DEGENERATE STATES AND THE PROBLEM OF QUANTUM MEASURMENT

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Abstract

In previous work the author has proposed a different approach to the problem of von Neumann measurement and wave function collapse. Here we apply it to the collapse of degenerate states. Our predictions differ from those of von Neumann and, separately, Lüders in significant ways. An experiment is suggested that might distinguish between the possibilities.

Keywords: Consciousness, Quantum Measurement, Degenerate States, von Neumann, Lüders.

Introduction.

In earlier work (1, 2) the author has tried to outline a new version of the von Neumann-Wigner Interpretation (consciousness brings about wave function collapse). We introduced the concept of admissible and inadmissible states. As an example, we considered a situation where an electron is introduced into a Stern-Gerlach apparatus. Initially the detector shines a blue light, indicating that no measurement has yet been performed. If the electron comes in spin-up, |+>, a green light is triggered. The state evolves from |B, +> into |G, +>. If it comes in |-> a red light is illuminated and the system becomes |R, ->. A conscious observer watches all this. If a $(|+\rangle + |-\rangle)/\sqrt{2}$ electron were to be introduced unitary evolution would have things evolve into $(|G, +\rangle + |R, -\rangle)/\sqrt{2}$ such that the observer would see a superposition of green and red. This state corresponds to no clearly definable conscious condition. Wigner called this situation "absurd." We call it *inadmissible*. Only states corresponding to definite conscious conditions are admissible. The system cannot enter into any inadmissible state. We require $\mathfrak{S}\,|\Psi(t)\rangle = |\Psi(t)\rangle$ always, where $\mathfrak{S}$ is a (non-linear) operator having some interesting properties:

I) If $|\Psi(t)\rangle$ is admissible it does nothing. The state is completely unaffected. Call the set of all admissible states $\{C_i\}$.

II) If $|\Psi(t)\rangle$ is not admissible it will look at all the amplitudes $\langle C_i|\Psi(t)\rangle$ for every $\langle C_i|$. It will square these amplitudes and, using these values as *relative* probabilities, convert $|\Psi(t)\rangle$ into one of the $|C_i\rangle$ at random.

Here, $|\Psi(t)\rangle$ and $|C_i\rangle$ do not represent wave functions as such but, rather, state vectors in a Fock space. In general, they represent all the particles in the universe. To keep things simple we will imagine they pertain only to the particle(s) being measured, the measuring device, and the conscious observer. $\mathfrak{S}$ functions as a projection operator taking mixed states (with respect to consciousness) into definite, admissible, states. Here we give up the idea of a unitary time-evolution operator. Such an operator has an inverse. We cannot go backwards in time according to $\mathfrak{S}$ since the decision how to go forward is made at random. This imparts a natural directionality to time. $\mathfrak{S}^2 = \mathfrak{S}$ and $\mathfrak{S}$ has no explicit time dependence. We allow for admissible null states of consciousness corresponding to the existence of no sensations at all. Readers who dislike the von Neumann-Wigner idea can, for purposes of this paper, replace it with other criteria (perhaps involving the size and/or complexity of the measuring device) that would serve to determine the admissibility of a state. We do like this interpretation since it provides a role for consciousness in physics. A universe populated only with Chalmers' zombies and Chalmers' zombie-animals (3) would not be the same as ours. It would evolve in a purely unitary fashion. We would not want to think of consciousness as nothing more than an epiphenomenal "innocent bystander" (although some readers probably would).

Now $|\Psi(t)>$ is in no way a function of the spatial coordinates. But it does contain all the information we can have regarding what is going on where. Specifically, we are interested in $<\Psi(t)|T_{\mu\nu}(\mathbf{x},t)|\Psi(t)>$ where $T_{\mu\nu}(\mathbf{x},t)$ is the stress-energy operator for our quantum field theory (2). To take account of Relativity we must make an additional stipulation: When an inadmissible state is projected into an admissible one that new state must be such that $<\Psi_{\text{new}}(t)|T_{\mu\nu}(\mathbf{x},t)|\Psi_{\text{new}}(t)>$ is altered only within the future light cone of the measurement event. Otherwise, physical information regarding the outcome of the measurement could travel faster than light. We cannot allow for this. If the state undergoes two or more measurements $<\Psi_{\text{new}}(t)|T_{\mu\nu}(\mathbf{x},t)|\Psi_{\text{new}}(t)>$ can only be altered within the union of the future light cones of the measurement events. And it must be altered consistently. The various observers must, ultimately, agree that they saw the same thing. This may look like "spooky action at a distance." It is.

Provided our apparatus is effectively shielded from all outside influences, and supposing that our measurement does not destroy the system, we will usually end up with a normal von Neumann measurement and the Born rule. An interesting problem arises if our system is degenerate with respect to eigenvalues that could be obtained through our measurements. What does the state collapse into then? Dirac (4), von Neumann (5), and Lüders (6) have tried to give us answers. We note that our above-described protocol gives predictions that differ very considerably from those of the above-mentioned authors. For recent discussions see (7, 8, 9, 10).

Simple Examples.

Suppose we have a system that can be characterized by two observables. Here we will use energy and spin. Suppose it exists in a Hilbert space spanned by three orthonormal basis states |I>, |II>, and |III> where:

1) $|\text{I}> = |\text{Energy} = E_1, \text{spin} = ->, \quad |\text{II}> = |\text{Energy} = E_2, \text{spin} = ->, \quad |\text{III}> = |\text{Energy} = E_2, \text{spin} = +>.$

Let there be a pure state we will describe as:

2) $|\Psi_0> = c_1\,|\text{I}> + c_2\,|\text{II}> + c_3\,|\text{III}>$ with $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1.$

Suppose we measure its energy. (We must find a definite value; a pointer cannot be perceived to point at two energy values simultaneously.) There is a probability, $|c_1|^2$, that we will find E_1. If we do, the state will be projected into |I>. If we, subsequently, measure the spin we are certain to find it **-**. (We assume that our states are eigenstates of the Hamiltonian and will only change by meaningless phases as time goes on.) On this point we and the other interpretations all agree. Suppose we find E_2. Then into what state is it projected? For Lüders the answer is simple: It is projected into $(c_2\,|\text{II}> + c_3\,|\text{III}>)/\sqrt{|c_2|^2 + |c_3|^2}$. We can then measure the spin. According to Lüders we have a probability $|c_1|^2$ of ending up in |I>, $|c_2|^2$ of ending up in |II>, and $|c_3|^2$ of ending up in |III>. There is nothing obviously unreasonable about this result. But our protocol leads to very different conclusions.

Suppose we measure E_2. *All* linear combinations of |II> and |III> are E_2 energy eigenstates and, therefore, admissible in terms of an energy measurement. The state could project into any of them in the manner suggested above. Let us write the most general combination as:

3) $\psi_a(\beta, \theta) = e^{i\theta}\sqrt{1-\beta^2}$ |II> + β |III> where β is real and between 0 and 1. $0 \le \theta \le 2\pi$.

The above retains an ambiguity since it can be multiplied by an arbitrary phase and nothing is changed. We do not have to worry about this as the same ambiguity applies to |I>. Since all we are interested in are the *relative* probabilities the effect of taking this into account nets out to zero. The total probability of measuring E_2 is given by:

4) $\int_0^1\int_0^{2\pi} \left| < \Psi_0 \right| \psi_a(\beta, \theta) > |^2 \; \mathcal{M}(\beta, \; \theta)\, d\theta\, d\beta$ where $\mathcal{M}\,(\beta, \theta)$ represent a kind of *measure* over the (β, θ) "space."

This gives us:

5) Total probability of $E_2 \; = \int_0^1\int_0^{2\pi} \left| c_2^* \, e^{i\theta}\sqrt{1-\beta^2} \; + \; c_3^* \, \beta \right|^2 \; \mathcal{M}(\beta, \; \theta)\, d\theta\, d\beta.$

If $\mathcal{M}\,(\beta, \theta)$ is assumed to be independent of θ the $e^{i\theta}$ containing terms integrate out and we are left with:

6) Total probability of $E_2 \; = 2\pi \int_0^1 \left(\left| c_2 \right|^2 \, \left(1-\beta^2\right) + \left| c_3 \right|^2 \, \beta^2 \right) \mathcal{M}(\beta)\, d\beta.$

But what to choose for $\mathcal{M}\,(\beta)$? Our measure represents the number of admissible states in the interval dβ dθ. Our (β, θ) "space" is, in polar coordinates, just the unit disk. The volume element here is β dβ dθ. This suggests we should try $\mathcal{M}\,(\beta) = \beta\, \mathcal{M}_0$.

7) Total probability of $E_2 \; = \int_0^1\int_0^{2\pi} \left| c_2^* \, e^{i\theta}\sqrt{1-\beta^2} \; + \; c_3^* \, \beta \right|^2 \; \beta\, \mathcal{M}_0 \, d\theta\, d\beta.$

If we set $\mathcal{M}_0 = 2/\pi$ we find that the total probability of ending up with E_2 is $|c_2|^2 + |c_3|^2$. The total probability of ending up in any state is $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$. This is exactly the result we want. We now measure the spin. The probability of ending up in |II> is:

8) $\int_0^1\int_0^{2\pi} \left| c_2^* \, e^{i\theta}\sqrt{1-\beta^2} \; + \; c_3^* \, \beta \right|^2 \; \beta \, \left(1-\beta^2\right) 2/\pi \, d\theta\, d\beta = \frac{2}{3} \; |c_2|^2 + \frac{1}{3} \left| c_3 \right|^2.$

The probability of ending up in |III> is:

9) $\int_0^1\int_0^{2\pi} \left| c_2^* \, e^{i\theta}\sqrt{1-\beta^2} \; + \; c_3^* \, \beta \right|^2 \; \beta \, \left(\beta^2\right) 2/\pi \, d\theta\, d\beta = \frac{1}{3} \; |c_2|^2 + \frac{2}{3} \left| c_3 \right|^2.$

The act of measuring the energy has cost us some information regarding the spin. We might have started with a state having $c_2 = 0$ and find the spin to be - after measuring E_2. This result differs from the predictions of Lüders and von Neumann. (The above results are relevant only if c_1 does not equal zero exactly and we will return to this point momentarily.) Another difference is the fact that, for us, the probabilities depend on the order of measurement — we would obtain a different result if we measured the spin first and, later, the energy. This is not the case for Lüders. We could imagine making our spin and energy measurements simultaneously. We would then recover the result of Lüders. One agreeable consequence of our method is that we are not obliged to use any particular basis for our degenerate Hilbert subspace. We have chosen the most convenient one but we could, for instance, call |II> (|A> + |B>)/$\sqrt{2}$ and |III> (|A> - |B>)/$\sqrt{2}$ or whatever we want. Our results will not change.

We could make our situation more complicated by allowing for a new state, |IV> = |Energy = E_2, spin = 0>. Our pure state can now be written:

10) $|\Psi_0\rangle = c_1\,|I\rangle + c_2\,|II\rangle + c_3\,|III\rangle + c_4\,|IV\rangle$.

Our most general E_2 eigenstate is now:

11) $\psi_a(\beta, \alpha, \theta, \delta) = e^{i\theta}\,\alpha\,|II\rangle + \beta\,|III\rangle + e^{i\delta}\sqrt{1 - \beta^2 - \alpha^2}\,|IV\rangle$ where β and α are real and between 0 and 1. $0 \le \theta, \delta \le 2\pi$.

Going through the same algebra (here we use $\mathcal{M}(\alpha, \beta, \theta, \delta) = \frac{6}{\pi^2}\,\alpha\,\beta$) gives us:

Probability of ending up in |II> = $\frac{1}{4}(2\,|c_2|^2 + |c_3|^2 + |c_4|^2)$

Probability of ending up in |III> = $\frac{1}{4}(|c_2|^2 + 2\,|c_3|^2 + |c_4|^2)$

Probability of ending up in |IV> = $\frac{1}{4}(|c_2|^2 + |c_3|^2 + 2\,|c_4|^2)$.

(If the degenerate subspace has dimension D our measure is given by $\frac{D!}{\pi^{D-1}}\,\alpha_1 ... \alpha_{D-1}$.)

Let us first measure the energy and, subsequently, the spin. We can examine the resulting change in the von Neumann entropy, - k Tr [ρ ln ρ]. For both us and Lüders it is, of course, positive. But, according to our update rule, the entropy increase is always greater than or equal to that predicted by Lüders. We might look at this as a good thing. Physical processes generally like to increase entropy to the greatest extent possible. On the other hand, our update rule requires that "more work" be done on the initial state. Lüders has the advantage here.

Discussion.

It must have struck the reader that there seems to be something strange about what we have suggested. Consider the state given in 2). If $c_1 \equiv 0$ we are already in an energy eigenstate. If the energy is measured it will certainly be found to be E_2. The state is, therefore, admissible and, according to our postulate I, $\mathfrak{S}$ will leave it unaffected. We recover exactly the result predicted by Lüders. But, if c_1 differs from zero in even the smallest way, we obtain the dramatically different results described above. This discontinuous change must seem very peculiar indeed. We could say that $\mathfrak{S}$ only projects the state if $|c_1|^2$ is above some critical value. This seems arbitrary and contrived, however. Another way of achieving a desirable result would be to say that, no matter how hard we try, we just cannot, as a practical matter, ever really produce a state having $c_1 \equiv 0$ — there will always be a contribution, however small, from c_1. This is probably the best way of looking at it.

There might, actually, be a way of testing this idea. Suppose we can construct a system consisting of two massive spin-1/2 particles, a and b. We are interested in their spins. Particle a will be measured at point A and b at B. Our most general initial state can be written as eq. 10) with |I> = $|+_a, +_b$>, |II> = $|+_a, -_b$>, |III> = $|-_a, +_b$>, and |IV> = $|-_a, -_b$>. For simplicity, assume that $c_1 = c_4 = 0$. We now measure our entangled state. If A and B are spacelike separated we can always choose to work in a Lorentz frame where they are simultaneous. In this case only |I>, |II>, |III>, and |IV> are admissible — both spins become known to the observers at the same time. Our system will be projected into either |II> or |III> with probabilities $|c_2|^2$ and $|c_3|^2$, respectively. We have, essentially, just re-performed something like the Aspect experiment (11) and we would get exactly his results. We think these results will always hold true provided that A and B are spacelike separated since we cannot imagine them depending on our arbitrary choice of a coordinate system. But suppose they are timelike separated with B lying inside the future light cone of A. (Aspect could not have looked at this case since his photons always travel at *c*. He might, however, have measured his first photon then, using a mirror, reflected the second photon back to the polarimeter to be measured later.) Suppose a is measured at A and found to be +. According to Lüders the state is now |II> and we are certain to find b - when it is later measured. For us the first measurement can project the state into any linear combination of |I> and |II> (as described above). Thus, when b is measured at B, there will only be a 2/3 chance of finding it -. It could be +.

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