Physical Possibility of the Aharonov-Bohm Effect

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Abstract

In much of recent literature, the debate surrounding the Aharonov-Bohm (AB) effect largely consisted of two main strands. On the one hand, the AB model has been viewed as an invitation to debate about the *ontology of electromagnetism*. Traditionally construed, the AB model would seem to pose a dilemma for one commonly accepted electromagnetic wisdom: the electric and magnetic fields are the fundamental physical entities of the theory while the scalar and vector potentials are mere mathematical over-descriptions. Motivated by the non-local action of the magnetic field in the AB model, the ontological debate has become somewhat inflated with accounts attempting to replace the field ontology. On the other hand, the recent philosophical work drew attention to the *idealizations* involved in the AB model and their significance for explanation. In this paper, I criticise one of the driving motivations behind the ontology debate and contrary to the orthodoxy, I conclude that the AB model does not present a problem case for the naïve realist about electric and magnetic fields. By analysing the representational capacity of the AB model I answer the following questions. **Q1**: Does the AB model represent a physical possibility? **Q2**: Does the AB model pose a dilemma for realism about electric and magnetic fields?

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1 Introduction

In much of recent literature, the debate surrounding the Aharonov-Bohm (AB) effect largely consisted of two main strands. On the one hand, the AB model has been viewed as an invitation to debate about the *ontology of electromagnetism*. Traditionally construed, the AB model would seem to pose a dilemma for one commonly accepted electromagnetic wisdom: the electric and magnetic fields are the fundamental physical entities of the theory while the scalar and vector potentials are mere mathematical over-descriptions. Motivated by the non-local action of the magnetic field in the AB model, the ontological debate has become somewhat inflated with accounts attempting to replace the field ontology. On the other hand, the recent philosophical work of Earman, Shech, and Dougherty [7, 8, 10] drew attention to the *idealizations* involved in the AB model and their significance for explanation. In this paper, I criticise one of the driving motivations behind the ontology debate and contrary to the orthodoxy, I conclude that the AB model does not present a problem case for the naïve realist about electric and magnetic fields.

The structure of the paper is as follows. In §2 I briefly review the dynamics of the AB effect and construct a sequence of solenoid models whose limit point is the idealized AB model. In §3 I formulate the thesis of AB scepticism, a sceptical thesis regarding the physical possibility of the possible world naïvely represented by the AB model, and I provide arguments in its favour. In §4 I show how this suggests a *careful* interpretative stance towards the AB model in the sense of Lehmkuhl [15] as opposed to a literal interpretative stance as suggested by Earman in [8]. In §5 I provide some additional reasons to reject the dilemma for field realism, and in §6 I comment on other idealizations of the AB effect which however do not seem to threaten the physical possibility of the associated possible world. Lastly, in §7 I conclude.

2 Dynamics of the Aharonov-Bohm Model

2.1 AB Setup

To setup the stage, consider a double-slit experiment in which a beam of electrons is directed at an impermeable wall with two slits and a detector behind.

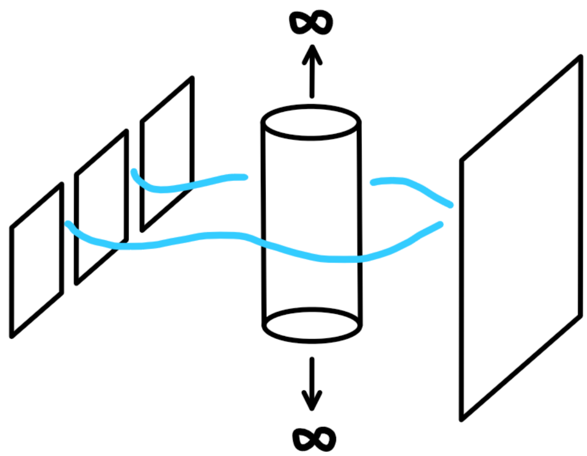

Figure 1: The AB setup including the idealized, infinitely long solenoid impenetrable to the incoming electrons with perfect confinement of the magnetic field.

Standard quantum mechanical machinery applied to this system suggests that an interference pattern characteristic of the quantum behaviour of electrons will form on the detector behind the wall. Now consider placing a solenoid behind the wall in between the two slits and let the current run through the solenoid. In order to add the bite characteristic of the AB effect, one needs to employ further assumptions. Earman and Shech correctly observe that the typical presentation of the AB effect involves the following idealizations:

(F1) An infinitely long cylindrical shaped solenoid S_∞.

(F2) When the current is turned on in the solenoid the magnetic field $\mathbf{B}_\infty$ generated is completely contained within the solenoid.

(F3) The solenoid is impenetrable to an external electron. [8, p. 1996]

Altogether, the idealized model then looks like the one depicted in figure 2.1. To set terminology for the rest of this paper, I will refer to this mathematical model as the *AB model*. By the *AB effect*, on the other hand, I will denote the actual shift in interference patterns observed in the laboratories with actual imperfect solenoids. While this terminology is perhaps slightly revisionary and disagrees with that of Earman in [8] wherein 'Aharonov-Bohm effect' refers to the idealized scenario itself, nothing in my argument ultimately hinges on adopting this linguistic convention. It should only help us to avoid unnecessary conflation of the mathematical models with actual experiments. The obvious consequence of my linguistic convention is that the AB effect has been observed in the laboratory. This is true *by definition* and uninterestingly so. Arguably, the philosophical interest has always been and should be directed to the idealized AB models. Note that alternative AB setups have been devised and experimentally tested. For example, Osakabe et al. [22] work with a toroidal solenoid while Caprez et al. [5] use a two-solenoid setup. These subtle differences will become important in our subsequent discussion.

In their seminal work, Aharonov and Bohm showed that when current is run through the solenoid, the interference pattern observed at the detector will shift depending on the value of magnetic flux inside the solenoid. Strikingly, the phase shift is predicted despite the electron never venturing into a region in which the magnetic $\mathbf{B}_\infty$ field is non-zero. This follows since by **(F1)** and **(F3)**, the configuration space of the electron is $\mathbb{R}^3 \setminus S_\infty$ and by **(F2)**, the magnetic field is supported only in S_∞. One way to cash out the dilemma for field realism would then be as follows. Either the magnetic field non-locally affects the electron without any overlap with its wavefunction; or it is non-fundamental and the physical content of electromagnetism is comprised by the four-potential which is conveniently non-zero in the region outside the solenoid. The nature of this non-locality has been extensively analyzed by Healey who on its behalf remarks that "[t]here have been numerous attempts to avoid this conclusion." [13, pp. 18–19]. Indeed, pernicious implications of non-locality led many to believe that the non-fundamentality of the electromagnetic fields is preferable. The fathers of the AB effect themselves shared this opinion in their seminal paper:

> It would therefore seem natural at this point to propose that, in quantum mechanics, the fundamental physical entities are the potentials, while the field are derived from them by differentiations. [1, p. 490]

More recently, Belot [3] comments sceptically on the fate of realism about the electromagnetic field in light of the AB effect:

> It is widely agreed that the subsequent experimental detection of the Aharonov-Bohm effect discredited the familiar way of understanding electromagnetism. One can maintain the traditional interpretation of the theory only by maintaining that fields act where they are not. But this flies in the face of the well-entrenched principle that classical fields act by contact rather than at a distance. It would seem, then, that the electric and magnetic fields cannot constitute the ontology of electromagnetism. [3, p. 532]

Jacobs, using the term F-realism for the same position, similarly notes that

> F-realism face two main problems. The first is the well-known fact that an explanation of the Aharonov-Bohm effect in terms of the Faraday tensor implies a violation of the principle of Local Action (Healey, 1997). Since there is no overlap between the electromagnetic field and the matter field, the former can only act on the latter at a distance. This is universally seen as a sufficient reason to reject F-realism. [14, p. 5]

A host of ontological proposals has been offered to replace F-realism a brief survey of which may be found in Jacobs. In this paper, I will not directly engage with the ontology debate despite its undeniable importance. Instead I pose and answer the following two questions:

Q1 Does the AB model represent a physical possibility?

Q2 Does the AB model pose a dilemma for realism about the electric and magnetic fields?

The dilemma of **Q2** is the one just presented: either magnetic fields act non-locally or they are non-fundamental. I argue that one should answer to **Q2** in the negative and to **Q1** in the positive with an additional proviso. As such, I take my conclusions to undermine the main motivation for the ontology debate surrounding the AB model which would be to relieve the 'ontological crisis' in light of the dilemma for field realism.

In §3 I consider **Q1** and argue that the possible world naïvely corresponding to the AB model is in fact not physically possible due to the idealized features of the AB model. I take this to suggest that one should take a more careful interpretational stance towards the AB model. Interpreted this way, the AB model indeed represents physical possibilities. I subsequently argue that this relieves the dilemma for field realism thus answering to **Q2** in the negative.

2.2 AB Dynamics

An interesting contribution to the dynamical analysis of the AB model has been worked out by de Oliveira and Pereira [21] and brought to attention of the philosophic community in the papers of Earman and Shech [8, 10]. I will now briefly revisit this aspect of AB dynamics.

It follows from Stokes' theorem that the magnetic flux through arbitary surface $\Sigma \subseteq \mathbb{R}^3$ is given by

$$\Phi(\Sigma) = \int_\Sigma \mathbf{B} \cdot d\mathbf{S} = \int_\gamma \mathbf{A} \cdot d\mathbf{l}, \tag{1}$$

where $\gamma = \partial\Sigma$ and $\mathbf{B} = \nabla \times \mathbf{A}$. Let us now set up a cylindrical coordinate system (ρ, ϕ, z) such that the z-axis aligns with the solenoid axis and assume the solenoid has radius a. Due to axial symmetry of the problem we may immediately infer that the vector potential takes the following form to be found in [8, p. 1997]:

$$A_z = A_\rho = 0 \tag{2}$$
$$A_\phi(\rho) = \begin{cases} \frac{\Phi}{2\pi\rho} \text{ for } r > a \\ \frac{\Phi\rho}{2\pi a^2} \text{ for } 0 \leq r \leq a, \end{cases}$$

from which it follows that that $\nabla \times \mathbf{A} = 0$ for $r > a$ yielding **(F2)**.

A difficulty which arises in the dynamical analysis of the AB model is the fact that $H_{AB} = \frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2$, the natural Hamiltonian for a particle with charge e propagating in a magnetic field, is not self-adjoint when acting on its natural domain.[1] Since $\mathbf{p} = -i\nabla$, the natural domain for H_{AB} would be that of smooth test functions with compact support $C_0^\infty(\mathbb{R}^3 \setminus S_\infty)$ which is dense in $L^2(\mathbb{R}^3 \setminus S_\infty)$, the space of wavefunctions. In the original treatment of the model by Aharonov and Bohm, the Dirichlet boundary conditions were imposed at the solenoid to extend H_{AB} to a self-adjoint operator $\bar{H}_{AB}$ and avoid this difficulty. However, as it turns out, there exist different possible self-adjoint extension of H_{AB} which correspond to different choices of boundary conditions at the solenoid each of which produces different admissible energy levels and different unitary dynamics. This fact has already been appreciated in the philosophical literature and so I don't dwell on it much further – the interested reader should refer to the cited papers.

The analysis of de Oliveira and Pereira then attempts to justify the choice of Dirichlet boundary conditions leading to the Aharonov-Bohm Hamiltonian $\bar{H}_{AB}$ by constructing a sequence of Hamiltonians which converges to $\bar{H}_{AB}$ in the appropriate limit. They consider a sequence of solenoids S_L of length $2L$ with the corresponding vector potential given by

$$\begin{aligned} A_{L,\phi}(\rho) &= \frac{\Phi}{4\pi^2 a} \int_{-L}^{L} dz' \int_0^{2\pi} d\phi \, \frac{\cos\phi'}{(\rho^2 + a^2 + z'^2 - 2a\rho\cos\phi')^{1/2}}, \\ A_{L,\rho} &= A_{L,z} = 0, \end{aligned} \tag{3}$$

and propose to implement the impenetrability assumption **(F3)** as follows. Let χ_L be the characteristic function of S_L defined by $\chi_L(x) = 1$ if $x \in S_L$ and $\chi_L(x) = 0$ else. Then we may add to the Hamiltonian the potential term $V_n = n\chi_L$ with $n \in \mathbb{N}$ which captures the idea of ever increasing degree of impenetrability as $n \to \infty$. The resulting Hamiltonian is then

$$H_{L,n} = \frac{1}{2m}(\mathbf{p} - e\mathbf{A}_L)^2 + V_n. \tag{4}$$

Remarkably, de Oliveira and Pereira prove that $\lim_{L,n\to\infty} H_{L,n} = \bar{H}_{AB}$, where the limit is understood in the *strong resolvent sense*.[2] Moreover, the order in which the limits are taken doesn't matter. As a result we have a 'lattice' of models $\mathcal{M}_{L,n}$ labelled by $L \in \mathbb{R}$ and $n \in \mathbb{N}$ which is depicted in figure 2.2. In some sense, the models $\mathcal{M}_{L,n}$ may be thought of as representing 'more realistic' systems consisting of actual-world solenoids of finite length and finite boundary potential. On the other hand, the model $\mathcal{M}_{AB}$ would naïvely seem to correspond to the idealized scenario characterized by **(F1)**–**(F3)**. The extent to which this is accurate will be revealed in §3 and §4 where we study the representational capacities of the lattice models.

[1] This is pathological for two reasons. Firstly, energy eigenvalues are not guaranteed to be real, but more importantly it means that H_{AB} is ill suited to generate unitary time-evolution. This is because by Stone's theorem, a strongly continuous family of unitaries is guaranteed to have a self-adjoint generator (see [12, thm. 10.15])

[2] If T_n is a sequence of operators on Hilbert space $\mathcal{H}$ and $R_z(T_n)$ with $z \in \mathbb{C}$ is the corresponding sequence of resolvents, we say that $T_n \to T$ in the strong resolvent sense if $|R_i(T_n)\psi - R_i(T)\psi| \to 0$ for any $\psi \in \mathcal{H}$. That is, $T_n \to T$ in the strong resolvent sense if the corresponding resolvents converge in the strong sense. See [20] for more detail.

$$
\begin{array}{ccccc}
 & & & & \mathcal{M}_{AB} \\
 & & & & \vdots \\
\mathcal{M}_{0,n} & \cdots & \mathcal{M}_{L,n} & \cdots & \mathcal{M}_{\infty,n} \\
\vdots & & \vdots & & \vdots \\
\mathcal{M}_{0,0} & \cdots & \mathcal{M}_{L,0} & \cdots & \mathcal{M}_{\infty,0}
\end{array}
$$

Figure 2: Lattice of models converging to the AB model $\mathcal{M}_{AB}$.

3 Aharonov-Bohm Scepticism

One may imagine a very naïve correspondence between the lattice models $\mathcal{M}_{L,n}$ and possible worlds $\mathcal{W}_{L,n}$ containing actual solenoids of length $2L$ and such and such impenetrability. Moreover, one could imagine a world $\mathcal{W}_{AB}$ containing a solenoid subject to **(F1)**–**(F3)**. While appealing initially, there is a sense in which this orthodox way of thinking according to which models directly correspond to possible worlds is simply wrong. In a forthcoming paper [6], Cudek criticises this common view and instead assumes the interpretive stance according to which models represent physically possible *situations*. He notes that

> [t]he orthodox way of thinking about the role of models is as follows: models of a theory $\mathcal{T}$ represent metaphysically possible worlds that conform to the laws of $\mathcal{T}$, which in turn provide an extensional analysis of modal propositions made relative to $\mathcal{T}$.
>
> [...] I will not say that models represent possible worlds so understood. Metaphysically possible worlds are maximal, in that they settle the truth-value of every proposition, whereas what is represented by models of spacetime theories [...], and what I shall henceforth call a 'possible (physical) situation [...]' need not be maximal in this sense. In particular, possible situations described by GR should only settle propositions that comprise the subject matter of GR. [6, p. 4]

I share Cudek's misgivings about the orthodox way of thinking; however, for the purposes of this paper I stick with the orthodox terminology and refer to $\mathcal{W}_{L,n}$ and $\mathcal{W}_{AB}$ as worlds. To my best knowledge, the conclusions of this paper don't hinge on the worlds being complete in the sense of settling the truth-value of every proposition and so the reader is free to use 'situations' and 'worlds' interchangeably throughout.

We are now in position to formulate the sceptical position of the AB sceptic which amounts to the simple denial of physical possibility of $\mathcal{W}_{AB}$.

AB scepticism: The world $\mathcal{W}_{AB}$ is not physically possible.

In the rest of this section I contrast AB scepticism with other sceptical responses to be found in philosophy of physics and then argue that there are good reasons to believe that the AB sceptic is justified.

3.1 Some Sceptical Traditions in Philosophy of Physics

Rejecting physical possibility of certain of models of physical theories has a longstanding tradition in philosophy of physics. Indeed, metaphysics of space and time furnishes several such examples:

- *Leibniz-Clarke Correspondence* [2]: Using the *principle of sufficient reason* (PSR) and the *principle of identity of indiscernibles* (PII) Leibniz attacks the possibility of shift-related models of Newtonian theory.
- *The Science of Mechanics* [16]: Ernst Mach's critique of the rotating bucket thought experiment and the two-globes thought experiment from Newton's *Principia*.
- *Physical Relativity* [4]: Harvey Brown's critique of Malament's alleged proof of non-conventionality of distant simultaneity in the special theory of relativity.
- Maudlin on the hole argument [18]: Maudlin's contention that spacetime points hold their metric properties essentially eliminates a class of possible worlds which naïvely correspond to the diffeomorphism-related models in the hole argument. According to Maudlin, these worlds are not even *metaphysically* possible.

In the following, I distinguish between *epistemic* and *metaphysical* scepticism and evaluate the prospects of vindicating AB scepticism in epistemic or metaphysical fashion.

3.2 Rejecting $\mathcal{W}_{AB}$ à la Mach

In *The Science of Mechanics*, Ernst Mach considers Newton's two-globe experiment and the rotating bucket experiment found in the *scholium* to Newton's *Principia* (see [2, Appendix A]). The two-globes experiment involves a counterfactual scenario in which two identical material spheres orbit each other in an otherwise empty universe connected by a cord. In his analysis of the two-globes experiment, Mach notes that

> [a]ll our principles of mechanics are, as we have shown in detail, experimental knowledge concerning the relative positions and motions of bodies. Even in the provinces in which they are now recognised

> as valid, they could not, and were not, admitted without previously being subjected to experimental tests. No one is warranted in extending these principles beyond the boundaries of experience. In fact, such an extension is meaningless, as no one possesses the requisite knowledge to make use of it. [16, p. 229]

Mach's response is sceptical in a proper sense of the word: it doubts one's *knowledge* of what would happen in the counterfactual two-globe scenario and warns before extending our theories 'beyond the boundaries of experience'. Machian scepticism is therefore distinctively *epistemic*.

Does epistemic scepticism à la Mach furnish the AB sceptic with means to reject the physical possibility of $\mathcal{W}_{AB}$? I argue it does not. By the very nature of epistemic scepticism, the epistemic sceptic will remain epistemically modest about features of $\mathcal{W}_{AB}$ denying our knowledge of any claims about physical features of $\mathcal{W}_{AB}$ such as outcomes of thought experiments set in $\mathcal{W}_{AB}$. However, the epistemic sceptic ultimately remains silent about physical possibility of $\mathcal{W}_{AB}$. They could reject it, endorse it or deny our knowledge of this matter of fact, all three options seem equally compatible with epistemic scepticism. Therefore, Machian epistemic scepticism does not provide a natural vindication for AB scepticism.

In spite of this, the epistemic sceptic can challenge the dilemma for field realism thus directly answering to **Q2**. Their argument would presumably run along the following lines:

> *Machian epistemic argument*: Our theories haven't been well-tested in experimental regimes akin to that of $\mathcal{W}_{AB}$, therefore we can't trust our theories in those regimes. Conclusions about locality in $\mathcal{W}_{AB}$ are therefore unjustified.

Arguably, for the Machian epistemic argument to be at least slightly convincing one first needs to specify what constitutes 'experimental regimes akin to that of $\mathcal{W}_{AB}$'. The epistemic argument thus puts its finger on two more general questions: (i) what constitutes an experimental regime? (ii) in which regimes can we trust the predictions of our theories?

Let me offer some mild suggestions in way of answering (i). Consider the set $\mathcal{O}$ of all operational procedures implementable in a laboratory of interest. This includes the set of all possible settings of the experimental apparatuses including manipulations which are sensible, non-sensible, quiescent, destructive as well any other. Some of these operational procedures correspond to all the possible respectable scientific experiments which one may carry out with those apparatuses. Let us denote this set by $\mathcal{P} \subseteq \mathcal{O}$. Moreover, some of the experiments in $\mathcal{P}$ will correspond to experiments which have been actually carried out in the laboratory. Let us denote this set by $\mathcal{E} \subseteq \mathcal{P}$. An *experimental regime* would then be any subset $\mathcal{R} \subseteq \mathcal{P}$. For illustration, consider Alice, an experimental high-energy physicist working with her particle accelerator. Suppose that the technological features of the accelerator allow Alice to prepare collisions of two beams of protons at arbitrary energy below E_0 electronvolts. The set of all operational procedures $\mathcal{O}$ available to Alice thus includes arbitrary manipulations with the accelerator including the less sensible ones such as placing inside of the accelerator the lunchbox of Alice's colleague Bob. The set of all respectable experimental procedures $\mathcal{P}$ consists in the case of Alice of all the settings of collision energy available to Alice, which is anything between 0 and E_0. However, suppose that for unspecified reasons, Alice has so far only probed scattering in the energy range up to $E_1 < E_0$. She is thus probing the scattering in a particular experimental regime $\mathcal{R}$ defined by the threshold energy E_1 and moreover $\mathcal{R} = \mathcal{E}$.

There is much missing from the Alice example. In more realistic examples, the space of all operational procedures would be much more complicated than the space available to Alice. Moreover, since science is a collaborative and global effort, when defining an experimental regime, one should presumably include the possible operational procedures in *all* laboratories in the world within $\mathcal{O}$. Moreover, one should presumably not be concerned only with operational procedures implementable in current actual laboratories but also in future possible laboratories as it is customary to hear, e.g. the high-energy physicists speculate about new physics to be found in future particle accelerators. Nevertheless, the main idea carries through more or less untouched. Experimental regime is defined by a specification of a subset of possible experiments some of which might have already been realized, some of which are yet to be and some of which never will.

Question (ii) may now be phrased in sharper terms. Given the set of performed experiments $\mathcal{E}$, which experimental regimes $\mathcal{R}$ with $\mathcal{E} \subseteq \mathcal{R}$ allow one to make justified claims about the outcomes of experiments in $\mathcal{R} \setminus \mathcal{E}$? Phrased this way, question (ii) is nothing less than a variant on Nelson Goodman's new riddle of induction[3] (see [11, ch. 3]) and so I refer the reader interested such issues to the extant literature. This connection allows us to rephrase the gist of Machian scepticism in terms of inductive predicate projectibility. Having tested Newtonian theory in our earthly regimes, why doesn't inductive extrapolation warrant conclusions about the two-globe world? Having observed the AB effect with ordinary actual-world solenoids, why doesn't inductive extrapolation warrant conclusions about the infinite case? The Machian sceptic could in both cases question the projectibility of our predicates in these hypothetical scenarios. However, then it runs the risk of collapsing into simple inductive scepticism. Explicating the difference between projectible and non-projectible cases and a direct engagement with the new riddle thus becomes a project of central importance to the Machian epistemic sceptic. Until they propose a solution to the new riddle in the context of experimental regimes, Machian scepticism is not a very compelling option.

[3]Thanks to Christian Wüthrich for pointing this out in a discussion.

3.3 Rejecting $\mathcal{W}_{AB}$ à la Leibniz

The *Leibniz-Clarke correspondence* [2] showcases a whole host of Leibniz's attempts to argue against implications of Newtonian theory set in absolute space, as defended by Clarke. This includes the physical possibility of kinematically shifted worlds which, despite being compatible with Newtonian theory, do not square with Leibniz's relational metaphysics. To attack the notion of there being such kinematically shifted worlds Leibniz employs his two metaphysical principles: the principle of sufficient reason (PSR) and the principle of identity of indiscernibles (PII). He notes that

> [t]o say that God can cause the whole universe to move forward in a right line, or in any other line, without making otherwise any alteration in it; is another chimerical supposition. For, two states indiscernible from each other, are the same state; and consequently, 'tis a change without any change. Besides, there is neither rhyme nor reason in it. But God does nothing without reason; and 'tis impossible there should be any here. Besides, it would be *agendo nihil agere*, as I have just now said, because of the indiscernibility. [2, p. 38].

As opposed to Machian epistemic scepticism, Leibniz's scepticism here is distinctively *metaphysical*. It prohibits the physical possibility of kinematically-shifted worlds using two metaphysical principles, PSR and PII and so the prospects for vindicating AB scepticism metaphysically à la Leibniz are much better. In fact, I argue that the following considerations drive the point home.

3.3.1 General problems with ∞-systems

It is a well-known fact highlighted for example by Norton's model of the infinite one-dimensional spring chain (see [19, pp. 27–29]) that systems involving an infinite number of degrees of freedom exhibit pathological behaviour such as indeterminism and energy non-conservation. Indeed, for the infinite spring chain, if x_n is the horizontal displacement from equilibrium of n-th point mass, one one may construct two *distinct* solutions compatible with the same initial condition $x_n(0) = 0$ and $\dot{x}_n(0) = 0$. An obvious solution corresponding to this initial condition is the quiescent one $x_n(t) = 0$ for all n and t; however, Norton shows that a second solution may be constructed which roughly corresponds to the time-reversal of two disturbances 'coming in from infinity' and destructively interfering such that equilibrium is reached. Existence of two distinct solutions compatible with the same initial condition demonstrates that the theory of the infinite spring chain is indeterministic. Moreover, the second solution blatantly violates energy conservation.

One may imagine constructing a similar scenario for $\mathcal{W}_{AB}$. The solenoid in $\mathcal{W}_{AB}$ is infinitely long and may be conceived of as consisting of small atoms making up the conductor. In first approximation, interactions between the constituent atoms could be represented by a mathematical model not unlike that of

Norton and so indeterminism and energy non-conservation would appear again. In analogy to Norton's example, one may imagine two material waves travelling through the solenoid from infinity, destructively interfering near the origin, and yielding the quiescent solution for future times. This scenario, once again, exhibits indeterminism and energy non-conservation and both of these features are somewhat pathological and unphysical. Pathology of indeterminism and enegy non-conservation could be contested. After all, the two indeterministic solutions are both perfectly good solutions to the equations of motion and therefore should be taken seriously as physical possibilities. However, just because a solution of a theory is mathematically admissible does not mean it is physically admissible. Moreover, indeterminism is generally perceived as a serious vice in philosophical debates, including the one surrounding the AB model. In classical electromagnetism, realism about the potentials regards gauge-equivalent potential configurations as physically distinct and so an indeterministic scenario may be constructed in fashion identical to the hole argument (see [9]). Indeterminism stemming from material waves in the solenoid is yet of a more serious kind as the two solutions corresponding to identical initial condition are observationally distinct! Readers who find indeterminism of the potentials disconcerting should therefore find determinism about material waves in the solenoid yet more severe. Readers who accept indeterminism as harmless will perhaps find my other arguments more convincing. Taken as a totality, these arguments hopefully paint a convincing picture that something is going wrong with $\mathcal{W}_{AB}$.

As noted in §2, not all versions of the AB model involve an infinitely long solenoid (such as the toroidal setup). They are therefore immune to the objections just presented and one is left wondering why this important distinction between the alternative experimental setups hasn't yet been highlighted by philosophers.

3.3.2 Problems specific to the AB effect

The solenoid in $\mathcal{W}_{AB}$ is infinitely long and therefore presumably has infinite resistance. How can it sustain electric current for any finite amount of time? On a straightforward model of resistivity, the resistance of a conductor is given by

$$R = \rho \, \frac{L}{A}, \tag{5}$$

where ρ is the resistivity, L the length, and A the cross-sectional area of the conductor. In the AB model we let $L \to \infty$ at constant ρ and A and so $R \to \infty$ which by Ohm's law implies that there is no current in the solenoid. Admittedly, the resistivity model captured by equation (5) and Ohm's law are both laws which hold only approximately; however, it is widely accepted that for generic metal conductors such as the ones making up the solenoid in $\mathcal{W}_{AB}$, the approximation holds. We therefore have a contradictory set of premises at hand:

1. S_∞ obeys the resistivity law (5).
2. S_∞ obeys Ohm's law.

3. There is non-zero current flowing through S_∞.

One line of response to the above problem case would be to adjust the properties of the material making up the solenoid such that the new properties allow one to reject 1. or 2. or both, while retaining 3. Indeed, materials which violate the resistivity law or Ohm's law while retaining the ability to conduct electric current exist. However, a closer inspection reveals that existence of such materials does not help to evade the inconsistency problem. While a particular material may not obey (5), it will still have a well-defined specific resistivity tensor ρ which may be determined experimentally and which in principle can take different values throughout the material. However, it would seem that $R \to \infty$ behaviour as $L \to \infty$ will be a generic feature of any material except perhaps for ideal superconductors and similarly, one would expect $I \to 0$ as $L \to \infty$ for any material with $\rho \neq 0$. While low-temperature superconductors are now part of the condensed matter folklore, it would be preposterous to suppose that their resistivity vanishes *exactly* and so changing the material of the solenoid isn't of much help. We also note that the toroidal AB model is once again immune from such considerations since only a finite length of wire is needed to construct a toroidal solenoid. However, the next problem applies to the toroidal AB model as much as to the linear one.

Setting solid state physics aside, let us now focus on the impenetrability assumption **(F3)**. Recall that in the dynamics of the AB model, the impenetrability assumption was implemented by setting up a sequence of potential barriers $V_n = n\chi_L$ which tends to infinity at the solenoid as $n \to \infty$. It is this infinite potential which I regard as unphysical and which in my view exposes that $\mathcal{W}_{AB}$ is not a genuine physical possibility. I outline three worries related to infinite potentials in order of increasing seriousness:

- *Formal worry.* From a mathematical standpoint, if a potential diverges at a point, it is not defined at that point. If we therefore adopt a naïve interpretive stance according to which the possible potentials admissible in the theory are in one-to-one correspondence with smooth functions in $C^\infty(\mathbb{R}^3)$ (or some other space of functions depending on one's taste), infinite potentials would not be physically possible since their mathematical counterparts do not satisfy the membership conditions of $C^\infty(\mathbb{R}^3)$.

- *Discontinuity worry.* It is a well-known fact that infinite potentials lead to discontinuities in the first derivative of the wavefunction. A simple demonstration of this fact can be found in the infinite well scenario. Consider a particle moving on the real line subject to the following potential:

$$V(x) = \begin{cases} 0 & \text{if } x \in [0, a] \\ \infty & \text{else,} \end{cases}$$

  for some constant a. The solutions of the time-independent Schrödinger equation are then given by the following set of piecewise modes

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a}) & \text{if } x \in [0, a] \\ 0 & \text{else,} \end{cases}$$

for $n \in \mathbb{N}$, which is clearly discontinuous in the first derivative at $x = 0$ and $x = a$. However, discontinuities are highly unphysical and when they feature in our mathematical models, this usually hints at behaviour and physical structure beyond the scope of the model under consideration. Indeed, physical 'boundaries' generally have microscopic structure which would upon closer inspection reveal that the potential is not actually discontinuous but only appears so in the coarse-grained model. This ties in with the last, and in my view most serious worry.

- *Physical realizability worry.* It seems difficult if not impossible to imagine an infinite potential to be physically realized by systems consisting of typical atomic matter with typical interatomic interactions. Let us suppose that an incoming electron in the AB model moves under the influence of Coulomb potentials $V(r) = -\frac{k}{r}$ of the elementary particles at the solenoid boundary. If the solenoid consists of a finite number of such particles (like in the toroidal model), then the finite superposition of such finite Coulomb potentials cannot produce infinite potential at any point except for the pathological points $r = 0$.

  However, if the number of particles is infinite like in the linear AB model, a setup which contains points at which the potential diverges may indeed be created. Simply consider the real line and place a charged particle at each integer location starting at $n = 1$. The Coulombic potential at 0 is then proportional to the harmonic series which is known to diverge. However, just because this mathematical exercise allows us to produce a pathological situation does not mean that this situation actually obtains in physical systems like $\mathcal{W}_{AB}$ purports to be. Indeed it seems highly unlikely that a mechanism akin to the one just discussed would be responsible for the potential barrier at the solenoid boundary.

One may imagine responding to the formal worry along two possible avenues: (i) we modify the mathematical space of potentials to include structures which capture the sense of potential being infinite at a point, e.g., by working with Dirac delta distributions; (ii) we simply exclude the points of infinite potential from the domain/configuration space of the particle thus yielding a well-defined potential on a smaller domain. Strategy (i) fails because it does not seem to capture all the infinite potentials that one encounters in physical models. While the delta distribution captures a sense of potential being infinite 'at a point', it cannot be used to construct infinite-wells and similar scenarios in which the potential diverges on more than just a finite set of isolated points. Moreover, by promoting potentials to distributions one loses algebraic properties since distributions (unlike functions) cannot be easily multiplied together – a problem well-known form quantum field theory. It is not clear to me that the multiplicative property of potentials is in any way essential to physics; however, this limitation deserves to be flagged. Strategy (ii) is perhaps more promising and is in fact explicitly employed in the dynamical analysis of the AB model when we restrict the configuration space of the electron to $\mathbb{R}^3 \setminus S_\infty$. But does this mathematical trick indeed capture the concept of infinite potential? Arguably, it doesn't – rather, it captures the possibility of removing points from space. A potential is an intrinsic property of spacetime points. If one removes from the domain the points which were supposed to instantiate the property of infinite potential, then one no longer has available the property bearers to instantiate this intrinsic property. The model with points removed thus does not capture what it means for a potential to be infinite at a point, rather it captures what it means for there to be a hole in space.

3.4 Section Summary

I've presented both epistemic and metaphysical reasons to reject the physical possibility of $\mathcal{W}_{AB}$. It was argued that epistemic reasons are insufficient for this purpose and that a vindication AB scepticism must proceed along metaphysical lines. Subsequently, it was argued that metaphysical considerations indeed provide good reasons to reject physical possibility of $\mathcal{W}_{AB}$.

I take physical possibility of $\mathcal{W}_{AB}$ as necessary condition for the field realism dilemma and consequently, I take my analysis to demonstrate that there is no such a dilemma. My analysis thus suggests that we should answer to **Q2** in the negative. But does it also suggest that we should answer to **Q1** in the negative? I argue it does not.

4 Interpreting the Aharonov-Bohm Model

4.1 Earman and Norton on Idealization

In his discussions of the idealizations involved in the AB thought experiment derivation, Earman distinguishes between two senses of idealization: idealization in the first sense and in the second sense. Roughly speaking, the first sense involves intentional use of falsehoods to allow for a simplified discussion of an actual world target system, whereas the second sense involves an appeal to counterfactual scenarios involving fictional systems while remaining compatible with fundamental physics. Earman opens by noting that

> [i]t seems fair to say, however, that almost all of the attention in the literature on idealizations in physics has focused on one sense of idealization: the target system is an actual system, which may be as simple as a hydrogen atom or as complex as the earth's climate system or even the entire cosmos; the purpose of the idealization is to further understanding of the hows and why of the behavior of the target system and/or to facilitate predictions about some aspect of

> its behaviour; and the idealization involves intentional use of falsehoods or distortions in representing (or modeling, if you prefer) the target system.
>
> [In the second sense] the target system is an idealization in the sense of a fictional system, a system which is compatible with what in the context of inquiry is taken to be a fundamental theory of physics, but which is not realized in the actual world. [8, p. 1992]

Earman's distinction may *prima facie* look similar to the one made by Norton in [19] between approximations and idealizations. Norton characterizes his distinction by the following:

> An *approximation* is an inexact description of a target system. It is propositional. An *idealization* is a real or ficticious system, distinct from the target system, some of whose properties provide an inexact description of some aspects of the target system. [19, p. 3]

Superficial similarity between Earman's and Norton's accounts leads one up the garden path. In fact, however, the distinctions made by Earman and Norton are different in an important sense. While Norton's distinction captures *how* modelling of the target system happens (either by propositional inexact descriptions or by reference to a distinct real/fictional system), Earman's distinction captures the actuality/fictionality of the target system itself. Moreover, any combination of the two properties seem admissible. Let us demonstrate this fact on the AB model itself. Suppose that we wish to model the target system $\mathcal{W}_{AB}$ by $\mathcal{M}_{AB}$. Since the target system $\mathcal{W}_{AB}$ is non-actual, and since we model by reference to a mathematical model $\mathcal{M}_{AB}$, this constitutes an idealization in the second sense. Moreover, following Norton, we may peform a *demotion* of this idealization to obtain an approximation in the second sense: "An idealiztion can be demoted to an approximation by discarding the idealizing system and merely extracting the inexact descritption" [19, p. 5]. Similarly, if instead of $\mathcal{W}_{AB}$ we model one of the actual-world-realizable systems $\mathcal{W}_{L,n}$ by either $\mathcal{M}_{AB}$ or $\mathcal{M}_{L,n}$ we obtain an idealization in the first sense. By performing demotion, we then obtain an approximation in the first sense. Thus, it seems that Earman's and Norton's distinction are perfectly compatible and capture various possibilities of how $\mathcal{M}_{AB}$ and its 'more realistic' cousins $\mathcal{M}_{L,n}$ can model the physically impossible $\mathcal{W}_{AB}$ and the actual-world-realizable systems $\mathcal{W}_{L,n}$. The upshot of this discussion is that Earman's subsequent contention that the AB model should be understood as an idealization in the second sense does not stand up to scrutiny. We take up this issue in the next section.

4.2 Literal vs. Careful Interpretations

In [15], Lehmkuhl considers the problem of motion of GR and suggest a particular interpretative stance towards the singularities involved in the vacuum derivation of the geodesic equation from the Einstein field equations. According to Lehmkuhl, the singularities occurring in the model should not be interpreted *literally* as genuine singularities in an empty space but rather *carefully* as placeholders for genuine material particles. The difference between literal and careful interpretations is further illustrated the following examples:

- **Schwarzschild solution**. Interpreted *literally*, the Schwarzschild model would seem to describe a rather unphysical scenario: a singularity in an empty universe. Alternatively, one may interpret the model *carefully* and take it to represent the gravitational field outside an actual stellar object such as the Sun or Sagittarius A*.
- **Fluid dynamics**. While in reality the fluid is a swarm of rapidly moving, nearly point-like particles, for the purposes of fluid dynamics, one assumes it to be well approximated and represented by velocity vector fields subject to the Navier-Stokes equation. Interpreting models of fluid dynamics *literally*, one would be thus led to believe that fluids genuinely have the structure of 'mereological gunk': an object all of whose parts have further proper parts. But this is plainly wrong and also goes very much against the spirit of kinetic theory. Interpreting the models *carefully*, one presupposes a separation of scales and takes the velocity vector at every point to represent the average velocity of fluid molecules at that point.

As previously noted, Earman believes we should understand the AB model as idealization in the second sense:

> [...] the target system in the AB effect is a fictional system, and there is no idealization in the usual sense—no distorted/false description of an actual world arrangement of magnets and electrons—but rather an accurate and precise description of an other-worldly arrangement. [8, p. 1993]

I argue this misses the point in an important sense. For one, it would seem to commit Earman to a rather literal interpretation of the AB model $\mathcal{M}_{AB}$. While in principle there is nothing wrong with maintaining that a model represents literally, usually this is a rather naïve standpoint. This is supported by the examples of literal and careful interpretations considered above as all of the literal interpretations correspond to rather unphysical situations. But more importantly, if the AB model, as Earman maintains is an idealization in the second sense, then it represents the 'other-worldly' arrangement $\mathcal{W}_{AB}$. But $\mathcal{W}_{AB}$ has just been rejected as physical impossible! This suggests that the literal interpretation of $\mathcal{M}_{AB}$ is not correct. After all, it is a very plausible minimal requirement to impose on the representation relation between models and worlds that the worlds must be physically possible. Instead, I propose a careful interpretive standpoint towards $\mathcal{M}_{AB}$ which is depicted in 4.2 together with the literal interpretation. For simplicity, only a subset of the original lattice of models and worlds is depicted.

On the careful interpretation, $\mathcal{M}_{AB}$ is used to model any of the actual-world-realizable systems $\mathcal{W}_{L,n}$. As such it may still deliver good approximations to the experimental data observed in the AB effect, especially if the actual solenoid

$$
\begin{tikzcd}
\mathcal{M}_{AB} \arrow[r] & \mathcal{W}_{AB} \\
\vdots & \vdots \\
\mathcal{M}_{\infty,n} \arrow[r] & \mathcal{W}_{\infty,n} \\
\mathcal{M}_{\infty,n-1} \arrow[r] & \mathcal{W}_{\infty,n-1}
\end{tikzcd}
$$

Figure 3: Literal interpretation of $\mathcal{M}_{AB}$. The arrows suggest 'interpreting literally'.

$$
\begin{tikzcd}
\mathcal{M}_{AB} \arrow[ddr, dashed] \arrow[dddr, dashed] & (\mathcal{W}_{AB}) \\
\vdots & \vdots \\
\mathcal{M}_{\infty,n} \arrow[r] & \mathcal{W}_{\infty,n} \\
\mathcal{M}_{\infty,n-1} \arrow[r] & \mathcal{W}_{\infty,n-1}
\end{tikzcd}
$$

Figure 4: Careful interpretation of $\mathcal{M}_{AB}$. The dashed arrows suggest 'interpreting carefully'.

is sufficiently long and impenetrable. Moreover, on the careful interpretation, $\mathcal{M}_{AB}$ should be thought of as idealization *in the first sense*! This is because the $\mathcal{W}_{L,n}$ are actual-world solenoid systems.

We are now in a position to answer **Q1**: does the AB model represent a physical possibility? The answer of course depends on the representation relation we choose. On the literal interpretation, $\mathcal{M}_{AB}$ represents $\mathcal{W}_{AB}$, which has been rejected as physically impossible and so the AB model does not represent a physical possibility. However, on the careful interpretation, $\mathcal{M}_{AB}$ represents some of $\mathcal{W}_{L,n}$ which are all genuine physical possibilities and actual-world-realizable solenoid systems. Therefore, on the careful interpretation the AB model represents a physical possibility. Moreover, it has been argued that the careful interpretation is preferable to the literal one and so we should answer to **Q1** in the positive with the proviso just described.

5 Rejecting the dilemma for field realism (again)

In §3, I already rejected the dilemma for field realism on the basis of physical impossibility of $\mathcal{W}_{AB}$. However, there are further independent reasons for why we should not take the dilemma too seriously which relate to the representation business just discussed.

Returning to Earman's two senses of idealization, let us note a further important feature of this distinction which Earman highlights:

> [o]n the first sense of idealization, where the target system is an actual world system, an effect is dismissed as a mere artifact of the idealization if it disappears when the idealization is made more realistic. One the second sense of idealization the center of interest is on effects that should, according to said theory, be exhibited by the

> fictional system. The goal of studying such effects is to illuminate the foundation of said theory and its relationship to predecessor and to competing theories. [8, p. 1992]

I argue that the non-local action of the magnetic field is precisely ‘a mere artifact of the idealization’ and subsequently connect this with scepticism regarding the temporality of the Malament world voiced by Brown in *Physical Relativity* [4].

Consider the lattice of models $\mathcal{M}_{L,n}$ and the lattice of propositions ‘the magnetic field acts locally in $\mathcal{M}_{L,n}$,’ which we denote by $P_{L,n}$. Moreover, consider $\mathcal{M}_{AB}$ and the proposition ‘the magnetic field acts locally in $\mathcal{M}_{AB}$,’ which we denote by P_{AB}. Note that $P_{L,n}$ is true for all $L \in \mathbb{R}$ and $n \in \mathbb{N}$ but, as traditionally argued in the AB debate, P_{AB} is false. We may then rerun the limiting procedures described in §2 and conclude that the limiting truth value of $P_{L,n}$ does not agree with the truth value of the limit property P_{AB}. Situations of this kind have usually been rejected as pathological. For example, according to Norton: “[i]n these cases, the infinite limit system fails to provide an idealization” [19, p. 12]. Moreover, according to Earman’s own criterion applied to $\mathcal{M}_{AB}$ as idealization in the first sense, as we make the system more realistic by considering the $\mathcal{M}_{L,n}$, the non-local action of the F-field disappears and therefore should be rejected as a mere artifact of idealization.

Let me now make a connection with the another kind of scepticism introduced in §3. In *Physical Relativity* [4], Harvey Brown raises doubts regarding certain counterfactual scenarios involving impoverished spacetimes of the special theory of relativity. Specifically, Brown criticises Malament’s alleged proof of non-conventionality of distant simultaneity (see [17]) which relies on the physical possibility of an impoverished world containing nothing but a single worldline of an inertial observer. Such “Malament worlds”, according to Brown, tell us very little about the metaphysics of space and time. I quote at length:

> The Malament world is so utterly different from ours, I think it is legitimate to ask whether it even contains time at all. It is not enough to say that being four-dimensional, the space-time manifold therein has time built into it. We are doing physics, not mathematics. [...] Time, at its most fundamental level, has something to do with change, and change is not and obvious feature of the Malament world.[4] [4, pp. 100–101]

Brown’s scepticism is also metaphysical akin to that of Leibniz but in a more local sense. Brown does not invoke grand metaphysical principles like PII and PSR to reject the physical possibility of the Malament world,[5] nor does he comment on our knowledge of the Malament world like a Machian sceptic would.

[4]In footnote accompanying this passage, the reader of Brown’s book is gently reminded to take care in assessing the meaningfulness of wildly counterfactual claims. To illustrate the point, Brown considers the felicitous: ‘If my grandmother had four wheels, she would be a bus.’ (see [4, p. 100])

[5]Brown does however echo Leibniz when he notes that different snapshots of the observer along the worldline are indistinguishable and represent identical situations.

Rather, he assumes a sceptical stance towards specific features of the Malament world, namely it's temporality, and towards metaphysical conclusions about non-conventionality of distant simultaneity drawn on the basis of there being such a world.

One important aspect of Brownian scepticism regarding Malament's proof is that that upon making the Malament world 'more realistic', e.g., by adding additional observers and material structure, non-conventionality no longer follows as the Malament construction may now be applied to any of the additional inertial worldlines leading to a proliferation of admissible simultaneity relations. It seems that this is yet another instance of a "mere artifact of the idealization" in Earman's sense. If we interpret Brown as rejecting the physical possibility of the Malament world, the corresponding Malament model must then represent as idealization in the first sense, if at all. However, since the desired metaphysical conclusions don't follow once the model is made more realistic, we reject these metaphysical conclusions as mere artifacts of idealization. This is of course conditional on the antecedent rejection of the Malament world as physically possible and so one cannot avoid the heavy-duty metaphysical work performed in full in Brown's book, but once the metaphysics has been settled, the analogy with the situation surrounding the AB model is complete.

6 Harmless Idealizations

Having completed the main task of this paper, I now review some additional idealizations involved in the AB model which, however, don't seem to threaten the physical possibility of $\mathcal{W}_{AB}$ and which would be present also in the less-idealized models $\mathcal{M}_{L,n}$. Both Earman and Dougherty comment on the use of the so-called 'bastardized theory' (see [8, p. 2016] and [7, pp. 12215–12221]) in the dynamical analysis of the AB model. In the bastardized theory, the electromagnetic field is treated as classical while the incoming particles enjoy quantum mechanical treatment. According to Earman,

> [...] the bastardized form of quantum electrodynamics in which the AB effect is usually discussed–an external unquantized electromagnetic field and an electron quantized in non-relativistic QM–is not a felicitous setting in which to try to resolve issues about locality vs. nonlocality. A more appropriate context would be relativistic quantum field theory (QFT). [8, p. 2016]

One should press Earman on his contention that the analysis should be rather carried out using relativistic QFT. Why? The AB thought experiment makes perfect sense in its current semi-classical setting and as such it purports to deliver a case *about classical electromagnetism.* That interpretation of false physical theories can bear philosophical fruit and even deliver insights about the actual world has been argued for by Belot in [3]. According to Belot, the AB effect is a paradigm example of such a situation and insofar as it constraints our beliefs about where the actual world might be located in the space of possible worlds, it delivers an insight about the actual world.

Sticking with Earman's reasoning, wouldn't we be led to conclude that *all* discussions should be carried out using QFT, our best theory of matter? Firstly, this would be computationally intractable, and secondly, we should note that many lessons may be learned by analyzing less-than-most-mature physical theories, albeit these lessons must be be understood as applying only contextually in the context of those theories. Dougherty also makes the following apt observation about quantum electrodynamics:

> Presumably the thought is that the AB effect is an issue in the foundations of physics, and therefore we should set our discussion in our most fundamental description of the system at issue. But on the topological view, the AB effect is part of the justification for our more fundamental theory of the electromagnetic interaction, because it constrains the classical limit of theory. And even if we forget about justification, QED presupposes that the principal connection interpretation is right and the field strength tensor interpretation wrong. QED lacks the ambiguity that Aharonov and Bohm set out to resolve, so we can't devise an experimental context in which the AB effect would be informative. [7, p. 12221]

In other words, quite independently of the AB thought experiment, QED privileges one ontological view over another. Analysing the AB effect in relativistic QFT would therefore be simply futile.

Lastly, it should be noted that the AB model ignores various kinds of background noise which would most certainly interfere with any actual-world experiment (see [7, p. 12207]). Such ignorance however is commonplace in physics in so far as the noise does not significantly affect outcomes of experiments and does not threaten the physical possibility of $\mathcal{W}_{AB}$.

7 Conlcusion

The conclusions of this paper undermine one of the main arguments against field realism: that it is a non-local ontology. The alleged dilemma for field realism – which could be seen as the driving force behind the various ontological proposals which preserve locality – is therefore not a dilemma after all. This is because the world $\mathcal{W}_{AB}$ which allegedly exhibits the non-local action of magnetic fields and which corresponds to the literal interpretation of $\mathcal{M}_{AB}$ is not physically possible. This already removes the alleged dilemma and also suggests that $\mathcal{M}_{AB}$ must be understood as idealization in the first sense contrary to what Earman proposes. Because of this the features of the model which disappear upon making it more realistic which include the non-local action of the magnetic field should be discarded as a mere mathematical artifact of the idealization. This is not to discard the rival ontological proposals or completely discredit the work done on this topic; however, the relevance of the AB effect for the ontology debate becomes tenuous in light of my findings.

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