Recalcitrant Anomalies, Ignorance, Insights, and Understanding: A Structuralist Approach

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Forthcoming in A.I. Mărăşoiu and M. Dumitru (Eds) *Understanding and Conscious Experience: Philosophical and Scientific Perspectives*, Routledge.

Abstract

Here, we aim at elucidating the epistemic grounds by which scientists traverse the path from ignorance to insight to scientific understanding within the realm of empirical sciences, specifically focusing on cases marked by the existence of *recalcitrant anomalies*. We contend that scientific understanding is structuralist. Furthermore, we contend that the partial overcoming of anomalies between scientific theory and observation resides in the finding of some structures that allow for reconciliation and integration of the resulting inconsistencies and explanatory gaps by providing novel inferential pathways and ways of making sense of dependency relations. Moreover, we claim that this type of understanding is gradual in that it begins with insight, understood in our view as a way of grasping what type of structures might be helpful in overcoming the above difficulties and leading eventually to a more full blown understanding involving systematic integration and subsumption of theoretical posits and empirical observations within theoretical structures.

Keywords: scientific understanding, structural realism, anomalies, ignorance, insight

1 Introduction

Scientific understanding —henceforth, 'understanding'— is frequently employed synonymously with the expression 'making sense'. Contemporary epistemologists have agreed that understanding is (mostly) about the successful identification of relations of dependence. So, '*understanding P*' allows one to make all kinds of correct inferences about it so that '*P* makes sense'.

In contemporary epistemology, understanding is thought to be composed of equally important subjective as well as objective elements. On the one hand, regarding the former, understanding has characteristic psychological indicators (i.e., the feeling of grasping and first-person authority). On the other hand, understanding's objectivity is often thought as resulting from its factivity; particularly from having grasped a *true* explanation of a subject matter (Cf. Trout 2002, 2007; Strevens 2011). This conception of understanding portrays it as the combination of different epistemic and psychological successes. It seems that only when one has succeeded significantly in previous tasks one can aim at understanding.

However, instances where understanding stems from secure foundations constitute the minority. In the majority of cases, understanding is built through extremely complex dynamics on less-than-perfect grounds, in which ignorance and uncertainty play key roles. In particular, when intellectual challenges motivate the pursuit of understanding, ignorance, uncertainty, and opacity, among others, are significantly present during the quest for understanding. For instance, in the empirical sciences, a particularly compelling motivation for the pursuit of understanding is the discovery of an anomaly; and more importantly, it is often the case that anomalies lead the way to the pursuit of understanding. An example of this might be found in the reaction from the physics community to the correct prediction delivered by General Relativity for the precession of Mercury's perihelion, which at the time was taken to be a (recalcitrant) anomaly *vis-a-vis* Newtonian gravitational theory. In this case, what started as an intriguing scientific puzzle, when solved, led to a new understanding of the world.[1]

Here, we aim at elucidating the epistemic grounds by which scientists traverse the path from ignorance to insight to scientific understanding within the realm of empirical sciences, specifically focusing on cases marked by the existence of *recalcitrant anomalies*. We do so by adopting a structuralist take on understanding (*modal understanding*), for which understanding consists of the positing of some structure (functions and relations) as holding for some objects as a means of making sense of the dependence relations between the unobservable and observable parts of a given subject matter.

We contend that scientific understanding is structuralist in the above sense. Furthermore, we contend that the partial overcoming of anomalies between scientific theory and observation resides in the finding of some structures that allow for reconciliation and integration of the resulting inconsistencies and explanatory gaps by providing novel inferential pathways and ways of making sense of dependency relations. Moreover, we contend that modal understanding is gradual in that it begins with insight, understood in our view as a way of grasping what type of structures might be helpful in overcoming the above difficulties and leading eventually to a more full-blown understanding involving systematic integration and subsumption of theoretical posits and empirical observations within theoretical structures.

To do so we proceed as follows. Section 2 addresses the basic elements of genuine understanding. Section 3 focuses on the connection between understanding and epistemic success and challenges it by appealing to both the gradability of understanding and the constant co-presence of ignorance and understanding. Section 4 focuses on the relation between ignorance, recalcitrant anomalies, and insight – as a particular instance of scientific understanding. Section 5 introduces our structuralist take on scientific understanding as an alternative to account for both the relation between ignorance and understanding as well as the dynamic character of understanding. Section 6 sketches a structuralist path from the detection of recalcitrant anomalies to ignorance, followed by insight and then by understanding, and illustrates it with a case study from physics. Finally, Section 7 draws some conclusions.

2 The basics of (genuine) understanding

We start with some preliminaries on the basic elements of understanding (Section 2.1). We explore how, when combined, these elements provide a (very optimistic) picture of understanding which combines "elements of rationalism and empiricism in a way that one

[1] We go back to this example in Sec. 4.2 and 6.2.

does not find with ordinary instances of knowledge" (Grimm 2018:4). We then explore the relation between understanding and truth, and, from a less optimistic perspective, we introduce some concerns regarding the impact of such a relation in the acquisition of understanding in scientific contexts (Section 2.2).

2.1 The subjectivity of making sense

In what follows, we focus only on cases of understanding concerning both scientific theoretical frameworks (theories/models) as well as scientific phenomena in the empirical sciences. Understanding has been traditionally seen as the neat identification of "relations of dependence. When one understands something, one can make all kinds of correct inferences about it" (Ylikoski 2013: 100). Broadly speaking, to understand something is to be able to make sense of it given one's background knowledge. This considered, it is thought that understanding surpasses the mere accumulation of discrete fragments of knowledge and that it is either a *sui generis* type of knowledge or even a different type of epistemic product.[2] One of the aspects that make understanding (as *making sense*) so special is its inner balance between subjective and objective features.

The subjective side of understanding refers to the active role that individuals take in constructing and recognizing their own understanding. There are two main intertwined elements of the distinctive psychology of understanding: transparency and the sensation of grasping.

- *Transparency*. Understanding is transparent in the sense that one is always in a position to recognize whether or not one can 'make sense of *P*' given a specific subset of one's background knowledge. Understanding requires a sort of first-person authority concerning one's capability of connecting the relevant data in a meaningful way.[3]
- *Sensation of grasping*. Such a sensation corresponds to a mental 'click' that signals a sudden realization of the subject at hand (Cf. Grimm 2018); in this sense, there is a significant level of transparency about having acquired the ability to 'make sense' of something.[4] In particular, this experience indicates an awareness of acquiring a newfound ability to connect fragments of information, revealing a coherence that is more evident when these elements are considered together rather than in isolation.

However, understanding cannot be characterized only by its psychological and phenomenological components. The main reason for this is that, though the psychology of

[2] It might be that the knowledge required for understanding some proposition or aspect of reality is fully general and *a priori*. This characterization is not meant to rule out those cases, nor is it necessary for us to take a stand on the sources of our knowledge of *a priori* truths.
[3] It is important to emphasize that this idea of transparency does not require the factivity of what is being understood as it only concerns the psychological side of understanding.
[4] Note that this psychological side of grasping is independent of the factivity of the subject matter.

understanding is unique when guided by their individual perceptions, agents often overestimate the detail, coherence, and depth of their understanding. This phenomenon is called *the illusion of depth of understanding* (Cf. Rozenblit and Keil 2002; Keil 2003; Mills and Keil 2004; Ylikoski 2013); and it is what motivates the seeking of objectivity when assessing one's achievement of understanding.

2.2 The objectivity of understanding

The objective side of understanding refers to the elements that delineate its distinctiveness from subjective individual perceptions – either by allowing for exhaustive critical revisions of its legitimacy or by testing its connection with mind-independent domains. In a sense, the objectivity of understanding responds to the possibility of individuals misjudging the reliability of their perceptions. For this reason, understanding is expected to include a grasping of a fragment of 'reality' – broadly constructed (Cf. Elgin 2007: 35). Some of the key elements that are behind the objective side of understanding are its structural character, an ordered and coherent basis, as well as the adequacy of what is being understood.

- *Structural character*. Broadly speaking a structure is a plurality of specific relationships and patterns that hold among objects/elements, independently from the particular properties of the objects/elements themselves. Understanding is, at its core, a structural task as it consists of organizing and interconnecting information; essentially, constructing a framework capable of satisfactorily relating sets of data to a particular subject matter (Cf. Macías-Bustos and Martínez-Ordaz 2023).

- *Ordered basis*. Order is a structure that indicates the arrangement of elements in a sequence or set, following a particular criterion, emphasizing regularity, sequence, or pattern inherent in that arrangement. It simplifies the analysis and manipulation of information. In the cases of understanding, elements are arranged in sequences or orders that reflect specific relationships (temporal and spatial connections, as well as causal and emergence relations, among others).

- *Coherent basis*. Understanding is partly the result of the exercise of appropriate epistemic virtues on the part of agents. In particular, coherence plays a key role in securing the reliability and stability of inferences that are carried out within an understanding-type structure. Coherence requires consistency, topical compatibility, and reinforcement (Cf. Elsamahi 2005). To understand something is to arrange the components of the subject matter coherently (Cf. Bengson 2018: 19).

- *Empirical adequacy*. Broadly speaking, a theory is empirically adequate if it is consistent, explanatory and broadly predictive with empirical measurements and observations. Some theories about complex empirical domains are limited in their predictive power but can at the very least give some broad forecasts as long as some antecedent conditions are fulfilled and can also explain past data. For example, climate models, cosmological models, and subjects such as macroeconomics have varying degrees of empirical success along those metrics.

Similarly, empirical adequacy of understanding also requires consistency, explanatory power, and predictiveness concerning the subject matter. The adequacy component is often seen as indicative of the factivity of understanding and it constitutes one of the key points of discussion regarding the nature of scientific understanding.[5]

2.3 The emergent features of understanding

When the combination of the subjective and objective sides of understanding is successful, five overall features of understanding are exhibited: gradability, epistemic robustness, intelligibility, a standing state, and praiseworthiness. It is important to note that these features are the result of a balanced satisfaction of all previous elements and that they allow for more accurate assessments of the understanding possessed by others.

- *Gradability*. Degree or extent to which an agent has understood a particular subject matter. It involves a relative scale or intensity of both the satisfaction of subjective and objective features. The interpretation of degrees of understanding can be context-dependent. The same grasping of the subject-matter may convey different degrees of understanding in different situations and depending on different demands of expertise. Gradability allows for the possibility of improving or deepening one's understanding.
- *Epistemic robustness*. Involves the resilience, strength, and reliability of understanding in the face of challenges. This supports the idea that once legitimate understanding is attained, it is challenging to find reasons to relinquish it.
- *Intelligibility*. Can be broadly defined as the epistemic virtue reflecting harmony between an agent's beliefs about P and their actions around P (see Chang 2009). Intelligibility is, in a sense, a result of combining coherence and order, extending the scope of understanding into a performative realm.
- *The standing state*. Understanding appears distinct from beliefs and knowledge. Understanding can influence epistemic elements by reinforcing or weakening them. When one understands something, it becomes possible to comprehend how certain epistemic elements interrelate to construct a more cohesive understanding.
- *Praiseworthiness.* Given its complexity, attributing understanding to an individual is not merely acknowledging a kind of success but is also a form of compliment or praise (Cf. Bengson 2018: 19).

[5] We are fully aware of the extensive discussions regarding the factivity of understanding (or lack thereof). However, we adopt a broader notion here to remain neutral concerning that particular issue. For extensive analyses of the state of the art in this particular regard see Lawler (2021) and Taylor (2023).

Summing up, intuitively, genuine cases of understanding should involve a sense of self-*transparency, a sensation of grasping, a structural character,* an *ordered and coherent basis*, and some kind of *adequacy*; as well as a display of *gradability, epistemic robustness, intelligibility, a standing state* as well as a significant level of *praiseworthiness*.[6]

From the outset, we want to be clear about the dialectic. Ongoing discussions persist regarding the necessity and sufficiency of these characteristics in the evaluation of genuine manifestations of understanding. However, there is an agreement about the fact that none of the elements described above, whether taken individually or in partial combinations, is thought to be sufficient, uniformly, to satisfactorily legitimize cases of understanding. In the large majority of instances, the partial combinations of these features might even be misleading or a source of the so-called *illusion of depth of understanding*. Our aim in the next section is to show, however, that the combination of these features might be too strict to account for non-ideal cases of the acquisition of understanding in scientific contexts, we focus particularly on those scenarios in which ignorance plays a crucial role.

3 Understanding and Success: the main problem

As some of its emergent features might suggest, understanding is in many ways the result of individual agents' epistemic success. In addition, the setup in which understanding is thought to take place is such that it also requires previous scientific success, at least in the sense of having revealed the subject-matter and its relations through the use of theoretical frameworks; leaving the useful information ready for agents to grasp it. The cases in which this is more evident are those in which young scientists learn to see and to explain the world through a well-established theoretical framework.

This section is devoted to exploring the demands of success that are made by understanding scientific contexts and challenging the plausibility of fulfilling them in all cases of genuine understanding.

3.1 Understanding as the result of success

Understanding is often seen as one of the most valuable goals of science, mainly because of how challenging the journey to understanding seems to be. Understanding requires the exhaustive gathering and evaluation of information, the structuring of such information, the reflection and later awareness of the resulting epistemic good(s), and finally, its exercise in different contexts. Combined, these result in the standing state of understanding as well as of its praiseworthiness.

Despite common agreement on its value in contemporary epistemology, there are still significant open debates about the nature of understanding that go as deep as to tell different stories about the type of road that leads to it. For instance, there is a view according to which understanding is a type of knowledge, and in that sense, it requires at

[6] For a similar characterization of the components of understanding, see Bengson (2018: 19-20).

least belief, truth, and justification. In contrast, there is also a view on which understanding does not require belief (therefore, knowledge cannot be an essential feature of understanding), because acceptance alone suffices in many cases. Yet another standpoint has it that understanding can sometimes be identified as an ability. This diversity of views also roots a (non-convergent) variety of possible ways to achieve understanding.

First, the view according to which understanding is a type of knowledge is a popular one (Cf. Woodward 2003; Grimm 2006, 2016; Lipton 2009; Kelp 2015, 2017; Sliwa 2015; Newman 2017; among others). This characterization takes understanding to be, broadly speaking, *knowledge about relations of dependence* – here, dependence is typically understood in a causal way. As with any other type of knowledge, it involves a belief, the truth of such a belief, and a justification for it. However, what makes understanding so special is the type of object that it targets: privileged relations. When assessing the type of relations that matter for the acquisition of understanding, much attention has been devoted to causal explanatory relations. This is motivated by the close connection expected to exist between explanation and understanding: "All scientific explanation … seeks to provide a systematic understanding of empirical phenomena by showing that they fit into a nomic nexus" (Hempel 1965: 488).

That said, if understanding is knowledge about relations of dependence, in particular, a kind of causal explanatory knowledge, it can be demonstrated either by one's successful interventionist interaction with the subject-matter, by one's ability to anticipate how the subject-matter behaves, or by exhibiting the ability to explain and communicate one's understanding (Cf. Ylikoski 2013).

According to this first view, the path to understanding goes from the adoption of a theoretical framework (*T*), to the accumulation of factual knowledge about different elements of a given subject matter (*P*), the interpretation of those elements of *P* given *T*, the identification of the causal links that hold between elements of *P* according to *T*, the construction of *T*-explanations grounded on such links, the awareness of such grasping, as well as the later manipulation of the subject-matter —at least, within the scope of *T*.

Second, there is another view, however, according to which *understanding might not require belief, but only acceptance* (Cf. Elgin 2004). Believing *p* means that "one is disposed to normally disposed to feel it true that *p* (and false that not-*p*) when one is attending to issues raised by *p*. By contrast, one accepts that *p* just in case one treats it as given that *p*" (Hannon 2021: 276); the difference might seem only in the strength of the endorsement, however, such difference can allow for the possibility of gaining a genuine understanding of a broader group of objects. This view on understanding neatly describes cases in which scientists explore how new theories might explain targeted phenomena before even considering candidates for (partial) truth. To illustrate this view, Dellsén (2017:249) discusses the case of theoretical physicists who might use string theory to explain empirical phenomena without necessarily taking it to be true, and more importantly, knowing that they lack the experimental resources to test the theory on any empirical domain. What is salient of scenarios like this one is that, despite any methodological reservations that scientists might entertain, they can still grasp phenomena with the help of the theory –

even if that grasping does not imply that the phenomenon is actually what the theory says it is.

Understanding phenomena through theories, especially if one doesn't believe them to be true (to obtain) or if one doesn't have the means to test their empirical reliability, requires an extremely high degree of knowledge about the theory in question up to the point at which one can reconstruct how the world would look if the theory were true. In this sense, if understanding doesn't require belief but acceptance, scientists still have to master the logical, mathematical, and conceptual apparatuses of the theory before being able to satisfactorily use it to understand anything else.

Third, the different ways in which one can gain and demonstrate one's understanding, leave the impression that understanding could be closer to an *ability* than to explanatory knowledge. The key element in the ability-based approach to understanding is 'grasping'. While the large majority of epistemologists of science have agreed on the fact that grasping plays a crucial role in understanding, there is no consensus on the specifics of its nature. For instance. according to some, grasping could require the *know-how* to wield a subject-matter to further one's epistemic ends (Elgin 2017:33). Others have seen understanding as an ability to answer counterfactual questions Grimm (2006; 2014) or to satisfactorily evaluate explanations (Khalifa 2013).

The ability-based approach posits an alternative road to understanding. More than theoretical explorations, it necessitates prior substantial cognitive engagement, fostering an intuitive proficiency that enables agents to aptly assess explanations or potentially use frameworks (theories, models, etc.)for other tasks. More importantly, because abilities do not inherently mandate explicit transparency concerning the origin of their dependability, the requisite level of expertise for agents surpasses that needed in scenarios where procedures are either explicit or, at a minimum, transparent.

The three outlined accounts of understanding underscore the notion that scientific understanding emerges as the outcome of exceptionally successful epistemic activities. Whether through the meticulous acquisition of explanatory knowledge, the apt navigation of theoretical frameworks through acceptance, or the intuitive proficiency fostered by ability-based approaches, each perspective highlights the demanding and triumphant nature of these intellectual endeavors. However, a question persists: to what extent do these idealized scenarios align with the complex realities of scientific practice? Exploring the application and efficacy of these epistemic activities within the dynamic landscape of scientific inquiry raises important considerations regarding the feasibility and prevalence of such extraordinary success in the pursuit of understanding.

While more could be said here, we hope it is clear that the most prominent accounts concerning the nature of understanding suggest that the praiseworthiness and the standing state of understanding are grounded in the prior achievement of great epistemic success.

3.2 The gradability of understanding and its connection with ignorance

As understanding is extremely valuable, it is also significantly gradual. The gradability of understanding stems from two sources. Firstly, individual cognitive limitations introduce a complexity that hinders agents from fully grasping a subject-matter in a singular cognitive endeavor. These inherent constraints necessitate repeated engagement in order to achieve a more comprehensive understanding. Secondly, the stage of scientific development at a given moment contributes to the gradability of understanding by imposing limitations on agents' access to pertinent information.

The epistemic endeavor of 'making sense' under uncertainty assumes heightened significance during pivotal instances of scientific discovery. These junctures are marked not only by the challenges presented through a lack of clarity and information on the subject-matter but also by the prior absence of mastery of the theoretical framework used by individual agents to tackle the target object of understanding. In particular, the so-called 'Eureka moment', emblematic of scientific discovery, applies to a scenario in which uncertainty and ignorance emphasize either the abruptness of understanding acquisition or the dynamic nature of it. And more importantly, it challenges the type of relationship that is expected to hold between understanding and success.

Such challenges can be summarized as follows:

1. Understanding is inherently praiseworthy and holds a standing state when compared to other epistemic products.

2. Prior exceptional epistemic successes are commonly seen as necessary conditions for understanding (as discussed in Section 3.1).

3. The *Eureka moment* is a peculiar instance of abduction that responds to well-recognized and targeted ignorance concerning a given subject-matter. The Eureka moment signifies emergent understanding within a triadic relation, connecting observed phenomena with explanatory hypotheses. This realization is embedded in the continuous process of inquiry for the satisfactory handling of ignorance and uncertainty, emphasizing the continuity of knowledge acquisition (Cf. Peirce 1992, Aliseda-Llera 2023).

4. Eureka moments, commonly considered exemplars of understanding, involve a composite interplay of psychological, objective, and emergent features, concomitant with the sudden realization of overcoming substantial intellectual challenges (in particular, ignorance).

5. The understanding witnessed in Eureka moments can be analyzed dichotomously: either as a sudden grasp of the subject-matter (focusing only on the final product of only successful inferential endeavors and the solution of intellectual challenges) or as the result of a complex process of information acquisition, handling of ignorance, evaluation of information, and interconnection.

6. If understanding, in Eureka moments, is construed as a sudden grasp, it risks undermining its praiseworthiness and standing state, potentially aligning it more with epistemic luck than a deliberate search for privileged relations.

7. If understanding in Eureka moments is framed as a dynamic product, continuously evolving through a gradual process, this preserves its praiseworthiness but at the same time weakens the traditionally stringent requisites for epistemic success.

∴ A dilemma arises concerning the nature of understanding in Eureka moments. On one horn, if understanding is sudden, it may forfeit a significant degree of its praiseworthiness; on the other horn, if it is gradual and dynamic, it risks compromising the robust prerequisites traditionally associated with epistemic success.

This dilemma invites a reevaluation of the philosophical underpinnings of understanding within the context of Eureka moments, challenging more conventional intuitions of both sudden insight and gradual gaining of understanding.

It is important to note that while Eureka moments constitute one extreme of a continuum, the gradability of understanding suggests that different degrees of the unknown (ignorance, uncertainty, etc.) should be considered when assessing mid-accomplished cases of understanding in scientific practice. The possibility of considering Eureka moments as legitimate instances of understanding, together with cases in which the road to understanding has been much clearer and straightforward, leads to the consideration of middle cases, those in which there is a gain of knowledge, and later understanding, but also the preservation of important levels of the unknown.

The analysis of the gradability of understanding naturally leads to the question of when an individual's grasp can be deemed to constitute understanding. This directs attention to the complex interplay between knowledge, ignorance, and unresolved puzzles within the cognitive and epistemic landscape of a scientific practice. To us, it seems quite clear that in many scenarios, understanding emerges not merely as an endpoint but as a dynamic process influenced by the continual negotiation between what is known, what remains unknown, and the recalcitrant presence of scientific puzzles.

Let's clarify the point we're making: we're not suggesting that understanding something is compatible with ignoring certain aspects of it. Our primary focus is on the idea that acknowledging one's ignorance, particularly after gaining an understanding of a specific subject, can provide valuable insights into the limitations of that understanding and suggest potential avenues for overcoming existing gaps in knowledge. In this way, ignorance can both spark the pursuit of understanding and, paradoxically, define its boundaries while offering clues for addressing similar intellectual challenges (ignorance, uncertainty, etc.).

To sum up, we have taken this section to show that, while the clearest/neatest cases of understanding give the impression that the road to understanding is paved with epistemic success, more ordinary instances lead a different way, that of ignorance, uncertainty, and other intellectual challenges.

What we think is at stake here is the conflict between the seemingly normative spirit of philosophical characterizations of understanding and the more descriptive nature of intuitive attributions, particularly in what concerns the role that success plays in coming to understand. With that concern in mind, in the rest of the paper, we tackle the question of what are the epistemic grounds by which scientists traverse the path from ignorance to scientific understanding within the realm of empirical sciences while still dealing with ignorance and uncertainty.

4 Ignorance, anomalies and insights

So far we have discussed the relationship that might exist between scientific understanding and epistemic success. We have also challenged the strength of such a relationship by appealing to both the gradability of understanding and the constant presence of ignorance and uncertainty in scientific activity.

Now we want to press that point further: this section is devoted to discussing the relation between ignorance, anomalies, and a particular instance of scientific understanding, insight. To do so, we start with some preliminaries on ignorance, then move to characterize scientific anomalies as instances of ignorance in science, and finally, we discuss cases of insight as cases of understanding.

4.1 (Preliminaries on) ignorance

In contemporary epistemology, ignorance has conventionally been construed as *lack of knowledge*.[7]

Ignorance as the absence of knowledge can be of different types:

- *absence of factual knowledge*: factual ignorance,

- *absence of objectual knowledge*: objectual ignorance,

- *absence of procedural knowledge*: procedural ignorance,

- *absence of answers to questions*: erotetic ignorance,[8]

- *absence of knowledge of (theoretical) structure*: ignorance of theoretical structure.

> When ignoring (the relevant parts of) the theoretical structure of a theory, scientists are not capable of grasping abstract causal connections between the propositions of their theory, they can neither identify the logical

[7] We are aware that important philosophical discussion regarding the intricate nature of ignorance is ongoing. Here, given the nature and scope of our proposal, we adopt the characterization of ignorance as lack of knowledge, recognizing, however, the existence of two predominant philosophical frameworks for probing into the nature of ignorance. On the one hand, the analytic epistemology stand encompasses rival views on ignorance as a lack or absence of knowledge and ignorance as a lack or absence of true belief (Le Morvan and Peels, 2016: 12). On the other hand, a naturalist view informed by cognitive sciences (Cf. Arfini 2019, Arfini and Magnani 2022) is not similarly constrained by a taxonomy reliant on epistemic status.

[8] For extensive analyses on this type of ignorance, see [Rescher 2009], [Nottelmann 2016], and [Peliš 2017].

consequences of the propositions that they are working with nor can explain under which conditions the truth value of such propositions will be false. (Martínez-Ordaz 2020: 8676)[9]

Intuitively, and regardless of whether or not one agrees to understanding being reduced to knowledge, all of these types of ignorance can, at least, weaken the possibility of achieving a legitimate understanding of a given subjectmatter. For instance, factual ignorance can affect the possibility of satisfying, at its best, the factivity condition, and at its worst, the empirical adequacy condition on understanding. Objectual ignorance of the subject-matter can undermine the possibility of determining the object of understanding, and therefore, the possibility of achieving any understanding of it in general. Procedural ignorance can either undermine some of the performative elements of understanding or even undermine the whole possibility of understanding in case that understanding is constituted by know-how. Erotetic ignorance can effectively undermine the possibility of achieving understanding if the questions that the agent is impeded from answering are either why-questions or questions about relations. Finally, ignorance of (theoretical) structure can, at its best, limit the agent's ability to draw correct inferences about the subject-matter, and at its worst, block the capability of an agent to grasp relations of dependence.

This said, it should be clear that, at least intuitively, ignorance and understanding seem incompatible – even if understanding is not at its core a privileged type of knowledge.

4.2 Recalcitrant anomalies as cases of ignorance

In the empirical sciences, the presence of ignorance is not always as clear nor so specifically localized as shown in the list provided above. In what follows, we focus on one particular presentation of ignorance: *scientific anomalies*. Anomalies have been traditionally characterized as consisting of the presence of a statement (generally some kind of observational outcome), *P*, such that when combined with a particular theory, *T*, and with a ceteris-paribus clause the statement becomes a potential falsifying statement for the theory (see Lakatos 1977: 40). Anomalies can be of two different types, the two most common are: information gaps, also called "lacunae", closely linked with abductive novelties (Cf. Kuipers 1999, 2000; Aliseda-Llera 2006), and logical contradictions (Cf. Laudan 1977, Priest 2002).

The former, lacunae, are epistemic gaps about specific phenomena. They indicate limitations in our knowledge and understanding of a given domain; they underscore instances where crucial information is absent or where uncertainties persist, which often leads to a reexamination of the scope and limitations of the theoretical framework used in

[9] This type of ignorance is frequently associated with the initial phases of scientific development – the building of new theories, the identification of new domains, etc. A complicating factor might be the holistic nature of the subject-matter combined with limited epistemic access to the theoretical framework's structure or the structural paths that link that framework with a target domain. In cases where internal or external holism is exceptionally salient, scientists may find it challenging to identify privileged inferential relations crucial for testing the truth-value of the premises involved, among other considerations.

tackling such a phenomenon. The latter, logical contradictions, are the result of a strong conflict between a prediction and an observational report, so strong that it cannot be solved by appeal to a margin of error or a human mistake or by *ad hoc* changes. This type of anomaly involves situations that appear to violate the logical coherence or consistency of a particular model or theory.

One of the most famous examples of anomalies in the history of science is that of the precession of the perihelion of Mercury. Planetary orbits in Newtonian cosmology are not fully elliptical but are however nearly so. The closest point to the Sun in a planetary orbit that's quasi-elliptical is its *perihelion*. Mercury's orbit is predicted to rotate in the direction of its motion, such that its perihelion shifts. According to Newtonian theory there should be a gravitational force acting on Mercury in relation to the Sun and other planets, however even taking that into account the theory does not give a precise prediction for the observed precession of Mercury's perihelion. Specifically, there's a discrepancy between theory and observation amounting to 43 arc-seconds (a unit of angle) per century.

It seems right to say that anomalies are complex instances of ignorance in science. Lacunae indicate that some important information is missing, either for the interpretation, explanation, prediction, or measurement of an observed phenomenon. In this sense, they can be compatible with any type of ignorance (factual, objectual, procedural, or erotetic). Contradictions, however, are likely to be instances of factual ignorance. In the large majority of cases, when scientist find a contradiction in their practice they appeal to ignorance to explain its presence. In those scenarios, scientists blame the presence of a contradiction on their ignorance about, at least, which of the mutually contradictory elements should be regarded as false (Cf. Bueno 1997, Brown 1990, Priest 2002). That is, in cases of contradictions, scientists lack knowledge of the truth values of each of the elements of the contradiction (Cf. Martínez-Ordaz 2020: Section 2). Given the close link between anomalies and ignorance, it seems fair to say that anomalies are indicative of a lack of understanding as well as of the challenging state of any future achievement of understanding – if the underlying issues that give rise to gaps or contradictions are not solved. But not all anomalies have the same effect in science. Here we focus on those that are called *recalcitrant*.

An anomaly is recalcitrant if it can survive the various attempts to solve it or explain it within a theoretical framework, to the point at which reasonable agreement between the theoretical framework and the observation cannot be reached — meaning that there is no accessible interpretation under which both can be true at the same time (Cf. Kuhn 1977). The most severe anomalies are those that despite the efforts invested in their harmonious solution remain persistently obtrusive up to the point of leading to drastic changes in the discipline. It is worth noticing that while scientists may recognize the presence of an anomaly during the investigative process, labeling it as "recalcitrant" typically comes after extensive efforts to explain or resolve it within the existing theoretical framework have failed. The identification of an anomaly as recalcitrant often occurs in retrospect, after attempts to reconcile the anomaly with the theoretical framework have proven unsuccessful.

This considered, one should look at recalcitrant anomalies as instances of high degrees of ignorance. There is an important sense in which a recalcitrant anomaly consists of more than just missing bits of information; when an anomaly becomes recalcitrant this suggests that scientists are ignorant of some of the most fundamental parts of either their theoretical framework or of the target phenomenon. Problems like the impossibility of efficiently deciding the truth-values of both components of an apparent contradiction, despite all efforts to find ways to do so, suggest that scientists lack a grasp of the inferential paths that could privilege one evaluation over another. Long-lasting conflicts emerge given the lack the information needed to interpret intended phenomena within a given framework, despite all attempts to close that information gap. Such ignorance may concern relations that hold either within the empirical domain where that phenomenon exists or between such a domain and a theoretical framework. This connects recalcitrant anomalies with ignorance of (theoretical) structure.

Let's illustrate this with the previous example. Once the anomaly in the orbit of Mercury was identified, different alternatives were provided to make sense of the anomaly within the Newtonian framework, ranging from trying to fill observational gaps (by searching for new planets that could be responsible for the perturbation) to changes made to the core of the theory. However, all proposed solutions fell short. The anomaly was later deemed 'recalcitrant' due to its persistent resistance to explanation within the established framework of Newtonian mechanics (Cf. Harper 2007). The solution only came after the adoption of a novel theoretical framework (Einstein's General Theory of Relativity), showing that what the anomaly signaled were the structural limitations of the theory when accounting for the curvature of space as well as the scientists' lack of clarity concerning the actual limits of the theory they used.

The major issue at stake here is not that recalcitrant anomalies could be indicative of a lack of clarity about the inference patterns that govern areas of a subject-matter, but that ignorance of (theoretical) structure might be responsible for important obstacles found when aiming at the identification of relations of dependence within a theory or a phenomenon. In this sense, the type of ignorance that underlies cases of recalcitrant anomalies might be the main challenge to the future achievement of scientific understanding. It is important to stress that even if the anomaly is the most salient issue caused by this ignorance, it is unlikely to be the only one.

If what has been said here is along the right lines, ignorance of theoretical structure when present in cases of recalcitrant anomalies might constitute the most important obstacle to understanding. [10]

4.3 Insights for the solution of recalcitrant anomalies

[10] For a detail analysis of the challenges that anomalies present for the scientists' capability of 'making sense' of new phenomena given their background knowledge, see Pérez-Ransanz (2018).

So far, this section has considered challenges for understanding, and when doing so, built a very dark picture that relates ignorance and anomalies with significant challenges to the achievement of understanding. Nonetheless, if ignorance were as dangerous as described above, science would (almost) never move forward once a recalcitrant anomaly is faced, and more importantly, in the presence of recalcitrant anomalies, understanding would be impossible. Here, we deal with a way in which recalcitrant anomalies get (partially) solved, and scientific understanding becomes attainable once again.

The "Eureka moment" is often synonymous with the concept of *insight*. Insight consists of a sudden realization or discovery of a solution path that allows one to solve a problem. The inferential foundation underpinning the construction of insights is inherently ampliative, yielding outcomes novel to the agent experiencing the insight.

"Insight" is employed to denote the resolution of a specific problem. This interpretation, predominantly addressing the process of insight, draws from Peirce's characterization and has garnered endorsement from philosophers and cognitive scientists. Peirce's seminal work has been instrumental in delineating the species of reasoning integral to the generation of insights — typically characterized by its ampliative, creative, and unconventional nature, often manifested within abductive contexts. Moreover, insights commonly evoke a sentiment of surprise.

Upon having insights, researchers frequently assert the discovery of newfound information perceived as efficacious, reliable, or instrumental in problem-solving, information acquired *via* a somewhat obscure trajectory. Some of the most characteristic features of insights include that:

- they are formed through an unclear/unrigorous process in response to a specific problem;
- they appear to be strong and robust enough to guide our acceptance or rejection of other beliefs;
- they are often seen as an indication of how we grasp a specific problem, object, domain, or phenomenon.

According to Peirce, insights play a crucial role in scientific understanding by driving the process of abductive reasoning. They combine moments of creative intuition, where scientists form new hypotheses to explain empirical data, with the constant evaluation of the problem and its possible solutions.[11]

Insights conspicuously embody characteristics associated with understanding. Firstly, the transparency and the sensation of grasping inherent in the "Aha!-moment" represent prominent facets, exemplifying some of the most salient subjective aspects of understanding. Secondly, insights also exemplify robust instances of a coherent basis. This is chiefly due to their primary objective, which is to furnish solutions for specific problems/puzzles, thereby assembling the components of a puzzle in a manner that

[11] A discussion of the epistemic merits of the products of insights-like processes can be found in Aliseda-Llera (2023).

enhances their meaningful coherence when integrated, as opposed to their isolation. In addition, part of evaluation of insights consists of the assessment of their reliability as satisfactory solutions to a given problem; when surviving such evaluation, they also exhibit a kind of empirical adequacy. Insights also illustrate epistemic robustness and lead to a standing state as they seem to be robust enough to inform the evaluation of surrounding beliefs and sets of information. Finally, and maybe even more poignantly than other features, insights also exhibit the praiseworthiness of understanding, given the complexity and relevance of insight-triggering problems, finding satisfactory solutions to them is all the more valuable.

The factors that make a problem interesting and trigger the pursuit of insights can vary, but several common elements that contribute to the compelling nature of a problem that stimulates insight-seeking are the problem's complexity, novelty, and relevance, among others. Given the superlative complexity of recalcitrant anomalies, they constitute salient motivators for insight. In acknowledging this, recalcitrant anomalies frequently find resolution or explanation through insights, which are commonly perceived as markers of understanding. Does this correlation imply that insights signify the triumph over the ignorance that lies at the basis of the anomaly? Regrettably, this is not the case, and this is mainly because insights are still heavily surrounded by ignorance. Nevertheless, insights play a pivotal role in propelling the pursuit of unraveling the more profound issues behind recalcitrant anomalies.

The lesson of this section is that the relationship between ignorance and understanding is so close that, on the one hand, ignorance has the power to prevent understanding. On the other hand, even when understanding is gained, it can still coexist with deep instances of ignorance. In the next section, we introduce a structuralist view of scientific understanding that can accommodate cases of high degrees of ignorance – in particular, cases of recalcitrant anomalies.

5 From ignorance to understanding: a structuralist perspective

In adherence to the idea that the ignorance that gives rise to recalcitrant anomalies is rooted in theoretical structures, we proffer an analogous perspective: a structuralist examination of scientific understanding. This section provides general remarks concerning such a view as well. It also brings to light the potential of a structuralist view to account for the gradability of understanding and capture the intricate dynamics inherent in non-ideal epistemic contexts marked by anomalies and insights.

5.1 A structuralist take on understanding

The basic idea behind a structuralist take is that the epistemic success achieved in certain tasks is attributed to grasp of an underlying structure. Particularly in the case of understanding, a structuralist view, broadly speaking, would consist in attributing the achievement of understanding to the identification of a privileged structure — where 'privileged' should be interpreted loosely, rather than as pertaining to gaining knowledge about all the elements posited in such a structure.

We take the notion of *structure* in the sense of mathematical structures (Cf. Bricker 1992/2020); in particular, we take them to be interpreted in a set-theoretical fashion.[12] With this in mind, we shift our attention from the analysis of specific objects and their properties to general inference relations or inference patterns.In terms of scientific understanding, this amounts to focusing on the salient relations of dependence that suffice to reconstruct the subject-matter as it is understood. This take allows understanding to coexist with having incomplete information about the *relata* of such relations (Cf. Reck and Price 2000).

For the sake of simplicity let us briefly mention what structural realism looks like when structures are taken to be understood in set-theoretic terms (Cf. Frigg and Votsis 2011). We can distinguish between concrete structures and abstract structures. A concrete structure is a set of objects D together with an indexed set of relations $R_1...R_i$. An abstract structure is a type, a set of concrete structures such that any two similar structures are members of it and nothing else is a member of it. Two concrete structures are similar *iff* for all $x_1...x_n$ in D under R_i there is a mapping f such that there are $f(x_1)...f(x_n)$ in $f(D)$ under R_i*, where R_i* is the relation that corresponds to R_i in $f(D)$, with the corresponding index. The structural realist claim, understood in this set-theoretic way, is that the world or some proper domain of it is structurally similar to the logico-mathematical structure posited of that domain in our best scientific theories.

From the time of Worrall (1989) onwards, the Structural Realist view has partly been motivated by the ambition to reconcile the explanatory power of Scientific Realism *vis-a-vis* scientific success with the worries raised by Laudan (1981) with regards to scientific change and the pessimistic induction argument. If Laudan is right, reference to theoretical terms is irrelevant for success, whereas if Scientific Realism is right the success of science is to be explained by the fact that successful scientific theories are approximately true. According to Structural Realism, the ontology posited by scientific theories is less important for success than the structure (functions and relations) posited by the theory, partly vindicating the lessons of Laudan and Scientific Realism in turn. As long as some structure is preserved through scientific change and this is connected to previously successful elements of past scientific theories, Structural Realism gets some support.

The original formal account of Structural Realism, couched in terms of the higher order logic of *Principia Mathematica* (which does not presuppose an ontology of sets or the iterative conception), goes as far back as Russell (1919, 1927) where it is couched in terms of *Principia Mathematica*.[13] As emphasized by Frigg and Votsis (2011) and as can be

[12] However, it is worth noticing that there might be underdetermination at the level of what either set-theoretical structures or metaphysical structures (e.g., pluralities of tropes, universals, *possibilia*) should be posited (if any) as the ontic ground of these mathematical structures. Here, we take no commitment on this question.

[13] On the logic of Principia and some of the controversies surrounding its interpretation see section 1 of Linksy's entry on Principia Mathematica. See also Klement's "PM Circumflex, Syntax and Philosophy of Types" (2013) for a detailed discussion on Russell's logic in defense

corroborated in Russell's work (1919; 1927; 1948) his defense of Structural Realism was less motivated by the history of science than by considerations involving perception, logical constructions, and structural preservation in causal chains. Both the historical and logico-empirical accounts are compatible and reinforce the other, however.

Bueno and Colyvan (2011) regard the isomorphism element, with regards to mathematics and an empirical domain, as a key ingredient in their account of the applicability of mathematics, the part which corresponds to "the mapping account". On the mapping account, a set of truths about an abstract mathematical structure holds of some specific empirical domain if that empirical domain is structurally similar to the mathematical structure. Structural equations on this account can be understood as summarizing how objects in the domain stand in some relation, whereas operations can be understood as functions defined over the domain.

Relatedly, we want to say that understanding is mainly modal in nature, in the sense that it ranges over logical and other modalities of possibility. Our view could also be characterized as a structural account of understanding since we hold that the crucial element in understanding is the positing of some structure (functions and relations) as holding for some objects as a means of making sense of the causal, logical, mathematical, constitutive and other dependence relations holding between the unobservable and observable parts of the domain. In our view, it is the hypothetical positing of such structures, as opposed to the categorical assertion that they hold, that allows agents and epistemic communities to build inferential networks for the domain under study. Since such positing can be revised in light of further evidence, it can be partial and accommodate different posits *vis-a-vis* ontology, ideology (the primitive notions of the theory) as well as underlying structure – be it causal, constitutive, or logico-mathematical.

Importantly, posited structures go beyond set-theoretic structures for the simple reason that, given a well-known argument set forth by Newman (1928) against Russell (1927), any structure can be imposed upon a set given cardinality constraints. Hence, on our view, understanding requires that a structure be distinguished but we take no stand concerning the source or nature of the specialness of posited structures, we only presuppose that they exist. This assumption plays a similar role *vis-a-vis* structures (functions and relations) that Lewis's natural properties (1983) play with regards to properties.

5.2 The dynamic character of understanding

Should we think of understanding as a dynamic or static phenomenon? Take the case of knowledge for the sake of analogy. On some views knowing P is a relation between the knower and the known that happens at a time so perhaps by analogy understanding P is a

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of the claim that the bound variables of PM's higher order logic take propositional functions as values and that this interpretation is substitutional. With regards to the interpretation of class in PM see Klement (2010) and on Russellian logicism see Landini (1998). The second author of this paper believes the resulting notion of structure in this logic differs in some respects from the set theoretic interpretation of Structural Realism, but since the topic is tangential to the aims of this paper we will not discuss it further here.

relation between the subject and the object of understanding at a time. That is the static view.

The static view allows for the possibility of refinement of understanding. We here do not aim to give necessary and sufficient conditions for understanding *P*, as understanding is a gradual relation. However, we say the subject *S* in fact understands *P* at t, if it is a necessary condition of *S* understanding *P* at t that it leaves *S* is in a position to rule out those regions of logical space that do not exemplify or afford instances of *P*'s structure. By an instance of *P*'s structure we mean some distinguished structure instantiated by some or all parts of *P*.

Intuitively however, when we understand something it is true of whatever we understand that we can refine our understanding of it just as we can refine our knowledge of whatever we happen to know. The structure, while distinguished, does not need to be fully preserved as the understanding of *P* increases, but the abstract structure as defined above needs to be partially preserved. In refining our understanding of *P* the candidate regions of logical space where we might find instances of *P* get further constrained. *S* does not rule out these regions as actual since *P* is not known to be the case, but merely as regions where any possible instance of *P* might be found.

There is, however, some force to the thought that understanding must necessarily involve these inferential or creative processes that lead to further refinements, since intuitively as soon as we have some understanding of *P*, we have some grasp of what we don't understand about *P* as well as some of the inferential or creative steps that led to our increased understanding. This seems to be true even if we leave room for some ignorance, epistemic opacity, uncertainty. If so, this favors the view of understanding as a process that does not grasp a phenomenon at a time and instead is a dynamic, unfinished task that partitions our grasp of the phenomenon into whatever is clearly understood, whatever is clearly not understood, and that which is vaguely understood or understood only opaquely. This partitioning in turn has vague boundaries as it does not take place at a specified set of times.

A process view of understanding nicely fits scientific practice but might leave metaphysicians who crave the possibility of understanding a total theory unsatisfied. We say that a metaphysician understands a total theory if they are in a position to rule out any structure that is not a natural model of the theory. If a natural model of a theory is a structure that satisfies the theory by realizing the natural pattern or objects and relations specified by the theory, this leaves out models that realize the structure by mere cardinality (Cf. Bricker 1992/2020). This structural realization of total theory does not need to be unique if the total theory for some reason cannot single out a unique structure, but it should strongly constrain the space of possible candidate structures. The process view discussed above does not rule out the possibility that some logically possible hyper-cognizant rational agent could grasp an upper limit of a sequence of refinements up to total understanding of global structure and the actual infinite totality of inferential strategies that led to such understanding as static truths about some logical or mathematical structures.

It is not necessary to endorse such a possibility for the sake of this discussion as metaphysical understanding is not the topic of this paper. The only reason to bring up this point is to illustrate that it cannot be fully ruled out by the process view of understanding as applied to how actual agents understand and refine their understanding of phenomena and structure, which is most relevant to the practice of science. In what follows, we use this view to tackle the road from ignorance to understanding when dealing with recalcitrant anomalies.

6 From anomalies to ignorance, to insights and to understanding

This section provides a structuralist take on how, when confronted with recalcitrant anomalies, scientists navigate a trajectory that extends from a lack of knowledge regarding (theoretical) structure to the attainment of insights or a partial resolution of this ignorance (intermittently), ultimately culminating in a state of enhanced understanding.

6.1 The road from ignorance to understanding

In what follows we use our framework to explain how scientists gain understanding in cases of recalcitrant anomalies. To do so, we focus on the role that structures play in the emergence of such anomalies as in their partial overcoming, and finally in the understanding of the initially anomalous subject-matter. The main claim here is that, in these cases, the ignorance that underlies anomalies is ignorance of theoretical structure and that the satisfactory tackling of such ignorance constitutes the first step into understanding.

As we have already anticipated (Section 4.2), in these cases, the anomalies' recalcitrance is a result of scientists lacking access to a satisfactory inferential path that could lead them to make sense of a given result, prediction, or observation within a theoretical framework. The identification of an inferential path (or a set of them) allows scientists to navigate the possibility-space created around the anomaly up to a point in which they can explain how the world should be for that anomaly to make sense — regardless of whether that characterization corresponds to the actual world or whether the selected path is temporary. The possibility of achieving any type of understanding requires, at least, the partial overcoming of scientists' ignorance.

The road that takes scientists from ignorance of (theoretical) structure to scientific understanding can be broadly described in the following steps:

(Step 1.) **Identification of an anomaly as significantly challenging.** The starting point, during the investigative process, is the identification of an anomaly as *significantly challenging* (and later candidate for recalcitrance). An anomaly consists in a persistent and significant discrepancy between the theoretical framework and the corresponding observations; such discrepancy can be in terms of either gaps or contradictions. In addition, despite multiple efforts, the anomaly resists satisfactory explanation or resolution. *Ad hoc* modifications or adjustments to the theoretical framework may be attempted but often fall short of fully addressing the anomaly. If an anomaly gets solved, is no longer counts as significantly challenging at that particular moment.

(Step 2.) **Acknowledgment of ignorance.** The next step requires that scientists identify the source of the anomaly in terms of ignorance. At this stage, it is important to differentiate between emergent instances of ignorance and a lack of knowledge about an underlying structure. This acknowledgment instigates an awareness of the anomaly being only a symptom of a larger and deeper problem; in this sense, it also hints that a solution for the anomaly is likely to not result from tackling solely the anomaly or the anomaly directly (*ad hoc*).

(Step 3.) **Partial overcoming of ignorance (1).** The third step is to postulate and evaluate tentative inference patterns that could perhaps save the theory being challenged. Such patterns can reconstruct and explain what is going on in specific cases where the theory is defective, as well as consider what the theory would entail if it were not defective – even before finding ways of fixing it and in the long run, allow for the tolerance and handling of the anomaly. Here it is also expected that a scientist can select particular inference patterns that promote safe reasoning around the anomaly and that help to either weaken its problematic character or, in the best cases, integrate it into the corresponding theoretical framework. It is important to notice that, at this stage, exploration is only tentative and that scientists are not required to pinpoint the *complete* or comprehensive structure of their subject-matter.

(Step 4.) **Insight.** When a scientist aptly identifies a privileged set of inference patterns to handle an anomaly more effectively than alternatives, a sudden realization occurs, indicating the scientist's understanding of *specific* relevant parts of the theoretical structure. This understanding suffices for recognizing and explaining under which circumstances such handling of the anomaly can be preserved.

While this realization exhibits elements of genuine understanding, it remains partially within the realm of the unknown, requiring further abductive reconstruction and evaluation of its success. Insights combine a level of understanding, one that is remarkable enough to solve, at least temporarily, a significant problem within the discipline, with the remaining triggering presence of ignorance regarding both the grounds of the anomaly as well as the mid- and long-term reliability of the solution. Furthermore, it is worth noticing that the value of the products of insight is seen through the light of the problems (ignorance, uncertainty, etc.) that they respond to. For all the above, this stage constitutes the first instance of (structural) understanding but it is still strongly linked to different ways ignorance is manifested in.

(Step 5.) **Partial overcoming of ignorance (2)** The abductive explanation and evaluation of the products of insight consist of the delimitation of *possibility spaces* given the selected structure – and in fortunate cases, also involve the enrichment of such structure.[14] This can be done by paying special attention to how the given solution undermines either the scope or the strength of the underlying ignorance, or by focusing on what effects such a solution has on the behavior of the anomaly in different relevant domains.

(Step 6.) **Modal understanding.** After identifyinga possibility space bounded by the products of insight, scientists gain a particular type of scientific understanding: *modal understanding*. One has a modal understanding of *P* when one knows how to navigate the *possibility space* associated with *P* (Cf. Le Bihan 2017).

At its core, modal understanding allows agents to grasp only possibilities, and while it is likely that with the refinement of one's modal understanding of a subject-matter one of the considered possibilities corresponds with the actual world, it is not clear that agents can effectively determine which of all possibilities, if any, does so.

Yet, when it comes to achieving modal understanding, it is evident that this accomplishment involves recognizing the partial and incomplete nature of the information used to characterize a specific subject and its dependent relationships. At the stage of gaining modal understanding, it becomes clear that ignorance has a dual role — it can initiate the quest for understanding and, paradoxically, set the limits of understanding while providing hints for tackling intellectual challenges.

This underscores an interaction between modal understanding and the partial overcoming of ignorance, all the while considering what epistemic limitations follow from the analysis and critical reassessment of previously attained understanding. In particularly challenging scenarios where scientists find themselves uncertain about avenues to enhance their understanding, modal understanding (accompanied by an implicit acknowledgment of ignorance) may guide them back to insights. From this juncture, there can be a partial overcoming of ignorance, leading either to a more nuanced form of modal understanding or, in the ultimate scenario, a comprehensive understanding of the subject-matter.

(Step 7.) **Total understanding of global structure.** This case, as it was explained in Section 5.2, should be treated only as a regulatory ideal or possible exclusively for an hyper-cognizant rational agent who could gain total understanding of a global structure and of the actual infinite totality of

[14] The *possibility space* addressed here is understood as a collection of possible worlds in which explanations (and other types of scientific statements) about a given subject-matter are the case. A possibility space for a subject matter *P* is a set of dependency structures in those possible worlds that give rise to any subset of *P* and the relations between those structures. The possibility space includes both the set of possible dependency structures for *P* and its subsets as well as the relationships between these structures. (Le Bihan 2017: 114)

inferential strategies that led to such understanding as static truths about some logical or mathematical structures.

That said, the general picture is the following:

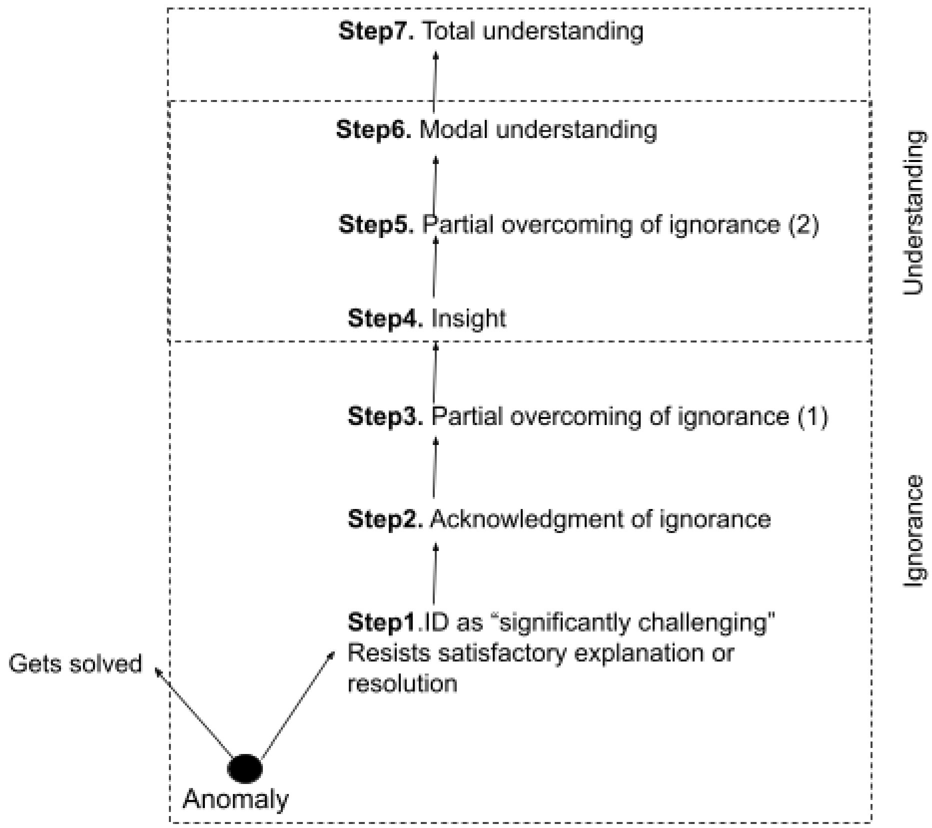

Figure 1. The road from recalcitrant anomalies to understanding.

In addition to all of the above, Figure 1 illustrates the interplay between ignorance and understanding in cases of recalcitrant anomalies (Step 4-Step 6) as well as the dynamic relations that hold between different ways to gain understanding and the handling of structural ignorance (Step 4-Step 7). It is important to say that the picture that we have provided here seeks to stress that acknowledgment of ignorance is not a setback but an essential step toward deeper understanding. In what follows, we illustrate this with the anomaly in the perihelion of Mercury.

6.2 Understanding the anomaly of Mercury's perihelion

Let's go back to the case of the anomalous precession of Mercury's perihelion. In 1859, when Le Verrier noted that Mercury's perihelion precession rate exceeded the predictions of Newtonian mechanics, physicists of the era believed that this anomaly could be elucidated by simply incorporating an additional piece of information into the existing theory.

Different proposals were considered. The most important were (a) the existence of a new planet orbiting very near the Sun, and (b) the modification or re-interpretation of Newton's law of gravitation (Cf. Giné 2008: 1004). The first challenged the understanding that scientists had about the empirical domain but reinforced the idea that the theory was trustworthy; while the second problematized how key elements of the theory were understood. Although these solutions initially seemed promising, they either lacked empirical support or appeared to be arbitrary. After four decades, the anomaly was considered a serious challenge for the theory and a symptom of a deeper problem in the Newtonian framework.

Physicists at the time struggled to pinpoint which assumptions of the theory were inaccurate or unreliable within that particular context, hampering their ability to grasp why the theoretical description of the empirical domain was flawed. In this sense, the ignorance underlying the anomaly was recognized as more complex than only missing bits of information.

Once having acknowledged their ignorance as lacking knowledge about structure, scientists followed two main paths to deal with it. The first one was about handling the anomaly. This was a pragmatic and inferential response that consisted in the positing of an inferential structure to block inferences around Mercury's orbit as they were unreliable (Davey 2014: 3018). The other path consisted of considering alternative theoretical structures for the domain to modify or substitute the Newtonian framework. These two paths showed ways to temporarily and *partially overcome the scientists' ignorance* so they could reason safely around the anomaly, yet not solve it.

Understanding the anomaly as a symptom of deeper limitations of the Newtonian theory took physicists to explore alternative frameworks. Einstein developed his Special and General theories of relativity in response to programmatic concerns related to the unification of several physical theories and concepts and not specifically to explain the above anomaly. However, having developed General Relativity, Einstein figured it could be tested to see if it gave a correct prediction for the precession of Mercury's perihelion. This resulted in the combination of both positing a tentative structure of the anomaly and discovering that such structure constituted a solution for it (Cf. Janssen and Renn 2022: Chapter 6). Einstein's theory accounted for the anomaly in a surprisingly accurate way – even from Einstein's perspective; involving a *sudden realization* of the strength of his theory given the anomaly. Einstein's positive surprise about this can be found in different writings like the letters to Hilbert, Zangger, and Nathanson, in late 1915 (Cf. Einstein 1998).[15]

As Maudlin (2012) points out, the spacetime of General Relativity stands to Minkowski spacetime in Special Relativity just as the space of Riemannian geometry stands to the space of Euclidean geometry. The intrinsic geometry of Minkowski spacetime is flat, just as the space in Euclidean geometry is flat whereas the spacetime of General Relativity is of variable curvature just as the space of Riemannian geometry is of variable curvature. In General Relativity, 4D spacetime is a smooth manifold of variable curvature, where the

[15] The first explanation of this result can be found in Einstein's "Explanation of the Perihelion Motion of Mercury from General Relativity Theory" (1915).

curvature is affected by the matter density. However, general relativity preserves a distinction between straight and curved worldlines just as much as Newtonian spacetime. Locally, if the effect of matter density on spacetime is small, Newtonian mechanics will work just fine as it has sufficient structure to physically model different phenomena in a precise enough way. It is only in unusual (for us) regimes where the different ontology and structures posited by General Relativity will enable the precise explanation and prediction of distinguished physical phenomena. However, this shift in ontology and structure still preserves distinguished topological and affine structures.

Indeed, the theory gives an accurate prediction for that quantity which is widely agreed as a very robust confirmation for it given its precision. In General Relativity the intrinsic curvature of spacetime in the Sun's neighborhood is predicted to affect more noticeably the trajectory of the nearest planet to the Sun, Mercury, and it is in proportion to that effect that a precise prediction for the precession of Mercury's perihelion was obtained. So, in General Relativity, spacetime structure is causal structure and hence the relations of dependence required for understanding the precession of Mercury's perihelion are in place.[16] This can be reconstructed as signaling a *modal understanding* of the anomaly and the corresponding domain through Einstein's theory.

In light of having accounted for the anomaly so successfully, Einstein revised his solution critically, and in his "The Foundations of the General Theory of Relativity" (1916), he presented what he called the "correct calculation of the bending of light" for which he referred to the perihelion of Mercury. He also explored some of the key consequences of his theory, which included the prediction of the deflection of light by the Sun and the gravitational redshift of light; which he presented as 'classical tests' of general relativity (Cf. Einstein 1919). This shows a larger exploration of the possibility space of the theory being posited in that empirical domain, which for a long time, while the predicted phenomena remained unobservable, was considered still only a possibility.

7 Final remarks

Here we have discussed that, according to the traditional literature, understanding is thought to be in many ways the result of having achieved different types of epistemic success (Sections 2 and 3). However, ordinary instances of scientific understanding, like insight, lead a different way, involving the triggering and constant presence of ignorance, uncertainty, and other intellectual challenges.

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[16] When we talk about the causal structure of spacetime we want to make it clear we are not taking a stand on any philosophically substantive views on the metaphysics and analysis of causation, but merely following the convention on how physicists and use the term "causal structure" in standard accounts of spacetime geometry. If spacetime has a light-cone geometry then for any point p, there will be spacetime regions which are time-like related to p (via the past lightcone and future lightcone); spacetime regions that are space-like related to p and spacetime regions that are light-like related to p. Spacetime regions that causally affect p are in its backward lightcone and the spacetime regions that can be causally affected by are in its future lightcone. See Manchak (2020).

Particularly, in the case of the understanding that is gained when dealing with recalcitrant anomalies, the type of ignorance that underlies the anomaly plays a key role in leading the pursuit of understanding as well as in defining its boundaries while offering clues for tackling similar intellectual challenges (Section 4). We identified this type of ignorance as *ignorance of (theoretical) structure* and we argued that its (partial) overcoming depends on the satisfactory identification of underlying structures that allow scientists to reconstruct and explain what is (possibly) going on in specific anomalous cases (Section 5). Finally, we illustrated this with the case of the anomaly in the precession of the perihelion of Mercury (Section 6).

Acknowledgments: The first author thanks Miguel Ángel Fernández-Vargas and Santiago Echeverri for discussions on the epistemology of understanding. The second author wants to thank Kevin Klement and Phil Bricker for valuable discussions on metaphysics and the philosophy of mathematics relevant to portions of this work. Thanks to the editors for their suggestions and critical comments, as well as for their assistance and constant understanding.
María del Rosario Martínez-Ordaz's research was supported by the Programa Nacional de Pós-Doutorado PNPD/CAPES (Brazil) and by the PAPIIT project IG400422 "Variables proposicionales y constantes proposicionales: aspectos lógicos, epistémicos y metafísicos".

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