The Traversal of the Infinite: Considering a Beginning for an Infinite Past

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Abstract

This paper offers a critical assessment of the Successive Addition Argument (SAA) in support of past finitism, i.e., the thesis that the past of the universe is finite in duration. This old philosophical argument, re-popularized by William Lane Craig in modern times, contends that the universe's past cannot be infinite because an infinite series cannot be formed by successive addition. I first address a recently popular objection to the argument, namely the Zeno Objection, showing that it can be easily dismissed once each addition is taken to have the same duration. Nevertheless, I contend that the onus of the proof lies on those who propose the SAA, and that their main argumentative strategies fail. Indeed, many of their arguments are based on the supposedly uncontroversial claim that one cannot traverse the infinite by starting somewhere. I argue that a complete traversal of the infinite, with a beginning infinitely far from its end, is logically and metaphysically possible. Other popular arguments against traversed infinities are based on thought experiments such as the backward counter or the Tristram Shandy thought experiments. I argue that, once infinitely far beginnings are granted, none of the arguments based on such thought experiments prove effective, so that the SAA must be rejected.

1. Introduction

According to the standard Λ CDM model of cosmology, our universe has a finite age $(13.799 \pm 0.021 \times 10^9 \text{ years})$. This appears to support *past finitism*, the thesis that the past of the universe is finite in duration.¹ Nevertheless, cosmologists have developed alternative models, with some denying past finitism.² As a consequence, the question of whether past finitism is correct remains unsettled to this day, which encourages some philosophers to revisit an old approach to the matter, i.e., determining the truth of past finitism as a matter of necessity, independently of empirical investigation.³

In this paper, I offer a critical assessment of an old philosophical argument in support of the necessity of past finitism which has gained renewed attention in recent years. I refer to it as the *Successive Addition Argument* (SAA). As Stephen Puryear noted, the contemporary debate on the argument has somewhat stalled in the reiteration of incompatible intuitions about infinity (Puryear 2014, p. 619). In this paper, I aim to contribute to overcoming this impasse by arguing that a crucial point has gone unnoticed by both the proponents and the opponents of the SAA: a complete traversal of the infinite, with a beginning infinitely far from its end, is not a contradictory concept. Once this point is taken, finitists end up in serious trouble. Indeed, many of their arguments are based on the supposedly uncontroversial claim that one cannot traverse the infinite by starting somewhere. If these arguments become unavailable to finitists, their key recourse remains in certain arguments based on thought experiments such as the 'backward counter' or the 'Tristram Shandy' thought experiments. However, I show that, once traversals with infinitely far beginnings are granted, none of these arguments are effective, and, missing a better one, the SAA must be rejected.

¹ This terminology allows this position to be distinguished, if necessary, from *temporal finitism*, the thesis that time is finite in the past.

² One finds cosmological models that could be interpreted as denying past finitism either among those based on our best empirically confirmed theories, i.e. *general relativity* and the *standard model of particle physics*, or among models based on quantum gravity. In the first family, a prominent example is *Conformal Cyclic Cosmology* by Roger Penrose (Penrose 2010; 2014). In the second family, some types of *bouncing cosmologies* from *string theory* or *loop quantum gravity* are suited to being associated with a past eternal universe. For an overview, see Brandenberger & Peter (2017).

³ This happens mainly, but not exclusively, within the context of theological debates. See note 5 for examples.

The SAA tradition traces back at least to the sixth-century Aristotelian commentator John Philoponus (*Contra Aristotelem*, Fr. 132). His proof for the temporal finitude of the universe became popular among the Islamic thinkers during the *Kalām* age (Craig 1979, p. 10), while in the thirteenth century Bonaventure employed it against Aquinas (*Sent II*, 1.1.1.2.). Notable later proponents include Ralph Cudworth and Richard Bentley, although the most famous instance can be found in the thesis of Kant's first antinomy (*CpR*, A429/B457).⁴ Nowadays, this argument has regained attention especially because of the work of William Lane Craig, who contends that the universe must be temporally finite in order to establish that it has a divine cause.⁵

One can vividly express the idea behind the argument with a question: "If the universe's past is infinite, how did we get to the present moment?". An infinite series of events finishing now seems to entail that an infinite series has just been "traversed", which, according to the proponents of the SAA, is an impossibility (Craig 2013, p. 12). Therefore, past finitism. More precisely, the argument can be presented as follows:

Successive Addition Argument (SAA):

- (1) If the past of the universe is infinite in duration, then the temporal series of past events is actually infinite.
- (2) An actually infinite series cannot be formed by successive addition.
- (3) The temporal series of past events is formed by successive addition.

Therefore (from (2) and (3)):

(4) The temporal series of past events cannot be actually infinite.

Therefore (from (1) and (4)):

(5) The past of the universe is finite in duration.

⁴ Kant understood the argument as only *apparently compelling* (Falkenburg 2013, p. 64) because it overlooks the possibility that space and time exist only dependently of the human mind, as it is the case from a transcendental idealist perspective (Stang 2023, p. 1).

⁵ See in particular Craig (1979, pp. 102–110; 1995, pp. 4–35; 2009). For recent works that build upon Craig's defense of the argument see Oderberg (2017), Erasmus (2018), and Loke (2014; 2017: 2022). Of course, there have also been many contemporary detractors. Among others, one finds Russell (1929), Bennett (1974), Popper (1978), Bell (1979), Mackie (1982), Sorabji (1984), Smith (1995), Sinnott-Armstrong (2004), Oppy (2002; 2006), Puryear (2014; 2016), Zarepour (2021; 2022), Morriston (1999; 2002; 2017; 2021), and Malpass (2022; 2023).

The SAA can be resisted on several grounds. To begin with, one could argue, in a Kantian fashion, that the argument is simply invalid insofar as (5) does not strictly follow. Indeed, (1) and (4) may allow one to infer that *it is not the case* that the universe's past is infinite, but this does not straightforwardly imply that the universe's past is finite. For instance, its duration may be *metaphysically indeterminate*.⁶ If they wish to rule out this option, proponents of the SAA should grant that denying the infinitude of the past is equivalent to affirming its finitude. Moreover, an additional commitment comes with premise (3), as it is widely acknowledged that this premise binds SAA's proponents to some dynamic view of time in which events are actualized one after another (Craig 1991a, p. 15; 2013, p. 13; Morriston 2021, p. 2).⁷ Even premise (1), apparently uncontroversial, has been recently met with reservations concerning the possibility of the past being a unique infinitely long event (Puryear 2014, p. 628; 2016, p. 1).

However, contemporary criticism has focused on the most dubious of the argument's premises: (2). Why should it be impossible for an actually infinite series to be formed by successive addition, that is, by adding its members one by one? Or, to put it somewhat more evocatively, why should it be impossible to traverse the infinite? This question must be answered by the finitists who propose the argument.⁸ As a result, the debate has centered on whether the answers provided by finitists are good. If the finitist cannot provide good reasons to believe that an actually infinite series cannot be formed by successive addition, one must reject the SAA.

In this paper I argue that this is precisely the situation. In the next section, I first clarify some terms within the SAA. In Section 3, I address a recently popular objection to the SAA related to Zeno's paradoxes to prevent potential misunderstandings. In Section 4, I substantiate my main point about infinitely far beginnings and show how it directly undermines many of the finitists' arguments. Additionally, I show its relevance in refusing an argument by Craig based on the backward-counter thought experiment. In Section 5, I illustrate that infinitely far beginnings can also provide a new solution to the

⁶ For a theory of metaphysical indeterminacy see Barnes & Williams (2011).

⁷ Loke (2017; 2022, pp. 212–214) is an exception.

⁸ Symmetrically, if one proposes an argument for a finite past, one must also argue for the plausibility of the argument's premises.

notorious Tristram Shandy Paradox. Lacking a better argument for premise (2), I conclude that the SAA must be rejected.

2. Preliminary remarks on the SAA

In this section, I clarify the terminology appearing in the SAA. I start from premise (2). What does it mean for something to be actually infinite? In this regard, the contemporary debate has entirely focused on Craig's conception. In line with the Aristotelian tradition, Craig distinguishes between *potential* and *actual* infinities. In Craig's understanding, potentially infinite are those multitudes with an indefinite (although finite) number of members, that "increase perpetually but never attain infinity" (Craig & Sinclair 2009, p. 105). Instead, actually infinite are those multitudes that possess an infinite number of elements (at least \aleph_0), where a multitude is, in general, just any grouping of things, be they mathematical objects (e.g., numbers, sets, and functions) or non-mathematical entities (i.e., entities that are not studied by pure mathematics). I refer to non-mathematical multitudes as 'collections'. By series I mean any ordered collection that can stand in a one-to-one correspondence with a sequence, that is, a *linearly* ordered and *discrete* set. The relevant notion of discreteness is the standard one, but it is important here to explicitly reiterate it: a set is discrete when it is composed of isolated elements, which informally means that no two elements can be arbitrarily close to each other. More formally, a set S is discrete in a larger topological space X if every point $x \in S$ has a neighborhood U such that $S \cap U = \{x\}$.

According to the standard conception, to form a series by successive addition one must add the elements one after another in a given order, so that for each element xthat belongs to the series there is a different element y such that the addition of ximmediately follows the addition of y (with the obvious exception of the first element, if there is one). To say that a series cannot be *formed* by successive addition is equivalent, in my terminology, to saying that the series cannot be *completed* in this manner, in the sense that there can be *no time* when all the elements of the series have already been added in one by one. As Morriston shows, the SAA's proponents would be better off adopting this interpretation of the term "formed", for a looser interpretation may lead to undesired results such as the unwelcomed exclusion of the possibility of an endless future (Morriston 2021, p. 4). Finally, the modality of (2) is taken to be logical and/or metaphysical, in accordance with the status of the debate.

Having clarified the terms of (2), I now address the other premises of the SAA. A *temporal* series is just a collection of temporally ordered elements. In the case of the temporal series of past events, the elements of the series are, indeed, events. It is not necessary here to provide a detailed account of events. It suffices to stick to the (relatively) standard and intuitive conception of events as happenings in time that entail changes. Examples of events are the foundation of Rome and the Moon Landing. Any such event has a finite temporal duration. Moreover, we can take events as always extending from a specific moment (its beginning) up to a different moment (its end). Now, just like the Moon Landing is an event, so it can be considered an event the collection of whatever happened in the universe *simultaneously* with the Moon Landing. Let us call this kind of event *maximally complex* (Smith 1995, p. 78).⁹ The temporal series of past events, then, is a series of distinct and non-overlapping past maximally complex events that includes *all* past events. The temporal series of past events is actually infinite if it contains \aleph_0 events.¹⁰

I define the past of the universe as having an infinite duration if, and only if, the temporal series of past events is infinite in duration. One might object to this definition by observing that the universe, understood as the maximal mereological sum of spatiotemporally related *things*, could have been unchanging for an infinite time before

⁹ To this definition one may object that, according to the Special Theory of Relativity, given the relativity of simultaneity, it is not possible to define a maximal complex of *absolutely* simultaneous events. However, one can recover this characterization thanks to the introduction of a preferred foliation, as it is done in some cosmological models. In any case, in order to deal with purely philosophical arguments for logical or metaphysical modal claims, one does not need to endorse preliminarily any particular physical theory (it may be logically possible while physically impossible for two events to be absolutely simultaneous).

¹⁰ The formulation of the SAA in terms of events is common, but it may lead to some perplexities, as it seems that one may prefer to include direct reference to periods of time, to accommodate those substantivalists who wish to propose the SAA. However, those substantivalists who think that an infinite temporal series cannot be formed by successive addition will agree that this is true both for temporal series of (ontologically independent) periods (given some a dynamic theory of time on which periods themselves "have actualized one after another") *and* for the temporal series of events occurring at those periods. Therefore, the SAA formulated in terms of events is compatible with a substantivalists position.

the occurrence of the first event/change, which happened only a finite time ago. I consider this a genuine metaphysical possibility.¹¹ However, in order to adhere to the common discussion of the SAA, by "universe" I mean here only the *changing* universe in which we live, i.e., the maximal mereological sum of spatiotemporally related *events*.¹² Consider, though, that one could easily reformulate the SAA in terms of finite periods of time rather than events, to also include, as a target, an infinitely unchanging universe before the beginning of change.

In general, a temporal series is infinite in duration or temporal length if the sum of all its element's durations can be expressed as a divergent mathematical series. For instance, suppose that a temporal series composed of \aleph_0 events is such that each of the events has the same temporal length: one temporal unit. To answer the question "What is the temporal duration of the whole series?", one must sum up all the singular values. This can be expressed as ' $\sum_{n=1}^{\infty} 1$ '. This mathematical series is divergent since the sequence of partial sums {1; 2; 3; 4;...} fails to converge to a finite limit. Therefore, the series at stake is infinite in duration. Conversely, I take that a temporal series of events has a finite duration or temporal length if the sum of all the elements' temporal durations is finite. Notice that a temporal series could have \aleph_0 events and still be finitely extended. Consider, for instance, a series with \aleph_0 events and a last event. Suppose that this event has a duration of half a temporal unit, and that each of the other events has half of the temporal duration of the following one. The sum of all the temporal lengths could be expressed as ' $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ '. This mathematical series converges to the finite limit one. Therefore, the series at stake is finite in duration.

Finally, to see why it is appropriate to talk about a traversal of the infinite, consider again a sequence containing \aleph_0 elements with a last element. If one were to count these elements one by one until reaching the last one, then one would have *formed*, in the sense specified, an actually infinite series (of utterances) by successive

¹¹ See Viglione (2022).

¹² For a criticism of the SAA based on the possibility of an infinitely unchanging universe before the first event, see Goetz (1989). Interestingly, Craig agrees with Goetz that the SAA cannot exclude this possibility (Craig 1991b, p. 106). However, he denies it for theological reasons (Craig 2001, p. 236). Oderberg makes instead the unorthodox suggestion that the passage of time itself may entail events, thereby excluding Goetz's scenario by the common formulation of the SAA (Oderberg 2017, p. 219).

addition. Similarly, imagine a path consisting of \aleph_0 tiles, also featuring a last tile. If one were to *traverse* this entire path one tile at a time, until reaching the last tile, then one would have *formed*, in the sense specified, an actually infinite series (of steps) by successive addition. This idea of a traversal can also apply, albeit somewhat figuratively, to the case of the temporal series of past events. Granted a dynamic view of time, if \aleph_0 events have actualized one after another throughout the history of the universe until the most recent past is reached, then again, an infinite series (of events) has formed by successive addition, and one could say that *something* (the nature of which depends on the details of the dynamic account) has just (temporally) traversed the infinite history of the universe. According to premise (2), no such things are possible.

3. Getting over the Zeno objection

Here is a first consideration that could lead someone to believe in premise (2) of the SAA. Consider the case of an infinite path. The SAA's proponent can claim that one could never succeed in traversing it because no matter how many steps they take, only finitely many tiles will have been stepped on. However, it follows from this a straightforward objection, which has been quite common in recent years: similar traversals happen literally all the time. This has been called the Zeno objection, from the notorious Zeno's paradoxes in support of the unreality of motion (Morriston 2002, p. 162; Sinnott-Armstrong 2004, p. 423; Puryear 2014, p. 622; 2016, p. 1). Consider for instance Achilles' paradox. To reach the tortoise, he must, in continuous space, traverse an actual infinity of distances, with each distance shorter than the previous one. If he were to reach the tortoise, he would ultimately form an actually infinite series (of traversals), providing a counterexample to (2). Therefore, on the basis of (2), one should deny, in agreement with Zeno's argument, that Achilles can ever reach the tortoise, along with the general possibility of motion. Since this is unreasonable, premise (2) must be rejected. Likewise, the Zeno objection goes, if time is continuous, then whenever some time elapses, or some event occurs, this results in the formation of an actually infinite temporal series through successive additions. This, once again, serves as a counterexample to premise (2).

The Zeno objection hinges on the assumption that any finite interval of space or time is composed of infinitely many parts. To counter the objection, SAA's proponents can simply reject this assumption. There are two ways in which this can be done. They might either deny the continuity of space and time and conceive every finite spatial or temporal interval as composed of a finite number of smallest parts; or, alternatively, they could maintain that finite intervals are indeed continuous but deny that this entails that they are composed of an actual infinity of parts (Morriston 2002, 162; Puryear 2014, pp. 623–624). The latter view is endorsed by Craig (Craig 1995, pp. 27–30; Craig & Sinclair 2009, pp. 112–113). He characterizes finite intervals of space and time as wholes that logically and ontologically precede any divisions that one can make of them, much like how a geometrical line may be seen as logically prior to any points one may specify on it. According to Craig, given that finite minds possess a limited capacity for division, one can assert that finite intervals of space and time possess only a *potentially* (never actually) infinite number of parts. This implies that events' occurrences or the passage of time itself do not entail a traversal of the infinite.

The adequacy of Craig's answer to the Zeno objection is still being discussed. Puryear (2014, 2016) has argued that, in adopting Craig's view, the SAA's proponents must conceive the infinitely extended past of the universe as one infinitely long event, with any smaller part of it being a mere conceptualization. This would contradict premise (1) of the SAA, according to which an infinitely long past entails an infinite number of events. To this, Loke (2016) and Dumsday (2016), defending the SAA, have answered that intervals of space and time could be continuous *and* naturally divide into finite smallest parts, with any *further* division being only conceptual. In this way, the Zeno objection would lose its grip while premise (1) would be vindicated: no finite interval would be composed of infinitely many sub-intervals, while infinite intervals would instead be composed of infinitely many sub-intervals.

Puryear has attempted a brief response to Loke's and Dumsday's proposal, raising the suspicion of incoherence (2016, pp. 3–5). However, regardless of whether the proposal entails a contradiction, I believe that, at this point, the debate has largely derailed from its intended course, so that I will not delve further into their suggestion. Indeed, even if the SAA's proponents could not save space and time continuity, they would always retain the option of denying it. Therefore, the whole discussion on the mereology of space and time seems to be beside the point. After all, the SAA is concerned with *infinitely long* intervals. All the finitist needs to realize is that (2) is too

strong for their purposes, and hence vulnerable to criticisms that they are not compelled to deal with in this context. Instead of defending (2) as it is, the finitist should propose a slightly revised argument. In the revised version, each successive addition is conceived as having *the same duration*.

Successive Addition Argument revised (SAAr):

- If the past of the universe is infinite in duration, then the temporal series of past events is actually infinite.
- (2)* An actually infinite series cannot be formed by successive additions *of equal (finite) duration.*
- (3)* The temporal series of past events is formed by successive additions *of equal (finite) duration.*

Therefore (from (2) and (3)):

(4) The temporal series of past events cannot be actually infinite.

Therefore (from (1) and (4)):

(5) The past of the universe is finite in duration.

This new argument is not susceptible to the Zeno objection. Consider again the case of Achilles and the tortoise. To make each new traversal, Achilles takes some time. Assume that each new traversal takes the same amount of time. Given that, by setting, each new traversal also covers less space, one can infer that Achilles is increasingly slowing down. However, for the paradox to arise, one must assume that Achilles *does not* slow down, for otherwise it would be perfectly reasonable to think that he may never reach the tortoise, especially if he comes to go slower than the tortoise. In other words, for the paradox to arise, one must assume that each new traversal covers a smaller distance *and* takes less time than the previous one. Therefore, in reaching the tortoise, Achilles does not execute an infinite number of *like* motions, i.e., motions of equal finite duration, which means that Achilles' reaching the tortoise does not provide a counterexample to $(2)^*$.

The same goes, in general, for all those cases where a finite interval of continuous space is traversed over a finite amount of time. Indeed, no such motions can be conceptualized as an infinite series of like motions (i.e., motions that take the same amount of time and *some* amount of time). This grants that no counterexample to $(2)^*$

can be derived from the fact that regular motions occur in continuous space. As for time, it seems straightforwardly incoherent to claim that finite periods are traversed through infinite successive additions of finite equal duration. As a result, premise (2)* is not susceptible to the Zeno objection: finite intervals of space or time, whether discrete or continuous, are not regularly traversed by infinite successive additions of finite equal duration.

Craig and those who follow his lead are generally detractors of infinities as being instantiated in the real world. But even if they disagree that finite intervals of space and time are composed of infinitely many parts, they do not need to establish this in order to infer the SAAr's desired conclusion. Once this point is taken, both finitists and their opponents can get over the problems related to the mereology of space and time, and focus on the problems linked with the idea of an *infinitely extended* past, the possibility of which is the true prize at stake.

Some perplexities may arise with respect to premise (3)* of the SAAr. For instance, SAA' proponents might fear that switching from (3) to the weaker (3)* will weaken their position, as they would be committed to claiming that there is nothing wrong with various scenarios involving infinite series formed by successive additions of different durations. One such example is the scenarios described in the formulations of the so-called Grim Reaper's paradox, which involves an infinite series of task performances of varying durations, executed by countably infinitely many Grim Reapers over a finite interval of time. I have two considerations regarding this perplexity. Firstly, if the SAA's proponent were to reject (3) in favor of (3)*, this would not mean that they must admit that there is nothing wrong with scenarios such as the Grim Reaper's. In fact, discussions on this scenario typically state that it is impossible because *inconsistent*, as it would entail both that some Grim Reaper has made a particular action and that no Grim Reaper has made that action by the end of the series (Koons 2014, p. 4). There are, therefore, other potential wrongs with the scenario that are not directly related to the presence of an infinite series formed by successive addition. Secondly, my claim is not that the SAA's proponents should reject (3) in favor of (3)*, nor that they would not encounter more trouble, in general, if they did. My point is simply that, concerning the desired conclusion of the SAA, all that is needed is

(3)*. Therefore, *in this context*, finitists can limit their defense to the weaker claim, which, as a weaker claim, should be, in principle, easier to defend.

A second problem regarding (3)* is that it seems more natural to think about the history of the universe as a series of events rather than a series of events of equal duration. However, it is only a matter of stipulation. After all, that an infinite number of events of equal duration could not have elapsed before the present moment was precisely the worry that belonged to many of the medieval proponents of the argument. Bonaventure, for instance, held that an infinite number of past celestial revolutions is impossible (Sent II, 1.1.1.2.). Each of the revolutions was, of course, intended to have the same temporal duration, so that it would take each successive revolution the same amount of time to add itself to the past series of revolutions. It seems, therefore, that, to avoid the Zeno objection, finitists could switch from the SAA to the SAAr while continuing to address their primary concerns and maintaining their core claim. However, both the SAA and the SAAr must face far more serious issues than the Zeno objection.

4. Infinitely far beginnings: logical and metaphysical possibilities

With the Zeno objection set aside, finitists will undoubtedly insist on the very consideration that initially raised the objection. They will argue that (2)* is justified by the fact that, regardless of how many additions of equal duration are made, only finitely many additions will have occurred at any given time. This point, consistently emphasized by Craig, can be summarized by the slogan: "One cannot count to infinity" (Craig 1979, pp. 103–104; Craig & Sinclair 2009, p. 117; Craig 2018, p. 310). In response, opponents have promptly argued that, although one certainly *cannot count to infinity by starting with some number*, the same does not hold in the case of a beginningless count (Morriston 1999, p. 8; 2021, p. 2). To use Craig's construction of the scenario (Craig 1992, p. 189), imagine approaching someone who is counting: "minus three, minus two, minus one...zero!". After being asked for clarification, they have been doing for all of the past eternity. Since *prima facie* there is no contradiction entailed in this scenario, opponents of the SAA(r) are apt to remark that, *in the case of a*

beginningless count, one may be able, in principle, to complete the count of infinitely many numbers (Morriston 2021, p. 4; Malpass 2023, p. 45).

From the above, one can deduce that participants in the debate agree on the following claim: by starting with a first addition, one cannot form an actually infinite series through equal successive additions. Let us refer to this as the "First Addition Claim" (FAC). For instance, Craig and Sinclair (2009, p. 117) assert that "the impossibility of the formation of an actual infinite by successive addition seems obvious in the case of beginning at some point and trying to reach infinity", while Loke contends that "it is impossible to traverse an actual infinite number of events [starting from] from event₁". On the other hand, opponents of the SAA such as Morriston (2021, p. 4) maintain that it is a "wholly uncontroversial claim that there can be no time at which an infinite series having a beginning has been formed by successive addition". Now, here is an appealing strategy for the SAAr's proponent: since FAC is supposed to be a "wholly uncontroversial claim", one could attempt to infer (2)* from FAC by arguing for some auxiliary assumption.

One such attempt was made by Whitrow, who held that, when one tries to form a series by successive addition, one *must* start with a first addition. This applies, therefore, even to the case of a series of order type ω^* , the standard order type of the negative integers $\langle \dots; -3; -2; -1 \rangle$.¹³ This auxiliary assumption, therefore, together with FAC, would allow us to infer (2)*: if by beginning with a first addition one can never form an actually infinite series by equal successive additions, and a first addition is mandatory, it follows that one cannot form an actually infinite series by equal successive additions. Another argument for (2)* that is based on FAC has been summarized by Puryear (2014, p. 621). He considers a sequence of order type ω , the standard order type of the natural numbers $\langle 0; 1; 2; 3; \dots \rangle$. By FAC, any series with this order type cannot be formed by equal successive additions. From this, it would be a first addition corresponding to the element "0". *By symmetry*, it is inferred that also a reverse ω -series cannot be formed by equal successive additions. From this, it would

¹³ The reason, according to Whitrow, would be that the only way in which one can *define* the infinite set of negative integers is via a successive addition that begins with -1' (Whitrow 1978, p. 42).

follow (2)*: no actually infinite series *at all* can be formed as such.¹⁴ The auxiliary assumption that is supposed to make to the hard work, here, is *the rule of symmetry*: a series can be completed by successive addition if and only if the reverse-series can be.

More arguments for (2)* (or equivalent claims) that are based on FAC, either explicitly or implicitly, can be found in recent literature (Loke 2014, p. 75; 2022, p. 206; Erasmus 2018, p. 117). Extensive criticism has already been leveled at these arguments (Morriston 2021, pp. 7-8; Malpass 2023). As Morriston correctly observed, the auxiliary assumptions endorsed in the arguments are frequently exposed to the criticism of being far too *ad hoc* (Morriston 2021, p. 5). However, a detailed evaluation of their plausibility does not serve the purpose of this section. My aim, instead, is to *directly criticize FAC*. Since the claim that one cannot form an actually infinite series by starting with a first addition is, as said, commonly accepted, finitists have thought they were on safe ground relying on FAC. I contend, however, that they were already building their arguments on sand. If this stands true, critics need not spend more effort challenging the auxiliary assumptions that finitists have maintained. To get over any argument based on FAC, all that is needed is to get rid of FAC.

The reason why FAC is commonly accepted is that it seems true by *logical necessity*: how could anyone ever finish counting infinitely many numbers by starting with a first number? However, consider a sequence that has two termini and \aleph_0 elements. This is not a common setting, but it can be done. Consider the set of all integers: one way of imposing an order on it is the standard one:

Another non-standard way of ordering them is as follows (Moore 2019, p. 122):

Call this order type ' $\omega + \omega^*$ '. The set of integers in the $\omega + \omega^*$ order is still discrete: all the elements that belong to it are isolated, according to the formal definition given above. The same may not hold, for instance, if we ordered the set as follows:

¹⁴ The inference is ungranted. One needs to assume that infinite series can only be of the order type ω or its reverse, which is false (as I explain below).

¹⁵ Notice that, technically, the integers are not well-ordered in either way (Moore 2019, p. 123).

<...;-3;-2;-1;1;2;...;0>, as in this case there would be at least one element between the element '0' and any other element of the set, so that '0' would not be isolated.¹⁶ Now, suppose that an actually infinite series has the $\omega+\omega^*$ order type. That is, each number of the ordering corresponds to an element of the series. Moreover, suppose that each of the elements has been added by (equal) successive additions, so that the additions in the series occur in that specified order and the overall length of the process is infinitely long. This infinite series, then, would have *a first element*, corresponding to the element '0' of the second ordering introduced above, and yet, it would have been formed by equal successive additions.

Of course, the coherence of the $\omega + \omega^*$ ordering of the integers does not, by itself, ensure that it is consistent to say that a series in one-to-one correspondence is formed by successive addition.¹⁷ However, recall the notion of successive addition: forming a series by successive addition typically means that the elements are added one after another. For each element in the series (except the first one), there is another element that was added immediately before it, with no addition in between. In an infinite series with the $\omega + \omega^*$ ordering, each element (except the first one) has an immediate predecessor. Therefore, it is consistent to suppose that this series is formed in the specified order by successive addition.¹⁸ This contradicts FAC, at least in the logical interpretation of its modality. Moreover, interestingly enough for the case against finitism, this means that one can postulate an infinitely extended temporal series of past events with a first event without falling into logical contradiction.

Even if it is to be accepted that FAC does not express a logical impossibility, there is still one move available for the defenders of this principle. One can claim, indeed, that FAC states a *metaphysical* impossibility (Morriston 1999, pp. 8–10). Since the modality of (2)* is taken to be either logical or metaphysical, this would be enough to defend the premise. However, in order to follow this path, one should bring an

¹⁶ Technically, '0' could be considered an *accumulation point*. However, in mathematics, the topology defined on a set usually does not depend on the order of the elements within the set. This raises the suspicion that one could still treat '0' as isolated, even though no element is immediately before it.

¹⁷ I thank anonymous reviewers for stressing this point.

¹⁸ This is not true for all non-standard orderings of course. For instance, it is inconsistent to suppose that a series in one-to-one correspondence with <...;-3; -2; -1; 1; 2;...; 0> has been formed by successive addition in this standard sense. This is because there is no element that immediately precedes '0'.

argument to the effect that an *essential property* of infinite series is that they cannot be formed by equal successive additions by starting with one of the members. No such argument has been proposed by finitists, and since the *onus* of the proof is on their side, strictly there would be no need to follow in the argumentation against FAC. Nonetheless, the following general considerations can be made.

The main reason why it appears impossible to begin counting all the \aleph_0 integers in their standard ordering now and complete the task later is that \aleph_0 does not belong to the set of integers. Now, it is true that, if I begin counting now, in any finite number of utterances I will have counted a finite number of integers. Similarly, if I start walking now, in any finite number of steps I will have made a finite number of steps. But all this entails is that there is no *determinate* step at which the number of my steps can become \aleph_0 , rather than that I cannot make \aleph_0 steps at all. Traversing the infinite by starting somewhere would require therefore no determinate step at which the total number of steps (or, in general, additions) becomes \aleph_0 .¹⁹ If one cannot find a reason to think that this is metaphysically impossible, then one must concede that traversing the infinite by starting somewhere may be possible. Such a start would simply be infinitely far behind the end of the series.

Another idea that could lead one to think it is impossible for a series formed by successive addition to be in one-to-one correspondence with the $\omega+\omega^*$ order of the integers, is that the structure of *time* may not allow this. In other words, one may think

¹⁹ Notice that even if I never started my walk, there is no determinate step at which my steps reach \aleph_0 in number: at whichever step, regardless of whether it is a finite or an infinite number of steps before the last step, the steps I made were already \aleph_0 . Let us label 'DET \aleph_0 ' a determinate addition at which the number of additions becomes \aleph_0 . Given the possibility of the non-standard order introduced above, one could say that, in both the beginningless and beginning cases, the possibility of forming an infinite series by equal successive additions depends on whether the existence of DET \aleph_0 is necessary for the series to be completed. Interestingly enough, if this claim is correct, then it is possible to justify the sort of rule by symmetry that finitists may be looking for: if it is impossible to complete an infinite count by starting somewhere because DET \aleph_0 is required, then it is impossible to complete it without starting somewhere (because DET \aleph_0 is required). However, given the nature of infinite sets and infinite series, it is more reasonable *not* to require DET \aleph_0 . A rule of symmetry based on DET \aleph_0 , therefore, would not be very useful in arguing for the impossibility of completing a beginningless count. Rather, it can be used to demonstrate, by contraposition, that if an infinite series can be completed without a first addition, then it can be completed with a first addition.

that, to allow a $\omega + \omega^*$ temporal series, one should defend the thesis that time has a nonstandard structure, for otherwise the left-side of the $\omega + \omega^*$ series would necessarily take up all of time. However, the orthodox view in the philosophy of time is that the overall structure of time should be addressed, and eventually settled, through empirical investigation (Le Poidevin 1993, p. 151). Since the target of my criticism is FAC, a modal claim, one does not need to defend the necessity (nor the plausibility) of any view regarding the structure of time. All is needed for my critic to FAC to be effective, is that (for all we currently know) the structure of time *could* be such to allow a $\omega + \omega^*$ series of successive additions.

Given the devaluation of FAC, finitists are left with only one viable option: to argue for the impossibility of completing a beginningless series independently on FAC. In Craig's words, one popular strategy goes as follows:

Suppose we meet a man who claims to have been counting [down to 0] from eternity and is now finishing. [...] The counter should at any point in the past have already finished counting all the numbers, since a one-to-one correspondence exists between the years of the past and the negative numbers.

(Craig 1991a, pp. 15–19)

According to Craig's reasoning, the counter cannot have finished counting all the negative integers in their standard order at any time. This is because, if it were possible to complete the count, then it would have always *already been completed*: whichever past time one considers, no matter how far back into the past, one does not find the counter counting. This, according to Craig, reveals that it is impossible to complete the count at all.

Here, the *sufficient condition* for the counter to have *finished* counting all the negative integers, is that they have already stated infinite utterances. Morriston questions this sufficient condition. He argues that the counter having had infinite time to backward count all the negative integers entails that they *could* have finished counting, not that they would have finished. Indeed, the \aleph_0 periods of time at stake *can* be put in one-to-one correspondence with the \aleph_0 utterances that constitute the count of the negatives in their standard ordering down to '0', as well as with the \aleph_0 utterances that

constitute the count of the negatives in their standard ordering down to '-1'. Given this, there could be a counter who would not be finished until next year or a hundred years from now, as well as a counter who would be finished now (Morriston 1999, p. 13; Morriston 2017, p. 77; Morriston 2021, p. 11).

I agree with Morriston's observations, but I would like to add a further point: given the possibility of infinitely far beginnings argued above, there could also be a counter that is infinitely far back in the count, and yet that is counting *a determinate number* at this very moment. This last point may appear minor in the case of the backward counter thought experiment, but it shows that counters can be "infinitely far back" in different ways, so that it makes perfect sense that not all counters are at the same point *after infinite utterances*. Morriston's already solid response to Craig's argument acquires therefore further plausibility.

The concept of an infinitely far beginning acquires essential relevance in relation to another well-known thought experiment, the so-called Tristram Shandy Paradox, to which finitists have often resorted in their arguments. As I shall show, it turns out that conceiving infinitely far beginnings provides a straightforward and reasonable answer to the arguments for (2)* based on this paradox.

5. A new answer to the Tristram Shandy Argument

The paradox, first proposed by Russell, goes as follows: Tristram Shandy, the famous fictional character, spent two years writing the history of the first two days of his life, which made him worry that, at that rate, he could never finish. Russell suggests that if Tristram Shandy were to pursue his task forever, no part of his biography would remain unwritten (Russell 1937, p. 358).

Here is the relevance of the paradox for the discussion. Consider a scenario where we find Tristram Shandy writing since infinite days. Let us call it the 'reversed Shandy scenario'. Now, the following conditional seems plausible: if an infinite series can be formed by equal successive additions, then (similarly to the backward counter scenario) the reversed Shandy scenario is possible. Notice that, for this scenario to be possible, Shandy needs not to be recording his last day at the end of the last year. Given that he collects 364 more unwritten days every writing year, this is straightforwardly

impossible. What is required is that he must be recording *some* day when we find him writing. But, the question arises, which one?

According to Oderberg (2017, p. 223), when we find Shandy writing, there is just *no possible day* he could be recording. If so, then this would grant a solid argument in favor of (2)*: if Shandy cannot be recording any specific day when we find him writing, then the infinite temporal series of his writing days cannot have formed successively. And since nothing is metaphysically peculiar about the temporal series of his writing days, we should conclude that an infinite series cannot be formed by equal successive additions at all.

To see Oderberg's point, consider the following: two subsets of days compose the series of Shandy's writing days, namely the written and the unwritten days.²⁰ In the paradox as formulated by Russell, at any time of Shandy's life, Shandy has been writing since a finite number of days, and the ratio between written and unwritten days is $\frac{1}{24}$. However, in the reverse Shandy scenario as understood by Oderberg, we must suppose both subsets to have the same number of elements as the total set of writing days: \aleph_0 . Just as all the \aleph_0 odd numbers plus the \aleph_0 even numbers equal the \aleph_0 integers, so the \aleph_0 written days and \aleph_0 unwritten days together equal the \aleph_0 writing days. However, dissimilarly from the case of the odd and even numbers, all of the written days must be before the unwritten days. Suppose that the series of unwritten days is in one-to-one correspondence with the negative integers in their standard ordering <...; -3; -2; -1 >, where '-1' corresponds to the day when we find Shandy writing (day-1). The reason why we may think that no specific day can be written about at day 1 is that all the way up the days associated with the negative integers, we do not find any written day, but only infinitely many unwritten days. And since the days associated with the negative integers are already infinite, it may seem that, so to speak, the written days can be "nowhen", so that the reversed Shandy's scenario is impossible.

However, consider the following non-standard discrete ordering of the integers, where the even negative integers are followed by the natural numbers which are followed by all the odd negative integers:

<...; -6; -4; -2; 1; 2; 3; ...; ...; -5; -3; -1 >.

²⁰ We suppose that Shandy has always been writing in order to simplify matters.

Given the considerations of the previous section, we can coherently suppose that the \aleph_0 unwritten days are in one-to-one correspondence with, say, all the numbers following '-6', so that there is still plenty of room for the written days. There is more: as a matter of fact, we could conclude that on day₋₁ Shandy is writing about the infinitely distant day₋₆, the last day of the series of days that are written about! One year before day₋₁, he was writing about day₋₈; after one year, he will be writing about day₋₄. Of course, nothing prevents us from postulating *a specific* last written day and, as a result, a specific one-to-one correspondence between the unwritten days and a subsequence of the non-standard ordering above. Just as the backward counter may complete the count now, next year, in a hundred years, and so on, Shandy's last written day may be day₋₆, day₋₃₇₂, day_{-9trillions}, and so on. If this is correct, then the argument for (2)* based on the Tristram Shandy thought experiment is not effective, certainly no more effective than any argument based on the thought experiment of someone counting backward the negative numbers.²¹

6. Final remarks on the SAA

The Successive Addition Argument proposed by finitists states that the past of the universe cannot be infinitely extended because this would entail an impossibility, namely, that events have been adding up since eternity, thus completing right now (as well as at any moment finitely far back from the present) the traversal of an infinite series. However, it is often noted, and correctly so, that there is nothing contradictory or metaphysically suspicious about a traversal of the infinite *that never began*. In this paper, I argued that the same holds for the concept of a traversal *with an infinitely far beginning*. This completely undermines most of the argumentative strategies in favor of

²¹ Discussions on different settings of the Tristram Shandy scenario, such as the one (very much discussed) provided by Robin Small (Small 1986), are beside the point. Small argues that, in order to make room for both written and unwritten days, one must imagine that Shandy is *planning* his life in advance instead of recording it. Certainly, the negation of (2)* may entail the possibility of Small's setting. However, it is highly plausible that the negation of (2)* should also entail the possibility of the reversed Shandy scenario in its classical setting (that is, unless one wants to maintain that something is metaphysically peculiar about this setting). Since Small does not provide a solution to the problems with the classical setting, my solution seems therefore preferable.

the SAA's critical premise. In particular, as shown in Section 4, it directly undermines those strategies based on the idea that one cannot traverse the infinite by starting somewhere. Moreover, as shown in Section 5, the concept of an infinitely far beginning suggests a solid solution to the Tristram Shandy Paradox, one of the last resources of the SAA's defenders. The possibility of infinitely far beginnings, therefore, deprives the SAA of the appeal it might have had left, and reveals that, over an infinite period, one could traverse the infinite not only once, but twice, three times, or even an infinity of times.

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