What Reference Frames Teach Us About Symmetry Principles and Observability

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Abstract

This paper is an exploration of the nuanced realm of reference frames within the framework of General Relativity. Our analysis exposes a violation of Earman's **SP1** principle in scenarios involving fields that are dynamically uncoupled, a common assumption for reference frames. Unlike other violations, we cannot foreclose it by eliminating background spacetime structure. Our analysis also leads us to challenge the conventional notion of partial observables as quantities that are associated with a measuring instrument and expressed within a coordinate system. Instead, we argue that a partial observable is inherently relational, even if gauge-variant, and needs dynamical coupling with other partial observables to form a *bona-fide*, gauge-invariant complete observables. This perspective allows us to distinguish between being relational and being gauge-invariant, two properties that are often conflated.

Contents

List of acronyms

- **URF**: (dynamically) Uncoupled Reference Frame

  - **IRF**: Idealised Reference Frame

  - **ARF**: Auxiliary Reference Frame

- **CRF:** (dynamically) Coupled Reference Frame

  - **DRF**: Dynamical Reference Frame

  - **RRF**: Real Reference Frame

- KPM: Kinematically Possible Model

- DPM: Dynamically Possible Model

- **SS**: Spatiotemporal Symmetry

- **DS**: Dynamical Symmetry

- **(RI)**: (point) Reshuffling-Invariance (i.e. relationality, or invariance under **SS**)

- **(DSI)**: Dynamical Symmetry-Invariance (i.e. invariance under **DS**)

- **(DET)**: Deterministic dynamical evolution

- **(GI)**: Gauge-Invariance $[\textbf{(GI)} \leftrightarrow \textbf{(RI)} \wedge \textbf{(DET)}]$

1 General Framework

1.1 Observables

When writing a physical theory in mathematical form, we must associate syntax with semantics: that is, an interpretation of the formalism. In particular, within the realm of mathematical objects constituting the theory's formalism, we are interested in those objects that represent something about the physical world we wish to describe (or represent) with our theory. These quantities are sometimes called '*observables*' of the theory, because they are to be associated with measurement outcomes.[1]

The distinction between a variable of a theory and an observable becomes imperative for a theory that has mathematical redundancy or gauge freedom. In the context of Hamiltonian theories this freedom or redundancy appears in the form of certain kinds of constraints that the variables have to satisfy, called 'first class constraints'.

Dirac (2001) (see also Henneaux and Teitelboim (1994)) formally defined observables for a theory with first-class constraints as quantities which commuted with all of the first-class constraints; or alternatively which assumed a single value for each set of gauge-equivalent states. The two characterisations coincide because in the Hamiltonian formalism the action of constraints on quantities via the Poisson bracket generates infinitesimal gauge transformations, and so commutation implies gauge invariance.

This is the formal definition; in practice explicit local observables may be hard to find. This is especially the case in vacuum General Relativity (GR). Given a three-dimensional foliation of spacetime, the first-class constraints of the theory are equivalent to spatial diffeomorphisms along the leaves of the foliation and to diffeomorphisms whose generators act in the normal directions to the leaves ('refoliations'). For spacetimes which satisfy the Einstein equations, there is a neat correspondence between these Hamiltonian symmetries and the four-dimensional spacetime diffeomorphisms of spacetime (Lee and Wald (1990)).

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[1]This pragmatic characterisation of what an observable is was famously championed by Bergmann, who took up the ideas of the young Einstein. According to Bergmann ''the equations of mathematical physics cease being mere mathematics to become honest physics only when one is able (a) to point to spatial quantities and expressions in the formalism and designate them as 'observable' and (b) to prescribe operational procedures by which such quantities may, in fact, be measured (observed), either by laboratory experiments or by astronomical measurements'' (Bergmann (1957)).

Since geometrical objects are dependent on the points of the manifold and the GR gauge group is thought to be the four-dimensional diffeomorphism group which shuffles the points, objects that are represented locally just in terms of the points are not gauge-invariant.

One way of addressing this problem was proposed by Rovelli in Rovelli (1991), Rovelli (2002b). By arguing that gauge invariant Dirac observables are not the only quantities of physical interest in GR, he proposes a distinction between two notions of observability in a general-relativistic context: *partial* gauge-variant observables and *complete* gauge-invariant observables.

The main idea is to relate different sets of gauge-dependent fields (partial observables) in a gauge-invariant manner, thus constructing a complete observable by composition. This construction implements the idea that the physical content of GR lies in the relations between dynamic quantities represented by partial observables. The idea is that we observe relational evolution between fields and not evolution with respect to some background unobservable structure, such as the 'bare' points of the manifold.

The mathematical formalism behind this idea was largely clarified by Dittrich in (Dittrich (2006),Dittrich (2007)). A partial observable is a physical quantity, written in some coordinate system, for which a measurement procedure can be established and which describes the 'phenomenology'. A complete observable is a physical quantity whose value (or probability distribution in the case of quantum theory) can be uniquely predicted by the relevant theory.

Since a complete observable is constructed from the relationship between two partial observables, Rovelli distinguishes between dependent partial observables and independent partial observables. Usually the role of the independent partial observables is played by the quantities giving the temporal localisation or the spatio-temporal localisation, whereas that of the dependent ones are given by the values that quantites, e.g. fields, take on those points.

1.2   Reference frames in GR

In what follows we largely quote a classification presented in Bamonti (2023) to which we refer for omitted details. However, this paper complements the classification by adding a fourth class of reference frames named '*Auxiliary Reference Frames*'. It further clarifies the distinction between coordinates and '*Idealised Reference Frames*' (see below), based on their different transformation

properties under active diffeomorphisms.

What is a reference frame in GR? Following Rovelli (1991), we define a reference frame, at the most basic level, as a set of variables representing a material system. The four possible classes of reference frames are:

1. The class of '*Idealised Reference Frames*' (**IRFs**), in which any dynamical interaction of the material system represented by the reference frame is ignored. In particular, two approximations are adopted:

    (a) In the EFEs, the stress-energy tensor of the matter field used as reference frame is neglected

    (b) In the system of dynamical equations, the set of equations that determine the dynamics of the matter field is neglected

    An **IRF** can be seen as an 'instantiated' coordinate system to which a physical referent can be assigned, but this referent is represented in an extremely approximated way.[2] The notion is distinct from that of a coordinate system, which assumes *no* instantiation. We will elaborate later (Section 3) that **IRFs** behave differently from a coordinate system under the action of active diffeomorphism: differently from **IRFs**, coordinates are not *necessarily* affected by reshufflings of manifold points. Furthermore, while coordinate systems are 'definitionally' dynamically uncoupled from the fields of the theory, satisfying no equations of motions (EOMs), **IRFs** obtain such a property via an approximation procedure.[3]

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[2]We are using the conceptual and semantic distinction between approximations and idealisations found in Norton (2012). In short, in the case of an approximation we do not introduce a novel system, as we do in the case of an idealisation of a target system. Thus, we see coordinates as idealisations, while **IRFs** are approximations (see also Bamonti (2023)).

[3]Since we will use this terminology often throughout the paper, let us specify what we mean by 'dynamically coupled fields'. In general, it is common to define the dynamical coupling relation between two fields as a relation meaning that: 'one field influences the dynamics of the other, *and viceversa*'. For example, this is how Bamonti (2023) understands the term coupling, which is also the way commonly used in physics: in terms of fields interacting with other fields. However, as we will elaborate in Section 2, we will use the term 'coupling' in a *different way*. According to our use of the term, in *every* spatiotemporal theory *any* field is coupled with *any* other, *via the common metric*. Hence, our notion is distinguished from that of influence (see Brown (2005); Brown and Lehmkuhl (2013);(James Read, 2023, p.43) and the so-called 'dynamical approach to spacetime theories' on some subtleties of the concept of influence regarding the role of the Minkowski metric in SR). If helpful, the term dynamical coupling used in this paper can also be referred to in terms of *correlation*, as distinct from influence.

2. We identify '*Auxiliary Reference Frames*' (**ARFs**) as a class of reference fields that extends over the spacetime manifold and thus they, like **IRFs**, are assumed to covary with reshufflings of points under an active diffeomorphism (a property also called equivariance). However, differently from an **IRF**, an **ARF** has EOMs describing its dynamics, but these EOMs are uncoupled from those of the object written with respect to the chosen **ARF**. Consider the case of four scalar fields $\phi^{(I)}, I = 1, \cdots, 4$, satisfying, *e.g.*, a Klein-Gordon dynamics, to be used as a reference frame for a Lorentzian metric g_{ab}.[4] Suppose, for instance, that the Klein-Gordon fields do not backreact on g_{ab}, and their dynamics is written in terms of an auxiliary Lorentzian metric h_{ab}, dynamically uncoupled from the 'main' metric g_{ab}. Such a reference frame will be dynamically uncoupled from the metric g_{ab}. This is because we need to choose initial data for the four scalar fields, and their evolution will depend on the specific form of the auxiliary metric h_{ab} and not on the metric g_{ab}, which we are assuming is the only dynamically relevant one for the theory. To be clear, the difference between **IRFs** and **ARFs** is that while the former are treated as purely kinematical fields, the latter have a well-defined (and not neglected) dynamics, but that dynamics is uncoupled from the dynamically relevant metric.

3. The class of '*Dynamical Reference Frames*' (**DRFs**) is one in which only approximation (a) above holds. In brief, the **DRF** is affected by the metric field (it is acted upon), but the metric field is not affected by the **DRF** (it does not act), so we are neglecting the backreaction of the frame on spacetime. A realistic example of a **DRF** is represented by the set of the so-called *GPS coordinates*, introduced in Rovelli (2002a). The idea is to consider the system formed by GR coupled with four test bodies, referred to as satellites, which are deemed point particles following timelike geodesics, meeting at some (starting) point O. Each particle is associated with its own proper time ϕ. Using light signals from the satellites, we can uniquely associate four numbers $\phi^{(I)}, I = 1, 2, 3, 4$ to each spacetime point P in the appropriate region. These four numbers represent the four physical variables that constitute the **DRF**. Physically they constitute the lightlike distance between the intersection points with the past lightcone of P and the starting point O. See Fig. 1.

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[4]Here, we are using the abstract index notation (see Penrose and Rindler (1987)) to stress that it is a geometrical object independent from the choice of a coordinate representation.
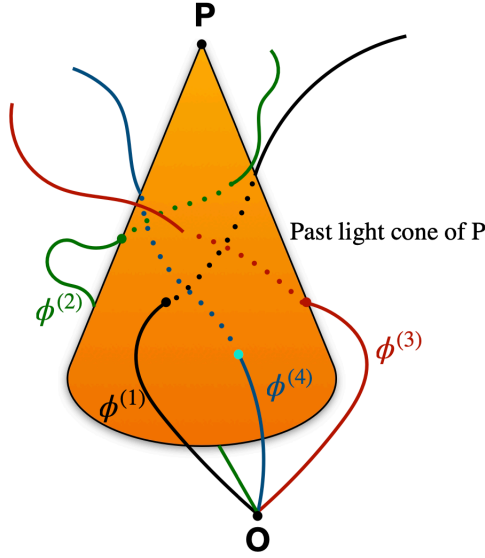
Figure 1: Construction of the set of GPS reference frames $\phi^{(I)}, I = 1, 2, 3, 4$.

4. Finally, the class of '*Real Reference Frames*' (**RRFs**) is one in which both the dynamics of the chosen material system and its energy-momentum tensor are taken into account. Example of **RRF**s are pressureless dust fields (Brown and Kuchař (1995); Giesel et al. (2010)) and massless scalar field (Rovelli and Smolin (1994); Domagała et al. (2010)).

For the sake of simplicity, we will group **IRFs** and **ARFs** under the label of '*Uncoupled Reference Frames*' (**URFs**), since they share the property of being fields defined in the manifold, but do not interact dynamically with the dynamical system under examination (such as the metric field). **DRFs** and **RRFs** will be grouped under the label of '*Coupled Reference Frames*' (**CRFs**).

1.3 Earman's SP principles

For the aim of this paper, which deals only with general-relativistic theories, we articulate Earman's SP principles in terms of internal and external parameters (cf. Earman (1992) or Gomes (2023a)). *External parameters* are the independent variables and, in our main case study, correspond to the points of the smooth manifold $\mathscr{M}$. *Internal parameters* are the value spaces, described by functions F (or dependent variables) of the independent variables. We assume all our models will share the same kind of description as $\langle \mathscr{M}, F_n \rangle$, where n denotes a generic index listing the functions. Given some domain of the functions F_n, and before the imposition of the dynamical, differential equations

of motion, we define models of the form $\langle \mathcal{M}, F_n \rangle$ as '*Kinematically Possible Models*' (KPMs). The only constraint that we will assume any definition of KPMs satisfy is that the F_n and their domain are only constrained algebraically: i.e. they are not (implicit) solutions of differential equations.[5] Among the models of the KPM's, there are the models that satisfy the equations of motion of the theory, a subset of KPMs known as '*Dynamically Possible Models*' (DPMs).[6]

In characterising symmetries of spatiotemporal theories, we need to distinguish between dynamical and background structures.[7] In the case of GR, we take this background structure to be the smooth manifold $\mathcal{M}$, or more precisely the smooth structure *induced* by the maximal atlas of compatible charts composing $\mathcal{M}$ (for such a 'chart-nominalist' way to define $\mathcal{M}$, see Lang (1999); a different viewpoint can be found in Kobayashi and Nomizu (1963), where $\mathcal{M}$ has an *intrinsic* smooth structure: see Wallace (2019) for a comprehensive discussion).[8] In more detail:

Spacetime Symmetries are a group of transformations that preserve the background structure of the base set of independent variables. In GR these are the automorphisms of the manifold $\mathcal{M}$. They form the group $G_S \equiv Diff(\mathcal{M})$ of (active) smooth diffeomorphisms, which are smooth reshufflings of points.

Dynamical Symmetries are a group of transformations that act on all the DPMs and preserve solutionhood of the dynamical equations. That is, they take solutions to solutions, and a non-solution to a non-solution. We assume that they form a group, G_D. For the Einstein field

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[5]This generally implies that such models can be represented locally as sections of some appropriate vector bundle.

[6]These two spaces can also be made more precise in terms of Anderson (1967)'s ideas about 'absolute' and 'dynamical' objects, but we will not need this specification here. Roughly, one partitions the structures of the models into a background, or fixed structure, which all of the models share, and into another, dynamical structure, which, for the DPMs, satisfies further constraints given by the equations of motion.

[7]Again, the definition is often given in terms of absolute objects, even if the notion of an absolute object is ambiguous (Anderson (1967); Friedman (1983); Pooley (2017)).

[8]We should point out that there is no common agreement on what to define as spacetime; whether it is: (i) the manifold $\mathcal{M}$; (ii) the pair $(\mathcal{M}, g)$ of the manifold and the metric field, representing gravitational field; (iii) the gravitational field g alone. In cases (i) and (ii), the difference is whether $\mathcal{M}$, seen as a stage on which the dynamical variables play out their roles, has independent ontological standing from the fields or not. The case (iii) sees $\mathcal{M}$ as a non-ontological mathematical tool and not as a non-dynamical stage with dynamical fields living over it: spacetime itself is a field (Rovelli and Gaul (2000); Rovelli (2006); Rovelli and Vidotto (2015); Einstein et al. (2015)).

equations, it is the case that $G_D \equiv Diff(\mathcal{M}).$[9]

Using these notions, Earman (1992) defines two principles about symmetries:

(SP1) Any dynamical symmetry is a spacetime symmetry

(SP2) Any spacetime symmetry is a dynamical symmetry

Jointly, the two principles require the dynamical symmetries to be just those induced by auto-morphisms of $\mathcal{M}$. As can be inferred from the above, in GR it is expected that the two principles are fulfilled. However, as we shall see in Section 2, if we drop some implicit assumption about GR, this can fail to be the case.

2 Breaking Earman's SP1 Principle

As will become apparent, at least two fields will be necessary to illustrate some of our claims about, or inspired by, **URFs**. So we introduce two generic dynamical fields $\Theta(p)$ and $\Psi(p)$ defined on $\mathcal{M}$. They may be two sets of scalar fields; or a metric field and a scalar field; or a generic tensorial field and a vector field, and so on. Their nature is irrelevant to the discussion, as long as they are sections of natural bundles meaning they admit a unique action of the diffeomorphisms.

In what follows, we propose a redefinition of dynamical symmetries (section 2.1) that will be used to violate Earman's **(SP1)** in section 2.2. The whole discussion will find a natural application in the case of **URFs**.

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[9]We draw attention to the distinction between symmetries of equations and symmetries of specific solutions. As we shall see below, dynamical symmetries preserve the *solutionhood*, but do not leave individual DPM's invariant. For example in GR, for each solution g_{ab} of the EFEs, any diffeomorphism d preserves solutionhood, but generically it is such that $[d^*g]_{ab} \neq g_{ab}$. The subgroup of $Diff(\mathcal{M})$ of the symmetries of g_{ab} that is connected to the identity is the Killing Group of transformations, and is generated by Killing vector fields, which for generic metrics is trivial, i.e. vanishing. (Pooley, 2022, p.121) argues that a dynamical symmetry ''preserves the *form* of the dynamical equations''. This could be misleading. Preserving the *form* is not sufficient for a transformation to define a dynamical symmetry. For example, we can write the general covariant form of Klein-Gordon dynamics in Special Relativity $\eta^{ab}\nabla_a\nabla_b\phi = 0$ and any $d \in Diff(\mathcal{M})$ will preserve its *form*, but will not preserve its *solutionhood* as Pooley (2022) himself sustains later (ivi, p. 250). This is because, in a strict understanding of Special Relativity, if $(\mathcal{M},\eta_{ab},\phi)$ is a DPM, then $(\mathcal{M},[d^*\eta]_{ab},d^*\phi)$ is a DPM *only* for those d such that $[d^*\eta]_{ab} = \eta_{ab}$, which define the Poincaré subgroup of $Diff(\mathcal{M})$.

2.1 Generalisation of Dynamical Symmetries

As far as we are aware, the redefinition of dynamical symmetries that we propose below may generalise the familiar ones in the literature (starting in Earman (1992), but encompassing the already mentioned Pooley (2022), Belot (2013), or the more recent Jacobs (2023)). (Earman, 1992, p.45) distinguishes between non-dynamical (A) and dynamical (P) objects. But only the latter would correspond to our Θ or Ψ; we are already assuming that the only background structure (A) is the smooth structure of the set $\mathscr{M}$. Earman (ibid.) then defines a dynamical symmetry as follows:

> Consider a model $M = \langle \mathscr{M}, A_1, A_2, \cdots, P_1, P_2, \cdots \rangle$ and let Φ be a diffeomorphism that maps $\mathscr{M}$ onto $\mathscr{M}$. Define $M_\Phi = \langle \mathscr{M}, A_1, A_2, \cdots, \Phi^* P_1, \Phi^* P_2, \cdots \rangle$. Now Φ will be said to be a dynamical symmetry just in case for any $M \in \mathbb{M}_T$, it is also the case that $M_\Phi \in \mathbb{M}_T$ [*here $\mathbb{M}_T$ represents the set of all DPMs*].

So in our notation:

(Standard) Dynamical Symmetry: $d \in G_D \subseteq Diff(\mathscr{M})$ such that (iff): $\langle \mathscr{M}, \Theta, \Psi \rangle$ is a DPM, *iff $\langle \mathscr{M}, d^*\Theta, d^*\Psi \rangle$ is.*

Thus, a dynamical symmetry is given by a *single* element $d \in G_D \subseteq Diff(\mathscr{M})$ acting on *every* dynamical field of the theory. Note, for future reference, that the definition allows $\langle \mathscr{M}, d^*\Theta, \Psi \rangle$ or $\langle \mathscr{M}, \Theta, d^*\Psi \rangle$ as DPMs; but in that case $d \in Diff(\mathscr{M})$ cannot be classified as a dynamical symmetry. We will attribute this restriction on what has been usually countenanced as a dynamical symmetry in the literature to the presumption that *all* the dynamical objects are dynamically coupled *to each other*. Such hidden assumption is a necessary condition to consider dynamical symmetries only as Standard ones. We will show below in Section 2.2 that it is easy to extend the formalism once this presumption is dropped

Recall that in section 1.3, we distinguished two types of symmetries: spacetime and dynamical. We now re-define them as follows in terms of models:

Spacetime Symmetry (SS): $d \in G_S \subseteq Diff(\mathscr{M})$ act as: $\langle \mathscr{M}, \Theta, \Psi \rangle \rightarrow \langle \mathscr{M}, d^*\Theta, d^*\Psi \rangle$, for all KPMs.

Dynamical Symmetry (DS): $d, f \in G_D \subseteq Diff(\mathcal{M}) \times Diff(\mathcal{M})$ such that (iff): $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, *iff* $\langle \mathcal{M}, d^*\Theta, f^*\Psi \rangle$ is.

DS is still defined in the broad spirit of section 1.3: it is any transformation that preserves solutionhood for Θ and Ψ and here, for convenience, we also require it to preserve the smooth background of $\mathcal{M}$. In order to recover **DS**, the passage in Earman's definition of (Standard) Dynamical Symmetries that would have to be modified lies in restricting G_D to be a subgroup of $Diff(\mathcal{M})$. In fact, this excludes the case in which e.g. $G_D = Diff(\mathcal{M}) \times Diff(\mathcal{M})$ and *different* elements of the G_D group act *independently* on the different dynamical fields, which is allowed in our more permissive definition.[10] [11]

At this point, we can introduce a notation that will be useful for the remainder of the paper. Just to keep the discussion as simple as possible we consider the case in which our fields Ψ and Θ are two sets of four scalar fields, constituting $\mathcal{M} \to \mathbb{R}^4$ maps, which we will call ϕ_1 and ϕ_2 (just for consistency of notation of the paper when referring to scalar fields). For any $d \in G_S \subseteq Diff(\mathcal{M})$, we require that: given $x \in \mathbb{R}^4$ and $(\phi_1)^{-1}(x) = p \in \mathcal{M}$, then $(d^*\phi_1)^{-1}(x) = d(p)$.[12] Thus $\phi_1(p) \equiv d^*\phi_1(d(p)) = x$. Since the same is true for $\phi_2(p)$, defining $\phi_2(\phi_1) := \phi_2 \circ \phi_1^{-1} : \mathbb{R}^4 \to \mathbb{R}^4$, we can conclude that:

$$\phi_2(\phi_1) \equiv d^*\phi_2(d_*\phi_1), \ \forall d \in Diff(\mathcal{M}). \tag{RI}$$

Here is a short proof:

Proof. $\phi_2(\phi_1)(x) = \phi_2(\phi_1^{-1}(x)) = \phi_2(p) = d^*\phi_2(d(p)) = d^*\phi_2((d^*\phi_1)^{-1}(x)).$ $\qquad\square$

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[10]In the case in which $d \equiv f \in G_D \equiv Diff(\mathcal{M})$ is a *single* diffeomorphic map, **DS** is used in the literature to define the so-called '*Diff-Invariance*' for a theory (Pooley (2017); Read (2016)). Using our definition of **DS**, '*Diff-Invariance*' can be generalised to the case in which d and f are two different elements of $G_D = Diff(\mathcal{M}) \times Diff(\mathcal{M})$. Our analysis suggests that Pooley's Diff-Invariance presumes dynamical coupling between fields of the theory, so our generalisation on G_D is *a priori* blocked and a *single* transformation ($d \in G_D \equiv Diff(\mathcal{M})$) acts in the same way on every dynamical field.

[11]The quoted passage would define a more encompassing view of dynamical symmetries if we interpret '*just in case*', not as an '*iff*', as it is probably intended, but as an '*if*'. While one verse of the implication is correct, that is: '*if* $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM iff $\langle \mathcal{M}, d^*\Theta, d^*\Psi \rangle$ is, *then* d gives a dynamical symmetry', the other verse: '*if* d is a dynamical symmetry, *then* $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM iff $\langle \mathcal{M}, d^*\Theta, d^*\Psi \rangle$ is, for $d \in Diff(\mathcal{M})$' excludes a range of other possible cases characterising d as a dynamical symmetry. In fact, there may be dynamical symmetries that map DPMs to DPMs and yet cannot be described with a single d, and so don't fit this format. In other words, there are dynamical symmetries that are not spacetime symmetries.

[12]$p = (\phi_1)^{-1}(x)$ is usually called a *dressed* point (Harlow and qiang Wu (2021); Goeller et al. (2022)), that is a spacetime point defined through four values some physical fields.

That is, *both fields change together by the same transformation, so that the relations between them do not change*.[13] Thus, invariance under **SS** will be referred to as '*(point) Reshuffling-Invariance*' **(RI)**. That is, **(RI)** is the requirement that nothing physically salient varies under smooth reshuffling of the manifold points. **(RI)** coincides with the demand that a quantity be relational, that is, **SS** preserves spatiotemporal relations, and so a quantity which is invariant under **SS** can also be called relational.

2.2 (SP1)* Principle

The validity of **(SP1)** principle presupposes the validity of the restriction of G_D group to a subgroup of $Diff(\mathcal{M})$. However, as we will show in this section, once we admit a generalisation of dynamical symmetries as in **(DS)** above, **(SP1)** can be broken without changing the background spatiotemporal structure.

As we already argued, the validity of such restrictions on G_D and, therefore, the validity of **(SP1)** hinges on the tacit assumption, taken as a necessary (but not sufficient) condition, that all the dynamical fields of the theory are dynamically coupled to the metric, and therefore, implicitly, to each other.[14] Such assumption has also a consequence, *qua* sufficient condition, on the allowed models, that is:

If all fields are dynamically coupled to each other, then:

if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, neither $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ nor $\langle \mathcal{M}, \Theta, d^*\Psi \rangle$ are, $\forall d \in G_D$.[15]

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[13]Even if we are not going to use it, we can also define invariance under **DS**, which will be referred to as '*Dynamical Symmetry-Invariance* **(DSI)**'. In the case of dynamically uncoupled fields, this condition reads as: $\phi_2(\phi_1) \equiv f^*\phi_2(d^*\phi_1)$ (or $d^*\phi_2(f^*\phi_1)$), $\forall d, f \in G_D \subseteq Diff(\mathcal{M}) \times Diff(\mathcal{M})$. Of course, when the quantity $\phi_2(\phi_1) \neq f^*\phi_2(d^*\phi_1)$, **(DSI)** is not met, but **(RI)** still is, just in virtue of the property of geometric fields to equivary with the action of a *single* diffeomorphism.

[14]It should be noted that in the context of GR this assumption *coincides* with one of the core tenets of the theory, namely, the universality of gravitational interaction which acts as a 'common cause' (we are echoing Reichenbach (1956)'s well-known 'common cause argument'). This assumption, therefore, is tacit for good reason. In the following, we are finding something strange when we foil this tacit assumption, which is the violation of **SP1**. This is consistent with a violation of the universality of gravitational coupling and it shows the connection between the two arguments. As we will see later, dropping this tenet is inspired by the possibility of **URFs**. Usually, reference frames' dynamics is approximated because, in practice, it is considered a good approximation not to include the influence of the reference frame, which is thus 'external', in the system under study.

Consequently, in order to preserve solutionhood, a dynamical symmetry acting on dynamically coupled fields *must* act equally on all fields and *cannot* act independently on one or the other. It is the case that **DS** derives from the dropping of the condition that all fields are dynamically coupled to each other, which results in the possibility of extending the dynamical symmetry group. That is: according to **DS** not only combinations as $\langle \mathcal{M}, \Theta, d^*\Psi \rangle$, or $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ are possible models (as in the case of (Standard) Dynamical Symmetry), but in such a case d is defined as a dynamical symmetry *for such uncoupled fields*:

If all fields *aren't* dynamically coupled to each other, then:

if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM, both $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ and $\langle \mathcal{M}, d^*\Theta, \Psi \rangle$ are, $\forall d \in G_D$.

However, also in this case it is still valid that:

If all fields are dynamically coupled to each other, then:

if $\langle \mathcal{M}, \Theta, \Psi \rangle$ is a DPM $\langle \mathcal{M}, d^*\Theta, f^*\Psi \rangle$ is not, for $d \neq f \in G_D$.

Keeping the working assumption made in the previous section to deal with two scalar fields in a general-relativistic setting, let's here consider the case where (ϕ_1, ϕ_2) are dynamically uncoupled. In that case, both models $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ and $\langle \mathcal{M}, d^*\phi_1, f^*\phi_2 \rangle, \forall d, f \in G_D = Diff(\mathcal{M}) \times Diff(\mathcal{M})$ are DPMs. Again without loss of generality, we restrict attention to the subset of transformations $\langle \mathcal{M}, \phi_1, \phi_2 \rangle \rightarrow \langle \mathcal{M}, d^*\phi_1, \phi_2 \rangle$.[16] Namely, since the fields are uncoupled, the G_D-group can act independently on *either* ϕ_1 or ϕ_2, while preserving solutionhood.

We now show why this breaks Earman's **(SP1)** symmetry principle.

In the case we introduced, both $\langle \mathcal{M}, d^*\phi_1, d^*\phi_2 \rangle$ and $\langle \mathcal{M}, d^*\phi_1, \phi_2 \rangle$ are related to $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ by **DS**. However, only $\langle \mathcal{M}, d^*\phi_1, d^*\phi_2 \rangle$ is related to $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ by **SS**, since spacetime sym-

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[15]Following what we stated in footnote 3 (i.e. that in *every* spacetime theory all fields are dynamically coupled with each other, due to the presence of a *common metric field*), also in SR, if $\langle \mathbb{R}^4, \eta_{ab}, \Theta, \Psi \rangle$ is a DPM, then e.g., the triple $([\Lambda^*\eta]_{ab}, \Lambda^*\Theta, \Psi)$ is not, $\forall \Lambda \in G_D \equiv Poin(\mathbb{R}^4)$. The same applies for Galilean transformations in Newtonian physics. One possible way to break the coupling between fields is to neglect the dynamics of at least one of them. In this regard, we would like to specify that **SP1** implies that all fields appearing in KPMs are dynamically coupled, according to our definition, not just dynamical ones.

[16]This case is not to be confused with the case where a dynamical symmetry d is an automorphism of ϕ_2, acting as $\langle \mathcal{M}, \phi_1, \phi_2 \rangle \rightarrow \langle \mathcal{M}, d^*\phi_1, d^*\phi_2 \rangle \equiv \langle \mathcal{M}, d^*\phi_1, \phi_2 \rangle$.

metries, by smoothly reshuffling the underlying spacetime points, require that the same, single diffeomorphism act on *every* internal parameter defined on $\mathscr{M}$, '*simultaneously*'. In other words, it is only **SS** that necessarily preserves all spatiotemporal relations. So here we have a dynamical symmetry which is not a spacetime symmetry, since it doesn't preserve all spatiotemporal relations and so it breaks **(SP1)**, as expected. Put differently, when we have a general-relativistic theory in which we use dynamically uncoupled fields, the dynamical symmetries are not ''just a reflection of the automorphisms of $\mathscr{M}$'' (Gomes (2023b)).

This allows us to advance a reformulation of the **(SP1)** principle:

(SP1)* Principle: every dynamical symmetry is a spacetime symmetry, *only for dynamically coupled fields*.

Earman (1992) sustains that:

> The theory that fails **(SP1)** is thus using more space-time structure than is needed to support the laws, and slicing away this superfluous structure serves to restore **(SP1)**.

A textbook counterexample of **(SP1)** is given by Newtonian mechanics. For Newtonian space-time has a standard of absolute rest, but this structure is not preserved by the boost symmetry of the EOMs. One realigns the structure to the laws, thus regaining **(SP1)**, by formulating an equivalent theory in Neo-Newtonian, or Galilean spacetime (Earman (1992)). However, in our case, $\mathscr{M}$ has very little structure: there is nothing to get rid of. The defining G_S-group of our base set of external parameters is, apart from arbitrary point permutations, or merely continuous but not differentiable transformations, the largest one, that is $Diff(\mathscr{M})$; but decreasing the structure of the manifold to only these automorphisms (e.g. continuous but not smooth) as spacetime symmetries would not help bridge the gap to the dynamical symmetries.[17] Consequently, Earman's argument to save **(SP1)** fails in the case we have illustrated here.

Earman's **(SP2)** principle, on the other hand, is preserved. In fact, given $\langle \mathscr{M}, \phi_1, \phi_2 \rangle$, every model $\langle \mathscr{M}, d^*\phi_1, d^*\phi_2 \rangle$ related by a spacetime symmetry is also related by a dynamically symmetry.

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[17]The approach of defining the structure of a space by the group of transformations that leave it invariant descends from the *Klein-Erlangen Program*. In the Kleinian sense, $\mathscr{M}$ is defined as the space structured by the group $Diff(\mathscr{M})$. See Wallace (2019).

It goes without saying that all the above finds a natural application when dealing with **URFs** in the theory. In fact, recognising the possibility of **URFs** motivated our redefinition of dynamical symmetries. This is a relevant argument in favour of the need for the exploration and critical analysis of the concept of reference frames.

In this regard, we should mention that **URFs** also include **IRFs**, which are non-dynamical reference frames, i.e. they do not have any dynamics entering the models of the theory. This is not a limitation to the discussion above, since given an **IRF** ϕ_2 and a DPM $\langle \mathcal{M}, \phi_1, \phi_2 \rangle$ — defined by taking into account *only* the dynamical equations of the dynamical fields ϕ_1 — we can obtain a DPM $\langle \mathcal{M}, d^* \phi_1, \phi_2 \rangle$ where d *only* acts on the dynamical field ϕ_1 and therefore does not represent a spatiotemporal symmetry transformation, thus violating **(SP1)**. Conversely, given a **(SS)** transformation d under which *all* the fields equivary, the model $\langle \mathcal{M}, d^* \phi_1, d^* \phi_2 \rangle$ will be a DPM (in which the solutionhood to be preserved only concerns the dynamic equations of ϕ_1 under reshufflings of points). Thus **(SP2)** is still preserved.

3 Two critiques on partial observables

3.1 Partial observables must be dynamically coupled to each other to form a complete, gauge-invariant observable

As we discussed in section 1.1, the standard way of introducing partial observables is to have a measuring procedure that can be associated with them. This makes them 'observables' in the ordinary sense. However, in the various practical examples of partial observables given in the literature (see e.g. (Rovelli, 2004, Sec. 2.3.1) or (Rovelli, 2002b, Sec.2)), the need for partial observables to be dynamically coupled in order to form a complete observable is never explicitly emphasised. On the contrary, Rovelli in his analysis of the significance of coordinates in GR, takes what we have classified as **IRFs** (which he calls undetermined physical coordinates) to be partial observables, in the same way as partial observables consisting, for example, of GPS coordinates. In fact, he states that:

> The coordinates are partial observables in (i) physical coordinates, with an interpre-
> tation as positions with respect to objects whose equations of motions are taken into

account and (ii) physical coordinates with an interpretation as positions with respect to objects *whose equations of motions are ignored* [Our italics].[18] (Rovelli, 2004, p.63)

As we will elaborate on below, the claim about (ii) is false. Not every pair of physical quantities to which measuring instruments can be associated can play the role of partial observables. *Bona fide* partial observables must be dynamically coupled to each other, in order for their relation to constitute a complete observable.

Suppose that we have a metric field g_{ab}, satisfying the EFEs, and four scalar fields $\{\phi^{(I)}\}$ to be used as a reference frame, each satisfying *e.g.* Klein-Gordon dynamics with distinct initial data so that their independent values describe a (local) diffeomorphism $U \to \mathbb{R}^4$ for some $U \subset \mathcal{M}$. To be clear, fields $(g_{ab}, \phi^{(I)})$ are a particular case of (Θ, Ψ) introduced in the previous section.

Let us assume that in fact $\mathcal{M}$ is diffeomorphic to $\mathbb{R}^4$, so that we can choose $U = \mathcal{M}$. This assumption is equivalent to the requirement that the set of scalar fields is invertible *everywhere*. Though this cannot be generically upheld, it serves to illustrate our claims in the remainder of the paper.[19] Note that, due to our assumptions, any two values for the scalar fields are related by a diffeomorphism of $\mathcal{M}$, since $(\phi^{(I)})^{-1} \circ \phi'^{(I)} \in Diff(\mathcal{M})$. Given *any* doublet $(g_{ab}, \phi^{(I)})$, the metric can be parametrised by the (four) values of the scalar fields, used as reference frames. Thus, we can write: $g_{IJ}(\phi) := g_{ab} \circ (\phi^{(I)})^{-1}$. Such a quantity is commonly defined a '*relational observable*'.

Let us summarise the requirements for a quantity to be a complete observable:

(RI): It must remain invariant under reshufflings of manifold points, via active diffeomorphisms; the quantity is relational

(DET): Its dynamical evolution must be deterministic.

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[18]We interpret the term 'ignored' as meaning that objects whose coordinates are used as 'physical coordinates' (which we call reference frames) are dynamically uncoupled from the other dynamical objects in the theory, through an approximation procedure. Also, but secondarily, this quote should be corrected, in light of what Rovelli writes on page 62: ''[the physical] coordinates X^μ are interpreted as the spacetime location of reference objects whose dynamics we *have chosen* to ignore.''. Thus the corrected quote in the text should read as: ''[...] physical coordinates with an interpretation as positions *of* objects whose equations of motions are ignored'', thus clarifying that we ignore dynamical equations *of* the reference objects.

[19]It is very difficult to think of a realistic situation in which a reference frame would cover the entire manifold. In fact, four Klein-Gordon scalars won't generally form a bijection to a four dimensional 'empirical manifold'. For example, they could end up having the same values everywhere on $\mathbb{R}^4$, thus representing only one (physical) point. Of course, tipically that is not going to happen, but the question is how do we know, based on initial values? Thus, in order to indicate viable reference frames, ϕ should be *at least locally* invertible, i.e. in some open set $U \subset \mathcal{M}$ and for a given chart, $\det(\partial \phi / \partial x^\mu) \neq 0$. Here, we choose to avoid stipulating a region $U \subset \mathcal{M}$, since stipulating the region would be like stipulating the reference of a spacetime point p.

These requirements also define what we will call the '*Gauge-Invariance*' **(GI)** property: **(GI)** $\leftrightarrow$ [**(RI)** $\wedge$ **(DET)**], where $\wedge$ is the logical conjunction stating that **(GI)** is true iff *both* **(RI)** and **(DET)** are true.[20]. Depending on whether $\phi^{(I)}$ is an **URF** or a **CRF**, we can conclude whether or not $g_{IJ}(\phi)$ is a complete, gauge-invariant observable. It is useful to analyse the two cases in terms of initial values and determinism, since we already showed that **(RI)** is always satisfied for $g_{IJ}(\phi)$ for any frame field.

i) As in our argument against **(SP1)** using uncoupled fields, in the case of **URFs**, any $\phi^{(I)}$ (whose Klein-Gordon equations of motion are now neglected) is still compatible with all the isometric metrics, i.e. $\langle \mathcal{M}, g_{ab}, \phi^{(I)} \rangle$ and $\langle \mathcal{M}, [d^*g]_{ab}, \phi^{(I)} \rangle$ are legitimate models for the dynamics, for all $d \in Diff(\mathcal{M})$.[21]

Moreover, since we are dealing with reference frames, while it is true that $g_{ab} \circ (\phi^{(I)})^{-1}$ is **(RI)**, so is $[d^*g]_{ab} \circ (\phi^{(I)})^{-1}$. And we have no reason to start with one g_{ab} rather than with an isomorphic metric, *even if we assume we have reason to start with one $\phi^{(I)}$ rather than any of its isomorphic distributions*. That is, even if we are given a field $\phi^{(I)}$ extending throughout spacetime, and even if we are also given initial data for the metric, we will not find a *unique* evolution for the (frame representation of the) metric. Consequently, **(DET)** is not met. The reason, as we mentioned above, is simply that any $\phi^{(I)}$ is compatible with any g_{ab}, including d^*g_{ab}. For each of the choices, $[d^*g]_{ab} \circ (\phi^{(I)})^{-1}$ is **(RI)**. But we still have effectively an action of $Diff(\mathcal{M})$ left, so we have the redundancy that we started off with, before choosing a reference frame representation. Thus a set of degrees of freedom that constitute an **URF** cannot be defined as partial observables, since by definition partial

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[20]More appropriately, we could use **(DSI)** (see footnote 13), instead of **(GI)**. Here is an important distinction: gauge symmetry could be construed as the automorphisms of a background structure, in which case they do not depend on the equations of motions, or they can be defined as dynamical symmetries that preserve the dynamics. For example, (Read, 2016, §2.2) accepts that KPMs could be gauge-related, thus dynamical equations need not to be involved. But in order to keep with the most common nomenclature on this corner of the philosophy of physics literature, we adopt **(GI)** as opposed to **(DSI)**.

[21]Note that the term isometry is being used here as a synonym for the induced isomorphism on the fields, through pull-back of the diffeomorphism d (see (Belot, 2017, p.959)). A distinction is therefore being made between isometries and automorphisms, only the latter leaving the metric invariant. This is not an obvious choice. In the physical literature, for example in (Landsman, 2021, p.71) an isometry is defined as a metric-preserving diffeomorphism: i.e. a diffeomorphism such that $[d^*g]_{ab} = g_{ab}$. Thus isometry and automorphism coincide. In other words, isometries are understood as flows of Killing fields (ibid. p.57). Here we advance the hypothesis that this distinction reflects the different understanding of the word isometry. In the first case one means *iso-(geo)metry*: 'same geometry', where a geometry is an equivalence class of diff-related metrics. In the second case *iso-metry*: 'same metric'.

observables could be used to construct a *unique* complete observable whose evolution is deterministic.

ii) In the case of **CRFs**, when $\langle \mathcal{M}, g_{ab}, \phi^{(I)} \rangle$ is a DPM, then $\langle \mathcal{M}, [d^*g]_{ab}, \phi^{(I)} \rangle$ is not, for a generic $d \in Diff(\mathcal{M})$. Here, a choice of $\phi^{(I)}$ (rather than any of its isomorphic distributions) and initial data give us a *unique* representation for $g_{IJ}(\phi)$. Thus $\phi^{(I)}$ and g_{ab} are *bona-fide* partial observables: not only is the composition $g_{ab} \circ (\phi^{(I)})^{-1}$ (**RI**), but generically *only* the diagonal action of the active diffeomorphism, relating $(g_{ab}, \phi^{(I)})$ and $([d^*g]_{ab}, d^*\phi^{(I)})$, preserves solutionhood. For this reason, here a choice for $\phi^{(I)}$ *fixes* the gauge for g_{IJ}. At most one of all of the distributions in the isometry class of g_{ab} is compatible with each distribution of $\phi^{(I)}$ and (**DET**) is ensured.

3.2   Partial observables are *relational*, but *gauge-variant* observables

A partial observable is formally defined in Rovelli (2002b) as a certain, measurable, gauge-variant quantity, expressed in a given coordinate system $\{x^\mu\} \in \mathbb{R}^4$. For consistency with the previous section, one example would be a metric field $g_{\mu\nu}(x^\rho)$. However, coordinates are often understood as mathematical artefacts, without any physical instantiation. In this understanding, how is it possible for partial observables to be measurable? In practice, all comparisons between theory and experiment rely on a coordinate system that is physically instantiated, that is, an **URF**. Thus we claim that it would be more in line with its claims about observability to define a partial observable as a relational quantity $g_{IJ}(\phi)$, where $\{\phi^{(I)}\}$ could be a set of four scalar degrees of freedom constituting e.g. an **IRF**. That way, we still save the distinction between partial and complete observables, since $g_{IJ}(\phi)$ is a partial, not a complete observable. But at the same time, we have a description that is closer to that of a measurement. What we measure is not a quantity written in an uninstantiated coordinate system, but a quantity in a certain reference frame $\phi^{(I)}$ which is being approximated to such an extent that it plays the exact same role as a set of coordinates x^μ, as far as the gauge-variance requirement of the observable is concerned. Said differently, an **IRF** is a frame approximated to such an extent that its dynamical significance has evaporated away, leaving behind only the standard notion of coordinate patches, plus the property of equivariance

under point reshufflings.[22]

Only by recognising that partial observables are relational, but *gauge-variant* quantities written in an **IRF**, is it possible to understand that we can associate to them a measuring procedure (possibly abstract, since **IRFs** are approximated representations of a frame: see footnote 2).[23]

It is worth expanding here on the link between relationalism, gauge-invariance and determinism.

The naive notion of a relational quantity as a spatiotemporal relation between degrees of freedom (here represented by physical fields) can be formalised by stipulating that a quantity is 'relational' *iff* it is **(RI)**. In fact, as shown above, a relational quantity written in terms of an **IRF** is **(RI)**, but not **(GI)**, since requirement **(DET)** above, necessary for **(GI)** is not met.

In the other direction, there is only one way to satisfy **(DET)** without satisfying **(RI)**: that is if the theory includes time-*independent* local reshufflings. Such transformations don't spoil determinism (Wallace (2002)), even if the theory includes quantities that vary under them. Thus, generally, **(DET)** is not sufficient for either **(GI)** or **(RI)**.

Consequently, according to our definitions, the following logical relations hold:

$$\text{Relational} \equiv \textbf{(RI)} \tag{L1}$$

$$\text{Relational} \nleftrightarrow \textbf{(DET)} \tag{L2}$$

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[22]**IRFs** are just a helpful approximation, but we only measure complete observables: **(RI)** quantities expressed w.r.t. some **CRF**. When we take measurements, we are necessarily dealing with physical quantities and all physical quantities, given the universality of gravitational interaction, are dynamically coupled. **IRFs** are approximations we adopt 'pretending the reference frame is not there'. We use them to construct (relational) partial observables, which model our measurement outcomes. However, strictly speaking, we should always use **CRFs** to model our measurements. This is because our measurements concern real physical fields, which **IRFs** cannot be. See (Bamonti, 2023, sec.4).

[23]This, moreover, substantiates the position held by Pooley and Read (2021) that empirical data are relational, thus giving rise to what they call 'underdetermination problem'. The problem consists essentially in the impossibility for an observer *making a measurement* to discriminate whether he is *measuring* $g_{ab}(p)$ or $[d^*g]_{ab}(p)$. However, if a partial observable — to which we can associate a measuring procedure —- can be written as $g_{ab}(p)$, as argued in Rovelli (2002b), then an observer can discriminate between the two measurements, since they are *different* partial observables. Nonetheless, from our point of view, the underdetermination problem is a well-posed problem, since what is ill-posed is the definition of a partial observable, as a non-relational quantity. Thus, since empirical data are relational (i.e. **(RI)**), an observer cannot empirically distinguish between $g_{ab}(p)$ and $[d^*g]_{ab}(p)$. Furthermore, our redefinition of partial observables as relational quantities supports the 'immanentist' view of empirical (in)equivalence ((Pooley and Read, 2021, p.10)), since the 'immanent observer' is in fact a reference frame.

$$\begin{cases} \textbf{(GI)} \rightarrow \textbf{(DET)} \\[2mm] \textbf{(GI)} \nleftarrow \textbf{(DET)} \end{cases} \qquad \text{(L3)}$$

$$\begin{cases} \textbf{(GI)} \rightarrow \text{Relational} \\[2mm] \textbf{(GI)} \nleftarrow \text{Relational} \end{cases} \qquad \text{(L4)}$$

The straightforward interpretation of complete observables is aligned with our notion of **(GI)**. This means that it is not straightforwardly true that relational observables are complete observables, a lesson that is not always acknowledged in the literature.

3.2.1 Point-coincidence argument: an operational stance

We now investigate two possible interpretations of what in the literature is called the point-coincidence argument (Einstein (1916); Stachel (1989)). We will show that, by further analysing Einstein's statements, it is possible to give an interpretation of the term '*relational*' that does not coincide with '*gauge-invariant*', as it is usually (tacitly) understood.

To a certain extent, we agree with Rovelli and others that any measurement in physics is performed in a given reference frame. In Anderson's words:

> All measurements are comparisons between different physical systems. (Anderson, 1967, p.128)

But see also (Rovelli, 1991, p.298):

> Any measurement in physics is performed in a given reference system.

Or (Landau and Lifshitz, 1987, p.1):

> For the description of processes taking place in nature, one must have a *system of reference.*

Einstein's early point-coincidence argument—stating that the physical content of a theory relies in spacetime coincidences of material point particles—can be construed similarly. In particular, he sustains that ''all our spacetime *verifications* invariably amount to a determination of spacetime

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coincidences [Our italics]'' Einstein (1916). That is, ''physical *experiences* [are] always assessments of point coincidences [Our italics]'' Einstein (1919). This supports our claim that we can only measure relational quantities and not coordinate dependent quantities. So, the output of a physical measurement of a metric will always take the relational form $g_{IJ}(\phi)$.[24]

However, as we have illustrated here, even if we stick to 'relational' quantities we do not guarantee unique evolution and determinism, which are necessary for the definition of complete observables.

Rovelli endorses an 'operational' reading of the point-coincidence argument—which understands *observability* in the sense of *measurability*. But Rovelli takes this reading to further support *observability in Dirac's sense*, and so to support *predictability* of complete observables. This is how he realizes the connection between relationism and gauge-invariance. For instance, in Rovelli (2004) he states that

> The GPS observables [which are complete ones] are [···] *precisely* Einstein's ''space-time coincidences''.[25]

According to our reading, on the other hand, the point-coincidence argument could also be understood as supporting the observability of partial observables written in relational terms in the sense of *measurability* only. Thus, we are not compelled to embrace, to the same degree, the connection between relationism and gauge-invariance.[26]

Thus, we partly concur with Rovelli about the significance of Einstein's point-coincidence argument: namely, about the connection between *verification* or *empirical experience* and rela-

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[24]A spatiotemporal point-coincidence is at least a 'five-party relationship', pace (Westman and Sonego, 2009, p.25) who claim that ''[···] in order to identify uniquely a point-coincidence [···] locally the values of four different quantities are enough''. What we mean is that a point $P \in \mathcal{M}$ can be *uniquely* defined using, e.g., four *independent* scalars: P is the point in which the value of scalar field $\phi^{(1)}$ has value x, $\phi^{(2)}$ has value y and so on. However, the point itself thus defined is not empirically measurable. A point-coincidence, that is an empirically measurable, *qua* **DI**, quantity, needs *five* quantities to be defined: four scalars that define the point P and a quantity on P. Therefore, the localisation of an '*event*' is defined by some relation like $g_{ab}(\phi)$: the event E is not happening at an apriori assigned place and time, but the place and time are defined by the 'happening' of the event.

[25]A minor criticism of this claim is that it should be correctly stated as: ''GPS observables are precisely *a type of* spatiotemporal coincidences''. The point is that GPS observables are spacetime coincidences (in Rovelli's sense), but not all spacetime coincidences are GPS observables.

[26]It is worth pointing out that to avoid a recursive prescription of the measurement concept – that is, we should define the measurement of the **IRF** $\phi^{(I)}$, understood as a partial observable written in some other **IRF** $\bar{\phi}^{(I)}$, and so on – one possible strategy we see is to argue that partial observables are not *independently* measurable. Partial observables should be intended as measurable only ''simultaneously'' in their definition of a spacetime coincidence of the kind $g_{IJ}(\phi)$.

tionism. Consequently, we take as a necessary condition for a quantity to be empirically accessible that it be **(RI)**, according to L1. However, Rovelli, takes an interpretation of observability 'in the sense of Dirac', which implies **(DET)**, to follow from this connection. And, as seen in L2, no such implication holds. It would hold only conditionally on the further assumption that all measurable quantities are dynamically coupled. Agreed: it is a defensible, often tacit, assumption. But given the fact that it is often dropped in the context of reference frames, it is one that needs to be made nonetheless. Therefore, according to our interpretation, relationalism does not imply logically gauge invariance; according to Rovelli's, it does.

To sum up, we showed that Rovelli's 2002b definition of a partial observable as ''a physical quantity to which we can associate a (measuring) procedure leading to a number'' (ibid.) may contain various sources of misunderstandings that require careful analysis, given the central role of the classification between partial and complete observables in the literature of the foundations of GR and quantum gravity.

4   Summary

The main results of this work are as follows:

In Section 1, we extended the classification found in Bamonti (2023) on possible reference frames in GR, identifying the class of **ARFs**: reference frames that are instantiated but whose dynamics are uncoupled to the metric, e.g. defined in terms of an auxiliary metric. Furthermore, we have proposed a coarser yet effective classification, which identifies reference frames as dynamically uncoupled from the geometry (**URFs**) or dynamically coupled to it (**CRFs**).

In Section 2, we defined reshuffling-invariance **(RI)** as the invariance property under spatiotemporal symmetries **(SS)**. We also introduced a more general definition of dynamical symmetry **(DS)** that includes the possibility of independent actions of symmetry group elements on dynamical objects (section 2.1). This more general definition, in the case of dynamically uncoupled fields, allowed us to highlight a violation of Earman's 1992 **(SP1)** principle, identifying dynamical symmetries that are not spatiotemporal symmetries (section 2.2). Moreover, this failure cannot be remedied by eliminating some background spacetime structure. Although this failure is avoided

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by an assumption of dynamical coupling between all the fields such an assumption is not mandatory either for mathematical consistency nor for physical coherence. Indeed, our results were motivated by recognising the possibility of **URFs**; a possibility that has been explored in the literature. In other words, the violation of **SP1** by uncoupled fields may only suggest that these fields should not be taken as physical fields in GR. Indeed, doing otherwise would violate a core principle of gravitational physics: universality. Nonethless, since in the literature dealing with quantum reference frames (see e.g. Kabel et al. (2024) and references therein), such uncoupled fields are frequently employed as reference frames thus, in these contexts, it is worthwhile to establish just which familiar principles of general relativity must be abandoned, and with which consequences. This section highlighted the fundamental importance of a critical analysis of reference frames, which has the potential to bring to light details of the foundations of space-time theories otherwise taken for granted.

We then pursued two criticisms of the notion of partial observables (Section 3). The first criticism concerns the possibility of two partial observables being combined in a relational manner to form a complete observable. Only in the case where the two partial observables are dynamically coupled to each other, and thus one of them is used as a **CRF**, is it possible to obtain a *bona-fide* complete observable: i.e. a **(RI)** quantity whose evolution is deterministic **(DET)**. The property of being **(RI)** and **(DET)** was called gauge-invariance **(GI)**. (Section 3.1).

The second criticism concerns the very formal definition of partial observables. For strictly speaking, coordinates hold no physical significance in GR since they are are not physically instantiated. Thus it is necessary to define a partial observable relationally, in terms of an **IRF**: partial observables are relational, but gauge-variant quantities. In this manner, we have also clarified the conceptual distinction between relationism and gauge-invariance, two terms that are often conflated. (Section 3.2).

These reflections led us to operationally interpret the point-coincidence argument. In accordance with this interpretation, the observability of spacetime coincidences refers to their relational character, not to the fact that they constitute gauge-invariant events and, therefore, observability does not imply *predictability*. The verifications of spacetime coincidences mentioned by Einstein (1916) are measurements of relational observables; but relational does not imply gauge-invariant.

(Section 3.2.1).

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