understand the axioms of infinity, one must first have developed set theory to a considerable extent* [italics are mine]. [8][19]

Here, the application of this positive context principle would look as follows: a new consciousness of the intrinsic value of a new axiom would emerge by means of merely *a posteriori* work within the formal framework. In particular, of course, it is possible that intrinsic naturalness may be obtained through the extrinsic features that the axiom may bear. Nevertheless, one should always provide the corresponding intentional analysis, which is the task originally demanded by Gödel's phenomenological program.

10. Reductionist temptations. We have presented one possible tool in order to provide a satisfactory intentional analysis of the 'naturalness' that some axioms may bear. This is not the same thing as believing that *all that can be*

[12] Also, Kripkenstein's skeptical argument: see [13].
[13] In [15].
[14] See [1].
[15] See [16].
[16] Again, see [1].
[17] Similar concerns may be found in [12].
[18] See [6]. There, Gödel also puts the example of the child's intellectual development.
[19] Note that Gödel refers, specifically, to axioms of infinity, but this is not essential for our purposes.

done must take this form. Indeed, we have seen the applicability of this schema of thought to the most general version of Gödel's program. But all along we have assumed the required specifications that enable such explanation and that will probably take more detailed and concrete forms. This, as we tried to argue before, is a purely phenomenological task, not mathematical: mathematical work is needed, that is granted, but it is not decisive for the deep conceptual clarification of the terms that are ubiquitous to it. Understanding the open task delineated by Gödel consists precisely in paying attention to delicate points as these.

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