# Reconstructing the Concepts of Physical Quantities and Physical Reality: From Classical Reality to Information-Theoretic ‘Reality’

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**Abstract**

This paper examines the transformation of the concepts of physical quantities and reality within the framework of quantum mechanics, with a particular focus on the philosophical shift from a materialistic universe rooted in classical local realism to an information-theoretic universe based on quantum theory. Classical physics, grounded in local realism, assumed that physical objects possess intrinsic properties independent of observation. However, quantum mechanics challenges this view, suggesting that physical quantities might not be attributes of physical objects, particularly in light of phenomena such as quantum entanglement and the violation of Bell’s inequality. Building on John Wheeler’s “*It from Bit*” hypothesis, this paper argues that ‘*It*’ emerges through informational processes, where the observer, the measuring apparatus, and the observed system interact. This shift in understanding critically examines local realism, rooted in philosophical foundations, in light of the empirical demands of quantum mechanics, leading to an interpretation of reality as *‘reality’*, an emergent phenomenon shaped by measurement and based on information. By reconstructing physical quantities within the framework of algebraic quantum theory, the paper highlights the necessity of moving beyond traditional philosophical debates and embracing a more relational and dynamic view of the universe.

## 1 Introduction

The nature of *physical reality* and the concept of *physical quantities* have been central concerns in the foundations of quantum theory and the philosophy of physics since the advent of quantum mechanics. Classical physics, grounded in *local realism*, assumes that physical objects possess *intrinsic properties* independent of the observer and that causal influences are limited by the speed of light. These concepts—*intrinsic properties* and *causality*—are rooted in the fundamental premises of Western philosophy and have profoundly shaped the development of physical theories. However, quantum mechanics raises fundamental doubts about this classical framework, particularly in light of phenomena such as *quantum entanglement*, the violation of *Bell's inequality (Bell-CHSH inequality)*, and the introduction of *probabilistic elements* into the description of physical systems. In response to these challenges, this paper proposes a shift from the classical concept of reality to an information-theoretic conception of *'reality'*, by examining the notions of *physical quantities* and *reality* in quantum mechanics from both mathematical and information-theoretic perspectives.

To this end, it is necessary to consider the historical context. This paper, therefore, examines both the physical significance of the work of Einstein, Podolsky, and Rosen (EPR), as well as the *Bell-CHSH inequality* and its violation, while addressing their broader philosophical implications. In doing so, it becomes evident that the foundations of local realism are deeply intertwined with the philosophical interpretation of physical quantities.

This paper explores the conceptual reconstruction of *physical quantities* within the framework of *algebraic quantum theory.* The focus is on the *relational nature* of these quantities, which depend on interactions between the observer, the measuring apparatus, and the observed system. This analysis paves the way for an *information-theoretic view of the universe*, aligning with John Wheeler's *'It from Bit' hypothesis.* Wheeler's hypothesis posits that *physical 'reality]* does not exist as a static, intrinsic structure, but rather emerges from *informational processes.*

In light of these challenges to *local realism* and classical assumptions about *physical reality*, this paper proposes a reconceptualization of *physical quantities* and *reality*, aiming to bridge *philosophical realism* with the empirical demands of *quantum mechanics.*

## 2 EPR

The 1935 paper by Einstein, Podolsky, and Rosen (EPR) had a profound impact on the philosophical and scientific understanding of quantum mechanics, particularly concerning the nature of *physical reality* and the completeness of quantum theory[6]. The central question posed by EPR was whether quantum mechanics offers a complete description of physical reality. According to EPR, for a theory to be considered complete, every element of physical reality must be accounted for within that theory. This is often referred to as the *completeness criterion* for a physical theory.

EPR introduced a *sufficient condition* for physical reality: if the value of a physical quantity can be predicted with certainty (i.e., with probability 1) without disturbing the system, then there exists an element of physical reality corresponding to that physical quantity. This sufficient condition aligns with *classical realism*, where physical properties exist independently of measurement.

However, the EPR paper critiques quantum mechanics for failing to meet this condition. In quantum mechanics, certain physical quantities cannot be predicted with certainty before measurement, which suggests that these properties do not exist until they are measured. EPR concluded that while quantum mechanics is successful in predicting experimental outcomes, it does not provide a complete description of reality.

At the heart of the EPR argument is the concept of *local realism*, which asserts that physical quantities possess definite values independent of observation, and that information cannot travel faster than the speed of light. *Locality* means that two spatially separated objects cannot instantaneously influence each other, while *realism* assumes that physical objects have intrinsic properties regardless of observation. The concept of *reality* that adheres to both *causality* and *locality* can be seen as a fundamental principle underlying Western philosophy.

Albert Einstein, advocating *local realism*, believed that quantum mechanics' failure to meet the sufficiency condition indicated that the theory was incomplete[1]. On the other hand, Niels Bohr defended quantum mechanics, arguing that its probabilistic nature did not undermine its completeness but instead reflected a deeper understanding of nature[4]. This incompatibility between *local realism* and quantum mechanics sparked philosophical debates about the nature of *reality*.

This incompatibility was regarded as a philosophical issue until 1964, and it was thought that physics would not resolve it. However, in that year, John Bell formulated the famous *Bell's inequality*, providing an empirical method to compare the predictions of quantum mechanics with the assumptions of local realism[3]. In the 1980s, experiments conducted by Alain Aspect and others strongly demonstrated that quantum mechanics violates *Bell's inequality*, challenging the concept of *local realism*[2]. Bell's work enabled physicists to directly test these assumptions, and as a result, many have concluded that quantum mechanics, with its non-local and probabilistic nature, offers a more accurate description of reality than *classical local realism*.

*Bell's inequality* encapsulates the assumptions of *local realism* in the form of an inequality. Therefore, if it is violated, this implies that *local realism* is not valid.The next section will explore this concept in detail and show how the *Bell's inequality* (*Bell-CHSH inequality*) can be derived to provide a quantitative test for *local realism*.

## 3 Local Realism and Bell's Inequality

Local realism, a concept deeply rooted in classical physics, asserts that physical properties exist independently of observation and that no information or influence can propagate faster than the speed of light. *Bell's inequality* offers a mathematical framework to explore and test this idea within the realm of quantum mechanics. Here, we will derive a specific form of Bell's inequality, known as the *CHSH inequality* (or *Bell-CHSH inequality*), using the

[1]It is undeniable that Einstein supported local realism, but the precise nature of the reality he envisioned remains unclear[7][10][11][12]. In fact, in a letter to Schrödinger dated June 19, 1935, Einstein spoke about the EPR paper as follows: 'For reasons of language this [paper] was written by Podolsky after much discussion. Still, it did not come out as well as I had originally wanted; rather the essential thing was, so to speak, smothered by the formalism [gelehrsamkeit][8].'

algebra of physical quantities[5]. This algebraic formulation is key to later applying tools from algebraic quantum theory, ultimately showing the limits of local realism.

Consider an experimental setup with two observers, Alice and Bob, positioned far apart. Each observer has access to two distinct measuring devices:

$$\text{Alice's devices: } A_1 \text{ and } A_2,$$
$$\text{Bob's devices: } B_1 \text{ and } B_2.$$

Pairs of entangled particles are emitted from a central source, traveling toward Alice and Bob, who are positioned equidistant from the source. The spatial separation ensures that no signal can travel faster than light between them, preserving the condition of *locality*.

During each trial, Alice and Bob randomly select which device to use. The result of any measurement, regardless of the chosen device, is always either $+1$ or $-1$. The key question that arises is what type of correlation exists between the outcomes observed by Alice and Bob.

The correlation between their outcomes is captured by the expected value of the product of their results. If both results are identical (either both $+1$ or both $-1$), the product is $+1$; if the results differ, the product is $-1$.

Let's denote the measurement outcomes as:

$$A_i \text{ for Alice's result when using device } A_i,$$
$$B_j \text{ for Bob's result when using device } B_j.$$

Both $A_i$ and $B_j$ take values in $\{+1, -1\}$. The relevant correlations are expressed through the following expectation values:

$$\langle A_1B_1\rangle, \quad \langle A_1B_2\rangle, \quad \langle A_2B_1\rangle, \quad \langle A_2B_2\rangle. \tag{1}$$

Next, we define the quantity $\langle S\rangle$ as:

$$\langle S\rangle := \langle A_1B_1\rangle + \langle A_1B_2\rangle + \langle A_2B_1\rangle - \langle A_2B_2\rangle. \tag{2}$$

Under the assumption of *local realism*, the outcomes of these measurements are determined by pre-existing local hidden variables, and no influence from one location affects the outcomes at the other.

Now, consider the expression for $S$ before taking expectation values:

$$S = A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2. \tag{3}$$

This can be reorganized and factored as follows:

$$S = A_1(B_1 + B_2) + A_2(B_1 - B_2). \tag{4}$$

Applying the *triangle inequality*, we obtain:

$$|S| = |A_1(B_1 + B_2) + A_2(B_1 - B_2)| \leq |A_1||B_1 + B_2| + |A_2||B_1 - B_2|. \tag{5}$$

Since $|A_1| = |A_2| = 1$, this simplifies to:

$$|S| \leq |B_1 + B_2| + |B_1 - B_2|. \tag{6}$$

Given that $|B_1| = |B_2| = 1$, we know that the maximum value of $|B_1 + B_2| + |B_1 - B_2|$ is 2.
Therefore, we conclude:

$$|S| \leq 2. \tag{7}$$

Finally, taking the expectation value of this inequality yields:

$$|\langle S \rangle| = |\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle| \leq 2. \tag{8}$$

Thus, we arrive at the *Bell-CHSH inequality*:

$$-2 \leq \langle S \rangle \leq 2. \tag{9}$$

If Alice and Bob's measurement results satisfy this inequality, it follows that local realism holds, as the inequality was derived from locally determined physical quantities. However, quantum mechanics predicts that this inequality can be violated under certain conditions. In the next section, we will examine the quantum mechanical framework and demonstrate the violation of the *Bell-CHSH inequality*.

## 4 The violation of Bell-CHSH inequality

The *Bell-CHSH inequality*, which encapsulates the principle of *local realism* within quantum mechanics, is famously violated according to the predictions of *quantum theory*. In other words, *local realism* does not hold in quantum mechanics.

In this section, we will demonstrate the violation of the *CHSH inequality* using methods from *algebraic quantum theory*[13]. We assume that the physical quantities measured by Alice and Bob, denoted by $A_1$, $A_2$, $B_1$, and $B_2$, satisfy certain conditions derived from *quantum theory*.

First, we impose the condition that the squares of the operators equal unity:

$$A_1^2 = A_2^2 = B_1^2 = B_2^2 = 1. \tag{10}$$

This condition reflects the fact that each measurement performed by Alice or Bob yields outcomes of either $+1$ or $-1$. Squaring these operators results in 1, consistent with the binary outcomes typical in *quantum measurements*.

Next, the relations between Alice's and Bob's operators are given by:

$$A_2A_1 = -A_1A_2, \quad B_2B_1 = -B_1B_2. \tag{11}$$

These relations indicate that Alice's and Bob's respective measurement operators do not commute. In *quantum mechanics*, non-commuting operators are central to the *uncertainty principle*, illustrating how Alice's measurement of $A_1$ affects her measurement of $A_2$, and similarly for Bob's measurements.

Finally, we assume that Alice's and Bob's measurement operators commute with each other:

$$A_1B_1 = B_1A_1, \quad A_1B_2 = B_2A_1, \quad A_2B_1 = B_1A_2, \quad A_2B_2 = B_2A_2. \tag{12}$$

These commutation relations ensure that Alice's and Bob's measurements are independent when they are space-like separated, which is crucial for maintaining *locality.* This independence is a critical condition in *Bell-CHSH experiments* that test whether *quantum mechanics* respects *local realism.*

Given these conditions, we now explore the possible values of the following expression, corresponding to equation (3).

$$S = A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2. \tag{13}$$

First, using conditions (10) and (11), we compute the following:

$$(B_1 + B_2)^2 = B_1^2 + B_1B_2 + B_2B_1 + B_2^2 = 1 + B_1B_2 - B_1B_2 + 1 = 2, \tag{14}$$
$$(B_1 - B_2)^2 = B_1^2 - B_1B_2 - B_2B_1 + B_2^2 = 1 - B_1B_2 + B_1B_2 + 1 = 2. \tag{15}$$

Thus, the operators $B_1 + B_2$ and $B_1 - B_2$ take values $\pm\sqrt{2}$.

Next, we consider the possible values of the product $A_1A_2B_1B_2$. Using (11) and (12), we find:

$$(A_1A_2B_1B_2)^2 = (A_1A_2)^2(B_1B_2)^2 = (-1)(-1) = 1. \tag{16}$$

Hence, $A_1A_2B_1B_2$ can take values $\pm 1$.

We can simplify (13) by grouping terms:

$$S = A_1(B_1 + B_2) + A_2(B_1 - B_2). \tag{17}$$

Next, we compute $S^2$ by expanding the square:

$$\begin{aligned} S^2 &= A_1^2(B_1 + B_2)^2 + A_1A_2(B_1 + B_2)(B_1 - B_2) \\ &\qquad + A_2A_1(B_1 - B_2)(B_1 + B_2) + A_2^2(B_1 - B_2)^2 && (18) \\ &= 2 + A_1A_2(B_1^2 - B_1B_2 + B_2B_1 - B_2^2) + A_2A_1(B_1^2 + B_1B_2 - B_2B_1 - B_2^2) + 2 && (19) \\ &= 2 + A_1A_2(1 - B_1B_2 - B_1B_2 - 1) - A_1A_2(1 + B_1B_2 + B_1B_2 - 1) + 2 && (20) \\ &= 4 - 4A_1A_2B_1B_2. && (21) \end{aligned}$$

Since $A_1A_2B_1B_2$ can take values $\pm 1$, the possible values of $S^2$ are:

$$S^2 = 4 + 4 = 8 \quad \text{or} \quad S^2 = 4 - 4 = 0. \tag{22}$$

Thus, the possible values of $S$ are:

$$S = \pm 2\sqrt{2} \quad \text{or} \quad S = 0. \tag{23}$$

Consequently, the maximum value of $S$ is $2\sqrt{2}$, and the minimum value is $-2\sqrt{2}$. Using the linearity of expectation, we derive the following inequality:

$$-2\sqrt{2} \leq \langle S \rangle \leq 2\sqrt{2}. \tag{24}$$

This result demonstrates that *quantum mechanics* allows values of $\langle S \rangle$ that exceed the classical limit set by the Bell-CHSH inequality, as shown in (9). Thus, the predictions of *quantum mechanics* fundamentally violate the classical bound, signifying a profound departure from the principles of *local realism.*

In fact, in the 1998 experiment conducted by Zeilinger and colleagues, the *Bell-CHSH inequality* was tested under conditions that closed the locality loophole, yielding a result of $|S| = 2.73 \pm 0.02$ [14]. The *locality loophole* refers to situations where particles could potentially influence one another at subluminal speeds, violating *Bell's inequality.* Zeilinger's team meticulously designed their experiment to ensure that no faster-than-light signals could be exchanged between particles, maintaining locality. As a result, they experimentally demonstrated that *local realism* is false.

The observed violation of the *Bell-CHSH inequality* necessitates a critical reassessment of the classical assumptions surrounding locality and realism, which have traditionally underpinned our understanding of physical quantities. If *local realism* fails to account for the behavior of quantum systems, how should we then redefine physical quantities? This calls for a reevaluation of the very foundations of physical quantities, particularly in the context of quantum measurement. Taking quantum mechanics into consideration, such a reconsideration is essential for advancing our understanding of the quantum realm.

## 5 Reconstruction of the Concept of Physical Quantities

In light of the challenges posed by the violation of *Bell's inequality* (*Bell-CHSH inequality*), it becomes essential to reassess how physical quantities are defined and understood, especially within the framework of *quantum mechanics.* Historically, physical quantities were regarded as *intrinsic properties* of objects, independent of observation. This notion reflects the deeply entrenched views in classical philosophy and physics, where objects were thought to *possess* fixed attributes, regardless of whether they were being measured. However, *quantum theory*, with its focus on the measurement process and the active role of the observer, challenges this classical perspective. It suggests that physical quantities *may* *not be* stable, intrinsic features of physical objects, but rather *emerge* from the interaction between the observer, the measuring instrument, and the system being observed.

When we look back at the history of philosophy, discussions on the *essence* of physical objects can be traced as far back as ancient Greece. Aristotle's concept of "*form*" ($\varepsilon\iota\delta o\varsigma$) represents the *essential qualities* inherent in all physical beings, giving structure and identity to matter. Aristotle's metaphysics, which sought to explain how physical entities embody their *essence*, underpinned much of medieval and early modern thought. In traditional Western philosophy, the view that physical entities must possess *essential qualities*, independent of human perception, was fundamental. This is reflected in the philosophical notion that something cannot exist as a "*substance*" without its *essence*.

However, over time, philosophers began to question which qualities were *essential* to an object and which were *non-essential*. In the modern period, John Locke made a pivotal contribution to this debate in his *Essay Concerning Human Understanding* (1690), where he distinguished between "*primary qualities*" and "*secondary qualities*." According to Locke, *primary qualities*—such as solidity, extension, motion, and number—are inherent in objects and exist independently of the observer. These qualities, Locke argued, are *properties of reality*, but ultimately, this leads to the assertion that the *physical quantities* addressed in classical physics are *real*. In contrast, *secondary qualities*, like color and taste, depend on the observer's perception and are not inherent to the object itself.

Locke's theory, however, was formulated before the advent of *quantum mechanics*, and thus his classification of physical quantities was based on a pre-quantum understanding of *reality*. His view aligns with the *deterministic nature* of *classical physics*, where objects are assumed to have *definite properties* independent of observation. But with the development of *quantum theory* in the 20th century, this classical view was upended. *Quantum mechanics* shows that physical quantities are *not* fixed attributes of objects but arise from the interaction between the observer and the system through measurement. In this framework, properties such as position and momentum, except in eigenstates, are not well-defined simultaneously until measured, casting significant doubt on the classical assumptions of *reality*.

If physical quantities are *not* intrinsic attributes of physical reality, as *classical physics* assumed, then key debates surrounding *local realism* and the *Bell-CHSH inequality*—which rest on the assumption that physical quantities are elements of reality—must be reconsidered. Quantum experiments that violate the *Bell-CHSH inequality* suggest that physical quantities cannot be treated as independent of the observer's role. *Local realism*, which posits that physical properties exist independently of observation and that information cannot travel faster than light, is incompatible with the results of these quantum experiments. Therefore, the classical notion of physical quantities as fixed, observer-independent properties is no longer tenable in the quantum context.

While physical quantities are still connected to physical reality, they cannot be viewed as attributes of a reality that is entirely independent of observation. Instead, these quantities are shaped by the dynamic relationship between the external world and the observer. In other words, physical quantities should be understood as relational properties that emerge through the process of measurement, reflecting the context in which the observation is made.

To address this conceptual shift, a method for mathematically defining physical quantities will be introduced. This method involves considering a set of measuring instruments, establishing an equivalence relation between them based on their measurement results, classifying these instruments, and defining the representative elements after classification as physical quantities. By adopting this approach, it becomes evident that viewing physical quantities as inherent attributes of physical reality is an oversimplification. A more nuanced definition, grounded in the interaction between observer and system, is necessary.

This reconceptualization of physical quantities as relational, shaped by the interactions between the observer, the measuring instrument, and the system, calls for a deeper examination of how probabilities are assigned to measurement outcomes. Probability emerges in the context of measurement during physical experiments. This brings us to the *frequentist* interpretation of probability, which provides a framework for understanding measurement outcomes over repeated trials in the quantum domain.

### 5.1 Probability based on frequentism

In the *frequentist interpretation* of probability, probability is understood as the long-term relative frequency of an event occurring across repeated trials. Unlike subjective or *Bayesian interpretations*, which focus on belief or degrees of certainty, the *frequentist approach* treats probability as an inherent characteristic of a physical system, grounded in observable data rather than personal judgment.

When an experiment or measurement is repeated under identical conditions, the proportion of a particular outcome stabilizes and converges to a fixed value. In the ideal case, as the number of trials approaches infinity, this stabilized value is taken to represent the probability of the outcome. Thus, *frequentist probability* emerges from the repeated observation of phenomena, providing a concrete basis for predicting future occurrences.

In physical experiments, where the same procedure can be replicated under controlled conditions, the *frequentist approach* is often more suitable than subjective methods. Physical systems, governed by laws of nature, tend to exhibit consistent behavior across repeated trials, making *frequentist probability* an objective and reliable framework for analyzing outcomes based on empirical data.

Let us now apply this framework to the measurement of a physical object. Suppose we measure an object $s$ using a measuring instrument $A$ over $N$ trials. If the measurement yields the value $a$ exactly $n_a$ times during these $N$ trials, the probability $\mathrm{Prob}_s^A(a)$ of obtaining the measurement value $a$ can be defined as:

$$\mathrm{Prob}_s^A(a) := \lim_{N\to\infty} \frac{n_a}{N}. \tag{25}$$

The *frequentist interpretation* offers a solid framework for understanding how measurement outcomes stabilize through repetition. By incorporating probability in repeated measurements, physical quantities can be rigorously defined within a consistent empirical framework.

### 5.2 The concept of classical physical quantities

In the process of measuring physical objects, we employ a wide range of instruments, from rudimentary tools such as rulers to sophisticated devices like gravitational wave detectors. These instruments vary in their precision—some offering high accuracy, while others may be less reliable, and some may even malfunction. Let us denote the set of all conceivable measuring instruments as $\mathcal{A}$.

To begin, we examine the notion of a *suitable measuring instrument* within the framework of classical theory. Let us represent the world of macroscopic objects—the classical world—by the set $W_c$.

Consider a classical object $c_i \in W_c$, measured by an instrument $A_j \in \mathcal{A}$. The probability that the measured value lies within a certain interval $\Delta_{v_{ij}} := [v_{ij} - \Delta/2,\, v_{ij} + \Delta/2]$, centered around $v_{ij}$, is given by

$$\mathrm{Prob}_{c_i}^{A_j}(\Delta_{v_{ij}}) = 1, \tag{26}$$

where $\Delta$ represents the measurement error, understood as a technical limitation. The smaller this interval, the more accurate the instrument.

Next, we consider the idealized case where the measurement error tends to zero, that is, $\Delta \to 0$. In this scenario, the interval $\Delta_{v_{ij}}$ converges to a single value $v_{ij}$, and the probability expression becomes

$$\mathrm{Prob}_{c_i}^{A_j}(v_{ij}) = 1. \tag{27}$$

When this condition holds, we describe $A_j$ as an *ideal classical measuring instrument.* For the sake of simplicity, we will refer to an ideal classical measuring instrument simply as a *classical measuring instrument.*

Now, let us consider an arbitrary classical object $c_i$ being measured by two classical measuring instruments, $A_1$ and $A_2$. Suppose the measured values are $v_{i1}$ and $v_{i2}$, respectively. We can express their measurement probabilities as

$$\mathrm{Prob}_{c_i}^{A_1}(v_{i1}) = 1, \quad \mathrm{Prob}_{c_i}^{A_2}(v_{i2}) = 1. \tag{28}$$

If the measurements agree (i.e., $v_{i1} = v_{i2} = v_i$), then we have

$$\mathrm{Prob}_{c_i}^{A_1}(v_i) = 1, \quad \mathrm{Prob}_{c_i}^{A_2}(v_i) = 1. \tag{29}$$

To formalize this relationship, we define an equivalence relation between $A_1$ and $A_2$ with respect to $c_i$ and $v_i$, denoted as $A_1 \overset{(c_i,v_i)}{\sim} A_2$. Similarly, if $A_2$ and $A_3$ provide the same measurement $v_i$ for $c_i$, we denote their equivalence as $A_2 \overset{(c_i,v_i)}{\sim} A_3$.

This relation $\overset{(c_i,v_i)}{\sim}$ satisfies the three essential properties of an equivalence relation:

Reflexivity:

$$A_j \overset{(c_i,v_i)}{\sim} A_j. \tag{30}$$

Symmetry:

$$A_i \overset{(c_i,v_i)}{\sim} A_j \Rightarrow A_j \overset{(c_i,v_i)}{\sim} A_i. \tag{31}$$

Transitivity:

$$A_i \overset{(c_i,v_i)}{\sim} A_j, \quad A_j \overset{(c_i,v_i)}{\sim} A_k \Rightarrow A_i \overset{(c_i,v_i)}{\sim} A_k. \tag{32}$$

We refer to instruments connected by this equivalence relation as *mutually equivalent classical measuring instruments*.

To ensure generality across all classical objects and their corresponding measurements, we extend this equivalence relation to encompass all pairs $(c_i, v_i)$ in $W_c \times \mathbb{R}$. Specifically, two instruments $A$ and $B$ are equivalent, denoted $A \overset{(c_i,v_i)}{\sim} B$, if for every $c_i \in W_c$ and corresponding $v_i \in \mathbb{R}$, they satisfy

$$\mathrm{Prob}_{c_i}^{A}(v_i) = 1 \quad \text{if and only if} \quad \mathrm{Prob}_{c_i}^{B}(v_i) = 1. \tag{33}$$

This generalized equivalence relation maintains reflexivity, symmetry, and transitivity across all classical objects and measurements, ensuring a consistent partitioning of $\mathcal{A}$ into equivalence classes.

The equivalence class of a classical measuring instrument $A_j$, with respect to all $(c_i, v_i)$, is defined as

$$[A_j]_c := \{A_l \in \mathcal{A} \mid A_l \overset{(c_i,v_i)}{\sim} A_j \text{ for all } (c_i, v_i) \in W_c \times \mathbb{R}\}. \tag{34}$$

Each equivalence class contains instruments that consistently measure classical objects with identical probability distributions. We select a representative element from each equivalence class, termed the *representative classical measuring instrument*, denoted as $A_j$. The selection of representatives can be performed using a well-defined criterion, such as choosing the simplest instrument in each class or selecting based on specific operational characteristics.

Let $\mathcal{A}_c$ represent the set of all representative classical measuring instruments. When the object $c_i$ is measured by a representative instrument $A_j \in \mathcal{A}_c$, we have

$$\mathrm{Prob}_{c_i}^{A_j}(v_i) = 1, \tag{35}$$

indicating that $A_j$ consistently returns the value $v_i$ for $c_i$. Thus, we can treat the representative classical measuring instrument $A_j$ as a function mapping objects in $W_c$ to real values:

$$A_j(c_i) = v_i. \tag{36}$$

The mathematical details of the algebra consisting of *representative classical measuring instruments* are provided in Appendix 1. This framework enables the interpretation of the set of equivalence classes formed from the set of classical measuring instruments $\mathcal{A}$ as $\mathcal{A}_c$, with the *representative classical measuring instruments* in $\mathcal{A}_c$ understood as *classical physical quantities*. Each physical quantity assigns a real number to a classical object $c_i \in W_c$, representing measurable properties such as mass, length, or charge.

The functional interpretation of representative classical measuring instruments facilitates the application of algebraic operations within $\mathcal{A}_c$. Specifically:

Addition of Physical Quantities:

> The sum $(\alpha A + \beta B)(c_i)$ represents a new physical quantity obtained by linearly combining the measurements of $A$ and $B$ with weights $\alpha$ and $\beta$, respectively.

Multiplication of Physical Quantities:

> The product $(A \cdot B)(c_i)$ corresponds to a physical quantity derived from the pointwise multiplication of the measurements of $A$ and $B$, such as calculating kinetic energy from mass and velocity.

These operations preserve the algebraic structure, ensuring that $\mathcal{A}_c$ remains closed under addition and multiplication. The commutative nature of the algebra aligns with the classical assumption that physical quantities can be simultaneously measured and combined without interference.

In classical physics, physical quantities are regarded as stable, deterministic properties that can be measured using classical instruments and are inherent to the reality of objects. Within this framework, these quantities are seen as inherent characteristics of the objects, independent of the observer. However, this interpretation becomes *unnatural* when physical quantities are defined in the context of measurement, highlighting that quantities cannot be fully defined without the act of measurement by the observer.

As we transition from the macroscopic classical world to the microscopic quantum realm, the relationship between objects and their measurable properties becomes increasingly ambiguous. In *quantum mechanics*, physical quantities no longer behave as intrinsic attributes of objects; rather, they emerge from the probabilistic nature of quantum interactions. Consequently, measurement plays a more fundamental role in defining physical quantities in *quantum theory*, necessitating a reconsideration of their conceptual foundation.

## 5.3 The concept of quantum physical quantities

Building upon the methods of Araki[1] in *algebraic quantum theory*, we revisit the concept of *quantum physical quantities*.

In this context, we explore the notion of a *suitable measuring instrument* within *quantum theory*. The set of all microscopic quantum objects is denoted by $W_q$, representing the *quantum world*. The set of all measuring instruments is denoted by $\mathcal{A}$.

Consider a quantum object $q_i \in W_q$ measured by a *measuring instrument* $A_j \in \mathcal{A}$. The probability that the measurement yields a value within a specific range $\Delta_v := [v - \Delta/2, v + \Delta/2]$ is given by:

$$0 \leq \mathrm{Prob}_{q_i}^{A_j}(\Delta_v) \leq 1. \tag{37}$$

Here, $\pm\Delta/2$ represents the measurement accuracy, with smaller $\Delta$ indicating higher precision, constrained by the *uncertainty principle*[2].

In *quantum mechanics*, particularly for measurements with continuous spectra, the concept of point probabilities must be handled carefully. Instead, we define the probability of obtaining a measurement outcome within an interval $\Delta_v$ using the spectral measure associated with the *measuring instrument* $A_j$. Specifically, the probability is given by:

$$\mathrm{Prob}_{q_i}^{A_j}(\Delta_v) = \omega_{q_i}(E^{A_j}(\Delta_v)) \in [0, 1], \tag{38}$$

where $E^{A_j}(\Delta_v)$ is the projection operator (or spectral projection) associated with the *measuring instrument* $A_j$ and the interval $\Delta_v$, and $\omega_{q_i}$ is the quantum state corresponding to the quantum object $q_i$.

Unlike classical measurements, in *quantum mechanics*, the outcome of a measurement is inherently probabilistic, and the probabilities depend on both the quantum system and the observable being measured.

We define a *quantum physical quantity* (observable) as an equivalence class of measuring instruments that yield the same probability distributions for all quantum systems. Specifically, two measuring instruments $A_1$ and $A_2$ are considered equivalent if:

$$\mathrm{Prob}_{q_i}^{A_1}(\Delta_v) = \mathrm{Prob}_{q_i}^{A_2}(\Delta_v), \quad \forall q_i \in W_q, \quad \forall \Delta_v. \tag{39}$$

Then the instruments $A_1$ and $A_2$ are considered equivalent with respect to $q_i$ and the probability, denoted by $A_1 \overset{(q_i,\mathrm{Prob})}{\sim} A_2$.

This relation $\overset{(q_i,\mathrm{Prob})}{\sim}$ satisfies the properties of an equivalence relation:

Reflexivity:

$$A_j \overset{(q_i,\mathrm{Prob})}{\sim} A_j. \tag{40}$$

Symmetry:

$$A_i \overset{(q_i,\mathrm{Prob})}{\sim} A_j \Rightarrow A_j \overset{(q_i,\mathrm{Prob})}{\sim} A_i. \tag{41}$$

[2] The limits imposed by the uncertainty principle must be considered when refining measurement accuracy.

Transitivity:

$$A_i \overset{(q_i,\mathrm{Prob})}{\sim} A_j, \quad A_j \overset{(q_i,\mathrm{Prob})}{\sim} A_k \Rightarrow A_i \overset{(q_i,\mathrm{Prob})}{\sim} A_k. \tag{42}$$

We refer to instruments related by $\overset{(q_i,\mathrm{Prob})}{\sim}$ as *mutually equivalent quantum measuring instruments.*

To generalize this across all quantum objects and measurement outcomes, we extend the equivalence relation to encompass all pairs $(q_i, \mathrm{Prob})$ in $W_q \times [0,1]$. Specifically, two instruments $A$ and $B$ are equivalent, denoted $A \overset{(q_i,\mathrm{Prob})}{\sim} B$, if for every $q_i \in W_q$ and for all probability values of the measurement outcomes,

$$\mathrm{Prob}^A_{q_i}(\Delta_v) = \mathrm{Prob}^B_{q_i}(\Delta_v). \tag{43}$$

This generalized equivalence relation maintains reflexivity, symmetry, and transitivity across all quantum objects and their measurements, partitioning $\mathcal{A}$ into distinct equivalence classes.

The equivalence class of a quantum measuring instrument $A_j$, encompassing all instruments equivalent to $A_j$, is defined as

$$[A_j]_q := \{A_l \in \mathcal{A} \mid A_l \overset{(q_i,\mathrm{Prob})}{\sim} A_j \text{ for all } (q_i, \mathrm{Prob}) \in W_c \times [0,1]\}. \tag{44}$$

Each equivalence class comprises instruments that yield identical probability distributions for all quantum objects and measurement outcomes. We select a representative element from each equivalence class, termed the *representative quantum measuring instrument*, denoted as $A_j$. The selection of representatives can be guided by criteria such as simplicity or specific operational properties.

Let $\mathcal{A}_q$ denote the set of all representative quantum measuring instruments. At the expense of some mathematical rigor, the expectation value $\omega(A_j)$ of a measurement outcome for $A_j \in \mathcal{A}_q$ is defined as follows:

$$\omega(A_j) = \sum_v v \mathrm{Prob}^{A_j}_{q_i}(\Delta_v), \tag{45}$$

where $\omega : \mathcal{A}_q \to \mathbb{R}$ is interpreted as a *quantum state.*

The mathematical details of the algebra, specifically a $C^*$-algebra, comprising *representative quantum measuring instruments*, are provided in Appendix 2. This framework allows us to interpret the set of equivalence classes formed from the set of quantum measuring instruments $\mathcal{A}$ as $\mathcal{A}_q$, with the *representative quantum measuring instruments* in $\mathcal{A}_q$ interpreted as *quantum physical quantities.* Each quantum physical quantity corresponds to a bounded operator on the Hilbert space $\mathcal{H}$, representing measurable properties such as spin, energy, or momentum.

The functional interpretation of representative quantum measuring instruments facilitates the application of algebraic operations within $\mathcal{A}_q$. Specifically:

Addition of Quantum Physical Quantities:

> The sum $\alpha A + \beta B$ represents a new quantum physical quantity obtained by linearly combining the measurements of $A$ and $B$ with coefficients $\alpha$ and $\beta$, respectively.

Multiplication of Quantum Physical Quantities:

> The product $A \cdot B$ corresponds to the composition of measurements of $A$ and $B$, capturing the non-commutative interactions intrinsic to quantum systems.

These operations preserve the $C^*$-algebraic structure, ensuring that $\mathcal{A}_q$ remains closed under addition, multiplication, and involution. The non-commutative nature of the algebra aligns with the quantum mechanical principle that the order of measurements affects the outcomes, reflecting the inherent probabilistic and operator-based structure of quantum theory.

In summary, the construction of the $C^*$-algebra $\mathcal{A}_q$ provides a robust mathematical framework for quantum physical quantities, enabling a systematic and rigorous analysis of quantum measurements and the algebraic relationships between quantum observables.

## 6 It from Bit: From Reality to 'Reality'

The notion of reality, particularly in the quantum realm, has undergone significant transformations since the advent of *quantum mechanics*. *Classical physics* treated physical entities as possessing definite, intrinsic properties, independent of observation. However, the violation of *Bell's inequality* and the rise of *quantum information theory* have challenged these classical intuitions. One of the most profound responses to these challenges came from John Wheeler, who famously introduced the phrase "*It from Bit*" during a lecture titled *Information, Physics, Quantum: The Search for Links* at the 3rd International Symposium on the Foundations of Quantum Mechanics in Tokyo in 1989[15]. This phrase resonates with Eastern philosophy and encapsulates Wheeler's revolutionary idea: reality itself arises from information, not the other way around.

Wheeler's "*It from Bit*" hypothesis suggests that the fundamental building blocks of the universe are not physical particles or forces, but *information* itself. He argued that every physical "it"—every object or phenomenon in the universe—derives its existence from binary choices, or *bits*, elicited through the process of measurement. As Wheeler explained in his own words:

> I, like other searchers, attempt formulation after formulation of the central issues and here present a wider overview, taking for working hypothesis the most effective one that has survived this winnowing: **It from bit**. Otherwise put, every **it** — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes or no questions, binary choices, **bits**.
>
> It from bit symbolizes the idea that every item of the physical world has at bottom — at a very deep bottom, in most instances — an immaterial source and explanation; that what we call reality arises in the last analysis from the posing of yes-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and this is a **participatory universe**.
>
> (Wheeler, 1990, pp.354-368)

This conceptual shift from a substance-based to an information-based view of *reality* aligns naturally with the peculiarities of *quantum mechanics.* In fact, the results of quantum experiments, particularly the *violation of Bell-CHSH inequalities*, suggest that reality does not adhere to the classical tenets of *local realism*, where properties exist independently of measurement, and causal influences are confined by the speed of light. Instead, quantum correlations defy these classical constraints, pointing to a deeper structure where *information* plays a central role in the manifestation of physical phenomena.

However, the term '*It*' used here can sometimes hinder our conceptual understanding of *reality.* Some philosophers might argue that Wheeler's phrase "*It from Bit*," meaning '*reality* arising from information,' is misleading and does not hold as a valid concept of *reality.* Indeed, such a claim may be considered a valid counterargument within traditional philosophical debates, but it is based merely on the worldview of Western philosophy.

One of the key issues with the concept of *reality* as traditionally discussed in philosophy is that it was treated as a metaphysical notion, independent of any observer's perspective. However, the conclusions drawn from *quantum mechanics* reveal that we do not have direct access to this Reality. Instead, what we experience is always a version of *'Reality'*—a reality shaped by our interactions with the world and our measurement processes.

This idea challenges traditional views of *objectivity* and *reality* in western philosophy and science. In *classical physics*, we assume that physical properties exist independently of whether we measure them. But *quantum mechanics*, and especially its informational interpretation, suggests that *'reality'* is more fluid. It is contingent upon the interaction between the observer, the measuring apparatus, and the system being measured. *Physical quantities* in *quantum mechanics*, as demonstrated earlier, are not *intrinsic properties* of objects but arise from the act of measurement itself.

This perspective requires careful wording. After all, what physics describes is an *emergent phenomenon*, not *reality* in the western philosophical sense. In other words, we have no choice but to treat *emergent phenomena* as *'reality'* in quotation marks. The importance of observation in experiments has already been discussed, but fundamentally, our human sensory organs can also be seen as devices for observing the external world, developed through the process of evolution. Through these innate observation tools, we gather *information* from the external world and construct the *'reality'* as it exists for us.

Wheeler's statement, "*It from Bit*," captures this conceptual shift: the world as we know it emerges not from physical "it" but from informational "bits." In *quantum theory*, *information* precedes *'reality'*. The violation of the *Bell-CHSH inequality* suggests that *classical local 'realism'* fails to fully capture *emergent phenomena*, that is, the *'reality'* as it exists for us. *Quantum mechanics* points to a world where *entanglement* and *non-local correlations* naturally arise from the underlying information structure. We should *straightforwardly* regard such a world as *'reality'* emerging from *information.* The problem lies in our use of the traditional concept of *reality* as something independent of us.

Indeed, the conceptual shift from *reality* to '*'reality'*'—the latter symbolizing a more dynamic and information-based understanding—is a philosophical challenge posed by *quantum mechanics.* In this view, the notions of *objectivity* and *reality* dissolve into something more contingent and emergent. Such a perspective would likely be unacceptable within the framework of conventional Western philosophy. However, just as a paradigm shift occurred from the geocentric to the heliocentric model, there should be *no issue* with a conceptual paradigm shift regarding *objectivity* and *reality*. *Quantum mechanics*, with its inherent probabilistic nature and dependence on measurement, invites us to move from a static, classical notion of reality to a more *relational and informational understanding.*

## 7 Conclusion

This paper has examined the conceptual changes in the notions of *physical quantities* and *reality* within the framework of *quantum mechanics*, focusing on the philosophical shift from *local realism* to an *information-theoretic* view of the universe. By reconstructing the concept of *physical quantities* through the lens of *algebraic quantum theory*, we have demonstrated the inadequacy of classical notions, which assume intrinsic, observer-independent properties as fundamental. *Quantum phenomena*, such as the violation of *Bell's inequality*, challenge these classical assumptions and prompt a deeper reconsideration of the nature of *reality.*

The central insight of this study is the understanding that *physical quantities* are not inherent attributes of objects but rather relational properties that arise from the interaction between the observer, the measuring apparatus, and the system being observed. This shift from a substance-based to an information-based concept challenges traditional realism and suggests that *quantum mechanics* does not support the classical view of an objective, fixed reality. By demonstrating that *physical quantities* can indeed be defined within the context of measurement, this study shows that the quantum mechanical perspective is, in fact, more natural.

John Wheeler's "*It from Bit*" hypothesis aligns with this analysis. We do not have access to the old notion of *reality*; only *'reality'*—'*It*' we experience—makes conceptual sense. Furthermore, this *'reality'* is participatory, shaped by the informational processes underlying measurement and observation. Rather than viewing *reality* as a static, pre-existing structure, Wheeler emphasizes that *'reality'* emerges for us through interactions with the world and the information we derive from it.

This approach differs from the idea of *reality* as having a fixed essence, a notion central to philosophical thought since Aristotle. It may be difficult for some to accept, but this is a matter of 'belief' rather than scientific attitude. Certainly, an independent *reality* may exist, but what emerges from *information* are the physical objects for us, the *'Reality'* as we experience it. Therefore, we should adopt a notion of *'reality'* that is meaningful to us.

The interpretation of *physical quantities* in this paper is also framed within the context of the *'reality'* that is meaningful to us. In fact, moving beyond the constraints of *local realism*, this paper proposes a more nuanced interpretation of *physical quantities*, one that fully embraces the relational and informational dimensions of *quantum theory.* This approach offers a pathway to bridge the gap between longstanding *philosophical debates* on realism and the empirical challenges posed by *quantum mechanics.* It also opens new avenues for exploring the deeper informational structure of the universe. From this perspective, the universe itself is understood as a dynamic interaction of *information*, reshaping our understanding of physical *'reality'* and the role of the observer within it.

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## Appendix 1: The Algebra of Classical Physical Quantities

This appendix provides the mathematical details of the algebra consisting of *representative classical measuring instruments*, allowing this algebra to be interpreted as the algebra of *classical physical quantities.*

To formalize the algebraic structure, consider $\mathcal{A}_c$ as a set of functions $A : W_c \to \mathbb{R}$. For $A, B \in \mathcal{A}_c$ and scalars $\alpha, \beta \in \mathbb{R}$, we define the following operations:

Addition:

$$(\alpha A + \beta B)(c_i) := \alpha A(c_i) + \beta B(c_i), \tag{A.1}$$

Multiplication:

$$(A \cdot B)(c_i) := A(c_i) \cdot B(c_i). \tag{A.2}$$

These operations are pointwise and ensure that both $\alpha A + \beta B$ and $A \cdot B$ remain within $\mathcal{A}_c$, provided that $A(c_i)$ and $B(c_i)$ yield real numbers for all $c_i \in W_c$.

Under these operations, $\mathcal{A}_c$ forms a vector space over the real numbers $\mathbb{R}$. Furthermore, the multiplication operation is distributive over addition and satisfies both associative and commutative properties:

Associativity:

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C, \quad \forall A, B, C \in \mathcal{A}_c, \tag{A.3}$$

Commutativity:

$$A \cdot B = B \cdot A, \quad \forall A, B \in \mathcal{A}_c, \tag{A.4}$$

Distributivity:

$$A \cdot (B + C) = A \cdot B + A \cdot C, \quad \forall A, B, C \in \mathcal{A}_c. \tag{A.5}$$

Additionally, if there exists a multiplicative identity $I \in \mathcal{A}_c$ such that

$$I(c_i) = 1, \quad \forall c_i \in W_c, \tag{A.6}$$

then $\mathcal{A}_c$ possesses a unit element, further solidifying its structure as a unital commutative algebra.

Thus, $\mathcal{A}_c$ possesses the structure of a commutative algebra, demonstrating that combinations of classical measuring instruments can be treated algebraically.

## Appendix 2: The Algebra of Quantum Physical Quantities

This appendix provides the mathematical details of the $C^*$-algebra consisting of *representative quantum measuring instruments*, enabling the interpretation of this algebra as the algebra of *quantum physical quantities*.

In *quantum mechanics*, observables are represented by self-adjoint operators acting on a Hilbert space $\mathcal{H}$. We formalize $\mathcal{A}_q$ as a set of bounded linear operators on $\mathcal{H}$ that form a $C^*$-algebra.

For $A, B \in \mathcal{A}_q$ and $\alpha, \beta \in \mathbb{C}$, we define the operations:

Addition:

$$(\alpha A + \beta B)\psi = \alpha A\psi + \beta B\psi, \quad \forall \psi \in \mathcal{H}. \tag{A.7}$$

Multiplication:

$$(AB)\psi = A(B\psi), \quad \forall \psi \in \mathcal{H}. \tag{A.8}$$

Involution ($*$-operation):

$$A^* \text{ is the adjoint of } A, \quad A^* \in \mathcal{A}_q. \tag{A.9}$$

These operations satisfy the properties of a $C^*$-algebra:

Associativity of Multiplication:

$$A(BC) = (AB)C. \tag{A.10}$$

Distributivity:

$$A(B + C) = AB + AC, \quad (A + B)C = AC + BC. \tag{A.11}$$

Bilinearity:

$$(\alpha A)B = A(\alpha B) = \alpha(AB). \tag{A.12}$$

Involution Properties:

$$(A^*)^* = A, \quad (AB)^* = B^* A^*. \tag{A.13}$$

$C^*$-Identity:

$$\|A^* A\| = \|A\|^2, \tag{A.14}$$

where $\|\cdot\|$ denotes the operator norm.
The set $\mathcal{A}_q$ is complete with respect to this norm, making it a $C^*$-algebra.
A *state* $\omega_{q_i}$ on $\mathcal{A}_q$ is a positive linear functional associated with the quantum system $q_i$:

$$\omega_{q_i} : \mathcal{A}_q \to \mathbb{C}, \tag{A.15}$$

satisfying:

Linearity:

$$\omega_{q_i}(\alpha A + \beta B) = \alpha \omega_{q_i}(A) + \beta \omega_{q_i}(B). \tag{A.16}$$

Positivity:

$$\omega_{q_i}(A^* A) \geq 0, \quad \forall A \in \mathcal{A}_q. \tag{A.17}$$

Normalization:

$$\omega_{q_i}(I) = 1, \tag{A.18}$$

where $I$ is the identity operator in $\mathcal{A}_q$.

The expectation value of an observable $A_j \in \mathcal{A}_q$ in the state $\omega_{q_i}$ is given by:

$$\omega_{q_i}(A_j) = \int_{\sigma(A_j)} v \, d\omega_{q_i}^{A_j}(v), \tag{A.19}$$

where $\sigma(A_j)$ is the spectrum of $A_j$, and $\omega_{q_i}^{A_j}$ is the probability measure on $\sigma(A_j)$ induced by the state $\omega_{q_i}$ and the spectral measure $E^{A_j}$:

$$\omega_{q_i}^{A_j}(\Delta_v) = \omega_{q_i}(E^{A_j}(\Delta_v)). \tag{A.20}$$

In cases where $A_j$ has a purely discrete spectrum, the expectation value simplifies to:

$$\omega_{q_i}(A_j) = \sum_{v\in\sigma(A_j)} v\, \omega_{q_i}(E^{A_j}(\{v\})). \tag{A.21}$$

Here, $E^{A_j}(\{v\})$ is the projection onto the eigenspace corresponding to the eigenvalue $v$.

The probabilities of measurement outcomes are determined via the *spectral theorem*, which associates projection operators with measurement outcomes.

By applying the *Gelfand-Naimark-Segal (GNS) construction*, we represent the abstract $C^*$-algebra $\mathcal{A}_q$ concretely on a Hilbert space $\mathcal{H}_{\omega_{q_i}}$, with the state $\omega_{q_i}$ inducing a cyclic representation[9][11]. In this representation, observables act on the Hilbert space, and states correspond to vectors or density operators in $\mathcal{H}_{\omega_{q_i}}$.

This framework provides a rigorous mathematical foundation for *quantum mechanics*, capturing the non-commutative nature of quantum observables and the fundamental role of states as positive linear functionals that encode the properties of quantum systems.

In contrast to *classical physical quantities*, which are represented by commutative algebras of functions (reflecting the deterministic nature of classical physics), *quantum physical quantities* form a non-commutative $C^*$-algebra, reflecting the inherent uncertainties and probabilistic outcomes in quantum measurements.

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